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Direct adaptive regulation in the vicinity of low damped complex zeros - Application to active vibration control

Abraham Castellanos Silva*, Ioan Doré Landau*,
Petros Ioannou†.

Abstract—The adaptive feedback approach is now widely used for the rejection of multiple narrow band disturbances with unknown and time varying frequencies in Active Vibration Control (AVC) and Active Noise Control (ANC). The approach is based directly or indirectly on the use of the Internal Model Principle and the Youla-Kučera parametrization combined with an adaptive law. All the algorithms associated with the approach make the assumption that the plant zeros are different from the poles of the disturbance model in order to achieve disturbance compensation. However in practice the problem is more intricate since it is not clear what happens if the plant have very low damped complex zeros (often encountered in mechanical structures) and the frequency of the disturbance is close to the anti-resonance frequency (the resonance frequency of the plant zeros). A recent international investigation on adaptive regulation in the presence of unknown time varying disturbances [16] has considered such a situation for a benchmark example. Several solutions have been proposed and the most successful has been based on the appropriate choice of the desired closed loop poles to be achieved by the Youla-Kučera central controller [5] using a Q FIR filter with the minimum number of parameters. Recently in [12] it was suggested that over parametrization of the Q (FIR) filter can enhance the robustness of the linear and adaptive scheme in the vicinity of plant complex zeros. The present paper compares these two approaches using the same benchmark example as in [16]. The results from simulations and real time experiments used to evaluate the two approaches are presented.

Index Terms—Adaptive Regulation, Active Vibration Control, Inertial Actuators, Multiple Narrow Band Disturbances, Youla-Kučera Parametrization, Internal Model Principle

I. INTRODUCTION

The basic problem in active vibration control (AVC) and active noise control (ANC) is the *strong* attenuation of multiple narrow band disturbances¹ with unknown and varying frequencies. An adaptive feedback approach (adaptive regulation) is now widely accepted as the most effective approach for solving this class of problems. The disturbance model is assumed to be either a function equal to the sum of sinusoids with unknown frequencies, amplitudes and phases or equivalently, a transfer function with unknown complex poles on the unit circle with white noise or a Dirac impulse as an input. In general, one can assess from data the structure for such *model of disturbance* (using spectral analysis or order estimation techniques) and assume that the structure does not change. However, the parameters of the model are unknown and may be time varying. This will require the use of an

adaptive feedback approach in order to adapt to changes in parameters.

It is also assumed that the plant model is stable and this property could be the result of a robust control design that is already incorporated in the system under consideration. The problem of disturbance rejection and adaptive regulation as defined above has been previously addressed in a number of papers ([4], [2], [24], [22], [8], [10], [11], [20], [14], [1], [7], [9], [3], [23], [6]) among others. [15] presents a survey of the various techniques (up to 2010) used as well as a review of a number of applications.

Among them, the *Internal Model Principle* implemented through a Youla-Kučera parametrization arises as a very attractive and efficient solution, since it allows to introduce the model of the disturbance in the controller without modifying the *desired* closed loop poles, defined by the designer [24], [20], [5]. This parametrization allows to obtain a direct adaptive scheme. The number of parameters to adapt is defined by the complexity of the assumed disturbance model. An international competition benchmark example on adaptive rejection of narrow band disturbances has been organized and the results are published in a special issue of the European Journal of Control [16].

A common challenge of all the up to date efforts and proposed methods is the following: the disturbance is considered to be periodic, i.e. the poles of the disturbance models are on the unit circle. All the adaptation algorithms make the assumption that the plant zeros are different from the poles of the disturbance model in order to achieve disturbance compensation. However in practice the problem is more intricate since it is not clear what happens if the plant has very low damped complex zeros (often encountered in mechanical structures) and the frequency of the disturbance is close to the anti-resonance frequency (the resonance frequency of the plant zeros). Obviously even in the linear case with known parameters the design of the controller in this region is difficult for robustness reasons. In the international benchmark example on adaptive regulation in the presence of unknown time varying disturbances [16] such a situation has been explicitly considered. Several solutions have been proposed and the most successful has been based on the appropriate choice of the desired closed loop poles to be achieved by the Youla-Kučera central controller [5] and by using a Q-FIR filter with the minimum number of parameters. Recently [12] it was suggested that

¹Called *tonal* disturbances in active noise control.

