

Tandem Cold Rolling Mill Modeling for Multi-variable Control Synthesis

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Abstract: A tandem cold rolling mill permits to reduce the thickness of a steel strip while ensuring the required mechanical properties. The tandem cold rolling could be modeled as a strongly non-linear differential-algebraic system including state-dependent delays. In order to reach the new products specifications, be inline with new challenges and also to optimize the process, advanced multi-variable control strategies are investigated. This paper will establish, as a main contribution, a model reaching a compromise between a good approximation of the process and its simplicity to control. A linearized model of the cold rolling process including state-dependent delays will be provided and validated numerically. A simulation with a simplistic multi-variable controller will illustrate the use of this model.

Keywords: Tandem Cold Rolling, Continuous Tandem, Mathematical Model, State Space Representation, differential-algebraic system.

1. INTRODUCTION

Metal rolling is a metallurgic process in which the thickness of a metallic strip is reduced. This process is important since the manufacturing of a wide types of products requires various thin and smooth metal sheets. Tandem Cold Mill (TCM) (Roberts, 1978) consists of consecutive stands where each one is usually a two high mill stand, which means that the stand contains a pair of work rolls. Writing down the mathematical formulation of all the subparts of TCM, we end up with a system having a large dimension consisting of non-linear differential equations and non-linear algebraic constraints. Moreover, the transportation of the strip with variable speed from one stand to the consecutive one implies the presence of state-dependent delays.

Due to the evolution of the technology, the specifications required for the steel sheets continuously change asking for thinner sheets with an increased performance assessment (mainly hardness, but also flatness...) (Jelali, 2007). In order to accomplish this objective one needs a grasp understanding of the process allowing the use of advanced control techniques, that can take into account the forthcoming introduction of possible new actuators. Moreover ArcelorMittal produces annually around 90 millions tonnes of crude steel. Any improvement of the global production cost by control theory, that is without changing the device, will induce significantly positive fallouts and will be at the heart of international competitiveness.

Due to its wide applications, the rolling process attracted the interest of many researchers and different mathematical models

have been proposed (Orowan, 1943; Bland and Ford, 1948, 1952; Bryant, 1973; Roberts, 1978). As detailed in the next section the modeling of complex systems occurring in the rolling process received also a lot of attention (see for instance (Pietrzyk and Lenard, 1991) for the hardness variation model or (Sims, 1954) for the force-mill characteristics).

Even if the control theory presents for industrial applications important evolutions during the last decades, as reported in the literature (O'Dwyer, 2000; Silva et al., 2005) more than 95% of the control-loop in the industry are controlled by (a set of) single input single output (SISO) Proportional-Integral-Derivative (PID) controllers. That can be explained by their ease of implementation and tuning. This observation holds true for the rolling process. Nevertheless, the evolution of the required steel sheet specifications asks for performances beyond the capacities of such a controller. It is thus necessary to adopt new control strategies aiming at: reaching the required specifications of the steel sheets; taking into account the complex interactions inside the system; reducing the energy cost of the rolling process and finally being ready for the further introduction of new actuators such as flexible lubrication (Laugier et al., 2011, 2015). We are entering together this way by developing new multivariable controllers for TCM.

Even if several techniques are dedicated to complex non-linear systems and have been applied on TCM (see (Pittner and Simaan, 2011)), a larger class of flexible tools is related to multivariable linear systems (Skogestad and Postlethwaite, 2005). In this paper, we propose to provide a rigorous linearization methodology taking into account the interstands transport delays to obtain an appropriate model for the design of multivari-

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able controllers. The proposed methodology being generic can be adapted to TCM with new additional actuators.

Our main motivation is given by the closed-loop performance improvements in presence of variable friction due to the flexible lubrication added in the process. The main contributions of this paper are thus the following:

- (1) we establish and linearize a Multi Input Multi Output (MIMO) model that takes in consideration a variable friction for the overall TCM system;
- (2) we validate the linearized model on appropriate variation intervals of various quantities occurring into dynamics;
- (3) we propose a preliminary multi-variable controller in absence of *a priori* fixed performance objectives.

