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# Optimal scaling of a generic point absorber WEC in a range of production sites

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**Abstract**—The primary aim of the study was to investigate the optimal scaling of a simple generic wave absorber for a range of operation sites characterized by their level of incoming wave power, annually averaged. The first step is to answer the simple question: “What is the optimal size of my device in a sea state characterized by a wave height and peak spectrum period?”. It is shown how to find the optimum scale starting from the power response operator of the reference device at unity scale. From this primary result, the method is extended to answer the same question but now considering an installation site with its own set of sea states given as a scatter diagram. In each case the answer could seem rather counterintuitive referring to the common practise in this matter.

**Index Terms**—Wave Energy Converter, WEC, point absorber, hydrodynamic optimal scaling

## I. INTRODUCTION

Basically the design of a wave energy converter (WEC) begins by the choice of a working principle among the tenth of proposals which can be found in the literature or in patents data bases, or simply from the imagination of an inventor. Immediately after, the second step is the choice of the length of the device, which in turn will determine the size, then the choice of all the sub-systems. A lot of criteria and constraints must be accounted for in this optimization process, including: efficiency (i.e mean annual energy production), building costs, availability of components and sub-systems, transportation and installation costs, CAPEX and OPEX, and some more. But one of the most important is certainly the hydrodynamic efficiency, which is certainly the main driver for lowering the Levelized Cost of Energy (LCOE). For the most populated family of devices consisting in large body(ies) moving in response to the excitation of the waves, it is well known that the optimal sizing in terms of hydrodynamics is “decided by the sea” itself, not by the developer ! At a given installation site, the energy production is not a monotonically increasing function of the length as in the case of wind or tidal turbines. In other word, the maximization of the energy by the maximization of the body response results from a process which tunes the impedance of the WEC system to the oscillatory wave excitation. In this paper, we will illustrate this process by working the very simple and basic example of the uncontrolled

“heaving can” generic WEC. This will permit to point out that the results of a rational approach eventually results in a scaling different from the intuitive approach generally adopted by developers. Starting from the hydrodynamic characteristics and energetic efficiency (production matrix) of a heaving cylinder (fig.1 given as the scale 1/1 basis, we will show how to find the optimal scaling of the device working: (i) in a two parameters sea state, then (ii) at a site characterized by its scatter diagram derived from the mean power level as a unique parameter.

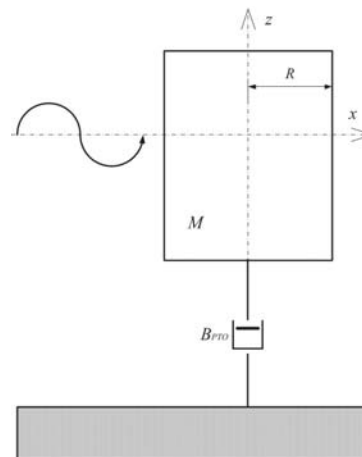


Fig. 1: the 1DoF generic wave absorber

## II. THE “HEAVING CAN” AS A GENERIC WAVE ABSORBER

### A. Basic results in frequency domain

We will consider here a floating cylinder free to move in heave only, and ideally restrained on all the other degree of freedom. The restoring force opposing the displacement is solely the hydrostatic vertical force  $K_H \zeta$ . The external Power Take Off (PTO) force is a linear damper proportional to the velocity  $\dot{\zeta}$ . This system constitutes one of the most simple one

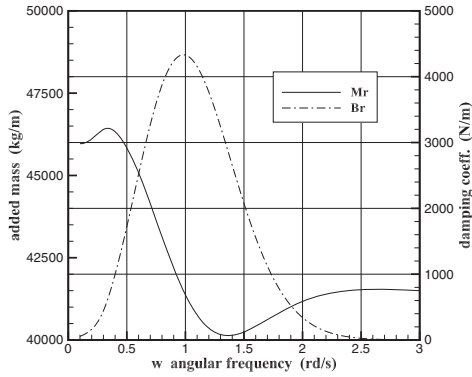


Fig. 2: Hydrodynamic coefficients of the WEC cylinder

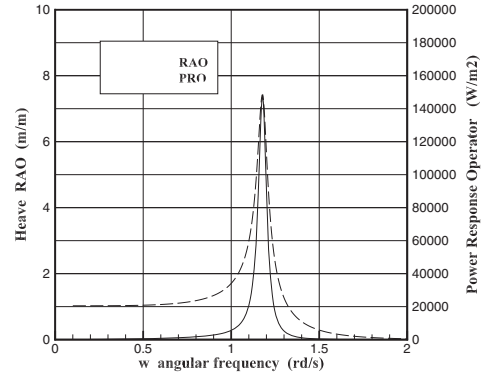


Fig. 3: Response Amplitude Operator (RAO) and Power Response Operator (PRO) of the WEC at scale 1/1.