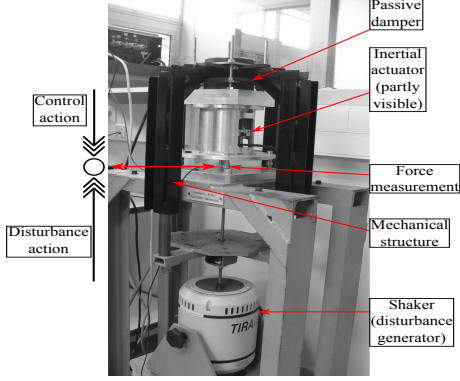


Fig. 1. Active vibration control using an inertial actuator (photo).

over parametrization of the Q -(FIR) filter can enhance the robustness of the linear and adaptive scheme in the vicinity of plant complex zeros². In this paper these two approaches are compared using the benchmark example and the simulation and real time results are presented.

II. AN ACTIVE VIBRATION CONTROL SYSTEM USING AN INERTIAL ACTUATOR

A. System structure

The photo of the active vibration control experimental set up used in this study is presented in fig. 1. Figure 1 also shows the description of the basic actions. The shaker acts as a disturbance source by introducing vibration forces and the inertial actuator can be used to counteract them by introducing vibrational forces in the opposite direction (inertial actuators use a similar principle as loudspeakers). This test bed was used in the international benchmark in adaptive regulation, whose results were published in [16]. The equivalent control scheme is shown in figure 2. The system input, $u(t)$ is the position of the mobile part (magnet) of the inertial actuator, the output $y(t)$ is the residual force measured by a force sensor. The transfer function ($H = q^{-d} \frac{C}{D}$), between the disturbance force, $\delta(t)$, and the residual force $y(t)$ is called *primary path*. In our case (for testing purposes), the primary force is generated by a shaker driven by a signal delivered by the computer. The plant transfer function ($G = q^{-d} \frac{B}{A}$) between the input of the inertial actuator, $u(t)$, and the residual force is called *secondary path*. The sampling frequency is $F_s = 800$ Hz.

Figure 3 gives the frequency characteristics of the identified parametric model for the secondary path (the excitation signal was a PRBS). The system itself in the absence of the disturbances features a number of low damped vibration modes as well as low damped complex zeros (anti-resonance). This makes the design of the controller difficult

²This idea has not been explored by the participants to the benchmark. Note that the over parametrization of the Q filter for robustness with respect to uncertainties in the plant model has been proposed in [24], [12], however, here the objective of over parametrization is different.

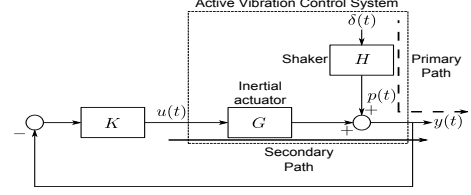


Fig. 2. Active suspension system (scheme).

for rejecting disturbances close to the location of low damped complex zeros (low or no system gain). The most significant are those near 50, 100 and 110 Hz (see the zoom of the frequency characteristics of the secondary path in figure 3). Note that the design of a linear controller for rejecting a disturbance at 95 Hz (as required by the benchmark) is difficult since this frequency is close to a pair of very low damped zeros (around of 0.005). The parametric model of the secondary path has a significant order, $n_A = 22$ and $n_B = 25$.

