

# A Reduction Technique for Mistuned Bladed Disks with Superposition of Large Geometric Mistuning and Small Model Uncertainties

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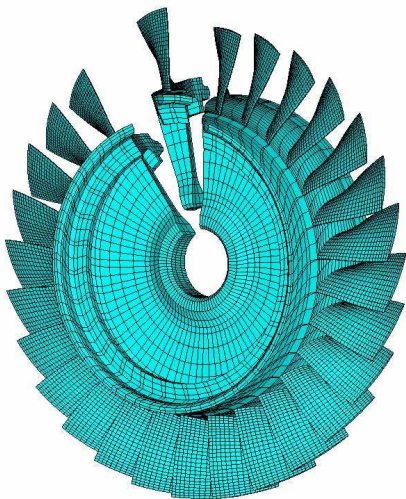


# Outline



- 1 Motivation
- 2 Proposed Method
- 3 Numerical Studies
- 4 General Conclusion

# Structures with cyclic symmetry



- ideal "tuned" bladed disks feature cyclic symmetry
- entire model can be created by revolution of a fundamental entity called sector
- block-circulant mass and stiffness matrices
- response can be calculated very efficiently using **a single sector** only

# Mistuning



- in practice differences are unavoidable among sectors
- cyclic symmetry analysis fail to predict accurately the response
- symmetry destroyed, full FEM analysis required
- mode localization, significant increase in maximum blade vibration amplitude and stress

Tuned disk response to EO 3 excitation

Mistuned disk response to EO 3 excitation

# Modeling of mistuned bladed disks



## reduced order modeling

- quantitative predictions of the dynamic response of real disks with ROM based on parent FEM
- statistical analysis of forced response using Monte-Carlo simulation

## previous research

- Component Mode Synthesis, Lim et al. (2003)
  - compact ROM, introduction of "measurable" small mistuning, ease of update in MC
  - "large" mistuning projection leads to slow convergence and high order ROM
- Static Mode Compensation, Lim et al. (2004)
  - compact ROM with "large" geometric mistuning
  - too expensive in MC loop for small mistuning projection, "non-measurable" small mistuning

# Modeling of mistuned bladed disks



## reduced order modeling

- quantitative predictions of the dynamic response of real disks with ROM based on parent FEM
- statistical analysis of forced response using Monte-Carlo simulation

## objective

- combine CMM and SMC to study small random parameter variations on geometrically mistuned disks in a computationally efficient way

# Projection-based Model Reduction



- Consider a large generalized Hermitian definite eigenvalue problem in  $\mathbb{R}^n$

$$\mathbf{K}\Phi = \Lambda\mathbf{M}\Phi$$

usually only small number of  $\lambda_i$  and  $\phi_i$  are desired

- Choose a  $k$ -dimensional subspace  $\mathcal{V}$  with  $k \ll n$  to find accurate approximations of eigenpairs in  $\mathcal{V}$
- That leads to the **projected** problem of significantly lower order

$$\mathbf{V}^T \mathbf{K} \mathbf{V} \mathbf{Y} = \Theta \mathbf{V}^T \mathbf{M} \mathbf{V} \mathbf{Y}$$

where  $\theta_i$  is approximate eigenvalue and  $\phi_i = \mathbf{V} \mathbf{y}_i$  eigenvector

# Projection-based Model Reduction



- Consider a large generalized Hermitian definite eigenvalue problem in  $\mathbb{R}^n$

$$\mathbf{K}\Phi = \Lambda\mathbf{M}\Phi$$

usually only small number of  $\lambda_i$  and  $\phi_i$  are desired

- Choose a  $k$ -dimensional subspace  $\mathcal{V}$  with  $k \ll n$  to find accurate approximations of eigenpairs in  $\mathcal{V}$
- Projected problem with mistuning

$$\mathbf{V}^T(\mathbf{K} + \mathbf{K}^\delta)\mathbf{V}\mathbf{Y} = \Theta\mathbf{V}^T(\mathbf{M} + \mathbf{M}^\delta)\mathbf{V}\mathbf{Y}$$



