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Risk Sharing in an Adverse Selection Model

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Abstract

We introduce risk aversion in a mixed moral hazard/adverse selection model. Under plausible assumptions, the effort level of the firm is distorted downward from the first best level of effort for both agent types. Thus, the traditional result of no distortion on the top does not hold with risk aversion. We also show that the effort level of the low-cost type may be distorted more than that of the high cost type. With an observable cost shock, an increase in exogenous risk may increase the effort level of the efficient firm and lower the expected cost of the project.

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1 Introduction

We introduce exogenous uncertainty into the standard adverse selection agency framework. Thus the agent's and the principal's utility functions depend not just on the physical allocation, the monetary transfer and the agent's type, but also on the realization of a random variable. To make the analysis interesting, we assume that both the agent and the principal are risk averse. In designing an optimal contract, the principal must then consider its effect on the level and distribution of risk between the contracting parties, as well as on the agent's incentives to truthfully reveal private information.

Our assumption that exogenous risk affects the outcome of an agency relationship appears to be well grounded in reality. In a regulatory context, the cost of the realization of a project often contains factors outside of the firms' control. Such factors include fluctuations in exchange rates or, more generally, variability in the price of any input. Overruns associated with the construction (and caused by poor evaluation) are assigned to the firm, exposing it to exogenous risk. On the other hand, costs overruns caused by "acts of nature" that are not insurable—but tend to plague large infrastructure projects—occurring during construction typically become the responsibility of the state. More generally, over the past few decades, the literature has identified a number of risks associated with large-scale projects and these include the risks associated with fluctuations in demand, sudden changes in the costs of production factors, setbacks in completion time, and the risk of competing programs sponsored by competitors (see, e.g. Akintoye, Beck, and Hardcastle, 2003). Typically (and unfortunately), these risks are not easy to allocate between the parties and their materialization has often led to substantial costs for the taxpayers (for a detailed discussion of these issues, see de Palma, Leruth, and Prunier, 2009). Similar examples exist in a labor context, where the output of a worker depends not just on the worker's effort and ability, but also on the random quality of materials used in producing output. Finally, in sharecropping, the size of the harvest clearly depends upon the whims of the weather.

While standard models of agency generally assume that either the principal, the agent, or both are risk neutral, our assumption on risk aversion on the part of both contracting parties appears to be equally well grounded in reality. In economics, firms, to start with them, are usually assumed to be risk neutral, but this assumption is not realistic in many circumstances, for example when they have limited access to financial markets (they are too small or lack experience). Note that this holds for risks that are not too large. For large risks, firms may consider that their liability will remain limited:

they may become risklovers and this may in turn create a problem for the principal. Similarly, in the regulatory context, internal agency problems such as managerial discretion can make regulated firms act in a risk averse manner. International experience suggests that the principal should also err on the side of caution when procuring goods or services and behave in a risk averse manner. To elaborate on the issue of exchange rate fluctuations, caution should be exercised even when the probability of going through exchange rates thresholds is very low (and the time during which it is outside the two bounds very short, making it hard to measure the risk). In Spain, private toll road operators called US\$ 2.7 billion of exchange rate guarantees (in the 1970's and the 1980's) when the currency unexpectedly and only briefly went through a threshold. Similarly, in Indonesia (in 2001), an exchange rate guarantee of US\$ 260 million was called in the context of an electricity project (see Irwin, 2004, for example). The same behavior is also evident in the labor and sharecropping examples. Finally, if the project represents a significant portion of the regulator's portfolio, and project risk is not easily diversified, the principal can be expected to have preferences that are risk averse (Lewis and Sappington, 1995).

Optimal risk sharing arrangements have been studied extensively in the context of moral hazard models. Adverse selection models, on the other hand, have focused almost exclusively on eliciting truthful information from the agents. Most agency relations involve aspects of both moral hazard and adverse selection, yet with few exceptions,¹ the literature has almost universally treated these problems separately. The reason for this dichotomy is entirely technical: models that include both adverse selection and moral hazard problems have generally proven intractable.²

We analyze the mixed adverse selection/moral hazard problem of regulation introduced by Laffont and Tirole (1986), and studied more extensively in their treatise on procurement and regulation (Laffont and Tirole, 1993). Laffont and Tirole assume that both the principal and the agent are risk neutral, thereby sidestepping any issues of optimal risk sharing. Under this assumption, they are able to prove that a menu of linear contracts, which is optimal in the case of pure adverse selection, also solves the problem when performance measures are noisy, so that true moral hazard is present.

Laffont and Tirole's analysis is interesting, but precludes any meaningful discussion of optimal

¹Notable exceptions include Laffont and Tirole (1986), Laffont and Rochet (1998), Picard (1987), and Caillaud, Guesnerie, and Rey (1992). More recently, Sung (2005) solved a dynamic mixed adverse selection/moral hazard model.

²A notable recent exception is Sung (2005), who studies a continuous time model in which the agent controls both the drift and diffusion coefficient of a Brownian motion. The process is interpreted as output, and observed by the principal at discrete moments in time. Sung's problem is somewhat special, however, because the adverse selection parameter does not enter the principal's objective function directly.

risk sharing. Instead, as already indicated, we assume that both the principal and the agent are risk averse. We further assume that the realization of the exogenous random variable is ex-post observable. This assumption allows us to eliminate the moral hazard component, and reduce the problem to an equivalent problem with pure adverse selection. Our assumption on the ex-post observability of the cost shock represents an empirically important phenomenon as is evident from the examples given above. The cost of the realization of a project often contains factors outside of the firms' control. We have already discussed fluctuations in exchange rates or, more generally, variability in the price of any input, are observed ex-post and expose both the firms and the consumers to exogenous risk. Besides, procurement contracts typically contain contingent clauses that are determined by the realization of a random variable. The realized value of these random variables is generally not known at the effort selection stage, but is easily observed ex-post. As an added bonus, this assumption allows us to move away from the generally intractable specification involving the simultaneous presence of adverse selection and moral hazard. Our formulation yields a tractable model in which the issue of risk sharing between consumers and the firm can be analyzed.

Our analysis reveals that the presence of such exogenous risk interacts with the adverse selection problem in ways that yield interesting results. For example, we are able to show that under plausible conditions, an increase in uncertainty, effectuated by a mean preserving spread, simultaneously reduces the effort level of the inefficient firm, and raises the effort level of the efficient firm. As a consequence, increased risk may reduce the expected cost of the project. Intuitively, this is because the presence of exogenous risk opens the door for renegotiation, allowing the principal to extract some surplus from a firm eager to obtain an insurance against that risk.

Another interesting result of our analysis is that, in the absence of exogenous uncertainty, the simultaneous risk aversion of agent and principal distorts all effort levels downward from their first best level. Distortions at the top can arise when there are type dependent outside options (see, e.g. Julien, 2000) or in rent extractions models with non-quasi linear utility functions, but the role of risk aversion in obtaining that result is new.

The paper is organized as follows: in Section 2, we present the model and we solve it in the absence of exogenous risk in Section 3. Section 4 is devoted to the presence of exogenous risk and Section 5 to the issue of risk sharing. Section 6 concludes the paper and discusses options for future research.

2 The Model

There are three types of economic actors in our model: consumers, a firm, and a government or regulator. The government acts as a benevolent principal, contracting with the firm for the provision of an indivisible public project. The cost to the firm of implementing this project is given by

$$C = \beta - e + \varepsilon. \tag{1}$$

The first component of cost, β , represents the firm's efficiency level. We assume that β is private information to the firm, and can take on two possible values, $\underline{\beta}$ and $\bar{\beta}$, where $\underline{\beta} \leq \bar{\beta}$. The prior distribution on β is specified as $\pi = \Pr(\beta = \underline{\beta})$, and is assumed to be common knowledge. The second component of cost, e , represents the firm's effort to reduce costs. We assume that e is unobservable to the principal. The final component of cost, ε , represents an exogenous shock to the cost of the project. The random variable ε can take on two possible values, ε_L and ε_H , where $\varepsilon_L \leq \varepsilon_H$. The prior distribution on ε is specified as $\alpha = \Pr(\varepsilon = \varepsilon_H)$.

Without loss of generality, we assume that $E(\varepsilon) = 0$. Neither the principal nor the firm know the realization of ε prior to the contracting and effort selection stages.

We assume that ex-post the principal can observe the realized cost value C and the realized shock ε . Contracts can therefore be conditioned on these realized values. However, the principal is unable to directly disentangle the remaining components of cost, even ex-post. These assumptions require some further explanation.

Ex-post verifiability of C limits adverse selection relative to the standard case where C is unobservable. This is most clear when there is no exogenous uncertainty, i.e. $\varepsilon_L = \varepsilon_H = 0$. A high cost type $\bar{\beta}$ wishing to mimic the low cost type $\underline{\beta}$ is then no longer free to select the profit maximizing level of effort. Instead she must select it such that the induced cost observation is identical to the equilibrium cost observation on the low cost type. Laffont and Tirole (1986), who introduced this assumption to the literature, elaborate further on this point.