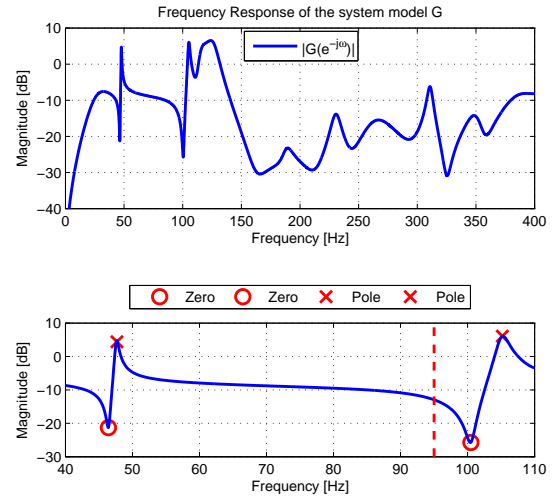


Fig. 3. Magnitude of the frequency response for secondary path model (top). Zoom at the low damped complex poles and zeros (bottom).

III. PLANT/DISTURBANCE REPRESENTATION AND CONTROLLER STRUCTURE

The structure of the linear time invariant discrete time model of the plant - the secondary path - used for controller design is:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})}, \quad (1)$$

with:

d = the plant pure time delay in number of sampling periods

$$A = 1 + a_1z^{-1} + \dots + a_{n_A}z^{-n_A};$$

$$B = b_1z^{-1} + \dots + b_{n_B}z^{-n_B} = z^{-1}B^*;$$

$$B^* = b_1 + \dots + b_{n_B}z^{-n_B+1},$$

where $A(z^{-1})$, $B(z^{-1})$, $B^*(z^{-1})$ are polynomials in the complex variable z^{-1} and n_A , n_B and $n_B - 1$ represent their orders³. The model of the plant may be obtained by system identification. Details on system identification of the models considered in this paper can be found in [21], [19], [18].

Since the control objective is focused on regulation, the controller to be designed (K) corresponds to a RS polynomial digital controller, ([17], [21] - see also figure 2). The controller is $K = \frac{R}{S}$, where $R(z^{-1})$ and $S(z^{-1})$ are polynomials in z^{-1} having the orders n_R and n_S , respectively, with the following expressions:

$$R(z^{-1}) = r_0 + r_1 z^{-1} + \dots + r_{n_R} z^{-n_R} = R'(z^{-1}) \cdot H_R(z^{-1}) ; \quad (2)$$

$$S(z^{-1}) = 1 + s_1 z^{-1} + \dots + s_{n_S} z^{-n_S} = S'(z^{-1}) \cdot H_S(z^{-1}) , \quad (3)$$

where H_R and H_S are pre-specified parts of the controller (used for example to incorporate the internal model of a disturbance or to open the loop at certain frequencies).

The output of the plant $y(t)$ and the input $u(t)$ may be written as:

$$y(t) = \frac{q^{-d} B(q^{-1})}{A(q^{-1})} \cdot u(t) + p(t) ; \quad (4)$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t) , \quad (5)$$

where q^{-1} is the delay (shift) operator ($x(t) = q^{-1}x(t+1)$) and $p(t)$ is the resulting additive disturbance on the output of the system.

We define the following sensitivity functions:

- Output sensitivity function (the transfer function between the disturbance $p(t)$ and the output of the system $y(t)$):

$$S_{yp}(z^{-1}) = \frac{1}{1 + GK} = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})} ; \quad (6)$$

- Input sensitivity function (the transfer function between the disturbance $p(t)$ and the input of the system $u(t)$):

$$S_{up}(z^{-1}) = \frac{-K}{1 + GK} = -\frac{A(z^{-1})R(z^{-1})}{P(z^{-1})} , \quad (7)$$

where

$$\begin{aligned} P(z^{-1}) &= A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \\ &= A(z^{-1})S'(z^{-1}) \cdot H_S(z^{-1}) + z^{-d}B(z^{-1})R'(z^{-1}) \cdot H_R(z^{-1}) \end{aligned} \quad (8)$$

defines the poles of the closed loop (roots of $P(z^{-1})$). In pole placement design, the polynomial $P(z^{-1})$ specifies the desired closed loop poles and the controller polynomials $R(z^{-1})$ and $S(z^{-1})$ are minimal degree solutions of (8) where the degrees of P , R and S are given by: $n_P \leq n_A + n_B + d - 1$, $n_S = n_B + d - 1$ and $n_R = n_A - 1$.

