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# On Green's correlation of Stokes' equation

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## Abstract

The derivation of Green's correlation naturally arises when identifying a linear propagation medium with uncontrolled random sources or ambient noise. As expected, this involves convolution of the well known Green's function with its time-reversed version. The purpose of this paper is to derive a general expression of the Green's correlation function of a linear visco-acoustic propagation medium, in which the pressure field satisfies Stokes' equation. From the expression obtained for a visco-acoustic medium, the Ward identity that was recently obtained for unbounded media is extended to the case of bounded propagation media. This extension appears necessary as the unbounded model is not valid in many practical cases, as for acoustic rooms for example. It is illustrated with both simulations and real-world aerial acoustics experimental data recorded in a closed room and in the framework of passive identification. In these experiments, Green's correlation is estimated by the classical coda-based approach, and the performances are studied in this new context.

*Keywords:* Stokes equation, viscous damping, Green's function, Green's correlation, Ward identity, acoustic codas.

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## 1. Introduction

Retrieving the parameters of a linear propagation medium is still an active research topic in many domains, such as wave celerity map estimation in medium tomography, impulse response estimation in wireless communications, and modal pulsation estimation in modal analysis of mechanical structures. Indeed, this list is not exhaustive. When controlled sources can be used, the Green's function of the explored propagation medium relates a known excitation source to the generated wave measured by a set of sensors. In this situation, received and emitted signals are processed to estimate the Green's function and then to extract the parameters related to the propagation medium and the sensor positions. In a passive context, ambient sources are used (see for example the tutorial [18]). Green's functions cannot be extracted directly, as no deterministic information on the source is available. To circumvent this lack of information, many studies have considered ambient (spatio-temporal) white noise (see [11, 2, 8, 17, 5, 4], for a non exhaustive list). For this type of ambient sources, the cross-correlations between two received signals are known as Green's correlation (and also called noise correlation). Green's correlation has a role similar to the Green's function, as it relates the auto-correlation of a source to the auto-correlation of the wave generated by this source.

The ambient noise that issues from uncontrolled white sources can be created by thermal noise [11], and by external random excitation [18]. Seismologists have shown a very interesting way to raise the number of 'useful' ambient sources [2]. They have shown experimentally that the cross-correlation of a coda generated by a propagated impulsion source (such as an earthquake) is a good approximation of white noise cross-correlation (the mathematical proof is still

an open problem; see section 6 of [5]). Briefly, and approximately, this can be explained through ergodic cavity theory. A localized impulse source snared in such a cavity will be 'converted' to a white source during the propagation inside the cavity, as after a certain time (i.e., the mixing time), the cavity 'randomizes' both the directions and the times of arrival. Therefore, the propagated source 'becomes' white after this mixing time, and the origin of the source disappears. This codas-based result is valid in the acoustic framework, and this significantly increases interest in the study of propagation media parameter retrieval from cross-correlations of uncontrolled white sources.

It has been shown that Green's correlation is the space-time convolution of the Green's function with its time-reversed version [19, 5]. This convolution exists (mathematically) only if the associated propagation equation contains a dissipation term. Many studies have used a constant dissipation model [11, 2, 13, 8, 5], which is not realistic in acoustics and elastic propagation media [4]. Indeed, in the presence of dissipation, Stokes' equation [14, 1] describes the propagation of acoustic waves, and, P-waves and S-waves in the elastic context [14]. Green's correlation of Stokes' equation was studied for an unbounded propagation medium in [4], and the Ward identity relating Green's correlation to the Green's function [19] was derived for that case. To the best of our knowledge, Green's correlation for Stokes' equation has never been studied in a general framework (see [17] for Green's correlation derivations in other practical physical contexts).

In this paper, we derive an exact expression of Green's correlation in the time domain. From this expression that is obtained through a modal decomposition, we extend the Ward identity for Stokes' equation obtained in [4] for unbounded

propagation media, to any bounded visco-acoustic propagation media. Furthermore, this expression is used to simulate the visco-acoustic Green's correlation, to improve its interpretation, as this is difficult to obtain directly from the equations. Then indoor aerial acoustics experiments with microphones are presented. These illustrate the possibility to retrieve the visco-acoustic Green's correlation with a coda-based approach. The results obtained in these experiments are compared with the theoretical ones developed in the first part of this paper.

This paper is organized as follows:

- Section 2 first introduces Stokes' equation. The associated dispersion relations and modes are computed. Then the visco-acoustic Green's function is derived from a modal decomposition. Codas are presented in the particular regime that combines semi-classical approximation (see part B of [10]) and low attenuation approximation (see section A of [1]). Finally, Green's correlation of Stokes' equation is introduced and an exact expression is derived in the time domain. From that expression, we establish the Ward identity for Stokes' equation in any bounded media in the presence of low attenuation. Simulations are presented to illustrate the expressions derived.
- Section 3 is dedicated to the experimental Green's correlation retrieval in visco-acoustic propagation media. We introduce the coda-based method, for which performances are established with real data and compared to simulations.
- Section 4 summarizes the results obtained in the previous sections and discusses the identified perspectives in Green's correlation interpretation, and passive parameters retrieval with a coda-based method.

## 2. Green's correlation of Stokes' equation

### 2.1. Stokes' equation.

Throughout this paper, we consider a three-dimensional (3D) visco-acoustic propagation medium denoted by  $X$  and assumed to be *linear* and *homogeneous*. We denote the surface of  $X$  by  $\partial X$  when this latter is bounded. Bounded and unbounded cases are considered.

We denote the pressure field by  $\mathbf{p}$  (fields are in bold). It is a function of time  $t$  and position  $\underline{x} \in X$  (vectors are underlined):  $\mathbf{p}(t, \underline{x})$  is the pressure at time  $t$  and position  $\underline{x}$ . In many applications, dissipation is discarded from the propagation model. However, this is no longer affordable for passive identification purposes [5, 19, 4]. Based on the results from [4], the viscous dissipation model is considered in the present study. This equation, called Stokes' equation, relates the pressure field generated by an excitation source field  $\mathbf{f}$  and writes [14, 1]:

$$\frac{\partial^2 \mathbf{p}(t, \underline{x})}{\partial t^2} - \alpha^2 \frac{\partial}{\partial t} \Delta_X \mathbf{p}(t, \underline{x}) - v^2 \Delta_X \mathbf{p}(t, \underline{x}) = \mathbf{f}(t, \underline{x}) \quad (1)$$

where  $v$  is the sound speed in the propagation medium,  $\alpha^2$  is the damping coefficient.  $\Delta_X$  is the Laplace operator defined on  $X$  and takes null values on its boundaries  $\partial X$ . To fully described the propagation of the pressure field, Equation (1) is completed by the initial conditions: causality of the pressure field and of its first derivative with respect to time.

### 2.2. Spatial modes of Stokes' equation.

We introduce the spatial modes of Stokes' equation. Modes formalism will be used to derive exact expressions of the visco-acoustic Green's function and of

the visco-acoustic Green's correlation. To derive the modes, we compute the dispersion relation of Stokes' equation. Before, let:

$$\hat{\mathbf{p}}(\omega, \underline{x}) = \int_{\mathbb{R}} \mathbf{p}(t, \underline{x}) \exp(i\omega t) dt \quad (2)$$

be the Fourier transform over the time variable  $t$  of field  $\mathbf{p}$ . In the Fourier domain, all of the fields will be capped by the symbol  $\hat{\cdot}$ . By considering the Fourier transform of Equation (1), we get:

$$-\omega^2 \hat{\mathbf{p}}(\omega, \underline{x}) - (i\alpha^2 \omega + v^2) \Delta_X \hat{\mathbf{p}}(\omega, \underline{x}) = \hat{\mathbf{f}}(\omega, \underline{x}) \quad (3)$$

The modes are the spatial solutions of Equation (3) when there is no source ( $\hat{\mathbf{f}} = 0$ ). They are the stationary responses of the propagation medium. Let  $\mathbf{m}$  be such a solution, from Equation (3), this satisfies:

$$\Delta_X \mathbf{m}(\underline{x}) = -k^2(\omega) \mathbf{m}(\underline{x}) \quad (4)$$

where:

$$k^2(\omega) = \frac{\omega^2}{v^2 + i\alpha^2 \omega} \quad (5)$$

Equation (5) is the dispersion relation associated to Stokes' equation: it relates the spatial frequency  $k$  to the temporal frequency  $\omega$ . Equation (4) shows that a mode is an eigenvector of the Laplace operator  $\Delta_X$ . When this latter is unbounded, there is an infinity of modes  $\mathbf{m}(\underline{x}) = \exp(i\underline{k}(\omega)^T \underline{x})$  where  $\underline{k}(\omega)^T \underline{k}(\omega) = k^2(\omega)$ ; here  $T$  is the transpose operator. When the medium is bounded with sufficiently regular boundaries (as it is the case for instance for moderately reverberating environments), the modes form a countable set  $(\mathbf{m}_n)_{n \in \mathbb{N}}$  associated to a countable set of eigenvalues  $(k_n)_{n \in \mathbb{N}}$ . Those two sets

depend only on the boundary  $\partial X$ .