degree-of-freedom (1DoF) wave absorber; it is used here just to exemplify the concept of hydrodynamical optimal scale on the simplest example.

The WEC data defining the scale 1/1 reference device are:

Diameter = 5m

Draft = 5m

Mass = 98174 kg (with  $\rho = 1000 \text{ kg/m}^3$ )

$K_H = 192619 \text{ N/m}$

Let  $\zeta$  measure the response of the cylinder to the incoming waves along the vertical axis  $z$  (positive upwards). In the present study, we will use the frequency domain approach, assuming the dynamic model of the WEC as fully linearized. Under this assumption, the equation of motion of the WEC derived from Newton's law can be written in the usual form:

$$(M + M_R(\omega))\ddot{\zeta} + B_R(\omega)\dot{\zeta} + K_H\zeta = F_{ex}(t) - B_{PTO}\dot{\zeta} \quad (1)$$

In this classical frequency domain approach, all time-varying variables are supposed being harmonic, of the form:  $x(t) = \Re[X(i\omega)e^{i\omega t}]$

$$\dot{\zeta}(t) = \Re[\dot{\zeta}(i\omega)e^{i\omega t}] \quad ; \quad \zeta(t) = \Re\left[\frac{1}{i\omega}\dot{\zeta}(i\omega)e^{i\omega t}\right] \quad (2)$$

$$\ddot{\zeta}(t) = \Re[i\omega\dot{\zeta}(i\omega)e^{i\omega t}] \quad ; \quad F_{ex}(t) = \Re[\tilde{F}_{ex}(i\omega)e^{i\omega t}] \quad (3)$$

leading to the frequency domain mechanical response equation:

$$\left[ i\omega(M + M_R(\omega)) + (B_R(\omega) + B_{PTO}) + \frac{K_H}{i\omega} \right] \dot{\zeta} = \tilde{F}_{ex}(i\omega) \quad (4)$$

Here  $M_R$  and  $B_R$  are respectively the wave radiation added mass and the wave radiation damping coefficients of the floating cylinder under investigation herein. For such a simple truncated cylinder, semi-analytical expressions for  $M_R$  and  $B_R$  are available in the literature, and have been used in this study. They are plotted in fig.2. From this later equation one

derives the complex transfer function  $\dot{\zeta}/\tilde{F}_{ex}(i\omega)$  linking the output (vertical velocity) to the input (wave excitation force) of this 1DoF WEC system :

$$\dot{\zeta}(\omega) = \frac{\tilde{F}_{ex}(\omega)}{\left[ (B_R(\omega) + B_{PTO}) + i\omega(M + M_R(\omega) - \frac{K_H}{\omega^2}) \right]} \quad (5)$$

In this simplified linear approach, the mean power recovered by the heaving WEC on regular waves at angular frequency  $\omega$  and amplitude of 1m is given by

$$\tilde{P}_{PTO}(\omega) = \frac{1}{2}B_{PTO} \left| \dot{\zeta}(\omega) \right|^2 \quad (6)$$

Without any added restoring force, the natural frequency of the WEC defined above is calculated as

$$\omega_0 = \sqrt{\frac{K_H}{M + M_R(\omega)}} \quad (7)$$

giving  $\omega_0 = 1.18 \text{ rad/s}$  as angular frequency, or  $T_0 = 5.32 \text{ s}$  as natural period.