The remaining part of the paper is organized as follows. Section 2 provides a detailed presentation of the subsystems consisting the dynamics of each stand of the TCM. The mathematical description of the overall TCM model as well as its linearization is presented in Section 3. This section also provides a numerical validation of the linearized system on appropriate variation intervals of the physical quantities occurring in the process. A preliminary multi-variable control scheme is designed and illustrated in Section 3. The paper finishes with some concluding remarks and perspectives.

2. TANDEM COLD MILL MODELING

The cold rolling tandem mill is typically made up of five stands, associated in cascade, traversed by the product steel strip which passes through their independent driven work rolls. The objective of such process is to reduce strip thickness and to obtain a desired product having certain thickness, rigidity and flatness. The structure of these stands is similar, thus the description of only one of them will be given in this section. The whole model concatenates the models of each part of the stand and the actuators, among them, several are phenomenological approximations and then not unique. First of all, let us present a block diagram representing the different functions and components of the chosen model representing the behavior of one mill stand in Figure 1.

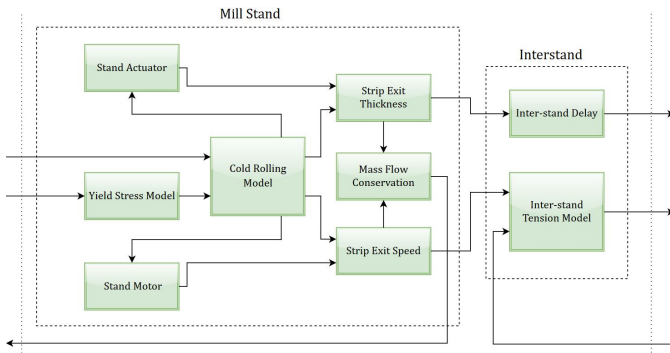


Fig. 1. Mill Stand Block Diagram

In the following, each block is briefly presented.

Stand Motor: Most of the literature (see (Geddes, 1998; Pitter and Simaan, 2010, 2011; Alves et al., 2011, 2012)) on the modeling of TCM assumes that the stand motor is a first order system. In this work, the motor is modeled as a second order to reflect the presence of an internal pre-controller.

Stand Actuator: This block is a high order non-linear system describing typically two main parts of the actuator: the servo valve and the hydraulic cylinder. An identified second order model was used to approximate and describe mathematically this actuator.

Yield Stress – SMATCH Model: Yield stress (flow stress, deformation resistance) model reveals the hardness variation of material during rolling process. The reduction of strip thickness after each stand increases the yield stress quantity. The relation describing the yield stress with the strain and the strain rate is complex and non-linear. It may be algebraic or dynamic. Several approximations more or less sophisticated are available in the literature. Among them, several models are obtained via implemented statistical methods interpolating the flow stress in function of the strain.

In this paper, SMATCH law (Ferreira et al., 2008; Dbouk et al., 2013, 2014) is considered in the process modeling and could be described as follows:

$$k(\varepsilon_p) = \lambda_0 + (\lambda_1 + \lambda_2 \varepsilon_p) (1 - \lambda_3 e^{-\lambda_4 \varepsilon_p}), \quad (1)$$

where λ_0 , λ_1 , λ_2 , λ_3 and λ_4 are constants determined statistically for each specific steel, $k(\cdot)$ is the equivalent yield stress and ε_p is the equivalent plastic strain.

Mass Flow Conservation: This law is simply conserving material volume (mass flow) between the input and output of each mill stand. Assuming that the strip width does not change, the product of strip thickness and speed is conserved during the rolling process.

Forward Slip: The rolled strip undergoes backward and forward slips respectively at the entry and exit of each roll bite. The backward slip is neglected while the forward slip at the exit is taken into consideration. The forward slip is the difference between the strip exit speed and work roll speed divided by the work roll speed.