From the dispersion relation (Equation (5)), we can derive the modal pulsations  $(\omega_n^\pm)_{n \in \mathbb{N}}$  of the propagation medium  $X$ . For a given integer  $n$ ,  $\omega_n^\pm$  is a solution of Equation (5) when  $k(\omega) = k_n$ ; *i.e.*:

$$\omega_n^\pm = id_n \pm q_n \quad (6)$$

where:

$$d_n = \frac{\alpha^2 k_n^2}{2} \quad (7)$$

$$q_n = \sqrt{v^2 k_n^2 - d_n^2} \quad (8)$$

Modal pulsations are fundamental in modal analysis of mechanical structures, and of propagation media in general. As discussed later here, they explicitly appear in the Green's function and Green's correlation expressions in the Fourier domain.

To finish this subsection, we introduce the example of an acoustic cubic room of length  $L$  *i.e.*,  $X = [0, L]^3$ . For this particular case, the modes are written:

$$\mathbf{m}_n(\underline{x}) = \frac{L^3}{8\pi^3} \sin\left(\frac{n_1\pi x_1}{L}\right) \sin\left(\frac{n_2\pi x_2}{L}\right) \sin\left(\frac{n_3\pi x_3}{L}\right) \quad (9)$$

$$k_n^2 = (n_1^2 + n_2^2 + n_3^2) \frac{\pi^2}{L^2} \quad (10)$$

$$d_n = (n_1^2 + n_2^2 + n_3^2) \frac{\pi^2 \alpha^2}{2L^2} \quad (11)$$

where  $\underline{x} = [x_1, x_2, x_3]^T$  and where  $n_1, n_2$  and  $n_3$  are integers. This simple example will be used throughout the paper.



*2.3. Green's function of Stokes' equation.*

In this subsection, we introduce the Green's function associated with Stokes' equation for a bounded visco-acoustic propagation medium. Using the modal decomposition, we derive exact expressions in the Fourier domain and then in the time domain. Thereafter, we consider the particular regime of low damping and high frequency (also called semi-classical). This case allows acoustic codas to be introduced, based on ergodic cavity analysis, which is fundamental in passive Green's correlation retrieval.

The Green's function of a propagation medium  $X$ , is denoted by  $\mathbf{G}$  and is the causal solution of Equation (1) when the source is a localized impulse source. Thus, the Green's function is the impulse response of the propagation medium seen as a generalized linear filter. This interpretation is enhanced by the fundamental relation:

$$\mathbf{p}(t, \underline{x}) = \int_{\mathbb{R} \times X} \mathbf{G}(t - s, \underline{x} | \underline{x}') \mathbf{f}(s, \underline{x}') ds d\underline{x}' \quad (12)$$

$$= (\mathbf{G} \otimes_T \otimes_S \mathbf{f})(t, \underline{x}) \quad (13)$$

where  $\otimes_T$  is the classical time convolution and  $\otimes_S$  is the generalized space convolution (Fredholm operator). [1] and [9] provide derivations and discussions of the Green's function of Stokes' equation for a free propagation medium.

As the Green's function is a solution of a partial differential equation with constant coefficients (with respect to time and space) when the source is a spatio-temporal impulse, its Fourier transform admits a modal decomposition

given by [5]:

$$\hat{\mathbf{G}}(\omega, \underline{x}|\underline{x}') = - \sum_{n \geq 0} \frac{\mathbf{m}_n(\underline{x})\mathbf{m}_n(\underline{x}')}{(\omega - \omega_n^+)(\omega - \omega_n^-)} \quad (14)$$

The inverse Fourier transform of Equation (14) leads to the exact Green's function in the time domain [5]:

$$\mathbf{G}(t, \underline{x}|\underline{x}') = H(t) \sum_{n \geq 0} \frac{\exp\left(-\frac{d_n t}{2}\right) \sin(q_n t)}{q_n} \mathbf{m}_n(\underline{x})\mathbf{m}_n(\underline{x}') \quad (15)$$

where  $H$  is the Heaviside distribution. Even if Equations (14) and (15) are exact, their interpretation remains difficult. However, these equations are interesting for at least two reasons: they allow simulation of the visco-acoustic Green's function (see Figure 1 for an example in an acoustic cubic room), and they allow the derivation of exact expressions of the visco-acoustic Green's correlation.

The visco-acoustic Green's function provides a representation simpler than Equation (15) when we consider the two following regimes: the high frequency regime (also called the semi-classical regime [10]) and the low dissipation regime (see Appendix 5.2 for a 'quantified definition' of this regime). To derive this simpler representation, we start by considering the case when there is no dissipation *i.e.*,  $\alpha = 0$ . With this condition, the Green's function satisfies [10]:

$$\mathbf{G}(t, \underline{x}|\underline{x}') = \sum_{n \geq 1} a_n(\underline{x}, \underline{x}') \delta(t - t_n(\underline{x}, \underline{x}')) \quad (16)$$

where  $\delta$  is the Dirac distribution. In this expression,  $t_n(\underline{x}, \underline{x}')$  is the  $n$ -th time-of-arrival received at  $\underline{x}$  if an impulse was emitted at  $\underline{x}'$  at time  $t = 0$ . Coefficient  $a_n(\underline{x}, \underline{x}')$  models the geometrical attenuation due to the propagation of the im-

pulsion in the medium. In particular:

$$t_1 = \frac{\|\underline{x} - \underline{x}'\|_2}{v} \quad (17)$$

where  $\|\cdot\|_2$  is the Euclidean norm, as the travel-time between points  $\underline{x}$  and  $\underline{x}'$ . The travel-time is a relevant example of a parameter retrieved in a passive context. Indeed, it allows passive tomography to be processed [3] and passive sensor network location [15]. We will briefly return to this parameter in the next sections.

Equation (16) shows that the Green's function can be related to the trajectory of a propagating wavefront. Thus, the bounded propagation medium acts as a billiard [10]. When this latter is ergodic, there exists a time - called the mixing time and denoted by  $t_H$  - after which an impulse source 'becomes' a propagated white noise. This interpretation, which is classical in indoor acoustics [6], allows the definition of acoustic codas that are part of the signal that starts after the mixing time. As indicated in the Introduction, it has been experimentally shown that the cross-correlation of codas generated by the same source and recorded at two different points is a good approximation of the cross-correlation of spatio-temporal white noise [2].

In the presence of low dissipation, the Green's function of Stokes' equation can be approximated by Equation (16), where the Dirac distributions  $\delta(t - t_n(\underline{x}, \underline{x}'))$  are substituted by Gaussian functions centered on the times of arrival  $t_n(\underline{x}, \underline{x}')$  (coefficients  $a_n(\underline{x}, \underline{x}')$  are also redefined; [1]). Other approximations have been proposed in the literature [1, 9]. In all cases, codas and the travel-time are valid limiting cases in the presence of low dissipation. This latter point is illustrated by Figure 1, where two examples of simulated visco-acoustic Green's functions

(obtained with the modal decomposition of Equation (15)) appear to be the sums of attenuated Gaussian functions centered on the times of arrival. This approximation of the visco-acoustic Green's function is no longer causal [1, 9]. We will return briefly to this consequence in the next subsection.