In all this study we shall consider the problem of an uncontrolled WEC, implying that the value of  $B_{PTO}$  will be held fixed with regard to the wave frequency, but will vary only in the scaling process as all other parameters; referring to the classical results for control of WECs (see e.g [5]), it will be chosen as the value of the damping coefficient at resonance:  $B_{PTO} = B_R(\omega_0)$ , namely, at reference scale 1/1:  $B_{PTO} = B_R(1.18) = 3862.45 \text{ N s/m}$

Applying these parameters to (5) and (6) gives the amplitude of the heave response (per meter of incident wave amplitude), and the related extracted power through the PTO (per square meter of incident wave amplitude), named respectively Response Amplitude Operator (RAO) and Power Response Operator (PRO), plotted here on fig.3.

B. Power absorbed by the reference WEC in an irregular sea state

From the result above, primarily from the PRO curve, and the assumptions of the linear theory which permit the modelling of the response in monodirectional irregular waves by combination of several regular plane wave, we will now look at the energy recovered in such a system of irregular plane waves.

The system of irregular wave is characterized here by its energy density spectrum. A Bretschneider form is chosen here for reasons of coherence with the work of Burger et al. [2] that will be used later on in this paper. The Bretschneider law [6] give the spectral density as a function of the wave angular frequency  $\omega$ , a significant wave height  $H_s$  and a peak period  $T_p$ :

$$S_B(\omega) = \frac{5}{16} H_s^2 \frac{\omega_m^4}{\omega^5} e^{-\frac{5}{4}(\frac{\omega_m}{\omega})^4} \quad (8)$$

with [6]

$$\omega_m = \left(\frac{3}{5}\right)^{\frac{1}{4}} \frac{2\pi}{T_p} = 5.530 \frac{1}{T_p} \quad (9)$$

where the total energy is obtained from the first moment of the spectrum:  $m_0 = \int_0^\infty S_B(\omega) d\omega = H_s^2/16$ , leading to:

$$H_s = 4\sqrt{m_0} \quad (10)$$

The computation of the power absorbed by a WEC on a sea state defined by its Bretschneider spectrum as defined above is then computed as:

$$P(H_s, T_p) = 4 \int_0^\infty \tilde{P}_{PTO}(\omega) S_B(H_s, T_p, \omega) d\omega \quad (11)$$

After discretization of the continuous spectrum into a set of  $N$  regular waves of circular frequency  $\omega_i$  and amplitude  $A_i$ , each of them carrying the equivalent energy contained in the band  $\delta\omega = \omega_i - \omega_{i-1}$ , the total absorbed power is then computed as:

$$P(H_s, T_p) = 4 \sum_{i=1}^N \tilde{P}_{PTO}(\omega_i) S_B(\omega_i) \delta\omega \quad (12)$$

with the amplitude of the  $i^{th}$  component  $A_i = 2\sqrt{S_B(\omega_i)\delta\omega}$ .

With the heaving cylinder defined above as our unity scale reference WEC, the power matrix fig.4 can be established by using these formulas for a large range of irregular sea states defined by the bin  $(H_s, T_p)$ . On this figure one can notice that the productivity of the device is at its best when the peak period coincide with its natural period ( $T_0 = 5.32s$ ), which is logical for such a narrow banded device. (see PRO curve in fig.3).

With this basic stuff in hand, we are now able to answer the first question developpers are faced to: "for a given sea state  $(H_s, T_p)$ , and a given reference device, what is the optimal length scaling which will maximize the production", at first considering only hydrodynamics properties to answer the question, and regardless all other constraints (PTO efficiency, construction cost, maintenance cost, etc..).

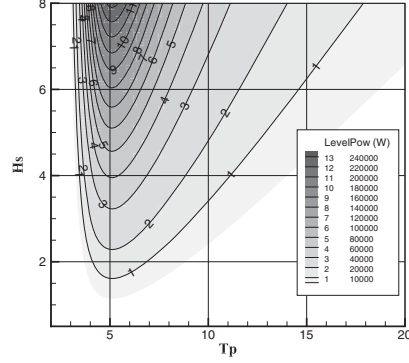


Fig. 4: Power matrix of the reference WEC at scale 1/1.