Our assumption on ex-post observability of the cost shock ε is empirically motivated. The cost of the realization of a project often contains factors outside of the firms' control. Fluctuations in exchange rates, or more generally variability in the price of any input, expose both the firm and the consumers to exogenous risk. The realized value of these random variables is generally not

known at the effort selection stage, but is easily observed ex-post. In addition, there is a technical reason for insisting on ex-post observability of exogenous risk factors: if ε were unobservable, the specification (1) would imply the simultaneous presence of adverse selection and moral hazard, and such models are generally intractable. Our formulation yields a tractable model in which the issue of risk sharing between consumers and the firm can be analyzed.

The government contracts with the firm on the provision of the public good, compensating it with a transfer t financed by consumers. The firm's net income or profit is defined by $\Pi = t - C$.

Effort provision is costly to the firm. We assume that the firm's utility U is additively separable in profit and effort, i.e. $U(\Pi, e) = u(\Pi) - \psi(e)$, where u and ψ are strictly increasing and concave functions. Thus the firm is risk averse with respect to monetary flows, and exhibits increasing marginal disutility of effort. We assume that $u(0) = \psi(0) = 0$, so that the firm's reservation utility equals zero. To ensure that the realized cost levels always remain positive, we require that $\lim_{e \rightarrow \underline{\beta} + \varepsilon_L} \psi(e) = \infty$. We also require that $\psi'(0) < u'(0)$, so that the first best effort level is positive.

There is a single consumer whose utility if the project is implemented and a transfer t is imposed equals $v(-t)$, where v is a strictly increasing and concave function.³ Thus the consumer is risk averse. To capture the idea that taxing consumers is socially costly, we require that $v'(-t) \geq u'(t - C)$ for all $C \geq 0$ and $t \geq 0$. Since u' and v' are decreasing functions, this is equivalent to the condition

$$\lim_{t \rightarrow \infty} v'(-t) \geq u'(0). \quad (2)$$

The government acts as a benevolent regulator, maximizing a weighted sum of the utilities of the consumer and the firm. For simplicity, we shall assume that the weights are equal. Thus the government selects a contract to maximize

$$E(u(\Pi) + v(-t)) - \psi(e), \quad (3)$$

where the expectation is taken with respect to the joint distribution of β and ε . Formally, a contract specifies a transfer t^* as a function of the observed cost C and the observed shock ε . Thus the expectation in (3) is taken w.r.t. (C, ε) . The contract is offered after the firm learns its type, but

³Equivalently, there could be a continuum of identical consumers.

before it selects effort. The timing structure in the model is therefore as follows:

1. The firm learns its type $\beta \in \{ \underline{\beta}, \bar{\beta} \}$;
2. The government offers a contract $t^*(C, \varepsilon)$, which the firm can either accept or reject;
3. If the contract is accepted, the firm selects its effort level e ;
4. The cost shock ε is realized, and the firm produces at the cost $C = \beta - e + \varepsilon$;
5. The government observes (C, ε) and effects the transfer $t^*(C, \varepsilon)$ from the consumer to the firm.

Rejection of the contract offer yields the firm its reservation utility of zero, and the consumer the reservation utility v_0 . To solve the contracting problem, we appeal to the revelation principle. According to this principle, in designing an optimal mechanism the regulator can restrict attention to truthful and obedient equilibria of a direct mechanism in which the firm reports its type β , and the regulator recommends an effort profile $e(\beta)$ as a function of the reported type. After observing the cost level and the cost shock, the regulator effects a transfer $t(\beta, C, \varepsilon)$ to the firm, in function of the type report and the regulator's observations. The timing structure implicit in the revelation game is as follows:

1. The firm learns its type $\beta \in \{ \underline{\beta}, \bar{\beta} \}$;
2. The government commits to a transfer $t(\tilde{\beta}, C, \varepsilon)$, and requests an effort level $e(\tilde{\beta})$, where $\tilde{\beta}$ is the type report of the firm when the offer is accepted;
3. If the contract is accepted, the firm reports its type $\tilde{\beta}$, and selects an effort level e ;
4. The cost shock ε is realized, and the firm produces at the cost $C = \beta - e + \varepsilon$;
5. The government effects the transfer $t(\tilde{\beta}, C, \varepsilon)$ from the consumer to the firm.

Application of the revelation principle assumes that the regulator has access to mechanisms more complicated than simple contracts of the form $t^*(C, \varepsilon)$. However, we shall show below that optimal direct mechanisms can always be implemented by such contracts.

To solve for the optimal direct mechanism, note first that the government observes the firm's expected cost $c = \beta - e$. This is because given observation of (C, ε) , the government can calculate

$C - \varepsilon = \beta - e$. By selecting $t(\tilde{\beta}, C, \varepsilon) = 0$ if $C - \varepsilon \neq \tilde{\beta} - e(\tilde{\beta})$, it can eliminate all incentive constraints associated with type report - effort choice combinations for which the expected cost does not belong to the set $\{\beta - e(\beta) | \beta \in \{\underline{\beta}, \bar{\beta}\}\}$. For example, if the high cost type wishes to report a low cost, and receive more than its reservation utility, it will have to select its effort level e such that $\bar{\beta} - e = \underline{\beta} - e(\underline{\beta})$. Only this effort level will lead the government to observe the expected cost level designed for the low cost firm, $\underline{\beta} - e(\underline{\beta})$.

To exhibit the remaining incentive constraints, let us simplify notation, as follows. Define $\underline{e} = e(\underline{\beta})$ and $\bar{e} = e(\bar{\beta})$ as the effort levels designed for types $\underline{\beta}$ and $\bar{\beta}$, respectively. Define $\underline{c} = c(\underline{\beta})$ and $\bar{c} = c(\bar{\beta})$ to be the associated expected cost levels, i.e. $c(\beta) = \beta - e(\beta)$. Let \underline{t}_L and \underline{t}_H denote the transfers to type $\underline{\beta}$ when it reports truthfully, selects the recommended effort level \underline{e} , and the cost shocks ε_L and ε_H are observed, respectively. The transfers \bar{t}_L and \bar{t}_H are defined analogously for type $\bar{\beta}$.

Let $U(\tilde{\beta}|\beta)$ denoted the expected utility of type β when it reports type $\tilde{\beta}$ and selects the effort level inducing the expected cost $c(\tilde{\beta})$, i.e. $e = e(\tilde{\beta}) + \beta - \tilde{\beta}$.

We may then calculate: $U(\underline{\beta}|\underline{\beta})$, $U(\bar{\beta}|\underline{\beta})$, $U(\bar{\beta}|\bar{\beta})$, $U(\underline{\beta}|\bar{\beta})$, where for example : $U(\underline{\beta}|\bar{\beta})$ is given, by $(1 - \alpha)u(\underline{t}_L - \underline{c} - \varepsilon_L) + \alpha u(\underline{t}_H - \underline{c} - \varepsilon_H) - \psi(\bar{\beta} - \underline{c})$.

The principal then selects $\{\underline{t}_L, \underline{t}_H, \bar{t}_L, \bar{t}_H, \underline{c}, \bar{c}\}$ to maximize

$$\pi \left((1 - \alpha)v(-\underline{t}_L) + \alpha v(-\underline{t}_H) + U(\underline{\beta}|\underline{\beta}) \right) + (1 - \pi) \left((1 - \alpha)v(-\bar{t}_L) + \alpha v(-\bar{t}_H) + U(\bar{\beta}|\bar{\beta}) \right) \quad (4)$$

subject to the incentive constraints

$$U(\underline{\beta}|\underline{\beta}) \geq U(\bar{\beta}|\underline{\beta}); \quad (5)$$

$$U(\bar{\beta}|\bar{\beta}) \geq U(\underline{\beta}|\bar{\beta}). \quad (6)$$

and the participation constraints

$$U(\underline{\beta}|\underline{\beta}) \geq 0; \quad (7)$$

$$U(\bar{\beta}|\bar{\beta}) \geq 0. \quad (8)$$

Before proceeding to solve this program, some comments are in order. First, in formulating the

revelation principle, we have restricted attention to deterministic mechanisms. This restriction is justified, because risk aversion and convexity of ψ render random mechanisms suboptimal (Laffont and Tirole (1993)). Second, for simplicity we assume that it is always optimal to implement the project, regardless of the type realization β . Exclusion of type $\bar{\beta}$ could be optimal if $\bar{\beta}$ were sufficiently high. In that case, type $\underline{\beta}$ would be asked to select the first best effort level, and would not receive any rent. Third, any optimal direct mechanism can always be implemented with a simple contract by selecting

$$t^*(C, \varepsilon) = \begin{cases} 0, & \text{if } C - \varepsilon \neq c(\beta) \text{ for some } \beta \in \{ \underline{\beta}, \bar{\beta} \} \\ t(\beta, c(\beta) + \varepsilon, \varepsilon) & \text{if } C - \varepsilon = c(\beta). \end{cases}$$

Finally, the special case of our model in which the consumers and the firm are both risk neutral, and in which there is no exogenous shock (i.e. $\varepsilon_L = \varepsilon_H = 0$), has previously been studied by Laffont and Tirole (1993).

To build up intuition for the solution of the optimal regulatory scheme, we will first analyze some special cases of our model.

3 Solution in the Absence of Exogenous Risk

In this section, we analyze the solution when there is no exogenous cost shock, i.e. $\varepsilon_L = \varepsilon_H = 0$. We consider two cases: complete information and incomplete information. The complete information case is characterized by the condition $\underline{\beta} = \bar{\beta} = \beta$. In this case, the government knows the firm's type β . Because the cost C can be observed ex-post, the government can then infer the level of effort e selected by the firm. Under incomplete information, we have $\underline{\beta} \neq \bar{\beta}$. While the government can still observe C , it can then no longer disentangle the relative contributions of β and e .