#### 2.4. Green's correlation and Ward identity of Stokes' equation.

Here, we introduce Green's correlation of Stokes' equation, and we derive an exact expression in the time domain, as for the Green's function in the previous subsection. Then we obtain a Ward identity for Stokes' equation in the presence of low attenuation. Simulations are presented to display the relations derived in this subsection.

Before introducing Green's correlation, we present first a generalized interference formula that involves cross-correlation of fields. This formula will be central to the interpretation of Green's correlation. The cross-correlation of a stationary pressure field  $\mathbf{p}$  is defined by:

$$\mathbf{C}_{\mathbf{p}}(t, \underline{x}|\underline{x}') = \mathbb{E}[\mathbf{p}(t + \tau, \underline{x})\mathbf{p}(\tau, \underline{x}')] \quad (18)$$

where  $\mathbb{E}[\cdot]$  is the expectation operator. Thus, if  $\mathbf{f}$  (defined in section 2.1) is a stationary source and if  $\mathbf{p}$  is the generated pressure field, Equations (1) and (18) imply (see Appendix 5.2):

$$\mathbf{C}_{\mathbf{p}} = \mathbf{G} \otimes_T \otimes_S \mathbf{C}_{\mathbf{f}} \otimes_T \otimes_S \mathbf{G}^- \quad (19)$$

where  $\mathbf{C}_{\mathbf{f}}$  is the cross-correlation of the source  $\mathbf{f}$  defined similar to Equation (18), and  $\mathbf{G}^-$  is the time-reversed Green's function:

$$\mathbf{G}^-(t, \underline{x}|\underline{x}') = \mathbf{G}(-t, \underline{x}'|\underline{x}) \quad (20)$$

Equation (19) is the so-called generalized interference formula (see for example, chapter 10 page 422 of [12]) that relates input/ output second-order statistics of a linear system. In this context, it relates the pressure field correlation to the source field correlation, and it can be interpreted as the "order 2" version of Equation (1).

Now, we define Green's correlation:

$$\mathbf{C} = \mathbf{G} \otimes_T \otimes_S \mathbf{G}^- \quad (21)$$

As recalled from the previous subsection, the Green's function is the medium response to an impulse source. This result can be transposed to Green's correlation through Equation (19). Indeed, Green's correlation is the cross-correlation of the medium response to a spatio-temporal white source of power unity *i.e.*, a source  $\mathbf{f}$  such that:  $\mathbf{C}_f(t, \underline{x} | \underline{x}') = \delta(t) \delta(\underline{x} - \underline{x}')$ . This result justifies the strong interest of acousticians and seismologists in Green's correlation of a propagation medium, as there are ambient sources (*e.g.*, codas generated by earthquakes) such that the cross-correlation provides good approximations of the cross-correlation of a white (in time and space) noise source.

It has been shown in a general framework [19, 5] (beyond the Stokes' equation framework) that Equation (21) is valid if and only if the medium is dissipative, eventually with a low dissipation coefficient. This has also been highlighted for particular damping models: for constant damping [8] and viscous damping [4].

Now, we derive exact expressions of Green's correlation for Stokes' equation in the Fourier domain and in the time domain. The Fourier transform of Equation

(21) leads to:

$$\hat{\mathbf{C}} = \hat{\mathbf{G}} \otimes_S \hat{\mathbf{G}}^* \quad (22)$$

where  $*$  is a complex conjugate operator. Using Equation (14) and the orthogonality of modes for the spatial convolution, we get:

$$\hat{\mathbf{C}}(\omega, \underline{x}|\underline{x}') = \sum_{n \geq 0} \frac{\mathbf{m}_n(\underline{x})\mathbf{m}_n(\underline{x}')}{|\omega - \omega_n^+|^2 |\omega - \omega_n^-|^2} \quad (23)$$

This exact expression reveals that the poles of Green's correlation are equal to the modal pulsation of the propagation medium. This result is interesting for passive modal analysis. In particular, this has already been highlighted in the context of structural monitoring, where the propagation equation is the discrete Navier equation for elastic solids (see for example [7, 16] where [7] is a pioneer article in this context).

The inverse Fourier transform of Equation (23) leads to an exact expression of Green's correlation in the time domain [5]:

$$\mathbf{C}(t, \underline{x}|\underline{x}') = \sum_{n \geq 0} \frac{\exp\left(-\frac{d_n|t|}{2}\right) (q_n \cos(q_n t) + d_n \sin(q_n|t|))}{d_n q_n k_n^2} \mathbf{m}_n(\underline{x})\mathbf{m}_n(\underline{x}') \quad (24)$$

From Equation (7), we note that Equation (24) diverges when  $\alpha$  goes to 0. This emphasizes the fundamental role of dissipation: Green's correlation does not exist if the dissipation is not taken into consideration. As recalled at the beginning of this subsection, this comment can be generalized beyond the visco-acoustic context.

As discussed in the previous subsection about the Green's function of Stokes'

equation, Equation (24) is similarly difficult to interpret. This difficulty is highlighted in Figure 2, where two examples are plotted of simulated Green's correlations obtained from the exact Equation (24) with the same parameters as Figure 1. In the presence of low dissipation, the interpretation can be facilitated through the derivation of a Ward identity, which relates Green's correlation to the Green's function, which is valid in this framework. It is important to recall that derivations of Ward identities are historically explained, as the Green's function was the parameter of interest in linear media identification, because only active methods were considered (*i.e.*, using controlled sources). Thus, for passive linear media identification, a step is missing to retrieve the Green's function from Green's correlation estimated with ambient noise only. When the propagation of the medium is modeled by a linear propagation equation that includes a dissipation operator (as Stokes' equation in the acoustic case), a Ward identity can be derived [19, 5]. Thus, the Green's function can be retrieved due to this identity and to the estimated Green's correlation.

The Ward identity generally used was obtained from a constant damping model (*e.g.*, valid for vibrating strings and membranes) [8, 19, 5]:

$$\frac{\partial \mathbf{C}}{\partial t} = -a^{-1} \text{Odd} \mathbf{G} \quad (25)$$

where  $a$  is the damping coefficient, and where:

$$\text{Odd} \mathbf{G}(t, \underline{x}, \underline{x}') = \frac{1}{2} (\mathbf{G}(t, \underline{x}, \underline{x}') - \mathbf{G}(-t, \underline{x}, \underline{x}')) \quad (26)$$

is the odd part of the Green's function. This relation is exact, but the constant damping model is not really valid in the 3D acoustic framework. When the viscous damping is considered, a Ward identity for Stokes' equation has been

derived [4] for a free propagation medium and in the presence of low attenuation:

$$\frac{\partial^3 \mathbf{C}}{\partial t^3} \approx \frac{v^2}{\alpha^2} \text{OddG} \quad (27)$$

Through Equations (24) and (15), we show in Appendix 5.2 that the Ward identity of Equation (27) is still valid for any bounded propagation media. One can observe in simulation the validity of this Ward identity for the two examples presented in Figures 1 and 2. Furthermore, the Ward identity of Equation (27) allows the interpretation of Green's correlation of Stokes' equation as the third-order primitive of (the odd part of) the Green's function of Stokes' equation. This latter admits a 'simple' representation Equation (16) in the high-frequency regime, as it is the sum of the attenuated times of arrival. Thus, as shown in Appendix 5.3, Green's correlation is the sum of piecewise quadratic functions. This result facilitates the understanding of this function, and in particular, of the examples plotted in Figure 2.

However, even if the viscous damping model is adapted in 3D acoustics, this involves a third time derivative to retrieve the Green's function. More generally, whatever the considered Ward identity, of Equations (25) and (27), or see [19], for a general derivation, Green's function retrieval is subject to uncertainties due to the derivatives in time and to the limit of the validity of the damping model used. A relevant improvement will be to avoid the use of a Ward identity in passive media parameter identification. Indeed, the passive identification process leads first to an estimation of Green's correlation of the studied propagation medium. Similar to the Green's function, Green's correlation contains information on the researched medium parameters (*e.g.*, as described in the subsection about modal pulsations). This appears to be relevant to retrieve these parameters directly from this function.