III. FROUDE SCALING LAWS

From the reference device, we will now vary systematically the length scale, while keeping constant the physical parameters of the problem:  $\rho, g$ . Let  $\sigma$  denotes the scale of the transformation (i.e  $L_\sigma = \sigma L_1$ , if  $L_1$  denotes for the reference length at scale 1). In this scaling, the ratio of the device length over the wavelength must indeed be kept constant, as being a non dimensional quantity. The wavelength is derived from the dispersion relation of free surface hydrodynamics:

$$\lambda = 2\pi g \frac{1}{\omega^2} \quad (13)$$

where  $\lambda$  is the wavelength. From this relation, the scaling law for time-like quantities is derived:

$$\frac{T_\sigma}{T_1} = \sigma^{\frac{1}{2}} \quad (14)$$

Once time and length ratio has been defined, all ratio for physical quantities of interest may be readily deduced; let us summarize them in a table:

Variable	unit	Scaling law
Angular displacement	rad	$\frac{\theta_\sigma}{\theta_1} = 1$
Angular velocity (frequency)	rad . s <sup>-1</sup>	$\frac{\omega_\sigma}{\omega_1} = \sigma^{-\frac{1}{2}}$
Angular acceleration	rad . s <sup>-2</sup>	$\frac{\alpha_\sigma}{\alpha_1} = \sigma^{-1}$
Area	m <sup>2</sup>	$\frac{A_\sigma}{A_1} = \sigma^2$
Force	N	$\frac{F_\sigma}{F_1} = \sigma^3$
Linear displacement	m	$\frac{L_\sigma}{L_1} = \sigma$
Linear velocity	m . s <sup>-1</sup>	$\frac{U_\sigma}{U_1} = \sigma^{\frac{1}{2}}$
Linear acceleration	m . s <sup>-2</sup>	$\frac{\gamma_\sigma}{\gamma_1} = 1$
Mass	kg	$\frac{m_\sigma}{m_1} = \sigma^3$
Moment	N . m	$\frac{M_\sigma}{M_1} = \sigma^4$
Power	W = N . m . s <sup>-1</sup>	$\frac{P_\sigma}{P_1} = \sigma^{\frac{7}{2}}$
Pressure	Pa	$\frac{p_\sigma}{p_1} = \sigma$
Time	s	$\frac{T_\sigma}{T_1} = \sigma^{\frac{1}{2}}$
Volume	m <sup>3</sup>	$\frac{V_\sigma}{V_1} = \sigma^3$

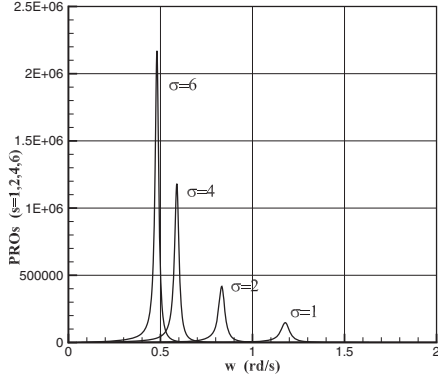


Fig. 5: Power Response Operator curve of the device at various scales:  $\sigma = 1, 2, 4, 6$ .

Now, from the Power Response Operator  $\tilde{P}_1(\omega)$  at scale 1 ( $= \tilde{P}_{PTO}(\omega)$  in eq.6), we can derive the PRO curve of the device at any scale  $\sigma$  directly with  $\tilde{P}_\sigma(\omega_\sigma) = \sigma^{\frac{3}{2}} \tilde{P}_1(\omega)$ .

Figure fig.5 shows the resulting Power Response Operators  $\tilde{P}_2, \tilde{P}_4, \tilde{P}_6$  for a short set of scales:  $\sigma = 2, 4, 6$ . From any of them we can establish a new scaled power matrix following the same steps as before for the reference scale 1/1 case.

Four examples of upscaled power matrix are shown in fig.6. One can easily check here that the wave period of optimal efficiency matches the natural period of the WEC as it grows in size ( $\propto \sigma$ ) and volume ( $\propto \sigma^3$ ).

Once this data base established, it is very easy to extract the best scaling of the WEC for a given sea state  $(H_s, T_p)$  by a one dimensional sweeping over the variable  $\sigma$ . Here, as an example, a table is drawn with the optimal scaling corresponding to several values of  $T_p$ , while keeping constant  $H_s = 2m$ .