3.1 Complete Information

Because the government can infer the level of effort e , there is neither moral hazard nor adverse selection, and the first best optimum can be achieved. Formally, the principal selects e and t to maximize $W = v(-t) + u(t - \beta + e) - \psi(e)$, subject to the participation constraint $U = u(t - \beta + e) - \psi(e) \geq 0$. Proposition 1 summarizes the solution for this benchmark case:

Proposition 1 *Under complete information and in the absence of exogenous risk, the agent receives no rent, and the effort level e^* is selected efficiently:*

$$\begin{aligned} U &= 0; \\ \psi'(e^*) &= u'(t^* - \beta + e^*). \end{aligned} \tag{9}$$

In addition, the effort level is independent of the firm's efficiency parameter β , and the transfer exactly compensates the firm for its monetary costs and disutility of effort. Formally, letting $g = u^{-1}$, we have

$$\begin{aligned} \frac{d}{de}g(\psi(e^*)) &= 1; \\ t^* &= \beta - e^* + g(\psi(e^*)). \end{aligned} \tag{10}$$

Because raising public funds is costly, as can be seen from (2), it is optimal to set t as low as possible; thus the agent is kept at its reservation utility. Efficiency dictates that effort be selected such that its marginal cost equals its marginal benefit. Because the net monetary transfer $t^* - C$ exactly compensates for the disutility of effort, the first order condition for optimal effort can be rewritten as $\psi'(e^*) = u'(g(\psi(e^*)))$. Thus e^* is independent of β . As emphasized by Laffont and Tirole (1993), this solution can be implemented by a fixed price contract, $t(C) = t^*$. The firm is then a residual claimant for its cost savings, and selects the optimal effort level e^* .

3.2 Incomplete Information

In the absence of exogenous risk, we have $\varepsilon_L = \varepsilon_H$ and hence $\underline{t}_L = \underline{t}_H = \underline{t}$ and $\bar{t}_L = \bar{t}_H = \bar{t}$. The principal's optimization problem then simplifies to

$$\max_{\{\underline{t}, \bar{t}, \underline{c}, \bar{c}\}} \pi [v(-\underline{t}) + u(\underline{t} - \underline{c}) - \psi(\underline{\beta} - \underline{c})] + (1 - \pi) [v(-\bar{t}) + u(\bar{t} - \bar{c}) - \psi(\bar{\beta} - \bar{c})] \tag{11}$$

subject to the incentive constraints

$$\underline{U} = u(\underline{t} - \underline{c}) - \psi(\underline{\beta} - \underline{c}) \geq u(\bar{t} - \bar{c}) - \psi(\underline{\beta} - \bar{c}); \tag{12}$$

$$\bar{U} = u(\bar{t} - \bar{c}) - \psi(\bar{\beta} - \bar{c}) \geq u(\underline{t} - \underline{c}) - \psi(\bar{\beta} - \underline{c}), \tag{13}$$

and the participation constraints

$$\underline{U} \geq 0; \tag{14}$$

$$\bar{U} \geq 0. \tag{15}$$

Define $\Delta\beta = \bar{\beta} - \underline{\beta}$ and for $e \geq \Delta\beta$ let $\Phi(e) = \psi(e) - \psi(e - \Delta\beta)$. Note that $\Phi(\cdot)$ is strictly increasing and strictly positive on its domain. Proposition 2 summarizes the solution of the principals's problem:

Proposition 2 *Under incomplete information and in the absence of exogenous risk, the efficient firm receives an information rent, and its effort level is chosen efficiently:*

$$\begin{aligned} \underline{U} &= \Phi(\bar{e}) > 0; \\ \psi'(\underline{e}) &= u'(\underline{t} - \underline{\beta} + \underline{e}). \end{aligned} \tag{16}$$

The inefficient firm is kept at its reservation utility, but the rent given up to the efficient firm is socially costly, which is compensated by a reduced level of effort for the inefficient firm. As a consequence, the marginal utility of effort exceeds the marginal cost:

$$\begin{aligned} \bar{U} &= 0; \\ \psi'(\bar{e}) &= u'(\bar{t} - \bar{\beta} + \bar{e}) - \frac{\pi}{1-\pi} \left(\frac{v'(-\underline{t})}{u'(\underline{t} - \underline{\beta} + \underline{e})} - 1 \right) \left(\frac{u'(\bar{t} - \bar{\beta} + \bar{e})}{v'(-\bar{t})} \right) \Phi'(\bar{e}). \end{aligned} \tag{17}$$

Contrary to the complete information case, the effort levels now depend on the efficiency parameters of the firm. In particular, the effort levels of both types of firm are below the complete information level, i.e. $\underline{e} < e^$ and $\bar{e} < e^*$. While the effort levels of the two firms are not unambiguously ranked, the cost and net transfers are: we have $\bar{c} > \underline{c} > c^*(\underline{\beta})$ and $\underline{t} - \underline{c} > t^*(\underline{\beta}) - c^*(\underline{\beta}) > \bar{t} - \bar{c}$.*

The incentive constraints (12) and (13) reveal that our mixed adverse selection - moral hazard model has the formal structure of a pure adverse selection model in which the firm reports its type β , and the government controls the allocation $\{t(\beta), c(\beta)\}$. Despite the fact that the effort level of the efficient firm is selected efficiently, in that it equates the marginal benefit of effort to its marginal disutility, we do not obtain the standard result from this literature, that \underline{e} equals the first best effort level. Risk aversion of the firm is the reason for this novel result. Comparing (16) to (9) we see

that the information rent $\Phi(\bar{e})$ raises the utility level of the efficient firm. Due to the concavity of the utility function $u(\cdot)$, the equilibrium marginal utility is then lowered. Since the marginal utility of effort equals the marginal disutility, the effort \underline{e} is then smaller than e^* . Figure 1 illustrates this effect.

[Insert Figure 1 here]

For the special case where the firm is risk neutral, this effect cancels. This result is in accord with Laffont and Tirole (1993), who analyze the special case of risk neutrality, and find that the efficient firm then selects the first best effort level.

To gain more insight into the nature of the solution, observe that at the optimum, only the incentive constraint of the low cost firm is binding. By mimicking the high cost firm, the low cost firm receives the same net transfer as the high cost firm, but need not expend as much effort, thereby saving the effort cost $\psi(\bar{e}) - \psi(\bar{e} - \Delta\beta)$. Thus the low cost firm always receives a positive information rent equalling $\Phi(\bar{e})$. The principal's problem can then be decomposed into two stages. In a first stage, the principal optimally selects \underline{t} and \underline{e} so as to provide the level of information rents $\Phi(\bar{e})$:

$$\begin{aligned} \max_{\{\underline{t}, \underline{e}\}} \pi [v(-\underline{t}) + u(\underline{t} - \underline{\beta} + \underline{e}) - \psi(\underline{e})] \\ \text{s.t. } u(\underline{t} - \underline{\beta} + \underline{e}) - \psi(\underline{e}) = \Phi(\bar{e}). \end{aligned}$$

To solve this problem, the principal will select \underline{e} to maximize $u(\underline{t} - \underline{\beta} + \underline{e}) - \psi(\underline{e})$, since this allows him to reduce \underline{t} as much as possible. This is beneficial, because transfers are socially costly. Hence we obtain (16).

In a second stage, the principal selects \bar{e} to determine the level of information rents of the efficient firm:

$$\max_{\bar{e}} \pi \{v(-\underline{t}(\Phi(\bar{e}))) + \Phi(\bar{e})\} + (1 - \pi) \{v(-\bar{t}) + u(\bar{t} - \bar{\beta} + \bar{e}) - \psi(\bar{e})\},$$

where $\underline{t}(\Phi(\bar{e}))$ is the solution in t to $r(t) = \Phi(\bar{e})$, and

$$r(t) = \max_e \{u(t - \underline{\beta} + e) - \psi(e)\}. \quad (18)$$

This form of the objective function immediately reveals that the principal will select \bar{e} inefficiently, in order to reduce the information rents earned by the low cost firm. This is because a higher information rent requires raising the transfer \underline{t} , which is socially costly. Hence we obtain (17).

Compared to the full information case, the Principal has two levers by which he can provide information rents: he can either offer a higher monetary transfer, or alter the level of effort required. To gain more information on the composition of information rents, note from envelope theorem that $r'(t) = u(t - \underline{\beta} + e(t)) > 0$. Since $r(t^*(\underline{\beta})) = 0$ and $r(\underline{t}) = \Phi(\bar{e}) > 0$, we see that $\underline{t} > t^*(\underline{\beta})$. At the same time, differentiating the first order condition associated with (18) reveals that $de(t)/dt = -u''/(u'' - \psi'') < 0$, so the principal selects $\underline{e} < e^*$. We conclude that in contrast to the case where the firm is risk neutral, the principal makes use of both levers: he raises the transfer, and lowers the required effort level.