To conclude this section, we briefly discuss the nonrespected causality of the approximated visco-acoustic Green's function in the presence of low attenuation (as mentioned in the previous subsection). The Ward identities of Equations (25) or (27) involve the odd part of the Green's function. The Green's function can be retrieved from its odd part if and only if it is a causal function. Thus, if this latter is not causal, the Ward identity is not sufficient to completely retrieve the Green's function from Green's correlation. However, in the presence of low attenuation, the causality is quasi-respected as the Gaussian functions, which substitute the Dirac distributions in Equation (16), admit a variance proportional to the damping coefficient  $\alpha$  [1] and thus exhibit a strong decay. When this latter goes to 0, Gaussian functions (which are not causal) converge to causal Dirac distributions.

### 3. Green's correlation retrieval

Green's correlation retrieval is the main step in passive identification. In the previous sections, we introduced codas that are a fundamental tool to provide a good approximation of white ambient noise. In this section, we describe the Green's correlation retrieval process from the use of codas. Then we present experimental results obtained with the acoustic signals recorded by microphones in an air room. The performances are discussed.

#### 3.1. Determination of the coda

The rigorous definition of a coda is difficult to explain (see [3] for a definition in the seismic context). In [3], and in many papers related to passive homogeneous media identification, the coda is defined as the tail of a propagated impulse signal. The beginning of the tail is the Heisenberg time (also called the mixing-time) of the medium seen as an ergodic cavity (if it is one). The tail stops when

its magnitude is at the same level as the magnitude of the experimental noise (including the electronic measurement noise). Figure 5 (top) shows a signal generated by the propagation of an impulse source (a localized snap in an acoustic room of size 5 m  $\times$  5 m  $\times$  2.5 m) and recorded by a microphone (sampling at 44.1 kHz on 16 bits). Figure 5 (bottom) shows the plot of the signal power evolution *versus* time.

Green's correlation is retrieved by cross-correlation of the codas extracted from different snaps. [3] presented the impact of coda extraction on Green's function retrieval in a seismic context. It is important to note that the beginning time of the extracted codas needs to be overestimated, and the end time of the extracted codas needs to be underestimated. If the beginning time is underestimated, a coherent part of the signal is considered. This introduces a bias in Green's correlation estimation. If the end time of the coda is overestimated, the signal-to-noise ratio of the cross-correlation estimate decreases under the influence of this part of the coda, which does not contain the signature of the propagation. The main difficulty relies on the estimation of the beginning of the coda, which is a well-known complex problem in the room acoustics domain (see [6], where they give an example of such an estimator).

We estimated the beginning time of the coda by considering a threshold in the decreasing part of the signal power (see Figure 5). For the experiments described below, the threshold is such that the signal is in the coda when it has lost a certain ratio, denoted by  $r_b$  ( $r_b < 1$ ), of its maximum power. The end of the coda is also determined by a threshold. The coda ends when its power is less than a certain ratio, denoted by  $r_e$  ( $r_e > 1$ ), of the experimental noise power measured on the record before the arrival of the snap excitation.

### 3.2. Determination of Green's correlation

Now, we describe the experimental results concerning the visco-acoustic Green's correlation extraction using a series of uncontrolled impulse sources generated in an *air* acoustic room and recorded by a pair of microphones. The protocol is as follows:

- The observation time window  $[-T, T]$  for Green's correlation is fixed. In practice, the value of  $T$  is conditioned by the application ( $T = 6$  ms in the presented experiments, as times of arrival lower than 6 ms are searched).
- For each impulse, the coda recorded by the two sensors is extracted, as described in the previous section, with the thresholds  $r_b$  and  $r_e$  ( $r_b = 1/2$  and  $r_e = 2$  in the presented experiments).
- All of the extracted codas are segmented into parts of length  $T$ .
- The sub-signals of the two sensors corresponding to the same coda part are cross-correlated, giving a 'coda correlation'.
- All of the coda correlations are normalized and averaged.

This protocol provides an estimation of the *normalized* visco-acoustic Green's correlation between two sensors. These experiments were carried out in different acoustic *air* rooms and for various distances between the sensors. We will limit our illustration to two realizations that are significant for all of the experimentation. Figures 6 and 7 show examples of the estimated Green's correlation obtained with this protocol and its third time derivative (obtained by a classical finite difference method). For those examples, sensors were placed in an acoustic 'uncontrolled' air room ('uncontrolled' indicates that the geometry of the room was not controlled). We present in the three following paragraphs the conclusions that emerged from these experiments.

We observed that the convergence of Green's correlation estimation is sensitive to the parameters  $r_b$ ,  $r_e$ ,  $T$ , and to the number of coda correlations averaged. The two first parameters,  $r_b$  and  $r_e$ , must be adjusted to ensure good coda extraction (as already mentioned in the previous paragraph). Experiments confirm that if 'useless' parts of the codas are extracted, the estimated Green's correlation cannot be exploited. The mixing of 'coherent' parts (too high a value of  $r_b$ ), and 'parasite experimental noise' (too low a value of  $r_e$ ) destroy the structure of the estimated Green's correlation (here, we have not tried to determine the optimal values of these parameters). The number of coda correlations (around 900 for the two examples presented) and the time window  $T$  impact on the signal-to-noise ratio. These averaging parameters need to be high in order to apply the Ward identity to retrieve the Green's function from the estimated Green's correlation. Such identities involve time derivatives so the signal-to-noise ratio on the estimated Green's function is drastically decreased, leading to nonexploitable estimations. This is illustrated by Figures 6 and 7, where the noise fluctuations of the higher magnitude of these appearing in the Green's correlation estimate (Figures 6a and 7a), appear in the estimated Green function (Figures 6b and 7b). The increasing magnitude of the fluctuations between Green's correlation and the Green's function is due to the time derivatives, and this can be critical if, *e.g.*, travel-time is the parameter to estimate. In practice, the number of coda correlations is limited by the number of snaps.

For the retrieved Green's correlation (Figures 6a and 7a), the conclusions are similar to those in the previous section: it is difficult to interpret this function and to deduce directly from it an estimator of the medium parameters (travel-time, speed propagation  $v$ , damping coefficient  $\alpha$ , and others). This lack of

interpretation makes it difficult to quantify the performances of Green’s correlation estimation in an experimental context. However, as discussed in the previous section, the retrieval quality can be measured by the use of Ward identity and a comparison with a reliable Green’s function model. This is the case for visco-acoustic and homogeneous propagation media with low attenuation (as in an acoustic air room), as discussed in section 2. This is illustrated in Figures 6b and 7b, where the third time derivative of the estimated Green’s correlation matches with the theoretical odd part of the Green’s function, which is, in this time window, a pure delay localized at the travel-time between the sensors ( $= 0.68$  ms for the first example,  $= 1.41$  ms for the second example). Furthermore, the retrieval quality can also be observed through simulated Green’s correlations if the modes of the acoustic room are known. In this context, Figure 8 shows the qualitative similarity between a simulated Green’s correlation (obtained through Equation (15) for a cubic acoustic air room) and the estimated Green’s correlation in Figure 6a (for an ‘uncontrolled’ real acoustic air room), for the same space between the microphones. This comparison shows that the structure of the two curves is identical, except around the delay 0. This discrepancy might be due to the nonperfect convergence of the coda-based method around the 0 delay. More precisely, we can illustrate with simulations that the quadratic shape of Green’s correlation around the 0 delay is difficult to retrieve, as it involves the first modes of the decomposition of Equation (24). In practice, the measured codas emulate white noise in a frequency band  $[f_{\min}; f_{\max}]$  where  $f_{\min} > 0$ . Missing low frequencies impact on the Green’s correlation retrieval, as clearly observed in Figure 7a for the times centered around 0, and lower than the time of flight. However, as mentioned in this paragraph, a part of the structure of this function is preserved as the time of arrival is well retrieved through the Ward identity (Figure 7b).

Finally, comparisons using simulations (for the estimated Green's correlation) and the Ward identity (for the third time derivative of Green's correlation) validate the efficiency of the coda-based Green's correlation estimation. A direct perspective is to extend the use of simulations into more complex rooms, to qualify more precisely the performances of passive Green's correlation estimation.