Using equations (4) and (5), one can write the output of the system as:

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) . \quad (9)$$

³The complex variable z^{-1} will be used for characterizing the system's behaviour in the frequency domain and the delay operator q^{-1} will be used for describing the system's behaviour in the time domain.

For more details on RS -type controllers and sensitivity functions see [21].

Suppose that $p(t)$ is a deterministic disturbance, so it can be written as

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) , \quad (10)$$

where $\delta(t)$ is a Dirac impulse and $N_p(z^{-1})$, $D_p(z^{-1})$ are coprime polynomials in z^{-1} , of degrees n_{N_p} and n_{D_p} , respectively. In the case of persistent (stationary) disturbances the roots of $D_p(z^{-1})$ are on the unit circle (which will be the case for this work). The energy of the disturbance is essentially represented by D_p . The contribution of the terms of N_p is weak compared to the effect of D_p , so one can neglect the effect of N_p .

IV. LOW DAMPED COMPLEX ZEROS

An important remark is that in order to be able to reject the disturbance introduced by the primary path, the secondary path has to provide enough gain. Looking at eq. (6), total rejection at a frequency ω is achieved when

$$S_{yp}(e^{-j\omega}) = 0 \quad \rightarrow \quad S(e^{-j\omega}) = 0, \quad (11)$$

nevertheless, in such case the modulus of the input sensitivity function (eq. (7)) becomes

$$|S_{up}(e^{-j\omega})| = \left| \frac{A(e^{-j\omega})}{B(e^{-j\omega})} \right|, \quad (12)$$

meaning that the robustness against additive plant model uncertainties is reduced and the stress on the actuator will be important if low damped complex zeros are located near or at the frequency ω . Therefore, the cancelation (or in general an important attenuation) of disturbance effect on the output should be done only in frequency regions where the system gain is large enough.

In [16], several approaches were used in the benchmark example to reject a disturbance at 95 Hz. It was found that stability and performance issues arises due the proximity of the low damped zeros; furthermore, if the gain of the input sensitivity function is not low enough (below -10 dB) above 100 Hz, important amplifications (and even instability) can appear on the real system. One of the best results was presented in [5], using a Yula-Kučera (YK) parametrization of the controller, a specific choice for the desired closed loop poles location and a direct adaptive approach.

V. DIRECT ADAPTIVE FEEDBACK REGULATION - A YOULA-KUČERA APPROACH

The YK-parametrization proposed is depicted in the fig. 4, where both fixed and adaptive parts are pointed out. For this paper a YK-parametrization using an equation-error disturbance observer is used, along with a finite impulse response (FIR) filter representation of the optimal Q filter

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + \dots + q_{n_Q} z^{-n_Q}. \quad (13)$$

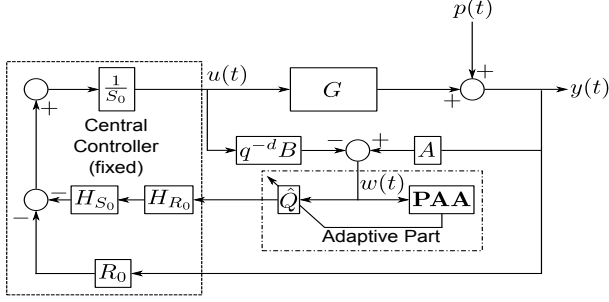


Fig. 4. Direct adaptive scheme using a YK-parametrization of the controller. Dashed box: fixed part, Point-dash box: adaptive part.

Using this parametrization, the controller polynomials R and S are defined by⁴

$$R(z^{-1}) = R_0 + H_{S_0}H_{R_0}QA \quad (14)$$

$$S(z^{-1}) = S_0 - H_{S_0}H_{R_0}Qz^{-d}B, \quad (15)$$

It is easy to show that for any arbitrary $Q(z^{-1})$, the closed loop poles remain unchanged; they are defined by

$$P(z^1) = A(z^{-1})S_0(z^{-1}) + z^{-d}B(z^{-1})R_0(z^{-1}). \quad (16)$$

A. Internal Model Principle

Looking at the output sensitivity function (eq. (6)) and the assumed model of the disturbance (eq. (10)), total rejection of a disturbance is possible if $S(z^{-1}) = S'(z^{-1}) \cdot D_p(z^{-1})$, meaning that the controller incorporates the model of the disturbance (internal model principle).