Sims' Gauge Model: Based on the experimental curve of force-mill stretch characteristics exhibiting a non-linear part for low forces and a linear one for high forces, an affine approximation is given to estimate the strip exit thickness in function of the work roll gap variation and the total rolling force (Roberts, 1978; Sims, 1954). Notice that the slope depends on the mill modulus.

Inter-stand Tension Stress: The passage of strip from one stand to another, accompanied by the variation of strip speeds at the entry and exit of each stand leads to the variation of the tension stress between each two consecutive stands. The tension stress is given by Hook's law applied to the strip. The elasticity of the strip depends on Young's Modulus for steel. Note that the non-linearity of both entry and exit speed models leads to strongly non-linear model of tension stress.

Rolling Mathematical Model – (Roll-Gap): Through the various research in the domain of cold rolling during the past few decades, several models were given and developed for expressing the required rolling force and torque in certain analytical models. Many physical quantities (tension stress, flow stress, strip speed, thickness, ...) at the entry and the exit of each stand intervene in this model as seen in Figure 2.

Several studies tried to reduce the complexity of Orowan model (Orowan, 1943; Pietrzyk and Lenard, 1991) by introducing some assumptions for cold rolling process. As a result, more tractable models such as Bland and Ford (Bland and Ford, 1948, 1952; Bland and Sims, 1953), Bryant and Osborn (Bryant, 1973), W. Roberts (Roberts, 1978), A. Tselikov (Pietrzyk and Lenard, 1991) were proposed.

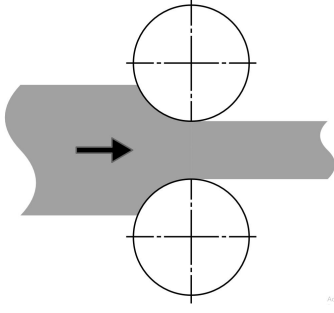


Fig. 2. Rolling Bite

ArcelorMittal Maizières with the help of academic partners has developed the so called Roll-Gap Model for cold rolling of steel which computes numerically the rolling force, torque and forward slip for a given rolling situation (Dbouk et al., 2013). Moreover it expresses high complexity and non-linearity. It is based on Slab method considering severe elastic deformation of work rolls. As a contribution in (Hitchcock, 1935), the flattened work roll radius was assumed to be elliptical. In Roll-Gap model, influence function technique is used to calculate the radial deformation of work roll radius due to rectangular compressive stresses. Inside the rolling zone, Roll-Gap model decomposes the rolled strip and work roll cylinder into finite elements and study the group of stresses exerted on each element. This permits to calculate precisely the elastic deformation of the work rolls and the plastic deformation of the strip. This model includes convergence algorithm with relaxation to validate the obtained values. Roll-Gap has been validated via large number of simulations and practical experiments. It has been simulated and compared to basic rolling algorithms.

Inter-stand Delay: The transportation of the rolled strip with variable speed from one stand to the consecutive one induces the transport delays, that are state-dependent and vary in time.

Exit Thickness Integral Action: One of the main variables to regulate is the exit thickness at the exit of each stand, because it is very sensitive and a source of perturbation for the whole system. To better regulate this variable we add an integral of the exit thickness. That will allow the multi-variable controller to have an integral action, similar to the integral term of the PID and to vanish the static position error.

3. TCM MULTIVARIABLE MODEL

3.1 Non-linear Form

As it has been shown, TCM consists of several physical systems describing all the process. Some of them are strongly non-linear especially for inter-stand tension stress and Roll-Gap models. Therefore by collecting all these dynamics, we can derive a state space non-linear multi-variable representation describing the whole TCM dynamics. This representation encapsulates all internal interactions and couplings between sub-systems. The obtained global model will depend on differential and algebraic states, input and output vectors as follows.