Because both \underline{e} and \bar{e} are now distorted below the first best level e^* , they can no longer be ranked unambiguously. Given any level of risk aversion for the firm, this ranking depends on the π , the probability that the firm is of the efficient type. If π is low, then reducing the rent of the efficient type is relatively unimportant, and \bar{e} will be close to e^* . At the same time, the efficient type's rent remains bounded away from zero, and hence \underline{e} remains bounded away from e^* as π is lowered. Hence for low π , we have $\bar{e} > \underline{e}$. At the same time, when π is increased, reducing the rent of the low type becomes increasingly more important, and the distortion in \bar{e} becomes very large. Thus if π is sufficiently large, we will have $\bar{e} < \underline{e}$. Figure 2 illustrates the ranking for the case where the agent's utility function exhibits constant absolute risk aversion, with parameter γ . In this case, there exists a curve $\pi(\gamma)$, such that $\bar{e} > \underline{e}$ for $\pi < \pi(\gamma)$, and $\bar{e} < \underline{e}$ for $\pi > \pi(\gamma)$. We conclude that agent risk aversion leads to powerful incentive schemes when π is low, and low powered incentive schemes when π is high.

[Insert Figure 2 here]

4 Solution under the Presence of Exogenous Risk

In this section, we analyze the solution when exogenous risk is present, i.e. $\varepsilon_L < 0 < \varepsilon_H$. We consider two cases: complete information and incomplete information. As before, under complete information the government can infer the level of effort selected by the firm. Under incomplete information it cannot disentangle the relative contributions of β and e .

4.1 Complete information

When there is no incomplete information, we have $\underline{\beta} = \bar{\beta} = \beta$ and hence $\underline{t}_L = \bar{t}_L = t_L$ and $\underline{t}_H = \bar{t}_H = t_H$. The government then solves

$$\max_{\{t_H, t_L, c\}} \{(\alpha [v(-t_H) + u(t_H - c - \varepsilon_H) - \psi(\beta - c)] + (1 - \alpha) [v(-t_L) + u(t_L - c - \varepsilon_L) - \psi(\beta - c)])\} \quad (19)$$

subject to

$$U = \alpha u(t_H - c - \varepsilon_H) + (1 - \alpha)u(t_L - c - \varepsilon_L) - \psi(\beta - c) \geq 0. \quad (20)$$

Our assumptions on u , v and ψ imply that this is a concave programming problem. The unique solution of this program, denoted by $\{\hat{t}_H, \hat{t}_L, \hat{c}\}$ is characterized by:⁴

Proposition 3 *Under complete information, the agent does not earn any rent, and both effort choice and risk allocation are efficient:*

$$U = \alpha u(\hat{t}_H - \hat{c} - \varepsilon_H) + (1 - \alpha)u(\hat{t}_L - \hat{c} - \varepsilon_L) - \psi(\beta - \hat{c}) = 0; \quad (21)$$

$$\psi'(\beta - \hat{c}) = \alpha u'(\hat{t}_H - \hat{c} - \varepsilon_H) + (1 - \alpha)u'(\hat{t}_L - \hat{c} - \varepsilon_L); \quad (22)$$

$$\frac{v'(-\hat{t}_H)}{v'(-\hat{t}_L)} = \frac{u'(\hat{t}_H - \hat{c} - \varepsilon_H)}{u'(\hat{t}_L - \hat{c} - \varepsilon_L)}. \quad (23)$$

The solution satisfies $\hat{t}_L < t^* < \hat{t}_H$ and $\hat{t}_H - \hat{c} - \varepsilon_H < t^* - c^* < \hat{t}_L - \hat{c} - \varepsilon_L$. Also, we have $\hat{e} < e^*$ whenever u exhibits decreasing absolute risk aversion (DARA), and $\hat{e} > e^*$ whenever u exhibits increasing absolute risk aversion (IARA).

The optimal level of effort $\hat{e}(\beta)$ is now no longer independent of the cost characteristic:

$$\text{sgn} \frac{d\hat{e}}{d\beta} = -\text{sgn} \left\{ \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \left(\frac{v''_H}{v'_H} - \frac{v''_L}{v'_L} \right) \right\}.$$

In particular, if both the firm and the taxpayers exhibit DARA, or if both exhibit IARA, then $\frac{d\hat{e}}{d\beta} > 0$.

If one of them exhibits DARA and the other one exhibits IARA, then $\frac{d\hat{e}}{d\beta} < 0$.

Because the firm can hide neither its type nor its effort, it cannot extract any rent from the relationship. The government can then select the level of effort so that its marginal benefit to the

⁴DARA and IRA utility functions are discussed in e.g. Eeckhoudt, Gollier and Schlesinger (2005).

firm equals its marginal cost. The government can also choose the transfers such that the marginal rates of substitution across the two states of the world are identical for both agents. Hence the solution is fully optimal.

Compared to the case without exogenous risk, optimal effort may either decrease, increase or remain constant, depending upon whether the firm exhibits decreasing, increasing or constant absolute risk aversion. The proof of this result is long and tedious.

To gain some intuition, let us start from the riskless case where $\varepsilon_L = \varepsilon_H = 0$, and increase the level of uncertainty. The principal could respond to the increased uncertainty by setting $t_H = t^* + \varepsilon_H$ and $t_L = t^* + \varepsilon_L$ and requiring the effort level e^* . This would fully insure the firm against exogenous risk, and hence keep keep the firm at its reservation utility level. Furthermore, productive efficiency would be satisfied, as

$$\alpha u'(t_H - c - \varepsilon_H) + (1 - \alpha)u'(t_L - c - \varepsilon_L) - \psi(e^*) = \alpha u'(t^* - c) + (1 - \alpha)u'(t^* - c) - \psi(e^*) = 0.$$

Unfortunately, taxpayers then bear all the exogenous risk, and so this solution does not achieve efficient risk sharing. More precisely, we would have

$$\frac{v'(-t_H)}{v'(-t_L)} = \frac{v'(-t^* - \varepsilon_H)}{v'(-t^* - \varepsilon_L)} > 1 = \frac{u'(t^* - c)}{u'(t^* - c)} = \frac{u'(t_H - c - \varepsilon_H)}{u'(t_L - c - \varepsilon_L)}.$$

To improve risk sharing, the principal will need to leave the firm with some risk, i.e. lower t_H and increase t_L . To keep the firm at its reservation utility and maintain productive efficiency, this will require changing the level of effort of the agent.

If the agent's utility function exhibits decreasing absolute risk aversion, selecting $t_H + \varepsilon_H < t^* < t_L + \varepsilon_L$ will raise the firm's expected marginal utility of effort. Indeed, from (21) we have

$$\alpha u'_H dt_H + (1 - \alpha)u'_L dt_L = 0, \tag{24}$$

and from (22) we have

$$d(\alpha u'_H + (1 - \alpha)u'_L) = \alpha u''_H dt_H + (1 - \alpha)u''_L dt_L. \tag{25}$$

Solving for $\frac{dt_H}{dt_L}$ in (24) and substituting into (25) yields

$$dE(u') = (1 - \alpha)u'_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) > 0,$$

where the inequality follows because u is of the DARA type. To maintain productive efficiency then requires raising the level of effort.

[Insert Figure 3 here]

Graphically, we may illustrate the effect of lowering effort while maintaining productive efficiency and keeping the firm at its reservation utility as follows (see Fig. 3). For fixed level of effort e , equation (21) describes an iso-utility curve in (t_L, t_H) space. This curve is downward sloping, since

$$\frac{\partial t_H}{\partial t_L} = -\frac{1 - \alpha}{\alpha} \frac{u'_L}{u'_H} < 0.$$

For fixed e , equation (22) specifies an iso-marginal utility curve in (t_L, t_H) space. It describes the transfer combinations that maintain productive efficiency, and is also a downward sloping:

$$\frac{\partial t_H}{\partial t_L} = -\frac{1 - \alpha}{\alpha} \frac{u''_L}{u''_H} < 0.$$

When u is of the DARA type, then as long as $t_H - \varepsilon_H < t_L - \varepsilon_L$, the iso-utility curve is steeper than the iso-marginal utility curve. The two curves intersect at the point $(t_L(e), t_H(e))$. Raising effort rotates the iso-utility curve clockwise through this intersection point, as can be seen from the expression: $\partial t_H / \partial e = \alpha u'_H + (1 - \alpha)u'_L - \psi'$.

It also shifts the iso-marginal utility curve down, as $\partial^2 t_H / \partial e^2 = \alpha u''_H + (1 - \alpha)u''_L - \psi'' < 0$. As a consequence, the intersection point moves to the southeast, i.e. t_H decreases and t_L increases.

By lowering t_H and increasing t_L , raising effort indirectly improves risk sharing. To complete the intuition, we need to also consider its direct effect. Raising effort does not directly affect the taxpayer's marginal utilities, but it does increase the firm's income in both states of the world. When the utility function is of the DARA type, this will lower the ratio u'_H/u'_L , since we have

$$\frac{\partial}{\partial e} \left(\frac{u'_H}{u'_L} \right) = \frac{u''_H}{u'_L} - \frac{u'_H}{(u'_L)^2} u''_L = \frac{u'_H}{u'_L} \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right) < 0.$$

The direct effect therefore widens the gap between the marginal rates of substitution. However, the following argument comes to the rescue. Note that the solution $(t_L(e), t_H(e))$ to (21) and (22) is unique, with $t_L(e^*) = t_H(e^*)$. Furthermore, the solution to (19) is unique, satisfies (21) and (22), and has $t_L > t_H$. Since $t'_L(e) > 0$ and $t'_H(e) < 0$, as e is increased from e^* , we eventually reach the unique level of effort solving (19).