#### 4. Discussion and conclusions.

Passive identification has been presented, and Green's correlation estimations of linear propagation media have been introduced. In the context of Stokes' equation, an exact expression of Green's correlation has been derived. These results have been used to simulate the Green's correlation function, the interpretation of which is complex. The use of the Ward identity of Equation (27), first derived by [4] for an unbounded visco-acoustic medium, has been extended for any bounded medium in the presence of low attenuation. Through this Ward identity, the third time derivative of Green's correlation is related to the Green's function of Stokes' equation. The Green's function can be more easily interpreted through ergodic billiards theory in the high frequency regime [10].

Green's correlation estimation based on coda correlations has been presented. The protocol and the performances have been discussed. The reliability of the approach has been reinforced from comparisons with simulations and from the use of the Ward identity derived in Equation (27). This study allows the validity to be established for the transposition in *homogeneous* acoustic propagation media of the coda-based Green's correlation retrieval initially introduced in seismology [2, 3].

Two main perspectives can be proposed.

First, travel-time estimation is today achieved using a Ward identity, which involves time derivatives, to estimate this parameter from the Green's function. A deeper study might avoid the use of a Ward identity to retrieve the time of flight directly from Green's correlation. This idea is reinforced by Figure 9, which shows with two examples that the travel-time can be extracted from the primitive of Green's correlation, which is a more robust estimation that needs fewer 'coda-correlations'. However, this empirical estimator needs to be theoretically justified (maybe from Equation (24)), by proving that the travel-time is a zero of Green's correlation. This question is still open.

Secondly, there is a more general question in passive media identification: the study of parameter estimation performances with respect to model uncertainties and to the 'performances' of the ambient sources used (where the concept of performances is here related to the capacity of the sources used to provide cross-correlation close to the cross-correlation of white noise). This study enters into the estimation theory domain. From our knowledge, no such study has been done even beyond the Stokes' equation.

## **5. Appendix.**

### *5.1. Proof of Equation (19).*

In this Appendix, we derive the generalized interference formula of Equation (19).

First, we expand the two terms appearing in Equation (18). From Equation (1), the first term is written as:

$$\begin{aligned}\mathbf{p}(t + \tau, \underline{x}) &= \int_{\mathbb{R} \times X} \mathbf{G}(t + \tau - s, \underline{x} | \underline{z}) \mathbf{f}(s, \underline{z}) ds d\underline{z} \\ &= \int_{\mathbb{R} \times X} \mathbf{G}(s, \underline{x} | \underline{z}) \mathbf{f}(t + \tau - s, \underline{z}) d\tau d\underline{z}\end{aligned}\quad (28)$$

Similarly, the second term is written as:

$$\mathbf{p}(\tau, \underline{x}') = \int_{\mathbb{R} \times X} \mathbf{G}(s', \underline{x}' | \underline{z}') \mathbf{f}(\tau - s', \underline{z}') ds' d\underline{z}' \quad (29)$$

Using Equations (18), (28) and (29), the cross-correlation of the pressure field is:

$$\begin{aligned}\mathbf{C}_{\mathbf{p}}(t, \underline{x} | \underline{x}') &= \int_{\mathbb{R} \times X} \int_{\mathbb{R} \times X} ds d\underline{z} ds' d\underline{z}' \times \\ &\quad \mathbf{G}(s, \underline{x} | \underline{z}) \mathbb{E}[\mathbf{f}(t + \tau - s, \underline{z}) \mathbf{f}(\tau - s', \underline{z}')] \mathbf{G}(s', \underline{x}' | \underline{z}')\end{aligned}\quad (30)$$

In this expression, only the source is stochastic, which explains why the expectation operates only on this, and not on the Green's function. The source is assumed to be a stationary process, therefore:

$$\mathbb{E}[\mathbf{f}(t + \tau - s, \underline{z}) \mathbf{f}(\tau - s', \underline{z}')] = \mathbf{C}_{\mathbf{f}}(t - s + s', \underline{z} | \underline{z}') \quad (31)$$

Now, Equation (30) is written as:

$$\begin{aligned}\mathbf{C}_{\mathbf{p}}(t, \underline{x} | \underline{x}') &= \int_{\mathbb{R} \times X} ds d\underline{z} \mathbf{G}(s, \underline{x} | \underline{z}) \int_{\mathbb{R} \times X} ds' d\underline{z}' \mathbf{C}_{\mathbf{f}}(t - s + s', \underline{z} | \underline{z}') \mathbf{G}(s', \underline{x}' | \underline{z}') \\ &= \int_{\mathbb{R} \times X} ds d\underline{z} \mathbf{G}(s, \underline{x} | \underline{z}) \int_{\mathbb{R} \times X} ds' d\underline{z}' \mathbf{C}_{\mathbf{f}}(t - s - s', \underline{z} | \underline{z}') \mathbf{G}(-s', \underline{x}' | \underline{z}') \\ &= \int_{\mathbb{R} \times X} ds d\underline{z} \mathbf{G}(s, \underline{x} | \underline{z}) (\mathbf{C}_{\mathbf{f}} \otimes_T \otimes_S \mathbf{G}^-)(t - s, \underline{z} | \underline{x}') \\ &= (\mathbf{G} \otimes_T \otimes_S \mathbf{C}_{\mathbf{f}} \otimes_T \otimes_S \mathbf{G}^-)(t, \underline{x} | \underline{x}')\end{aligned}\quad (32)$$



The second equality was obtained using the transform  $s' \mapsto -s'$  in the second integral. In the third equality, the definitions of Equation (20) of  $\mathbf{G}^-$  and of the operator  $\otimes_{\mathcal{T}} \otimes_{\mathcal{S}}$  were used. The proof is complete, as Equation (19) matches with Equation (32).

*5.2. Proof of Equation (27) for bounded propagation media.*

In this Appendix, we derive approximate expressions of the Green's function and Green's correlation of Stokes' equation for a bounded medium and in the presence of low attenuation. From these approximations, we extend to any bounded propagation media the Ward identity of Stokes' equation established in [4] for an unbounded propagation medium.

The sums appearing in Equations (15) and (24) can be approximated by finite sums ending at a certain mode of index denoted by  $N$ . This index is such that each sum indexed from this  $N$ -th mode to infinity is negligible with respect to the same sum indexed from 0 to this  $N$ -th mode. Then the low dissipation approximation is formalized by the following approximation:  $q_n \approx vk_n$  valid for all modes  $n$  between 0 and  $N$ . Note that this approximation is equivalent to the following one:  $\alpha^2 \ll 4v^2k_N^{-2}$ . With this approximation, Equation (15) of the visco-acoustic Green's function is now written as:

$$\mathbf{G}(t, \underline{x}|\underline{x}') \approx H(t) \sum_{n \geq 0} \frac{\exp\left(-\frac{\alpha^2 k_n^2 t}{2}\right) \sin(vk_n t)}{vk_n} \mathbf{m}_n(\underline{x}) \mathbf{m}_n(\underline{x}') \quad (33)$$

Similarly, Equation (24) of the visco-acoustic Green's correlation is now written as:

$$\mathbf{C}(t, \underline{x}|\underline{x}') \approx \sum_{n \geq 0} \frac{\exp\left(-\frac{\alpha^2 k_n^2 |t|}{2}\right) \cos(vk_n t)}{v\alpha^2 k_n^4} \mathbf{m}_n(\underline{x}) \mathbf{m}_n(\underline{x}') \quad (34)$$

Thus, by considering the third time derivative of this expression, and using at each derivation the approximation  $q_n \approx vk_n$ , we obtain the Ward identity for Stokes' Equation (27) for any bounded propagation medium.

*5.3. Interpretation of Green's correlation of Stokes' equation in the presence of low attenuation and in the high-frequency regime.*

In this Appendix, we show that the visco-acoustic Green's correlation can be decomposed into a sum of piecewise quadratic functions in the presence of low attenuation and in the high-frequency regime.