$H_s$ m	$T_p$ s	$\sigma_{opt}$	$P(\sigma_{opt})$ kW	$D$ m	$T_0$ s
2	7	2.550	50	12.8	8.50
2	9	4.279	109	21.4	11.00
2	11	6.386	200	31.9	13.44
2	13	8.890	331	44.5	15.86
2	15	11.880	510	59.4	18.34

Now, for any given sea state  $(H_s, T_p)$ , we are able to determine the optimal scale of the WEC, and from it by following the scaling rules recalled in section above, its diameter, volume, natural period and optimal power corresponding to this scale. Considering now the table above, and fig.7 where the natural period of the optimally scaled WEC is plotted against the peak period  $T_p$  of the Bretchneider spectrum, one can answer the question posed at the beginning and derive the first conclusion of this study as: the natural period of the optimally scaled WEC is not equal but always larger (22% in the present case) than the peak period of the spectrum.

This could be attributed to the fact that none of the functions involved in eq.11 is symmetric with respect to its apex.

This partial conclusion is however rather counter-intuitive, and should be known by WEC developpers at the very first stage of design. Conversely, a lot of examples can be found in the related literature where authors base the sizing of their WEC precisely on the matching of these two periods.

Nevertheless, although the interest of the warning, this conclusion remains partial as long as we have considered only one sea state, given that a WEC is never optimized for a single sea state, but rather for a large set of sea states weighted by their probability of occurrence during the lifetime of the device at a given site.

#### IV. OPTIMAL SCALING FOR A GIVEN INSTALLATION SITE

##### A. From the sea state to the scatter diagram

This set of probability of occurrence of sea states  $(H_s, T_p)$  at a given site is generally called the *scatter diagram*. It is established by analysing the statistics of wave measurements by buoys moored at the site, if any, and require several years of data to reach a sufficient operational accuracy. When these data are required at a site where no data from measurement buoy is available, scatter diagrams are established by exploiting sea states hindcast data bases which are computed by running metocean wave computer models like Wave Watch III which compute these parameters on a grid from the long history of recorded meteorological conditions [7].

Whatever the method used to obtain it, let us consider from now on that we know a function  $f$  of  $H_s$  and  $T_p$  giving the density of probability of occurrence of the associated sea state  $(H_s, T_p)$ . From it, the number of hours with such a sea state in a year  $h(H_s, T_p)$  will be obtained by multiplying  $f dH dT$  by 8766 which is the number of hours a year (here corrected to account for leap years). Finally the production of energy

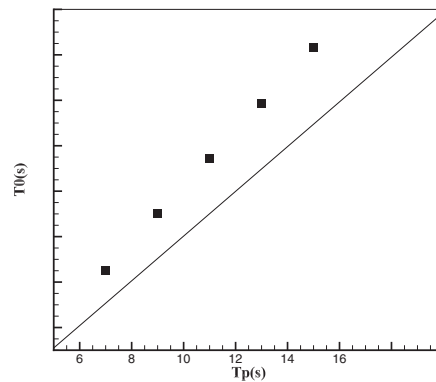


Fig. 7: natural period  $T_0$  of optimally scaled WECs versus peak period of the Bretchneider spectrum.