Our formal proof proceeds along a somewhat different line, by considering the effect of a mean preserving spread on the solution. It establishes that effort increases when u is of the DARA type, and conversely when u is of the IARA type. It also proves that the solution to (19) satisfies $t_L > t_H$.

4.2 Incomplete Information

We finally consider the full problem, in which the principal observes neither the firm's type nor its effort level. The optimal contract then solves (4), and is characterized as follows:

Proposition 4 *Under incomplete information and in the presence of exogenous risk, the efficient firm receives an information rent, and its effort level is chosen efficiently:*

$$\begin{aligned} \underline{U} &= \Phi(\bar{e}) > 0; \\ \psi'(\underline{e}) &= Eu'(\underline{t} - \underline{\beta} + \underline{e} - \varepsilon). \end{aligned} \tag{26}$$

The inefficient firm is kept at its reservation utility, but the rent given up to the efficient firm is socially costly, which is compensated by a reduced level of effort for the inefficient firm. As a consequence, its marginal utility of effort exceeds its marginal cost:

$$\begin{aligned} \bar{U} &= 0; \\ \psi'(\bar{e}) &= Eu'(\bar{t} - \bar{\beta} + \bar{e} - \varepsilon) - \frac{\pi}{1-\pi} \left(\frac{v'(-\underline{t}_L)}{u'(\underline{t}_L - \underline{\beta} + \underline{e} - \varepsilon_L)} - 1 \right) \left(\frac{u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L)}{v'(-\bar{t}_L)} \right) \Phi'(\bar{e}). \end{aligned} \tag{27}$$

The exogenous risk is still shared efficiently, i.e. the marginal rates of substitution of the taxpayers and the firm are equal for each realization of the efficiency parameter β :

$$\begin{aligned} \frac{v'(-\underline{t}_H)}{v'(-\underline{t}_L)} &= \frac{u'(\underline{t}_H - \underline{c} - \varepsilon_H)}{u'(\underline{t}_L - \underline{c} - \varepsilon_L)}, \\ \frac{v'(-\bar{t}_H)}{v'(-\bar{t}_L)} &= \frac{u'(\bar{t}_H - \bar{c} - \varepsilon_H)}{u'(\bar{t}_L - \bar{c} - \varepsilon_L)}. \end{aligned} \tag{28}$$

The solution satisfies $\underline{c} < \bar{c}$, $\bar{e} < \hat{e}(\bar{\beta})$ and $\underline{e} < \hat{e}(\underline{\beta})$. Increased uncertainty has an ambiguous effect

on efficiency: there exists constants $A > 0$ and $B > 0$ such that

$$A \frac{de}{d\varepsilon_L} + B \frac{d\bar{e}}{d\varepsilon_L} \geq 0 \text{ as } \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right) \geq 0.$$

In particular, if u is of the CARA type, then $\frac{de}{d\varepsilon_L}$ and $\frac{d\bar{e}}{d\varepsilon_L}$ have opposite sign.

The solution to the screening problem with exogenous uncertainty shares some characteristics with those of the screening problem without exogenous uncertainty. In particular, the efforts of both the efficient and the inefficient firm are below their respective complete information levels, $\widehat{e}(\beta)$ and $\widehat{e}(\bar{\beta})$. The intuition is the same as for the case where exogenous uncertainty is absent. The principal reduces the effort of the inefficient firm in order to limit the information rents earned by the efficient firm, as the transfers necessary to pay those rents are socially costly. He also reduces the effort level of the efficient firm, as this further lowers those transfers.

Intuitively, the condition for the optimal effort of the inefficient firm can be derived as follows. Increasing \bar{e} by one unit has two effects. First, it raises the expected utility of type $\bar{\beta}$ by $\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})$. Because it is optimal to keep this type at its reservation utility, the transfers \bar{t}_H and \bar{t}_L will be lowered to offset this change. Hence we will have

$$\alpha \bar{u}'_H d\bar{t}_H + (1 - \alpha) \bar{u}'_L d\bar{t}_L = -(\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})).$$

The decreased transfers in turn increase the utility of the taxpayer, by the amount

$$-(\alpha \bar{v}'_H d\bar{t}_H + (1 - \alpha) \bar{v}'_L d\bar{t}_L) = -\left(\alpha \frac{\bar{v}'_H}{\bar{u}'_H} \bar{u}'_H d\bar{t}_H + (1 - \alpha) \frac{\bar{v}'_L}{\bar{u}'_L} \bar{u}'_L d\bar{t}_L \right).$$

Optimal risk sharing implies $\bar{v}'_H/\bar{u}'_H = \bar{v}'_L/\bar{u}'_L$, and hence

$$-(\alpha \bar{v}'_H d\bar{t}_H + (1 - \alpha) \bar{v}'_L d\bar{t}_L) = -\frac{\bar{v}'_L}{\bar{u}'_L} (\alpha \bar{u}'_H d\bar{t}_H + (1 - \alpha) \bar{u}'_L d\bar{t}_L) = \frac{\bar{v}'_L}{\bar{u}'_L} (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})).$$

The probability that the firm is of type $\bar{\beta}$ equals $(1 - \pi)$, so that the expected increase on social welfare when type $\bar{\beta}$ is realized equals

$$(1 - \pi) \frac{\bar{v}'_L}{\bar{u}'_L} (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})). \quad (29)$$

Second, raising \bar{e} raises the rents earned by the efficient type by the amount $\Phi'(\bar{e})$. To finance this increase in rents, the transfers \underline{t}_H and \underline{t}_L must increase. The magnitude of the change in transfers is such that

$$\alpha \underline{u}'_H dt_H + (1 - \alpha) \underline{u}'_L dt_L + (\alpha \underline{u}'_H + (1 - \alpha) \underline{u}'_L - \psi'(\underline{e})) = \Phi'(\bar{e}).$$

Productive efficiency now requires that $\alpha \underline{u}'_H + (1 - \alpha) \underline{u}'_L - \psi'(\underline{e}) = 0$, so we have

$$\alpha \underline{u}'_H dt_H + (1 - \alpha) \underline{u}'_L dt_L = \Phi'(\bar{e}).$$

The increased transfers in turn lower the utility of the taxpayer, by the amount

$$\alpha \underline{v}'_H dt_H + (1 - \alpha) \underline{v}'_L dt_L = \left(\alpha \frac{\underline{v}'_H}{\underline{u}'_H} \underline{u}'_H dt_H + (1 - \alpha) \frac{\underline{v}'_L}{\underline{u}'_L} \underline{u}'_L dt_L \right).$$

Using the optimal risk sharing condition then yields

$$\alpha \underline{v}'_H dt_H + (1 - \alpha) \underline{v}'_L dt_L = \frac{\underline{v}'_L}{\underline{u}'_L} (\alpha \underline{u}'_H dt_H + (1 - \alpha) \underline{u}'_L dt_L) = \frac{\underline{v}'_L}{\underline{u}'_L} \Phi'(\bar{e}).$$

Since the probability of type $\underline{\beta}$ equals π , the expected decrease in social welfare when type $\underline{\beta}$ is realized equals

$$\pi \left(\frac{\underline{v}'_L}{\underline{u}'_L} - 1 \right) \Phi'(\bar{e}). \quad (30)$$

Equating (29) to (30) then yields (27).

The solution also shares some characteristics from the problem where only exogenous uncertainty is present. In particular, an increase in exogenous uncertainty, brought about by a mean preserving spread in ε , raises a weighted sum of the efficient and inefficient type' effort levels when the firm's utility function exhibits DARA, and lowers it when its utility function exhibits IARA.

[Insert Figure 4 here]

To understand the intuition for this result, let us consider the effect of such a mean preserving spread on the efficient firm's expected utility, while keeping the effort levels \underline{e} and \bar{e} fixed, and maintaining optimal risk sharing and productive efficiency. Define $\underline{y}_H = \underline{t}_H - \underline{c} - \varepsilon_H$ and $\underline{y}_L =$

$\underline{t}_L - \underline{c} - \varepsilon_L$. Then for fixed ε_L the optimal risk sharing equation

$$\frac{v'(-(\underline{y}_H + \underline{c} + \varepsilon_H))}{v'(-(\underline{y}_L + \underline{c} + \varepsilon_L))} = \frac{u'(\underline{y}_H)}{u'(\underline{y}_L)} \quad (31)$$

traces out a contract curve in $(\underline{y}_L, \underline{y}_H)$ -space. This contract curve is upward sloping, and a mean preserving spread shifts the curve to the right (see Fig. 4). For fixed \underline{c} , the productive efficiency condition for the low cost firm traces out a downward sloping curve. Indeed, implicit differentiation of the first order condition $\alpha u'(\underline{y}_H) + (1 - \alpha)u'(\underline{y}_L) = \psi'(\underline{c})$ yields

$$\alpha u''(\underline{y}_H)d\underline{y}_H + (1 - \alpha)u''(\underline{y}_L)d\underline{y}_L = 0. \quad (32)$$