First, we consider the case of an infinite visco-acoustic propagation medium. The general case will be easily deduced from this first study. From Equation (16), the odd part of the Green's function is written as:

$$\text{Odd}\mathbf{G}(t, \underline{x}|\underline{x}') = a_0\delta(t - \frac{r}{v}) - a_0\delta(t + \frac{r}{v}) \quad (35)$$

where  $a_0$  is the attenuation coefficient and  $r = \|\underline{x} - \underline{x}'\|_2$ . Due to the Ward identity of Equation (27), we can compute Green's correlation by integrating Equation (35) three times. The first integration leads to:

$$\frac{\partial^2 \mathbf{C}(t, \underline{x}|\underline{x}')}{\partial t^2} = b_0 H(t - \frac{r}{v}) - b_0 H(t + \frac{r}{v}) \quad (36)$$

where  $b_0 = a_0 v^2 \alpha^2$ . The second time derivative of Green's correlation appears to be a door function. The second integration leads to:

$$\frac{\partial \mathbf{C}(t, \underline{x}|\underline{x}')}{\partial t} = b_0(t - \frac{r}{v})H(t - \frac{r}{v}) - b_0(t + \frac{r}{v})H(t + \frac{r}{v}) + \frac{b_0 r}{v} \quad (37)$$

The constant  $\frac{b_0 r}{v}$  ensures that  $\partial \mathbf{C} / \partial t$  is an odd function with respect to time. This property is justified by Equation (24), which shows that Green's correlation

is an even function with respect to time. Thus, the derivative of an even function is an odd function. Finally, the third integration leads to:

$$\mathbf{C}(t, \underline{x}|\underline{x}') = b_0(t - \frac{r}{v})^2 H(t - \frac{r}{v}) - b_0(t + \frac{r}{v})^2 H(t + \frac{r}{v}) + \frac{b_0 r t}{v} + \beta \quad (38)$$

The constant  $\beta$  is difficult to determine. It can be deduced by considering  $t = 0$  in Equation (24). In Equation (38), Green's correlation appears to be a piecewise quadratic function.

We can generalize the results obtained in the previous paragraph through Equation (16) of the Green's function. In a bounded propagation medium, the odd part of the Green's function is the sum of the terms of Equation (35), where the coefficients are  $b_n = a_n v^2 \alpha^2$  and the times of arrival are  $t_n$ , both of which were introduced in section 2. Thus, the second time derivative of Green's correlation is a sum of piecewise constant functions (illustrated by Figure 4). Then the first time derivative of Green's correlation is a sum of piecewise linear functions (illustrated by Figure 3). Finally, Green's correlation is a sum of piecewise quadratic functions (illustrated by Figure 2).

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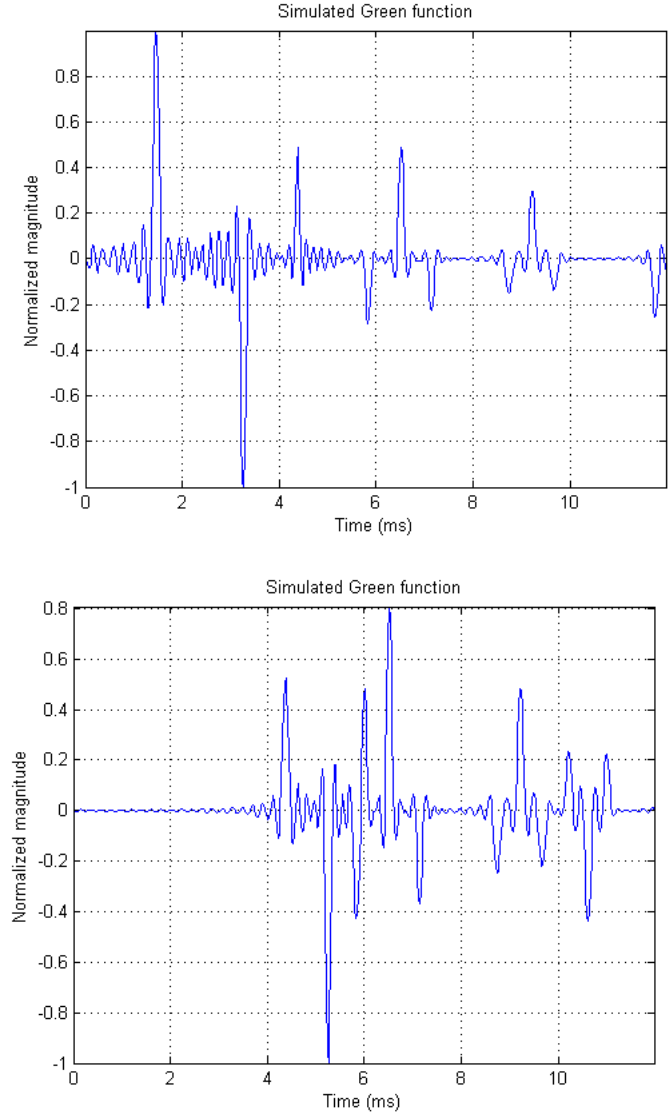


Figure 1: Simulated visco-acoustic Green's functions between two points (1a:  $\underline{x} = [2; .75; 2]$  and  $\underline{x}' = [2; 1.25; 2]$ ; 1b:  $\underline{x} = [2; .75; 2]$  and  $\underline{x}' = [2; 2.25; 2]$ ) of a cubic room of length  $L = 2.5$  m, speed propagation  $v = 343$  m.s<sup>-2</sup>, and damping coefficient  $\alpha^2 = 0.1$  m<sup>2</sup>.s<sup>-1</sup>. Time is sampled at 44.1 kHz. In both configurations, the Green's functions obtained are the sum of the attenuated times of arrival, which can be fully deduced from the geometrical configuration. These simulations involve a truncation at the 50th term (mode) of the infinite sum appearing in Equation (15). This truncation corresponds to low pass filtering, which explains the oscillations that appear around the times of arrival.

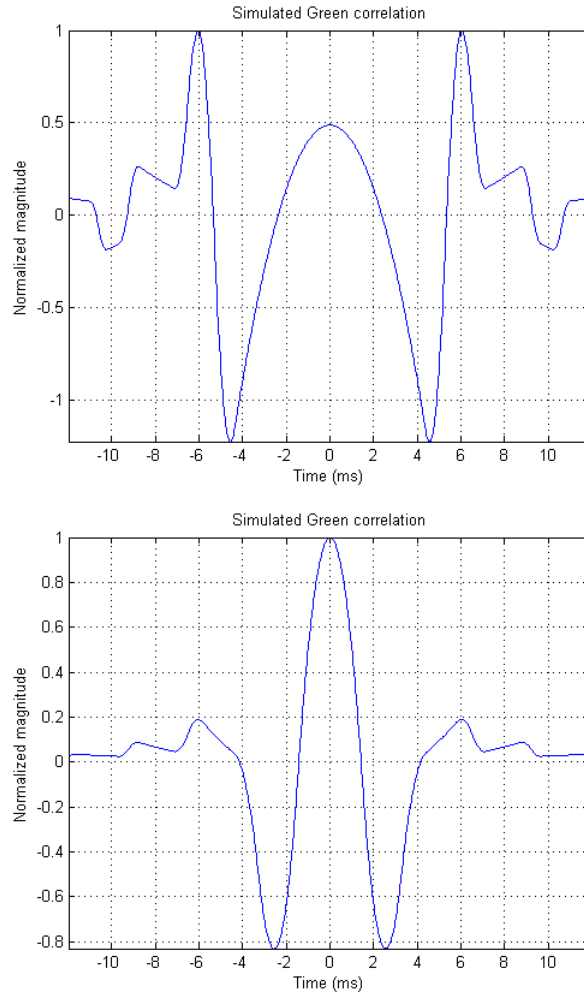


Figure 2: Simulated visco-acoustic Green's correlations between two points (the same configuration as Figure 1). These simulations involve a truncation at the 50th term (mode) of the infinite sum appearing in Equation (24). Through the Ward identity of Equation (27) and the approximation of Equation (16) of the visco-acoustic Green's function in the presence of low attenuation and in the high-frequency regime, these Green's correlations appear to be sums of piecewise quadratic functions.