$$\begin{aligned} \frac{d}{dt}x(t) &= F(x(t), u(t), z(t), \mu(t), d(t), h_{en}(t)), \\ 0 &= G(x(t), u(t), z(t), \mu(t), d(t), h_{en}(t)), \\ y(t) &= L(x(t), u(t), z(t), \mu(t), d(t), h_{en}(t)), \end{aligned} \quad (2)$$

where the *control vector* $u(t) \in \mathbb{R}^{10}$, including motor and actuator inputs. The *differential state vector* $x(t) \in \mathbb{R}^{29}$, which contains dynamics of inter-stand tension stress, motor and actuator dynamics, exit thickness integral actions. The *algebraic state vector* $z(t) \in \mathbb{R}^{15}$ includes the rolling forces, rolling torques and forward slips. The *exogenous vector* $d(t) \in \mathbb{R}^3$ includes the entry annealed thickness, and TCM entry and exit tension stresses. The *output vector* $y(t) \in \mathbb{R}^{50}$, which includes all needed physical quantities in the TCM process such as tension forces, rolling forces, work roll speeds... etc. *Entry thickness vector* $h_{en}(t) \in \mathbb{R}^4$ is the vector representing the entry thicknesses for all stands except first one, which is an exogenous input. *Friction vector* $\mu(t) \in \mathbb{R}^5$, which includes the frictions between strip and work rolls in all stands.

The non-linear function F in the first equation represents the evolution of the system differential dynamics, while the non-linear function G in the second one represents the algebraic constraints of the model. The last function L represents the system outputs.

Modeling the TCM process aims at approaching as closely as possible the behavior of the real tandem. The class of non-linear multi-variable models has a strong capacity to represent high order and complex industrial processes. Nevertheless, dedicated non-linear control strategy may imply very sophisticated tools, like State-Dependent Riccati Equations (SDRE) (Pitner and Simaan, 2011), sliding modes or flatness. Here our approach is to linearize the TCM non-linear model by taking into account the presence of state-dependent delays to permit the use of the wide range of linear control tools. This method will allow to reach a compromise between good approximation and availability of tools. Linearization will permit to consider one of the linear multi-variable control strategies dedicated to such types of systems containing state and time dependent delays. Furthermore these strategies are robust and can be adjusted to certain industrial specifications booklet.

3.2 Linearized Form

In this paper, the linearization will be carried out in the neighborhood of a nominal operating point which is the steady state of the non-linear differential-algebraic system given by Equation (2). In other words, the nominal operating point is a solution of the following set of algebraic equations.

$$\begin{aligned} 0 &= F(x_{eq}(t), u_{eq}(t), z_{eq}(t), \mu_{eq}(t), d_{eq}(t), h_{en,eq}(t)), \\ 0 &= G(x_{eq}(t), u_{eq}(t), z_{eq}(t), \mu_{eq}(t), d_{eq}(t), h_{en,eq}(t)), \\ y_{eq}(t) &= L(x_{eq}(t), u_{eq}(t), z_{eq}(t), \mu_{eq}(t), d_{eq}(t), h_{en,eq}(t)). \end{aligned} \quad (3)$$

Solving the above equations may have many solutions. The user in rolling process precises the nominal operating values of some physical quantities (inter-stand tension stress between stands, exit thickness in each stand, forward slips and others) for each rolled product and thus this limits the number of solutions. The solution depending on the rolling scenario is defined for certain product.

During rolling process and due to several internal and external perturbations, the system deviates from its rolling operating point. In the vicinity of the steady state, the linearized form of the system governs the evolution of these deviations, which should be rejected by the controller. The linearization of the set

of Equations (2) consists in obtaining the expansion of Taylor series of first order at the operating point.

The validation of the linearization answers if the size of the vicinity where the difference between the non-linear model and the linearized one is enough small for the application. The linearization phase of the non-linear Roll-Gap model is numerically validated by varying quasi-statically one input of the Roll-Gap algorithm and fixing the others one. It is then possible to compute the partial derivative by the rate of differences.