It follows that a mean preserving spread lowers \underline{y}_H and raises \underline{y}_L . At first sight, the effect of such a change on the expected utility of the low cost firm thus seems ambiguous. However, equation (32) requires that the change in \underline{y}_H be proportional to $(1 - \alpha)u''(\underline{y}_L)$ and that the change in \underline{y}_L be proportional to $\alpha u''(\underline{y}_H)$. Thus we will have $d\underline{y}_H = -\kappa(1 - \alpha)u''(\underline{y}_L)$ and $d\underline{y}_L = \kappa\alpha u''(\underline{y}_H)$ for some $\kappa > 0$. The induced marginal change in expected utility then equals

$$dE\underline{u} = \alpha \underline{u}'_H d\underline{y}_H + (1 - \alpha) \underline{u}'_L d\underline{y}_L = \kappa\alpha(1 - \alpha) \underline{u}'_H \underline{u}'_L \left(\frac{\underline{u}''_H}{\underline{u}'_H} - \frac{\underline{u}''_L}{\underline{u}'_L} \right).$$

In order to maintain the incentive constraint $\underline{U} = \Phi(\bar{e})$, \underline{c} and \bar{e} will then have to be adjusted so that

$$dE\underline{u} = d(\psi(\underline{c}) + \Phi(\bar{e})) = \psi'(\underline{c})d\underline{c} + \Phi'(\bar{e})d\bar{e}.$$

Hence a weighted sum of $d\underline{c}$ and $d\bar{e}$ is negative whenever u is DARA, and positive whenever u is IARA.⁵

Despite sharing some characteristics with the solution of the complete information case, a new phenomenon presents itself in the incomplete information case: the sign of $\frac{d\underline{c}}{d\varepsilon_L}$ and $\frac{d\bar{e}}{d\varepsilon_L}$ need not be the same. This raises the possibility that increases in uncertainty could decrease the expected cost of the project.

⁵Of course, a change in \underline{c} will have a feedback effect through the productive efficiency requirement in (26). Our proof in the appendix shows that even when this feedback effect is fully accounted for, the same conclusion obtains.

An investigation of this question is complicated by the fact that a mean preserving spread changes both the complete and incomplete information effort levels. To simplify, we will therefore concentrate on the case where both u and v are of the CARA type. The complete information effort levels are then independent of the degree of uncertainty and the efficiency parameter β . In other words, we then have $\widehat{e}(\beta) = \widehat{e}(\bar{\beta}) = e^*$. We have:

Proposition 5 *Suppose that u and v are both of the CARA type. Then if either $\psi''' \geq 0$, or if π is not too large, we have:*

$$\frac{d\bar{e}}{d\varepsilon_L} < 0.$$

Since a mean preserving spread lowers ε_L , it follows that under the conditions of the Proposition the effort level of the low efficiency type increases. When it is sufficiently likely that the firm is of the inefficient type, i.e. when π is low, an increase in risk will then lower the expected cost of the project, $\pi\underline{c} + (1 - \pi)\bar{c}$.

The intuition behind Proposition 5 is rather subtle. Consider the effect of a mean preserving spread in ε , keeping the effort levels of both firms constant. As explained above, this would lower \underline{y}_H and raise \underline{y}_L . The impact on the net income of the inefficient firm is similar: it lowers \bar{y}_H and it raises \bar{y}_L . Indeed, the optimal risk sharing equation

$$\frac{v'(-(\bar{y}_H + \bar{c} + \varepsilon_H))}{v'(-(\bar{y}_L + \bar{c} + \varepsilon_L))} = \frac{u'(\bar{y}_H)}{u'(\bar{y}_L)}$$

traces out an upward sloping contract curve in (\bar{y}_L, \bar{y}_H) space, and a mean preserving spread shifts the contract curve to the right. At the same time, for fixed level of effort \underline{e} the IR constraint traces out a downward sloping curve in (\bar{y}_L, \bar{y}_H) . Thus \bar{y}_H decreases and \bar{y}_L increases. It remains to consider the impact on the optimal effort condition

$$\alpha u'(\bar{y}_H) + (1 - \alpha)u'(\bar{y}_L) - \psi'(\bar{e}) = \frac{\pi}{1 - \pi} \left(\frac{v'(-(\underline{y}_L + \underline{c} + \varepsilon_L))}{u'(\underline{y}_L)} - 1 \right) \left(\frac{u'(\bar{y}_L)}{v'(-(\bar{y}_L + \bar{c} + \varepsilon_L))} \right) \Phi'(\bar{e}). \quad (33)$$

Because \bar{y}_L and \bar{y}_H were adjusted so as to keep the gross utility $\alpha u(\bar{y}_H) + (1 - \alpha)u(\bar{y}_L)$ constant, and because u exhibits constant absolute risk aversion, the term $\alpha u'(\bar{y}_H) + (1 - \alpha)u'(\bar{y}_L)$ remains unchanged. Since \bar{e} is being held constant, the left side of (33) therefore remains unchanged.

An increase in \underline{y}_L and \bar{y}_L will decrease the term $\frac{\bar{u}'_L}{\bar{v}'_L}$ but increase the term $\frac{\underline{u}'_L}{\underline{v}'_L}$. Our proof implicitly establishes that when u and v are both of the CARA type, then both terms change by the same amount, so that the right hand side of (33) increases. The conditions in the Proposition guarantee that a change in \bar{e} affects the left hand side of (33) by more than the right hand side (either through its impact on $\Phi'(\bar{e})$, or through feedback effects). To maintain productive efficiency then requires raising the left hand side of (33), i.e. raising \bar{e} .

5 Risk Sharing

There are two sources of risk facing the participants in our model: the risk associated with the realization of the firm's type (the efficiency parameter β) and the risk associated with the realization of the cost shock ε . As is the case in all adverse selection models, the risk associated with the firm's type is entirely borne by consumers. This is because at the time of contracting, the firm knows its efficiency parameter. The only risk to be shared is therefore the exogenous uncertainty stemming from the cost shock ε . Our model can thus shed light on two questions: (i) How is this risk shared between the firm and the consumers?; and (ii) How does the risk sharing differ between the efficient and inefficient firm types?

Unfortunately, question (i) has a somewhat mundane answer. Our assumption on the ex-post observability of the cost shock essentially eliminates the moral hazard component from the model. Thus the traditional trade-off between risk sharing (which suggests that the firm's profit should not depend too strongly on the cost realization) and incentives for cost reduction (which induce the principal to condition the firm's profit on the cost realization) is absent. This property of our model is reflected in the fact that exogenous uncertainty is always shared efficiently (see equation (28)). This equation shows that (conditionally on the firm's cost realization) this risk is shared according to the parties' ability to absorb risk, as measured by the coefficient of absolute risk aversion. If this coefficient is constant, then the relationship is exact, as can be seen from equation (72):

$$(t_H - t_L) = \frac{\rho_u}{\rho_u + \rho_v} (\varepsilon_H - \varepsilon_L).$$

Otherwise, the same relationship still holds approximately when the exogenous risk is small.

Question (ii) has a more interesting answer, as shown in the following result.

Proposition 6 *Suppose the firm has constant absolute risk aversion. Then introducing a small amount of informational asymmetry raises the risk of the inefficient firm and the consumers in that efficiency state, and lowers the risk of the efficient firm and consumers in that efficiency state, i.e.*

$$\frac{d(\bar{t}_H - d\bar{t}_L)}{d\Delta\beta} > 0 > \frac{d(\underline{t}_H - dt_L)}{d\Delta\beta}.$$

More generally, with a sufficiently small amount of informational asymmetry, then whenever the firm has decreasing absolute risk aversion, the inefficient firm and the consumers in this efficiency state bear more risk than the efficient firm and the consumers in that efficiency state, i.e.

$$\frac{d(\bar{t}_H - d\bar{t}_L)}{d\Delta\beta} > \frac{d(\underline{t}_H - dt_L)}{d\Delta\beta}.$$

The opposite conclusion holds if the firm exhibits increasing absolute risk aversion.

6 Conclusion and suggestions for further research

In this paper, we examine the effect of risk aversion in a mixed moral hazard/adverse selection model of regulation. We show that when the agent's utility is additively separable in effort and money, the effort level of the firm is distorted downward from the first best level of effort for both agent types. Thus, the traditional result of no distortion on the top does not hold. Note that we obtain this as a consequence of risk aversion and not of the existence of type dependent outside options (e.g. Julien, 2000) or other mechanisms. Furthermore, we show that the effort level of the low-cost type may be distorted more than that of the high cost type.

We then introduce an observable cost shock to study risk sharing between the taxpayers and the agent. Relative to the complete information case (absence of adverse selection), we show that the effort levels of both types of agents are still distorted downwards. However, exogenous uncertainty interacts with adverse selection in such a way that the more efficient firm will bear more or less risk depending on certain condition related to risk aversion. In particular, increased exogenous uncertainty (measured by a mean preserving spread in the cost shock), has an opposite effect on the effort choices of the two agent types whenever the agent has constant absolute risk aversion. If taxpayers also have constant absolute risk aversion, then, under plausible conditions, the effort level

of the efficient firm increases. As a consequence, exogenous uncertainty may improve the expected efficiency of the project.