# Algorithm formulation



## combined SMC with CMM

- 1 Use SMC method to obtain a set of compensated basis vectors  $(\Phi_r^S - \Psi^{S,Q} \mathbf{G}_r^Q)$ .
- 2 Project large mistuning  $\mathbf{K}^{\delta_L}$  and  $\mathbf{M}^{\delta_L}$  onto the subspace spanned by the basis vectors.
- 3 Calculate mode participation factors of CMM method using  $(\Phi_r^S - \Psi^{S,Q} \mathbf{G}_r^Q)$ .
- 4 Project small mistuning  $\mathbf{K}^{\delta_S}$  and  $\mathbf{M}^{\delta_S}$ .
- 5 Assemble total reduced order model mass and stiffness matrices  $\mu_L^{syn} + \mu_S^{syn}$  and  $\kappa_L^{syn} + \kappa_S^{syn}$ .
- 6 Solve the reduced system.

Note that only steps 4 to 5 are repeated in Monte-Carlo loop.

# Calculation of compensated basis vectors



## Modal Acceleration Method

$$\Phi^S = ((\mathbf{K} + \mathbf{K}^{\delta_L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta_L}))^{-1} \mathbf{F}(\omega) + \sum_{i=1}^N \left( \frac{\omega^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - \omega^2} \hat{\Phi}_i$$

and equivalent to large mistuning excitation matrix is

$$\begin{aligned} \mathbf{F}^Q &= (-[\Lambda^S - \omega_c^2 \mathbf{I}][\mathbf{M} + \mathbf{M}^{\delta_L}] + [\mathbf{K} + \mathbf{K}^{\delta_L}] - \omega_c^2 [\mathbf{M} + \mathbf{M}^{\delta_L}]) \Phi^S \\ &= \left\{ \begin{array}{l} 0 \\ (-[\Lambda^S - \omega_c^2 \mathbf{I}] \mathbf{M}^{\delta_L} + \mathbf{K}^{\delta_L} - \omega_c^2 \mathbf{M}^{\delta_L}) \Phi^S \end{array} \right\} \end{aligned}$$

# Calculation of compensated basis vectors



Compensated "tuned" normal modes

$$\Phi^S - ((\mathbf{K} + \mathbf{K}^{\delta_L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta_L}))^{-1} \mathbf{F}(\omega) = \sum_{i=1}^N \left( \frac{\omega^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - \omega^2} \hat{\Phi}_i$$

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# Calculation of compensated basis vectors



Compensated "tuned" normal modes

$$\Phi^S - ((\mathbf{K} + \mathbf{K}^{\delta_L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta_L}))^{-1} \mathbf{F}(\omega) = \sum_{i=1}^N \left( \frac{(\omega)^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - (\omega)^2} \hat{\Phi}_i$$

$\omega$  correspond to "tuned" system natural frequencies  
and equivalent to large mistuning excitation matrix is

$$\begin{aligned} \mathbf{F}^Q &= (-[\Lambda^S - \omega_c^2 \mathbf{I}][\mathbf{M} + \mathbf{M}^{\delta_L}] + [\mathbf{K} + \mathbf{K}^{\delta_L}] - \omega_c^2 [\mathbf{M} + \mathbf{M}^{\delta_L}]) \Phi^S \\ &= \left\{ \begin{array}{l} 0 \\ (-[\Lambda^S - \omega_c^2 \mathbf{I}]\mathbf{M}^{\delta_L} + \mathbf{K}^{\delta_L} - \omega_c^2 \mathbf{M}^{\delta_L}) \Phi^S \end{array} \right\} \end{aligned}$$