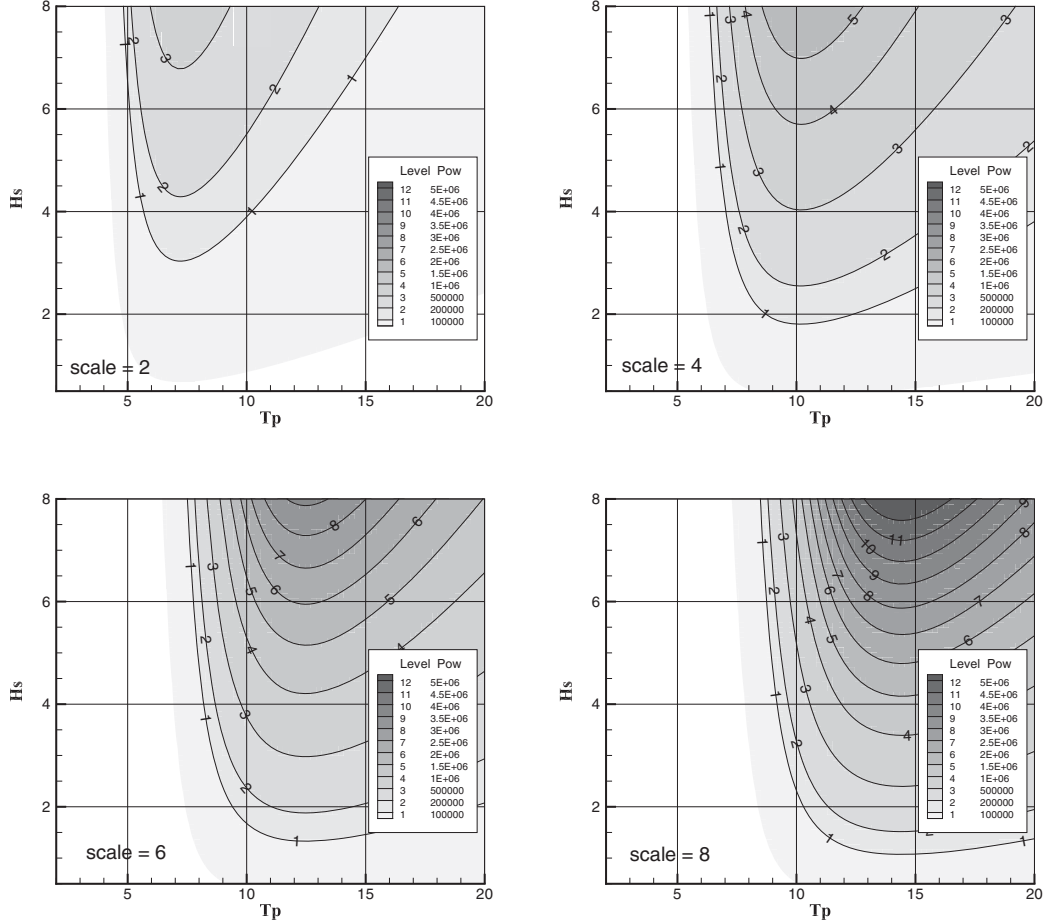


Fig. 6: Evolution of the power matrix with upscaling :  $\sigma = 2, 4, 6, 8$

for a WEC over one year on the site will be simply given by:

$$E_\sigma = 8766 \int_0^\infty dH \int_0^\infty P_\sigma(H, T) f(H, T) dT \quad (15)$$

where  $\sigma$  is to indicate that the device and then its production matrix is dependent on the scaling, here considered as a parameter.

So the second question we'll try to answer now is: given a site with a known scatter diagram, what is the scaling which will optimize the yearly averaged energy production? As previously, the optimum is to be understood in a hydrodynamical sense only, and still in the framework of linear modelling without other constraints.

### B. The Ochi's bivariate probability function

In this methodology study, we did not seek answering this question for a particular site (with a particular scatter diagram), but rather find a way to generate as many scatter diagram as necessary from an analytical model. We choose the model proposed by Ochi [1] and applied by Burger et al. [2], giving the joint probability density function for  $H_s$  and  $T_p$  as a bivariate log-normal model:

$$f_O(H_s, T_p) = \frac{1}{2\pi H_s T_p \delta_{H_s} \delta_{T_p} \sqrt{1 - \rho^2}} \exp\left(-\frac{1}{2(1 - \rho^2)} q\right) \quad (16)$$

with

$$q = q_1 + q_2 + q_3$$

and

$$q_1 = \left( \frac{\log T_p - \lambda_{T_p}}{\delta_{T_p}} \right)^2$$

$$q_2 = -2\rho \frac{(\log T_p - \lambda_{T_p})(\log H_s - \lambda_{H_s})}{\delta_{H_s} \delta_{T_p}}$$

$$q_3 = \left( \frac{\log H_s - \lambda_{H_s}}{\delta_{H_s}} \right)^2$$

The parameter  $\rho$  here is the linear correlation coefficient between the two variables, and is given by:

$$\rho = \frac{cov(\log T_p, \log H_s)}{\delta_{H_s} \delta_{T_p}} \quad (17)$$

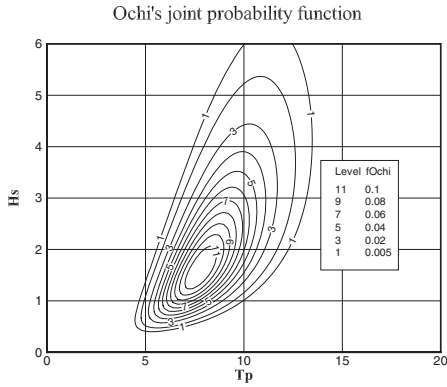


Fig. 8: Ochi's joint probability density function  $p_O(H_s, T_p)$  for an average power level of  $\kappa = 50 \text{ kW/m}$ .