Figure 3 shows the non-linear variation of the non-linear specific rolling force with respect to the exit thickness (blue curve) around its nominal value for an arbitrary stand i . The straight line is the linear approximation of the non-linear model. As it is clear, for a large variation of the exit thickness (up to $\pm 8\%$ of the nominal value), the linear approximation (black line) remains acceptable with a small error less than 3% at the extremities. This means that the Roll-Gap linear model can be considered as enough correct.

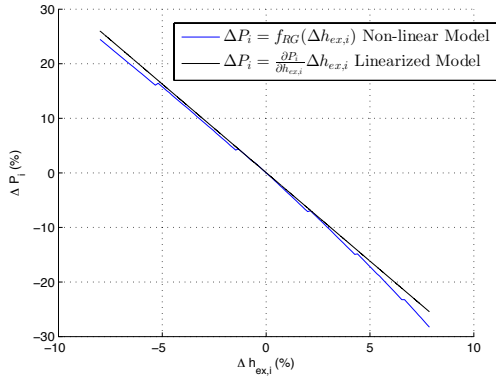


Fig. 3. Comparison between Specific Force Roll Gap and its linearized model in function of exit thickness.

To go deeper into the validation, the same method is applying by varying quasi-statically two inputs of the Roll-Gap algorithm and fixing the others ones. Figure 4 shows the variation of the specific rolling force with respect to both entries, exit thickness (up to $\pm 8\%$) and friction (up to $\pm 20\%$). The non-linear surface (blue) shows slight curvilinear aspect at the extremities. Friction varies in a wide range, since in cold rolling there is no determined variation of friction between work rolls and the strip. The plane representing the linear surface (green) shows a very good approximation of the non-linear one (multi-colored). In order to better evaluate and validate this approximation, it is important to introduce the gap between both surfaces. Figure 5 shows that the difference is less than $\pm 4\%$ with high friction variation (up to $\pm 20\%$). The error is small comparing to the inputs wide variations.

As it has been precised, the linearized model will be operated around its steady state operating point, and any variation or deviation will take place in its neighborhood. So it is better and easier to express any quantity X which have an equilibrium value X_{eq} by its variation value denoted $\Delta X = X - X_{eq}$. This variation is due to the external perturbation and variation in the exogenous components, the model will deviate relative to its equilibrium point. See Equations (5)–(7).

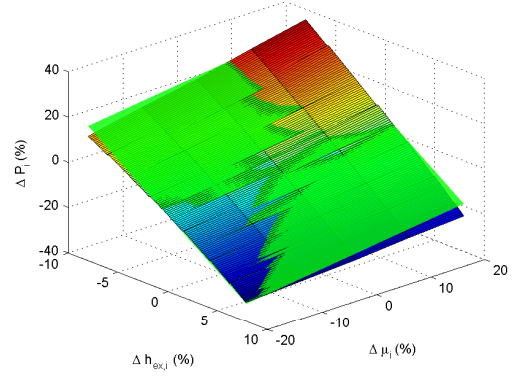


Fig. 4. Comparison between specific force and its linearized model in function of exit thickness and friction.

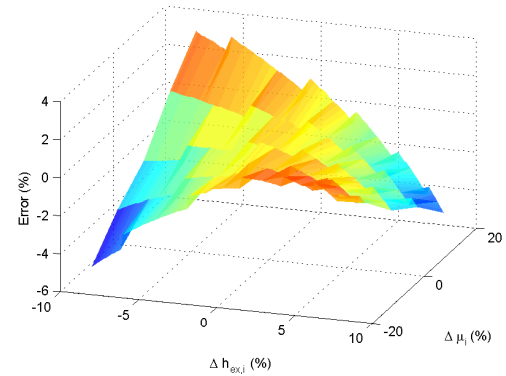


Fig. 5. Error between non-linear and linearized specific force surfaces.