# Calculation of compensated basis vectors



Compensated "tuned" normal modes

$$\Phi^S - ((\mathbf{K} + \mathbf{K}^{\delta L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta L}))^{-1} \mathbf{F}(\omega) = \sum_{i=1}^N \left( \frac{\omega^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - \omega^2} \hat{\Phi}_i$$

and equivalent to large mistuning excitation matrix is

$$\begin{aligned} \mathbf{F}^Q &= (-[\boldsymbol{\Lambda}^S - \omega_c^2 \mathbf{I}][\mathbf{M} + \mathbf{M}^{\delta L}] + [\mathbf{K} + \mathbf{K}^{\delta L}] - \omega_c^2 [\mathbf{M} + \mathbf{M}^{\delta L}]) \Phi^S \\ &= \left\{ \begin{array}{c} \mathbf{0} \\ (-[\boldsymbol{\Lambda}^S - \omega_c^2 \mathbf{I}]\mathbf{M}^{\delta L} + \mathbf{K}^{\delta L} - \omega_c^2 \mathbf{M}^{\delta L}) \Phi_{\Gamma}^S \end{array} \right\} \end{aligned}$$

# Calculation of compensated basis vectors



Compensated "tuned" normal modes

$$\Phi^S - ((\mathbf{K} + \mathbf{K}^{\delta_L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta_L}))^{-1} \mathbf{F}(\omega) = \sum_{i=1}^N \left( \frac{\omega^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - \omega^2} \hat{\Phi}_i$$

and equivalent to large mistuning excitation matrix is

$$\begin{aligned} \mathbf{F}^Q &= (-[\mathbf{\Lambda}^S - \omega_c^2 \mathbf{I}][\mathbf{M} + \mathbf{M}^{\delta_L}] + [\mathbf{K} + \mathbf{K}^{\delta_L}] - \omega_c^2 [\mathbf{M} + \mathbf{M}^{\delta_L}]) \Phi^S \\ &= \left\{ \begin{array}{c} \mathbf{0} \\ (-[\mathbf{\Lambda}^S - \omega_c^2 \mathbf{I}]\mathbf{M}^{\delta_L} + \mathbf{K}^{\delta_L} - \omega_c^2 \mathbf{M}^{\delta_L}) \Phi_\Gamma^S \end{array} \right\} \end{aligned}$$

taking advantage of number of non zero terms in  $\mathbf{M}^{\delta_L}$  and  $\mathbf{K}^{\delta_L}$

$$\begin{aligned} (\mathbf{K} + \mathbf{K}^{\delta_L} - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta_L}))^{-1} &= \\ [\mathbf{K} - \omega_c^2 \mathbf{M}]^{-1} (\mathbf{I} + [\mathbf{K}^{\delta_L} - \omega_c^2 \mathbf{M}^{\delta_L}][\mathbf{K} - \omega_c^2 \mathbf{M}^S]^{-1})^{-1} & \end{aligned}$$

# Calculation of compensated basis vectors



Compensated "tuned" normal modes

$$\Phi^S - ((\mathbf{K} + \mathbf{K}^{\delta_L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta_L}))^{-1} \mathbf{F}(\omega) = \sum_{i=1}^N \left( \frac{\omega^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - \omega^2} \hat{\Phi}_i$$

and equivalent to large mistuning excitation matrix is

$$\begin{aligned} \mathbf{F}^Q &= (-[\mathbf{\Lambda}^S - \omega_c^2 \mathbf{I}][\mathbf{M} + \mathbf{M}^{\delta_L}] + [\mathbf{K} + \mathbf{K}^{\delta_L}] - \omega_c^2 [\mathbf{M} + \mathbf{M}^{\delta_L}]) \Phi^S \\ &= \left\{ \begin{array}{c} \mathbf{0} \\ (-[\mathbf{\Lambda}^S - \omega_c^2 \mathbf{I}]\mathbf{M}^{\delta_L} + \mathbf{K}^{\delta_L} - \omega_c^2 \mathbf{M}^{\delta_L}) \Phi_\Gamma^S \end{array} \right\} \end{aligned}$$

taking advantage of number of non zero terms in  $\mathbf{M}^{\delta_L}$  and  $\mathbf{K}^{\delta_L}$

$$\begin{aligned} (\mathbf{K} + \mathbf{K}^{\delta_L} - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta_L}))^{-1} &= \\ [\mathbf{K} - \omega_c^2 \mathbf{M}]^{-1} (\mathbf{I} + [\mathbf{K}^{\delta_L} - \omega_c^2 \mathbf{M}^{\delta_L}][\mathbf{K} - \omega_c^2 \mathbf{M}^S]^{-1})^{-1} & \end{aligned}$$