In this Ochi's model,  $f_O(H_s, T_p)$  is a function of two variables and five parameters:  $\lambda_{T_p}, \lambda_{H_s}, \delta_{T_p}, \delta_{H_s}$  and  $\rho$ . In their approach to define a synthetic joint PDF for a large oceanic region, Burger et al. [2] established, by regressive optimization, several sets of the parameters for four different areas of the near Atlantic Ocean. They used all scatter diagrams available in the WERATLAS database in each delimited area, and proceeded by error minimization. The five parameters  $\lambda_{T_p}, \lambda_{H_s}, \delta_{T_p}, \delta_{H_s}$  and  $\rho$  valid for the near coastal Atlantic area from Spain, France, Ireland to Iceland are given as linear functions of the yearly average power level  $\kappa$  (in kW/m) considered as the continuous input variable. The ten coefficients (one slope and one offset for each of the five parameter) are given in [2]. So, with this set of coefficients, we implemented a routine giving analytically the joint probability  $p_O(\kappa, H_s, T_p)$ , derived with  $\kappa$  the mean power level as a single input variable. There is a great advantage for future optimization to have such a mean to generate continuous joint probability function from a single continuous variable. An example is plotted in fig.8 for  $\kappa = 50 \text{ kW/m}$ .

From it, the number of hours a year featuring a sea state in a given range  $(\delta H_s, \delta T_p)$  around  $(H_s, T_p)$  will be given as:  $8766 p_O(\kappa, H_s, T_p) \delta H_s \delta T_p$ , and the annual production of the WEC at scale  $\sigma$ , at this site by summing:

$$E(\sigma, \kappa) = 8766 \sum_{i=1}^{n_H} \sum_{j=1}^{n_T} P_\sigma(H_i, T_j) p_O(\kappa, H_i, T_j) \delta H_s \delta T_p \quad (18)$$

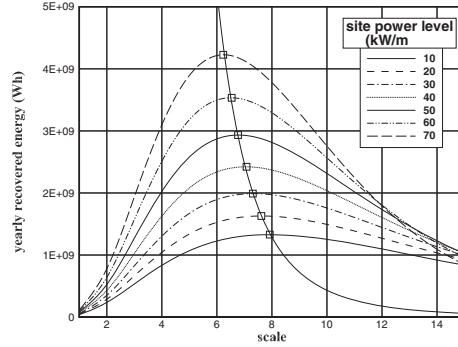


Fig. 9: Annual energy recovered as a function of scale  $\sigma$  for different site power levels  $\kappa = 10, \dots, 70$ .

A set of curves  $E(\sigma, \kappa)$  is plotted here in fig.9, for  $\sigma = 1, \dots, 15$  (or in other words for a WEC diameter from 5 m to 75 m) and power levels ranging from 10 kW/m (mild wave climate) to 70 kW/m (very severe wave climate). One can observe that, as expected, for a given site the energy recovered begins as an increasing function of scale, reaches a maximum, then decreases for oversized wave absorbers. This is simply a confirmation that, as was said in introduction, the optimal scale of a device is "decided by the sea" itself, not by the developer; too small means too few power density due to small applied forces, when too big means no motion, then no energy linked to the response amplitude of the floating body. It can be observed also that the optimal scale is a mildly decreasing function of the power level. The table below gives the coordinates of the square dots which mark the maximum of each curve in fig.9. An exponential fitting for the locus of these maxima, shown as a solid curve in the figure is provided by the function:

$$\tilde{E} = \exp(-4.896 \ln(\sigma) + 31.16) \quad (19)$$

Power level kW/m	opt. scale	annual energ. GW	diameter m	mass T
10	7.923	1.33	39.6	48827
20	7.615	1.63	38.0	43351
30	7.307	1.99	36.5	38301
40	7.077	2.42	35.4	34797
50	6.769	2.93	33.8	30448
60	6.538	3.53	32.7	27436
70	6.231	4.22	31.2	23750