Consider eq. (9) and eq. (15), then the output of the system can be expressed as follows:

$$y(t) = \frac{A[S_0 - H_{S_0}H_{R_0}Qq^{-d}B]}{P} \cdot p(t). \quad (17)$$

In order that the numerator polynomial contains the model of the disturbance, the following diophantine equation has to be solved

$$S'D_p + H_{S_0}H_{R_0}Qz^{-d}B = S_0 \quad (18)$$

where D_p , H_{S_0} , H_{R_0} , d , B and S_0 are known, and S' and Q are unknown. Eq. (18) has a unique and minimal degree solution for S' and Q with $n_{S_0} \leq n_{D_p} + n_B + d - 1$, $n_{S'} = n_B + d + n_{H_{R_0}} + n_{H_{S_0}} - 1$ and $n_Q = n_{D_p} - 1$.

Remark: It is assumed that D_p and B do not have common factors but nothing is said of the feasibility of the solution if some complex zeros of D_p are very close to some low damped complex zeros of B (of course the Bezout equation to be solved will be "ill conditioned" as we approach cancellation).

From eqs. (17) and (18), and using a standard parameter adaptation algorithm (PAA) as explained in [21], a direct adaptive algorithm for the rejection of multiple unknown

⁴The arguments (z^{-1}) and (q^{-1}) will be omitted in some of the following equations to make them more compact.

TABLE I
FREQUENCY DOMAIN RESULTS IN SIMULATION AND REAL TIME

Closed Loop Poles	Case 1 Plant Poles + 12 real poles					Case 2 Plant Poles + 2 pairs of resonant poles + 4 real poles	
	a)		b)			a)	b)
	$n_Q =$					1	5
RT	1	2	3	4	5	21.0	22.0
	SIM	9.8	15.6	16.1	16.1	17.0	25.4
RT	1	2	3	4	5	39.4	38.0
	SIM	9.5	15.2	15.6	15.9	16.8	39.8
RT	1	2	3	4	5	8.1	7.2
	SIM	21.0	16.9	15.7	16.5	15.4	10.0

RT: Real time, SIM: Simulation, GA: Global attenuation, DA: Disturbance attenuation, MA: Maximum amplification, a): minimal solution and b): augmented solution.

time-varying narrow band disturbances can be developed, by considering an adaptive \hat{Q} filter of the form

$$\hat{Q}(z^{-1}) = \hat{q}_0 + \hat{q}_1z^{-1} + \dots + \hat{q}_{n_Q}z^{-n_Q}. \quad (19)$$

The details of the adaptation algorithm are given in [18] and a stability analysis is provided in [20].

VI. ADAPTIVE REGULATION IN THE VICINITY OF LOW DAMPED ZEROS

Eq. (18) has a unique and minimal solution for $Q(z^{-1})$ when the roots of $D_p(z^{-1})$ are not contained in $B(z^{-1})$, nevertheless the modulus of the output sensitivity function $S_{yp}(z^{-1})$ may become larger, specially when $B(z^{-1})$ has roots close to those of $D_p(z^{-1})$, e.g. presence of low damped zeros in the system at frequencies where attenuation is introduced. To overcome such situation, in [12] the augmentation of the order of the polynomial $\hat{Q}(z^{-1})$ is proposed, claiming that if the solution of (18) is not unique, and an infinity possible values for the coefficients $Q(z^{-1})$ exist to have the internal model as a factor, then there is a structural freedom to choose the optimum set of coefficients that provide the best performance by minimizing the output sensitivity function, (e.g. the modulus margin will be minimized)⁵. Note that the controller considered in [12] uses an "output error" type Youla-Kučera disturbance observer⁶.