# Calculation of compensated basis vectors



Compensated "tuned" normal modes

$$\Phi^S - ((\mathbf{K} + \mathbf{K}^{\delta L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta L}))^{-1} \mathbf{F}(\omega) = \sum_{i=1}^N \left( \frac{\omega^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - \omega^2} \hat{\Phi}_i$$

and equivalent to large mistuning excitation matrix is

$$\begin{aligned} \mathbf{F}^Q &= (-[\mathbf{\Lambda}^S - \omega_c^2 \mathbf{I}][\mathbf{M} + \mathbf{M}^{\delta L}] + [\mathbf{K} + \mathbf{K}^{\delta L}] - \omega_c^2 [\mathbf{M} + \mathbf{M}^{\delta L}]) \Phi^S \\ &= \left\{ \begin{array}{c} \mathbf{0} \\ (-[\mathbf{\Lambda}^S - \omega_c^2 \mathbf{I}]\mathbf{M}^{\delta L} + \mathbf{K}^{\delta L} - \omega_c^2 \mathbf{M}^{\delta L}) \Phi_{\Gamma}^S \end{array} \right\} \end{aligned}$$

block-wise inversion using cyclic symmetry

$$\tilde{\Psi}_{\Gamma}^{S,Q} = \tilde{\mathbf{B}} \text{diag} [(\tilde{\mathbf{K}}_h - \omega_c^2 \tilde{\mathbf{M}}_h)_{\Gamma}^{-1}]_{h=1, \dots, H}$$



# Calculation of compensated basis vectors



Compensated "tuned" normal modes

$$\Phi^S - ((\mathbf{K} + \mathbf{K}^{\delta L}) - \omega_c^2 (\mathbf{M} + \mathbf{M}^{\delta L}))^{-1} \mathbf{F}(\omega) = \sum_{i=1}^N \left( \frac{\omega^2 - \omega_c^2}{\hat{\lambda}_i - \omega_c^2} \right) \frac{\hat{\Phi}_i^T \mathbf{F}(\omega)}{\hat{\lambda}_i - \omega^2} \hat{\Phi}_i$$

and equivalent to large mistuning excitation matrix is

$$\begin{aligned} \mathbf{F}^Q &= (-[\boldsymbol{\Lambda}^S - \omega_c^2 \mathbf{I}][\mathbf{M} + \mathbf{M}^{\delta L}] + [\mathbf{K} + \mathbf{K}^{\delta L}] - \omega_c^2 [\mathbf{M} + \mathbf{M}^{\delta L}]) \Phi^S \\ &= \left\{ \begin{array}{c} \mathbf{0} \\ (-[\boldsymbol{\Lambda}^S - \omega_c^2 \mathbf{I}]\mathbf{M}^{\delta L} + \mathbf{K}^{\delta L} - \omega_c^2 \mathbf{M}^{\delta L}) \Phi_{\Gamma}^S \end{array} \right\} \end{aligned}$$

then the equivalent force can be expressed as

$$\mathbf{G}_{\Gamma}^Q = \left\{ \begin{array}{c} \mathbf{0} \\ (\mathbf{I} + [\mathbf{K}^{\delta L} - \omega_c^2 \mathbf{M}^{\delta L}] \Psi_{\Gamma}^{S,Q})^{-1} \mathbf{F}_{\Gamma}^Q \end{array} \right\}$$