Although risk sharing is discussed here in the strict context of Principal-Agent theory and optimal regulation, it can also be applied to other situations, for example to risk sharing between two agents with different negotiation power (see Browning, Chiappori and Weiss, 2014). One other interesting application involves optimal income taxation, pioneered by Mirlees (1971). In this case, the government observes the individual income of its citizens. The adverse selection parameter is then the marginal productivity of effort β . Assuming that the utility function is additively separable in effort cost and money, the utility of a type β individual is then given by:

$$u(\beta e + \varepsilon - t) - \psi(e),$$

where t is the net tax paid to the government, and ε is an ex-post observable shock to income. The government's objective is to redistribute income, so we allow the tax to be negative. Thus, it maximizes expected welfare, subject to the budget constraint

$$\pi \underline{t} + (1 - \pi) \bar{t} \geq 0,$$

where t denotes the tax levied on low productivity individuals, and \bar{t} denotes the tax on high productivity individuals. It can then be shown that the effort choices of both types will be distorted downwards from the first best. Furthermore, increased exogenous uncertainty will improve expected output when agents have constant absolute risk aversion.

Our formulation produces a tractable model, while at the same time permitting interesting conclusions on how exogenous uncertainty impacts environments where adverse selection is present. It would be very interesting to extend our model to allow for the case of unobservable cost shocks. This could lead to a more interesting moral hazard problem where risk sharing between agent and principal may also be distorted. Unfortunately, such mixed problems have generally proved intractable.

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7 Appendix

Proof of Proposition 1

The principal’s problem is

$$\max_{e,t} (v(-t) + u(t - \beta + e) - \psi(e)) \quad \text{s.t. } U = u(t - \beta + e) - \psi(e) \geq 0.$$

Let $g = u^{-1}$. Because $t = \beta - e + g(\psi(e) + U)$, the above problem is equivalent to

$$\max_{U,e} (v(e - \beta - g(\psi(e) + U) + U) + U) \quad \text{s.t. } U \geq 0.$$

Assumption (2) implies that the latter objective is strictly decreasing in U , so it is optimal to set $U = 0$. Maximizing w.r.t. e then yields the remainder of (10).

Proof of Proposition 2

We prove the result in a sequence of lemmata.

We begin by characterizing the feasible set for the program (11), i.e. the set of transfers and effort levels $\{\underline{t}, \bar{t}, \underline{e}, \bar{e}\}$ satisfying (12)-(15).

Lemma 1 *On the feasible set, the constraint $\underline{U} \geq 0$ is never binding.*

Proof. It follows from (12) and (15) that

$$\underline{U} \geq u(\bar{t} - \bar{c}) - \psi(\bar{\beta} - \underline{c}) = \bar{U} + \Phi(\bar{c}) \geq \Phi(\bar{c}) > 0.$$

It is a standard exercise to show that on the feasible set the inequality $\underline{c} \leq \bar{c}$ holds. We now show that if this inequality holds, and (12) holds with equality, then (13) is satisfied. ■

Lemma 2 *If $\underline{c} \leq \bar{c}$ and (12) holds with equality, then (13) is satisfied.*

Proof. If (12) holds with equality, then we have

$$\begin{aligned} u(\bar{t} - \bar{c}) - \psi(\bar{\beta} - \bar{c}) &= u(\underline{t} - \underline{c}) - \psi(\underline{\beta} - \underline{c}) + \psi(\underline{\beta} - \bar{c}) - \psi(\bar{\beta} - \bar{c}) \\ &= [u(\underline{t} - \underline{c}) - \psi(\bar{\beta} - \underline{c})] + [\psi(\bar{\beta} - \underline{c}) - \psi(\underline{\beta} - \underline{c}) + \psi(\underline{\beta} - \bar{c}) - \psi(\bar{\beta} - \bar{c})]. \end{aligned}$$

To show that (13) holds it therefore suffices to prove that $\underline{c} \leq \bar{c}$ implies the inequality

$$\psi(\bar{\beta} - \underline{c}) - \psi(\bar{\beta} - \bar{c}) \geq \psi(\underline{\beta} - \underline{c}) - \psi(\underline{\beta} - \bar{c}).$$

This inequality is equivalent to showing that the function $h(\beta) = \psi(\beta - \underline{c}) - \psi(\beta - \bar{c})$ is nondecreasing in β . This is immediate, because the convexity of ψ and $\underline{c} \leq \bar{c}$ imply

$$h'(\beta) = \psi'(\beta - \underline{c}) - \psi'(\beta - \bar{c}) \geq 0.$$

■

We will therefore omit the constraints (14) and (13), and show that in the solution to this less constrained problem the constraint (12) holds with equality, and $\bar{c} > \underline{c}$. Omitting the mentioned constraints, it is immediate that (12) and (15) must hold with equality, as the objective function is strictly decreasing in \underline{t} and \bar{t} . Solving these equations for \underline{t} and \bar{t} yields

$$\begin{aligned} \underline{t} &= \underline{\beta} - \underline{c} + g(\psi(\underline{c}) + \Phi(\bar{c})); \\ \bar{t} &= \bar{\beta} - \bar{c} + g(\psi(\bar{c})). \end{aligned}$$

Substituting these expressions into the objective then yields the problem

$$\max_{\{\underline{e}, \bar{e}\}} \pi (v(-[\underline{\beta} - \underline{e} + g(\psi(\underline{e}) + \Phi(\bar{e}))]) + \Phi(\bar{e})) + (1 - \pi) (v(-[\bar{\beta} - \bar{e} + g(\psi(\bar{e}))])).$$

Differentiating w.r.t. \underline{e} and \bar{e} then produces (16) and (17). We now show that at this solution, the inequality $\bar{e} \geq \underline{e}$ is indeed satisfied.

Lemma 3 *At the optimum, we have $\bar{e} > \underline{e}$ and $\bar{t} - \bar{e} < \underline{t} - \underline{e}$.*

Proof. Suppose to the contrary we had $\underline{e} \geq \bar{e}$. Because (12) holds with equality, we would then have

$$u(\underline{t} - \underline{e}) = u(\bar{t} - \bar{e}) + \psi(\underline{\beta} - \underline{e}) - \psi(\underline{\beta} - \bar{e}) \leq u(\bar{t} - \bar{e}).$$

This would imply that $\bar{t} - \bar{e} \geq \underline{t} - \underline{e}$. Since u' is strictly decreasing, we would then have

$$u'(\bar{t} - \bar{e}) \leq u'(\underline{t} - \underline{e}) = \psi'(\underline{\beta} - \underline{e}) < \psi'(\bar{\beta} - \underline{e}) \leq \psi'(\bar{\beta} - \bar{e}),$$

where the equality follows from (16). Hence we would have $u'(\bar{t} - \bar{e}) < \psi'(\bar{\beta} - \bar{e})$, contradicting (17).

We conclude that we must have $\bar{e} > \underline{e}$.

Because (12) holds with equality and because $\bar{e} > \underline{e}$, we then have

$$u(\underline{t} - \underline{e}) = u(\bar{t} - \bar{e}) + \psi(\underline{\beta} - \underline{e}) - \psi(\underline{\beta} - \bar{e}) > u(\bar{t} - \bar{e}),$$

implying $\bar{t} - \bar{e} < \underline{t} - \underline{e}$. ■

We now show that \underline{e} and \bar{e} cannot be unambiguously ranked. To see that this is the case, note first of all that $\bar{e} \rightarrow e^*$ as $\pi \rightarrow 0$, but that \underline{e} remains bounded away from e^* . However, if $\pi > 0$ and $u'' \rightarrow 0$, then $\underline{e} \rightarrow e^*$ while $\bar{e} < e^*$.

Finally, let us show that $\underline{t} - \underline{e} > t^* - c^*(\underline{\beta}) > \bar{t} - \bar{e}$. Define the function $e(\Phi)$ as the solution in e to the implicit equation

$$g'(\psi(e) + \Phi)\psi'(e) = 1,$$

and let

$$\tau(\Phi) = g(\psi(e(\Phi)) + \Phi).$$

It follows from (16) that $\underline{e} = e(\Phi(\bar{e}))$ and that $\underline{t} - \underline{c} = \tau(\Phi(\bar{e}))$. Using the implicit function theorem, we may calculate

$$\begin{aligned}\frac{de}{d\Phi} &= -\frac{g''\psi'}{g''(\psi')^2 + g'\psi''} < 0; \\ \frac{d\tau}{d\Phi} &= g' \left(\frac{g'\psi''}{g''(\psi')^2 + g'\psi''} \right) > 0.\end{aligned}$$

Since τ is strictly increasing in Φ , and since $\tau(0) = t^*(\underline{\beta}) - e^*(\underline{\beta})$, we obtain $\underline{t} - \underline{c} > t^*(\underline{\beta}) - e^*(\underline{\beta})$.

Next, observe from (17) that

$$\bar{t} - \bar{c} = g(\psi(\bar{e})).$$

Since $t^*(\underline{\beta}) - e^*(\underline{\beta})$ and since $\bar{e} < e^*$, we conclude that $\bar{t} - \bar{c} < t^*(\underline{\beta}) - e^*(\underline{\beta})$.