In [5] it was shown that using the plant model information (frequency characteristics), it is possible to keep the modulus of $S_{yp}(z^{-1})$ under an imposed maximum value by choosing appropriately the desired closed loop poles in $P(z^{-1})$. In this approach the minimal degree for the polynomial $\hat{Q}(z^{-1})$ is maintained and an "equation error" Youla-Kučera observer is used.

The objective will be subsequently to compare the two approaches in the context of the international benchmark on adaptive regulation. The experiments were conducted in

⁵ $|S_{yp}(e^{-j\omega})|_{\max}$ corresponds to the H_∞ norm of the output sensitivity function.

⁶For a definition of the various types of Youla-Kučera disturbance observers see [16].

the international test bed proposed in [16], where a single sinusoidal disturbance at 95 Hz will be introduced.

VII. COMPARISON OF THE TWO APPROACHES - SIMULATION AND REAL TIME RESULTS

The comparison of the two approaches has been done on the active vibration control system described in Section II. Two main cases have been considered, with two options each:

- 1) $P(z^{-1})$ contains the stable poles (**SP**) of the plant model along with 12 real poles (**RP**) (This will reduce the modulus of $S_{up}(z^{-1})$ in high frequencies. Without these poles the control signal is saturated in the real-time application due to the presence of the harmonics of the disturbance).
 - a) With the minimal solution $n_Q = n_{D_p} - 1$.
 - b) With an augmented solution $n_Q > n_{D_p} - 1$.
- 2) $P(z^{-1})$ has the stable poles (**SP**) of the plant along with some auxiliary poles (**AP**): 2 pairs of low damped complex poles and 4 real poles.
 - a) With the minimal solution $n_Q = n_{D_p} - 1$.
 - b) With an augmented solution $n_Q > n_{D_p} - 1$.

Simulations (**SIM**) and real-time (**RT**) experiments were conducted using both approaches. The results were classified in frequency and time domain. Time domain results are provided only for real-time experiments.

Table I summarizes the results in frequency domain (measured in dB) obtained for simulations and real-time experiments. The objective is to strongly attenuate the disturbance with a limited amplification of the other frequencies. To evaluate the performance three indicators have been defined together with their target values according to [16]: Disturbance Attenuation (**DA**) (min = 40 dB), Global Attenuation (**GA**) (min = 25 dB), and Maximum Amplification (**MA**) (max = 6 dB)⁷. The effects of the vicinity of the low damped complex zeros is noted for the *Case 1 a)*, where the global attenuation is minimum and a significant amplification was found, both for RT and SIM. For *Case 1 b)* with $n_Q = 5$, the improvements of the augmentation of the size of $\hat{Q}(z^{-1})$ are evident. Better attenuation and decreasing of the unwanted amplification are obtained⁸. When AP are used with a minimal solution (*Case 2 a)*), the results are significantly improved for the three specifications. It is important to observe that for the *Case 2 b)*), augmenting the size of the Q -filter improves further all the performance indicators in simulation. In real time the performance is improved by reducing the maximum of the unwanted amplification and augmenting the global attenuation, however the disturbance attenuation is slightly lower with respect to the case of minimal order for the $\hat{Q}(z^{-1})$.

⁷GA and MA give indication about the quality of the control which is supposed to introduce a very limited amplification at frequencies different from the frequency of the disturbance

⁸Due to the presence of harmonics of the disturbance in real time experiments, differences arise between the RT and SIM results for the *Case 1* since the gain of $S_{up}(z^{-1})$ above 100 Hz is not low enough.

TABLE II
TIME DOMAIN RESULTS

$n_Q =$	Case 1 SP+RP					Case 2 SP+AP	
	a)		b)			a)	b)
	1	2	3	4	5	1	5
TE %	0	97.21	96	72.84	97.78	100	100
CT μ s	2.8	3.0	3.5	3.8	4.5	2.8	4.2

TE: transient behaviour indicator (desired = 100%). CT: computation time.