# Geometric mistuning projection



The reduced order stiffness matrix due to geometric mistuning

$$\kappa_L^{syn} = (\Phi^S - \Psi^{S,Q} \mathbf{G}_r^Q)^T (\mathbf{K} + \mathbf{K}^{\delta_L}) (\Phi^S - \Psi^{S,Q} \mathbf{G}_k^Q)$$

Mass matrix

$$\mu_L^{syn} = (\Phi^S - \Psi^{S,Q} \mathbf{G}_r^Q)^T (\mathbf{M} + \mathbf{M}^{\delta_L}) (\Phi^S - \Psi^{S,Q} \mathbf{G}_r^Q)$$

Those products can be taken on a sector-by-sector basis in a memory efficient way

# Small mistuning projection



Reduced stiffness matrix

$$\begin{aligned}\kappa^{syn} &= \kappa_L^{syn} + (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q)^T \mathbf{K}^{\delta_S} (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) \\ &= \kappa_L^{syn} + \sum_{n=1}^N \mathbf{q}_n^{K^T} \mathbf{U}_n^{K^T} \mathbf{K}_n^{\delta_S} \mathbf{U}_n^K \mathbf{q}_n^K\end{aligned}$$

# Small mistuning projection



Reduced stiffness matrix

$$\begin{aligned}\kappa^{syn} &= \kappa_L^{syn} + (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q)^T \mathbf{K}^{\delta_S} (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) \\ &= \kappa_L^{syn} + \sum_{n=1}^N \mathbf{q}_n^{K^T} \mathbf{U}_n^{K^T} \mathbf{K}_n^{\delta_S} \mathbf{U}_n^K \mathbf{q}_n^K\end{aligned}$$

Reduced mass matrix

$$\begin{aligned}\mu^{syn} &= \mu_L^{syn} + (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q)^T \mathbf{M}^{\delta_S} (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) \\ &= \mu_L^{syn} + \sum_{n=1}^N \mathbf{q}_n^{M^T} \mathbf{U}_n^{M^T} \mathbf{M}_n^{\delta_S} \mathbf{U}_n^M \mathbf{q}_n^M\end{aligned}$$

# Small mistuning projection



Reduced stiffness matrix

$$\begin{aligned}\kappa^{\text{syn}} &= \kappa_L^{\text{syn}} + (\Phi_r^S - \Psi^{S,Q} \mathbf{G}_r^Q)^T \mathbf{K}^{\delta_s} (\Phi_r^S - \Psi^{S,Q} \mathbf{G}_r^Q) \\ &= \kappa_L^{\text{syn}} + \sum_{n=1}^N \mathbf{q}_n^{K^T} \mathbf{U}_n^{K^T} \mathbf{K}_n^{\delta_s} \mathbf{U}_n^K \mathbf{q}_n^K\end{aligned}$$

Reduced mass matrix

$$\begin{aligned}\mu^{\text{syn}} &= \mu_L^{\text{syn}} + (\Phi_r^S - \Psi^{S,Q} \mathbf{G}_r^Q)^T \mathbf{M}^{\delta_s} (\Phi_r^S - \Psi^{S,Q} \mathbf{G}_r^Q) \\ &= \mu_L^{\text{syn}} + \sum_{n=1}^N \mathbf{q}_n^{M^T} \mathbf{U}_n^{M^T} \mathbf{M}_n^{\delta_s} \mathbf{U}_n^M \mathbf{q}_n^M\end{aligned}$$

where the blade portion of compensated system normal modes

$$(\Phi_r^S - \Psi^{S,Q} \mathbf{G}_r^Q) = \tilde{\mathbf{B}} \text{diag} [\mathbf{U}_n] \mathbf{q}_{h=1, \dots, H}$$