The entry thickness of the each stand $h_{en,i}$ is the delayed exit thickness of the previous stand.

$$\Delta h_{en,i+1}(t) = \Delta h_{ex,i}(t - \tau_{i,i+1}), \quad (4)$$

where $h_{ex}(\cdot)$ is the exit thickness vector containing all exit thicknesses $h_{ex,i}(\cdot)$. $\tau_{i,i+1}$ represents the time delay between stand i and $i + 1$. The exit thickness is delayed due to the transportation of the steel over the inter-stand distance and then enters the next stand. Thus a part of the vector y will be delayed and looped as an additional input of the TCM system as shown in Figure 6.

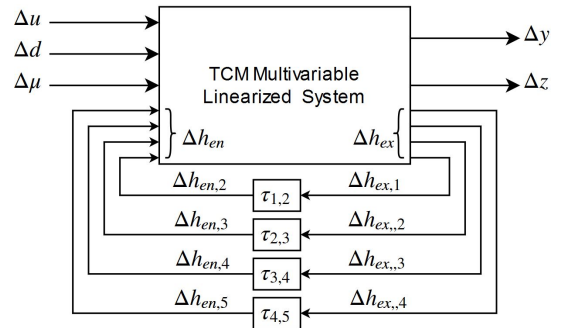


Fig. 6. TCM Linearized System looped by delayed Exit Thickness

As shown in Equation (4), the delayed exit thicknesses are the entry thicknesses. Depending on Sims' Gauge Model (Roberts,

$$\frac{d}{dt}\Delta x(t) = A\Delta x(t) + B_z\Delta z(t) + B_\mu\Delta\mu(t) + B_d\Delta d(t) + B_h\Delta h_{en}(t) + B_u\Delta u(t), \quad (5)$$

$$0 = H_x\Delta x(t) + H_z\Delta z(t) + H_\mu\Delta\mu(t) + H_d\Delta d(t) + H_h\Delta h_{en}(t), \quad (6)$$

$$\Delta y(t) = C\Delta x(t) + D_z\Delta z(t) + D_\mu\Delta\mu(t) + D_d\Delta d(t) + D_h\Delta h_{en}(t). \quad (7)$$

1978), the exit thickness is function of work roll position (differential dynamic in vector x) and total rolling force (algebraic dynamic in vector z). So, the entry Thickness vector Δh_{en} can be expressed by certain combination of delayed state differential and algebraic vectors.

$$\Delta h_{en}(t) = \sum_{i=1}^4 \Lambda_{x,i}x(t - \tau_{i,i+1}) + \Lambda_{z,i}z(t - \tau_{i,i+1}), \quad (8)$$

where $\Lambda_{x,i}$ and $\Lambda_{z,i}$ are constant matrices of appropriate dimensions. Replacing Equation (8) in (5)–(7) and regrouping all matrices in Equation (5)–(7), the obtained form is given by Equations (9)–(11), where $\tau_{0,1} = 0$, \tilde{A}_i , $\tilde{B}_{z,i}$, $\tilde{B}_{d,i}$, $\tilde{B}_{\mu,i}$, \tilde{C}_i , $\tilde{D}_{d,i}$, $\tilde{D}_{\mu,i}$ are matrices of appropriate dimensions. This form introduces the state dependent delay effect on all physical quantities. Note that in other studies (Pittner and Simaan, 2010, 2011; Alves et al., 2011, 2012) delays are usually approximated by transfer functions following the padé approximation or multiple first order lags, by assuming that the delay is constant to allow a frequency domain representation. Here such an assumption is not considered to emphasize the fact that the delays are state-dependent.

3.3 Simplistic Multi-variable Control

In order to highlight the efficiency of the model and also to outline the perspectives of our research, a simulation is presented with a typical variation of the strip entry thickness at the first stand. To have a sense, it is nevertheless necessary to associate a control law with the obtained model. A simplistic Linear Quadratic Regulator (LQR) (Anderson and Moore, 1971) is chosen for illustration purposes and to show the feasibility of our method. The LQR strategy leads to a state-feedback controller solution of an optimization problem with a quadratic cost function (specifying the requirements) under the linear constraint characterizing the system. It is well known by its well robustness and performance. The class of controllers and their tunings will be tackled during the following steps of our research. In that conditions, the following simplifying assumptions are imposed:

- the full state of the system is accessible;
- the weights of quadratic cost function are fixed to very simple values;
- the linear system is controllable; in fact, this is verified numerically on the linear system;
- only for the LQR synthesis, the delays are imposed to be trivial.