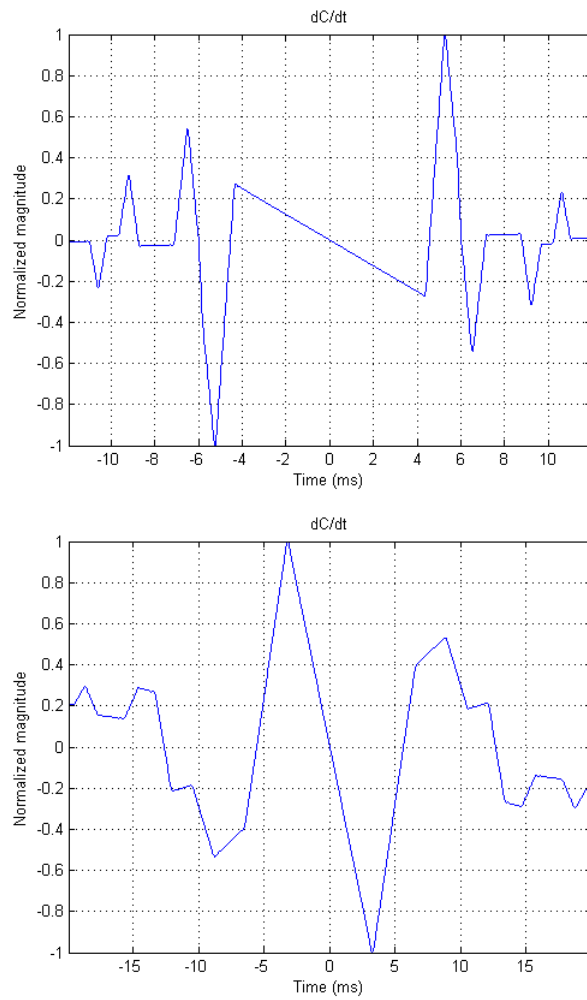


Figure 3: Time derivative of the simulated Green's correlations (the same configuration as Figure 1). Similar to Green's correlations shown in Figure 2, these functions appear to be sums of piecewise linear functions.



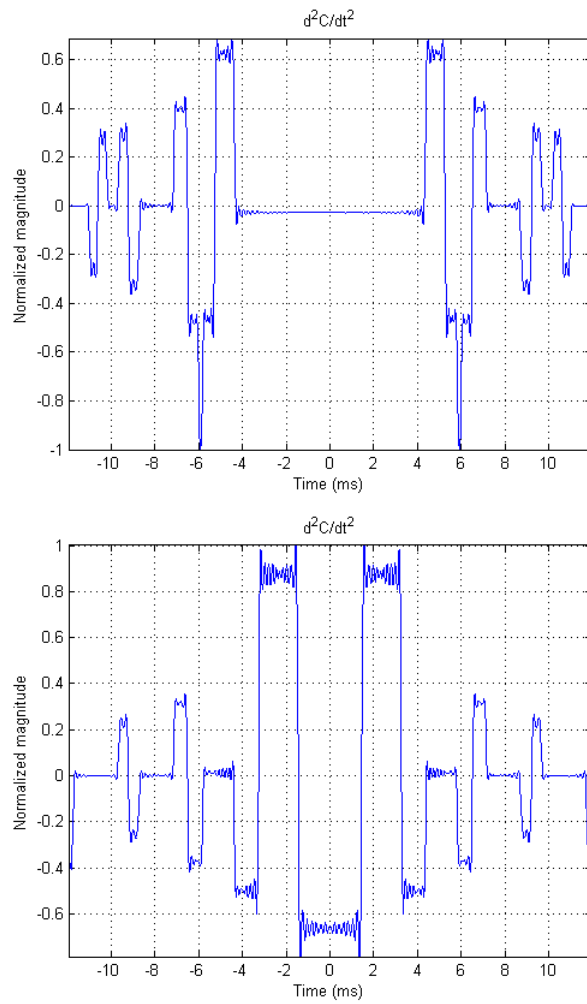


Figure 4: Second time derivative of the simulated Green's correlations (the same configuration as Figure 1). Similar to Green's correlations shown in Figure 2 and their first derivative shown in Figure 3, these functions appear to be sums of piecewise constant functions.

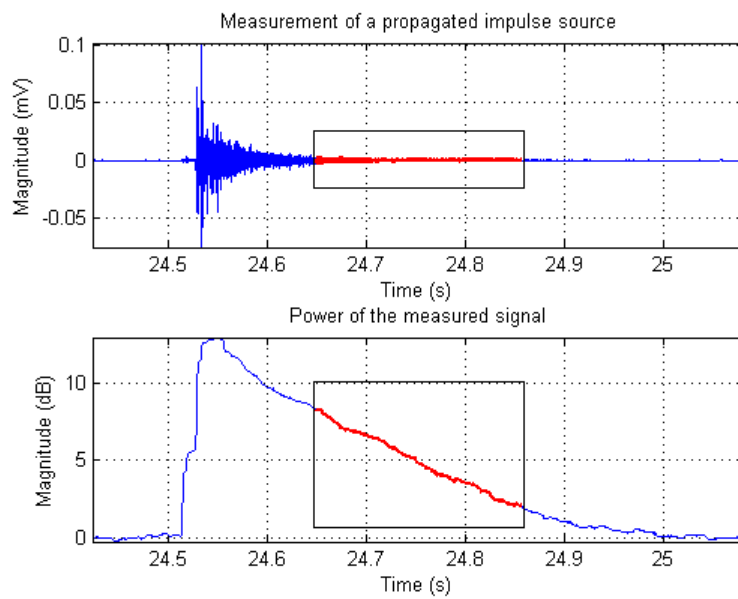


Figure 5: Top: Signal measured using a microphone of a propagated impulse source. The rectangle indicates the sub-signal corresponding to the extracted coda. Bottom: Signal power in which an increasing part appears, and then a decreasing part. The coda is localized in this second part, and it is extracted due to the two thresholds introduced in this subsection (here,  $r_b = 1/2$  and  $r_e = 2$ ).

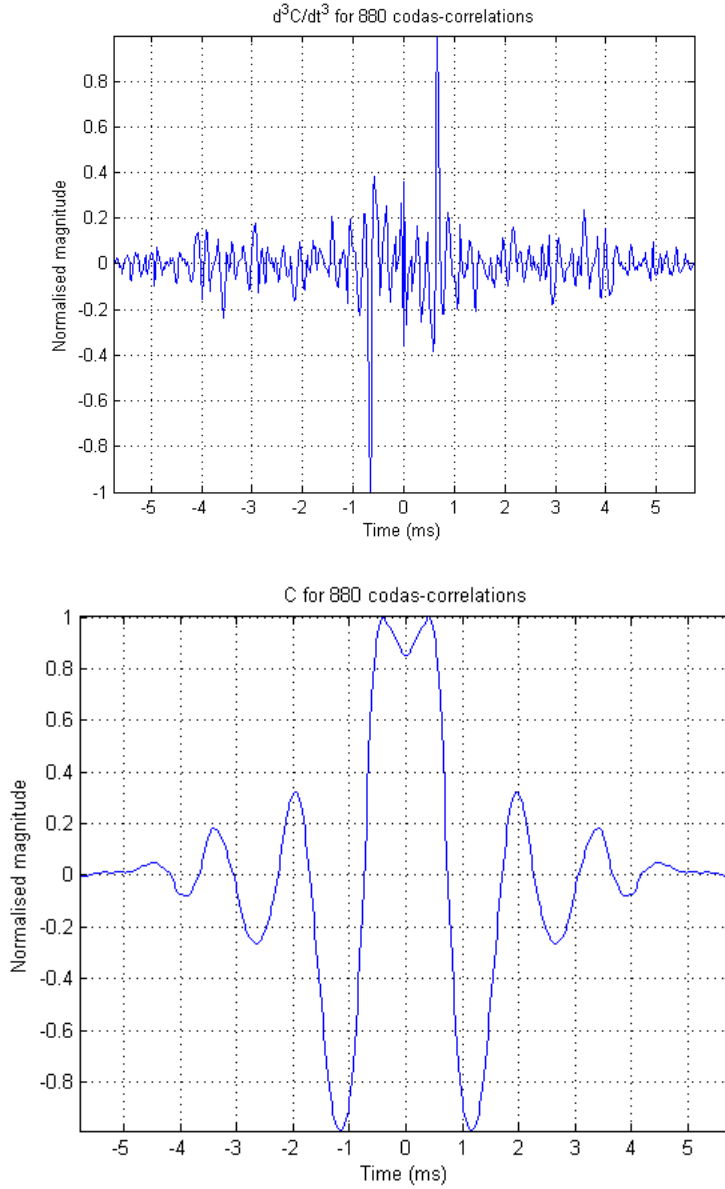


Figure 6: 6a: Retrieved Green's correlation with the coda-based method when the sensors are spaced at 23 cm. 6b: Third time derivative of the retrieved Green's correlation. The peaks are localized at  $|t| = 0.68$  ms, which corresponds to the travel-time between the sensors spaced at 23 cm in an air room where the sound speed is  $v = 343 \text{ m}\cdot\text{s}^{-1}$  at  $20^\circ\text{C}$ .