# Small mistuning projection



Reduced stiffness matrix

$$\begin{aligned}\kappa^{\text{syn}} &= \kappa_L^{\text{syn}} + (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q)^T \mathbf{K}^{\delta_S} (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) \\ &= \kappa_L^{\text{syn}} + \sum_{n=1}^N \mathbf{q}_n^{K^T} \mathbf{U}_n^{K^T} \mathbf{K}_n^{\delta_S} \mathbf{U}_n^K \mathbf{q}_n^K\end{aligned}$$

where the blade portion of compensated system normal modes

$$(\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) = \tilde{\mathbf{B}} \text{diag} [\mathbf{U}_n] \mathbf{q}$$

$h=1, \dots, H$

is described by **cantilevered-blade normal modes**

$$\mathbf{U}_n^K = \begin{bmatrix} \Phi_n^B & \Psi_n^{B,K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \Phi_n^B - (\mathbf{K}_{n,\Gamma_{ii}} + \mathbf{K}_{n_{ii}}^{\delta_L})^{-1} (\mathbf{K}_{n,\Gamma_{ib}} + \mathbf{K}_{n_{ib}}^{\delta_L}) & \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

# Small mistuning projection



Reduced stiffness matrix

$$\begin{aligned}\kappa^{\text{syn}} &= \kappa_L^{\text{syn}} + (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q)^T \mathbf{K}^{\delta_S} (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) \\ &= \kappa_L^{\text{syn}} + \sum_{n=1}^N \mathbf{q}_n^{K^T} \mathbf{U}_n^{K^T} \mathbf{K}_n^{\delta_S} \mathbf{U}_n^K \mathbf{q}_n^K\end{aligned}$$

where the blade portion of compensated system normal modes

$$(\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) = \tilde{\mathbf{B}} \text{diag} [\mathbf{U}_n] \mathbf{q}$$

$h=1, \dots, H$

is described by cantilevered-blade normal modes + boundary modes

$$\mathbf{U}_n^K = \begin{bmatrix} \Phi_n^B & \Psi_n^{B,K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \Phi_n^B - (\mathbf{K}_{n,\Gamma_{ii}} + \mathbf{K}_{n_{ii}}^{\delta_L})^{-1} (\mathbf{K}_{n,\Gamma_{ib}} + \mathbf{K}_{n_{ib}}^{\delta_L}) & \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

subscripts i and b indicate that each n-th blade component is partitioned into internal and boundary DOFs

# Small mistuning projection



Reduced stiffness matrix

$$\begin{aligned}\kappa^{syn} &= \kappa_L^{syn} + (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q)^T \mathbf{K}^{\delta_S} (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) \\ &= \kappa_L^{syn} + \sum_{n=1}^N \mathbf{q}_n^{K^T} \mathbf{U}_n^{K^T} \mathbf{K}_n^{\delta_S} \mathbf{U}_n^K \mathbf{q}_n^K\end{aligned}$$

where the blade portion of compensated system normal modes

$$(\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) = \tilde{\mathbf{B}} \text{diag} [\mathbf{U}_n] \mathbf{q}$$

$h=1, \dots, H$

a set of precalculated modal participation factors

$$\mathbf{q}_n^K = \begin{bmatrix} \mathbf{q}_{\Phi,n}^K \\ \mathbf{q}_{\Psi,n}^K \end{bmatrix} = \begin{bmatrix} \Lambda_n^{B^{-1}} \Phi_n^{B^T} (\mathbf{K}_{n,\Gamma} + \mathbf{K}_n^{\delta_L}) (\Phi_{n,\Gamma}^S - \Psi^{S,Q} \mathbf{G}_{n,\Gamma}^Q) \\ (\Phi_{n,\Gamma}^S - \Psi^{S,Q} \mathbf{G}_{n,\Gamma}^Q)_b \end{bmatrix}$$

subscript  $b$  denotes blade boundary DOF partition of compensated modes



# Small mistuning projection



Reduced stiffness matrix

$$\begin{aligned}\kappa^{syn} &= \kappa_L^{syn} + (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q)^T \mathbf{K}^{\delta_s} (\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) \\ &= \kappa_L^{syn} + \sum_{n=1}^N \mathbf{q}_n^{K^T} \mathbf{U}_n^{K^T} \mathbf{K}_n^{\delta_s} \mathbf{U}_n^K \mathbf{q}_n^K\end{aligned}$$