The obtained LQR is applied on the linearized system with (non trivial) state-dependent delays. The exogenous signals vary near their nominal values, the annealed entry thickness (strip entry thickness at the first stand) varies about 8%. The TCM entry traction stress varies around 2% to 3%, while the TCM exit traction stress varies about 1%. The friction in each stand varies as a noise around a certain value. Figure (7) shows the normalized variation of the exit thickness of each stand, with respect to the steady exit thickness.

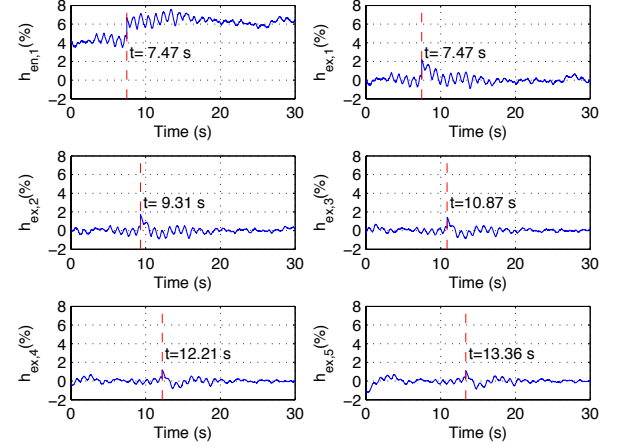


Fig. 7. TCM stands' exit thicknesses $\Delta h_{ex,i}(t)$ with a simplistic LQR

The percentage error at the exit of the first four stands is less than 2%, while at the tandem exit is less than 1%. The delay effect is clear in the strip propagation within the presented figures. This controller synthesis did not take into account the presence of inter-stand delays, though it shows a good feasibility when tested on the delayed system. The control of TCM is going to be the main objective in the future work. It should consider all system complexities and different aspects. All delays are variable, same as friction in stands are not constant. Thus the control method has to be robust enough and should follow certain specifications. Different MIMO control methods are available such as robust control (Geddes and Postlethwaite, 1994; Postlethwaite and Geddes, 1994; Pittner and Simaan, 2011).

4. CONCLUSION

The present work has proposed a global mathematical modeling of the TCM process. The elaborated MIMO model is strongly non-linear with state dependent delays. Moreover this modeling takes into account stands' friction variation in the process during rolling. It is linearized around an operating point depending on the rolled product. The linearization was validated for sufficient variation around the steady state. The linearized model is completed by the presence of state-dependent delays. Future works will focus on the design of MIMO control strategies that consider robustness, performance industrial specifications and also variable frictions.

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$$\frac{d}{dt}\Delta x(t) = \sum_{i=0}^4 \tilde{A}_i \Delta x(t - \tau_{i,i+1}) + \sum_{i=0}^4 \tilde{B}_{z,i} \Delta z(t - \tau_{i,i+1}) + B_\mu \Delta \mu(t) + B_d \Delta d(t) + B_u \Delta u(t), \quad (9)$$

$$0 = \sum_{i=0}^4 \tilde{H}_h \Delta x(t - \tau_{i,i+1}) + \sum_{i=0}^4 \tilde{H}_z \Delta z(t - \tau_{i,i+1}) + H_\mu \Delta \mu(t) + H_d \Delta d(t), \quad (10)$$

$$\Delta y(t) = \sum_{i=0}^4 \tilde{C}_i \Delta x(t - \tau_{i,i+1}) + \sum_{i=0}^4 \tilde{D}_{z,i} \Delta z(t - \tau_{i,i+1}) + D_\mu \Delta \mu(t) + D_d \Delta d(t). \quad (11)$$

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