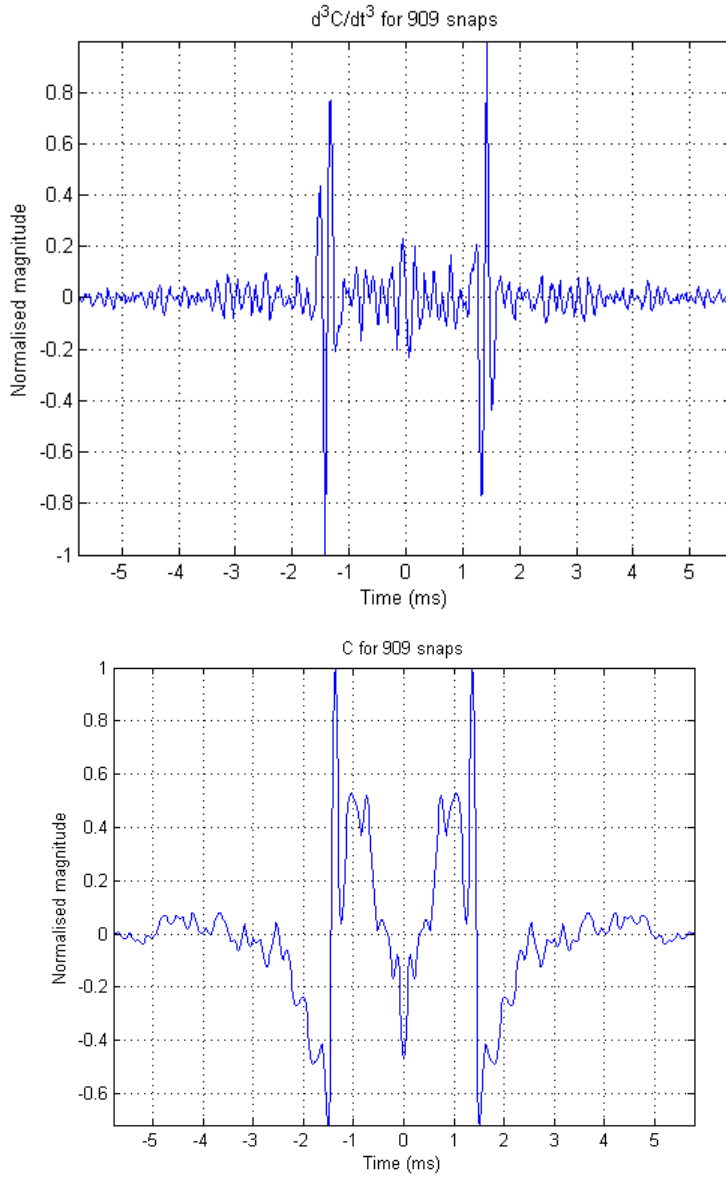


Figure 7: 7a: Retrieved Green's correlation with the coda-based method when the sensors are spaced at 48.5 cm. 7b: Third time derivative of the retrieved Green's correlation. The peaks are localized at  $|t| = 1.41$  ms, which corresponds to the travel-time between the sensors spaced at 48.5 cm in an air room where the sound speed is  $v = 343 \text{ m.s}^{-1}$  at  $20^\circ\text{C}$ .

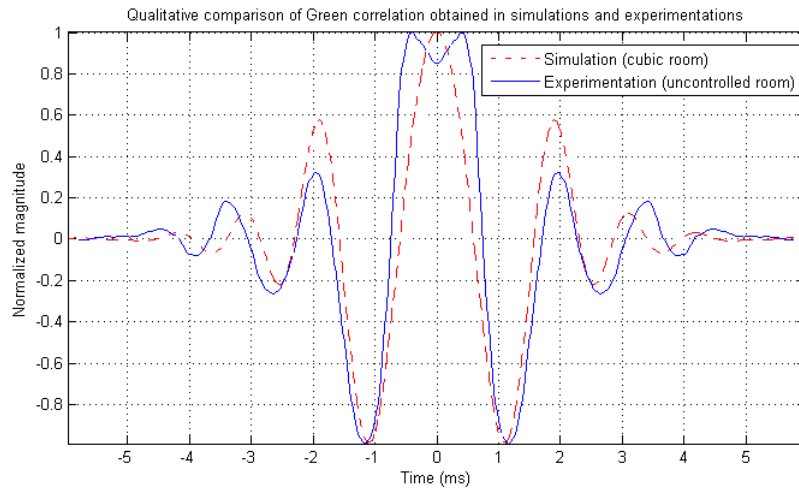


Figure 8: Comparison between Green's correlation obtained with simulations in a perfect cubic acoustic air room (dotted line) and Green's correlation obtained with real data in a real and uncontrolled acoustic air room (plain line).

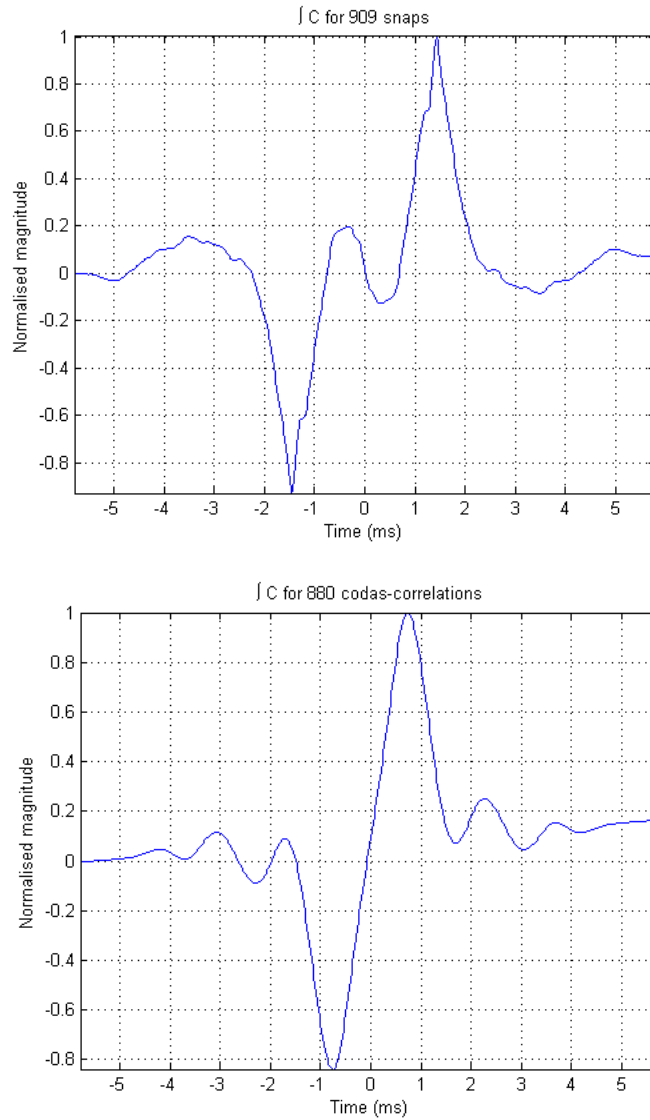


Figure 9: Primitive of estimated Green's correlations in Figures 6 (9a) and 7 (9b). Maximum arguments are localized at times  $|t| = 0.68$  ms and  $|t| = 1.41$  ms, respectively. These times correspond to the travel-times of an acoustic air impulse between sensors spaced at 23 cm and 48.5 cm, respectively.