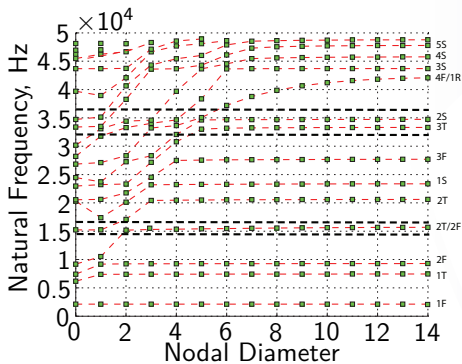
where the blade portion of compensated system normal modes

$$(\Phi_\Gamma^S - \Psi^{S,Q} \mathbf{G}_\Gamma^Q) = \tilde{\mathbf{B}} \text{diag} [\mathbf{U}_n] \mathbf{q}$$

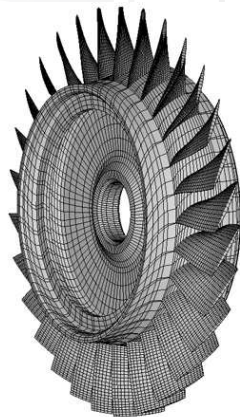
$h=1, \dots, H$

$\mathbf{U}_n^M$  and  $\mathbf{q}_n^M$  for mass mistuning are computed separately in a similar way

# Test Case: Industrial rotor

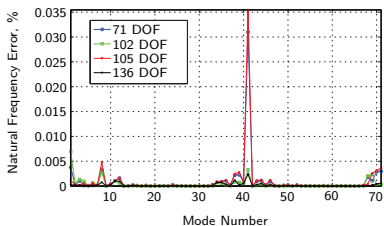


Tuned disk mode characterization

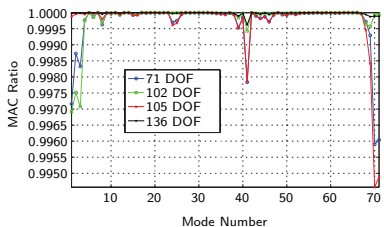


29 blades, 126846 DOF

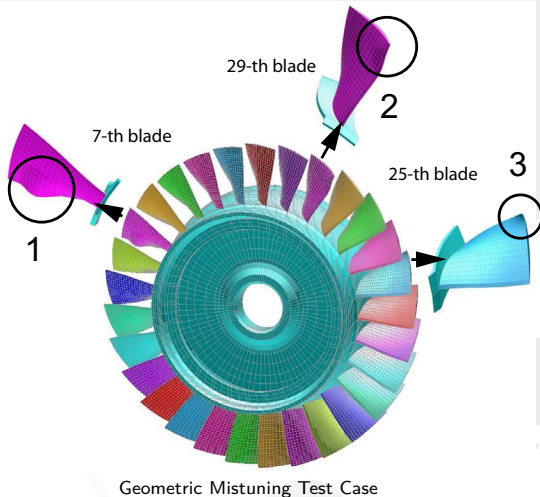
# Test Case: Industrial rotor



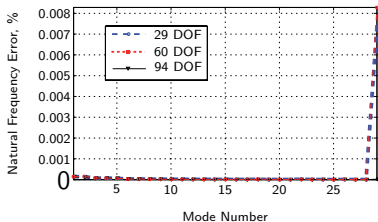
Natural frequency errors for 33 – 36 KHz region