Table II summarizes the results obtained in real-time with respect to the transient performance and computation time. Two specifications were considered according to [16]: a Transient Evaluation (**TE** in %) and the Computation Time (**CT** in μ s). The transient evaluation criterion establishes that the transient duration when a disturbance is applied, has to be smaller than 2 s. A percentage was established for the fulfilment of this criterion. TE = 0% indicates a transient duration of 4 s and TE = 100% a transient duration smaller than 2 s. The detailed computation formulas can be found in [16].

The computation time is calculated from the Task Execution Time evaluated in the MATLAB[®]'s xPC-Target environment. The computational time only consider the closed loop calculations⁹.

The use of **SP+AP** shows its efficiency since 100% fulfilment of the transient evaluation criterion is achieved. The results are slightly less good when augmentation of the size of the $\hat{Q}(z^{-1})$ is considered. As was expected, the increase of the number of parameters (order of \hat{Q}) implies an increase in the computation time, but this effect is not very significant.

Figure 5 shows the disturbance attenuation comparison between the two cases with the minimal ($n_Q = 1$) and highest order ($n_Q = 5$) solution. The figure shows real-time experimental results. As can be seen in the figure, some improvements are introduced when the order of Q is increased, such as lower amplifications in high frequencies and a larger attenuation (differences between the dashed blue line and dotted red line). Nevertheless, choosing appropriately the fixed closed loop poles (*Case 2, a) and b)*), the adaptive scheme introduce a significant attenuation with a minimum amplification. This can be seen also in the resulting output sensitivity function calculated with the estimated parameters of $\hat{Q}(z^{-1})$ for each case, from the real-time experiments. Figure 6 displays the modulus of each sensitivity function. It is noticed that passing from the *Case 2 a)* to *Case 2 b)* the characteristics of the output sensitivity function remains almost unchanged in high frequencies, while the maximum amplification is reduced. keeping the *water bed effect* bounded in the frequency zone of interest [5].

VIII. CONCLUDING REMARKS

Careful selection of the closed loop poles for the design of the central controller combined with a minimum order

⁹The CT for an open loop test is 12.9 μ s.

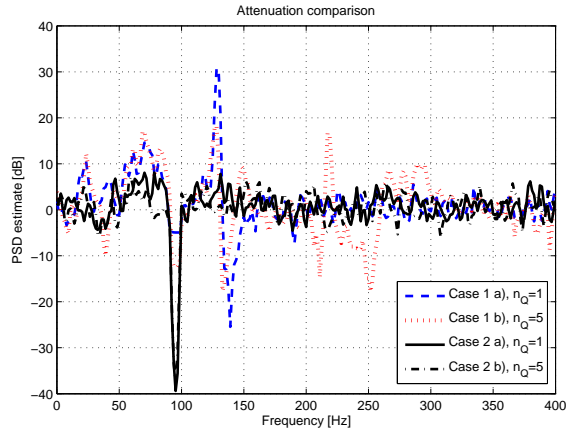


Fig. 5. Disturbance attenuation comparison between the three cases, real-time results.

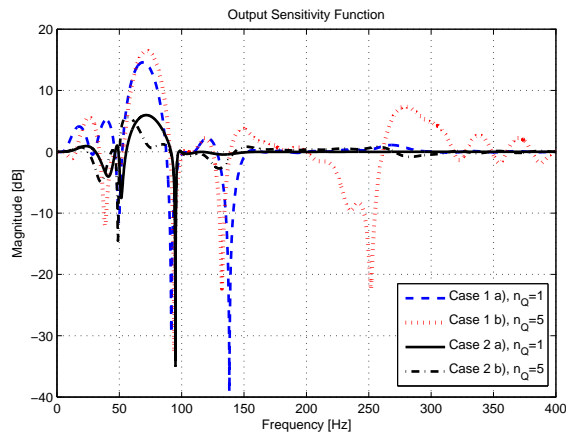


Fig. 6. Output Sensitivity Function comparison between the three cases, real-time results.

adaptive Q filter or over parametrization of the adaptive Q filter are two interesting solutions for improving the performance of adaptive regulation schemes in the vicinity of low damped complex zeros. The two approaches can also be combined. However over parametrization of the Q filter will require to use robust parameter estimation in order to avoid parameter drift. This case has been considered in a recent paper [13].

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