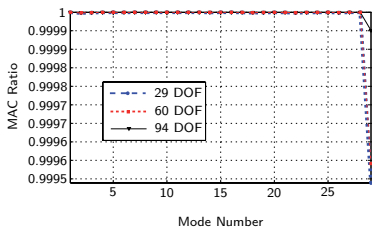
MAC for 33 – 36 KHz region



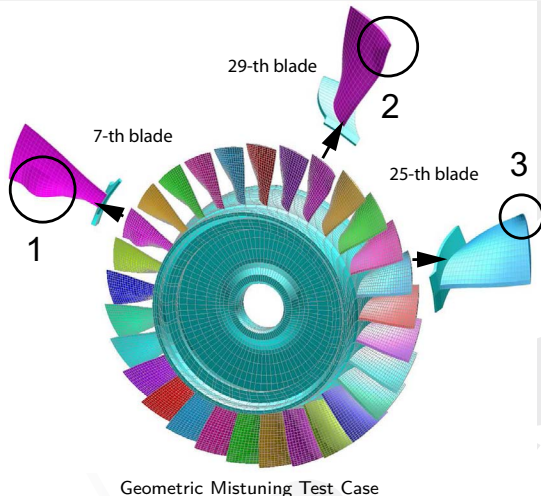
# Test Case: Industrial rotor



Natural frequency errors for 14 – 16 KHz region



MAC for 14 – 16 KHz region

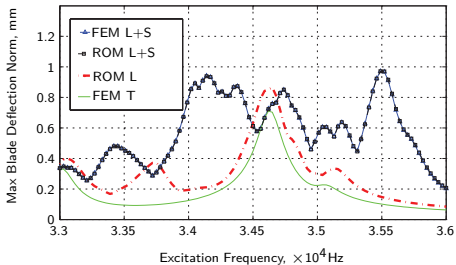


# Tuned and Mistuned Forced Response

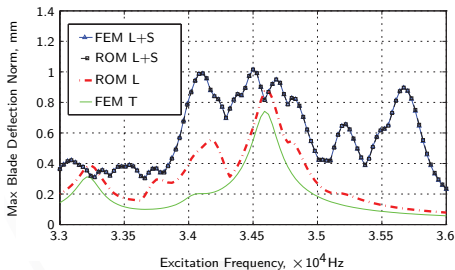


introduction of additional small stiffness mistuning  
(proportional mistuning)

- Nominal Young's modulus of the  $n$ -th blade mistuned as  $E_n = E_0(1 + \delta_n^e)$



Forced response in 33 – 36 KHz EO 5  
excitation



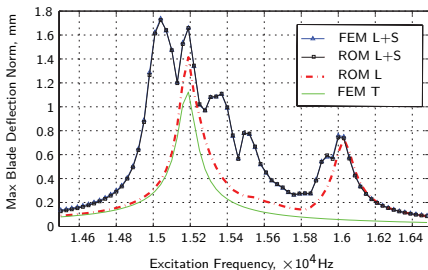
Forced response in 33 – 36 KHz EO 3  
excitation

# Tuned and Mistuned Forced Response

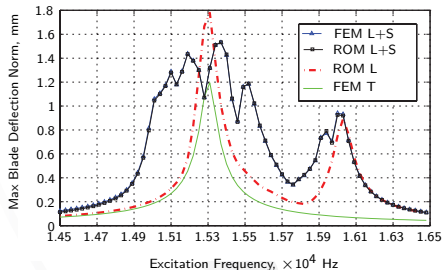


introduction of additional small stiffness mistuning  
(proportional mistuning)

- Nominal Young's modulus of the  $n$ -th blade mistuned as  $E_n = E_0(1 + \delta_n^e)$



Forced response in 14.5 – 16.5 KHz EO 2  
excitation



Forced response in 14.5 – 16.5 KHz EO 3  
excitation

# Concluding remarks and future work



## conclusion

- SMC is extended to non-symmetric multiple mistuned sector case and effectively coupled with CMM
- Computational efficiency of FORTRAN implementation with Intel MKL library ( $\sim 45$  sec ROM vs.  $\sim 5$  min ANSYS full test case model analysis)
- Demonstrates accuracy of approximation

## future work

- Quantify the error of approximation
- Improve algorithm since it suffers from slow convergence for large amplitude perturbations
- Study the effect of large and small random mistuning on bladed disks (intentional mistuning, design optimization)