Proof of Proposition 3

We begin by establishing the optimality conditions (21)-(23). Let μ denote the Lagrange multiplier associated with (20). Differentiating the Lagrangean w.r.t. t_H and t_L yields

$$\begin{aligned}(1 + \mu)u'(t_H - c - \varepsilon_H) &= v'(-t_H); \\ (1 + \mu)u'(t_L - c - \varepsilon_L) &= v'(-t_L).\end{aligned}$$

Eliminating $(1 + \mu)$ from these equations produces (23). Differentiating the Lagrangean w.r.t. c immediately yields (22). If the constraint (20) were not binding, we would have $\mu = 0$, and hence $\frac{\partial L}{\partial t_H} = u'(t_H - c - \varepsilon_H) - v'(-t_H) < 0$ for all t_H . It would then be optimal to set $t_H = c + \varepsilon_H$. Similarly, it would be optimal to set $t_L = c + \varepsilon_L$. Observe now that (22) implies that $e > 0$. Hence we would have $U = u(0) - \psi(e) < 0$, violating (20). Hence (20) is binding, and (21) holds.

Let us now consider the solution to (19) as a function of ε_L , while preserving the mean

$$\alpha\varepsilon_H + (1 - \alpha)\varepsilon_L = 0.$$

The variable ε_L serves as a shift variable, with a decrease in ε_L effecting a mean preserving spread.

Lemma 4 *For all $\varepsilon_L \neq 0$ we have $t_L < t_H$ and $t_H - c - \varepsilon_H < t_L - c - \varepsilon_L$.*

Proof. Observe that

$$\frac{v'(-t_H)}{v'(-t_L)} \geq 1 \Leftrightarrow t_H \geq t_L$$

and

$$\frac{u'(t_H - c - \varepsilon_H)}{u'(t_L - c - \varepsilon_L)} \geq 1 \Leftrightarrow t_H - c - \varepsilon_H \leq t_L - c - \varepsilon_L.$$

Since marginal rates of substitution must be equated at the optimum, it therefore suffices to show that we cannot have

$$\frac{v'(-t_H)}{v'(-t_L)} = \frac{u'(t_H - c - \varepsilon_H)}{u'(t_L - c - \varepsilon_L)} \leq 1.$$

If this inequality held, we would have $t_L - c - \varepsilon_L \leq t_H - c - \varepsilon_H$ and $t_H \leq t_L$, implying $t_L - c - \varepsilon_L \leq t_H - c - \varepsilon_H \leq t_L - c - \varepsilon_H$. This is equivalent to $\varepsilon_H \leq \varepsilon_L$, contradicting that $\varepsilon_L < 0$ and $\varepsilon_H > 0$, and establishing the result. ■

We now study the effect of marginal variations in ε_L on the solution $(t_H(\varepsilon_L), t_L(\varepsilon_L), e(\varepsilon_L))$.

Lemma 5 *For all ε_L , we have*

$$\begin{aligned} \operatorname{sgn}\left(\frac{de}{d\varepsilon_L}\right) &= \operatorname{sgn}\left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L}\right); \\ \frac{d(t_L - \varepsilon_L)}{d\varepsilon_L} &< 0; \\ \frac{d(t_H - \varepsilon_H)}{d\varepsilon_L} &> 0. \end{aligned}$$

Thus if u is of the DARA type, then $\frac{de}{d\varepsilon_L} < 0$, and if u is of the IARA type, then $\frac{de}{d\varepsilon_L} > 0$.

Proof. Totally differentiating (21), we obtain:

$$\alpha u'_H \left(\frac{dt_H}{d\varepsilon_L} - \frac{dc}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) + (1 - \alpha) u'_L \left(\frac{dt_L}{d\varepsilon_L} - \frac{dc}{d\varepsilon_L} - 1 \right) + \psi' \frac{dc}{d\varepsilon_L} = 0.$$

Using (22), this simplifies to

$$\alpha u'_H \left(\frac{dt_H}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) + (1 - \alpha) u'_L \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) = 0. \quad (34)$$

Totally differentiating (22) yields:

$$\alpha u_H'' \left(\frac{dt_H}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) + (1 - \alpha) u_L'' \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) - (\alpha u_H'' + (1 - \alpha) u_L'' - \psi'') \frac{dc}{d\varepsilon_L} = 0.$$

From (34) we have:

$$\alpha \left(\frac{dt_H}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) = -(1 - \alpha) \frac{u_L'}{u_H'} \left(\frac{dt_L}{d\varepsilon_L} - 1 \right). \quad (35)$$

Substituting into the previous equation yields:

$$(\alpha u_H'' + (1 - \alpha) u_L'' - \psi'') \frac{dc}{d\varepsilon_L} + (1 - \alpha) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) u_L' \left(\frac{u_H''}{u_H'} - \frac{u_L''}{u_L'} \right) = 0. \quad (36)$$

Totally differentiating (23) yields:

$$-v_H'' u_L' \frac{dt_H}{d\varepsilon_L} + v_H' u_L'' \left(\frac{dt_L}{d\varepsilon_L} - \frac{dc}{d\varepsilon_L} - 1 \right) + v_L'' u_H' \frac{dt_L}{d\varepsilon_L} - v_L' u_H'' \left(\frac{dt_H}{d\varepsilon_L} - \frac{dc}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) = 0.$$

Using the fact that $v_H' u_L' = u_H' v_L'$ holds, we may rewrite the latter equation as

$$0 = \left(-\frac{v_H''}{v_H'} \frac{dt_H}{d\varepsilon_L} + \frac{u_L''}{u_L'} \left(\frac{dt_L}{d\varepsilon_L} - \frac{dc}{d\varepsilon_L} - 1 \right) \right) + \left(\frac{v_L''}{v_L'} \frac{dt_L}{d\varepsilon_L} - \frac{u_H''}{u_H'} \left(\frac{dt_H}{d\varepsilon_L} - \frac{dc}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) \right). \quad (37)$$

We will now solve system (34)-(37). From (34) we have:

$$\frac{dt_H}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} = -\frac{(1 - \alpha) u_L'}{\alpha u_H'} \left(\frac{dt_L}{d\varepsilon_L} - 1 \right).$$

Substituting into (37) produces:

$$\begin{aligned} 0 &= \left(\frac{(1 - \alpha) u_L'}{\alpha u_H'} \left(\frac{v_H''}{v_H'} + \frac{u_H''}{u_H'} \right) + \left(\frac{v_L''}{v_L'} + \frac{u_L''}{u_L'} \right) \right) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) \\ &\quad + \left(\frac{v_L''}{v_L'} - \frac{v_H''}{v_H'} \frac{d\varepsilon_H}{d\varepsilon_L} \right) + \left(\frac{u_H''}{u_H'} - \frac{u_L''}{u_L'} \right) \frac{dc}{d\varepsilon_L}. \end{aligned} \quad (38)$$

From (36) we obtain

$$(1 - \alpha) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) u_L' \left(\frac{u_H''}{u_H'} - \frac{u_L''}{u_L'} \right) = -(\alpha u_H'' + (1 - \alpha) u_L'' - \psi'') \frac{dc}{d\varepsilon_L}. \quad (39)$$

Multiplying (38) by

$$(1 - \alpha)u'_L \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right)$$

and using the previous equation yields:

$$\begin{aligned} 0 = & - \left(\frac{(1 - \alpha)u'_L}{\alpha u'_H} \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) + \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) \right) [\alpha u''_H + (1 - \alpha)u''_L - \psi''] \frac{dc}{d\varepsilon_L} \\ & + (1 - \alpha)u'_L \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right) \left(\frac{v''_L}{v'_L} - \frac{v''_H}{v'_H} \frac{d\varepsilon_H}{d\varepsilon_L} \right) + (1 - \alpha)u'_L \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right)^2 \frac{dc}{d\varepsilon_L}. \end{aligned} \quad (40)$$

Observe that

$$\begin{aligned} & - \left(\frac{(1 - \alpha)u'_L}{\alpha u'_H} \left(\frac{u''_H}{u'_H} \right) + \left(\frac{u''_L}{u'_L} \right) \right) [\alpha u''_H + (1 - \alpha)u''_L] + (1 - \alpha)u'_L \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right)^2 \\ & < - (1 - \alpha)u'_L \left(\frac{u''_H}{u'_H} \right)^2 - (1 - \alpha)u'_L \left(\frac{u''_L}{u'_L} \right)^2 + (1 - \alpha)u'_L \left(\frac{u''_H}{u'_H} \right)^2 + (1 - \alpha)u'_L \left(\frac{u''_L}{u'_L} \right)^2 = 0. \end{aligned}$$

Hence in the term multiplying $\frac{dc}{d\varepsilon_L}$ in (40) has a negative sign. Observe also that

$$\frac{v''_L}{v'_L} - \frac{v''_H}{v'_H} \frac{d\varepsilon_H}{d\varepsilon_L} = \frac{v''_L}{v'_L} + \frac{v''_H}{v'_H} \frac{1 - \alpha}{\alpha} < 0.$$

It follows that

$$\text{sgn} \left(\frac{dc}{d\varepsilon_L} \right) = -\text{sgn} \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right).$$

The equations (39) and (35) then imply the remainder of the desired result. ■

Thus as ε_L is lowered, e increases when u is of the DARA type, and decreases when u is of the IARA type. We now investigate how the solution to the problem with exogenous uncertainty depends on the cost characteristic of the agent. Fix $\varepsilon_L > 0$ and denote the solution by $t_H(\beta)$, $t_L(\beta)$, $e(\beta)$.

Lemma 6 *We have*

$$\text{sgn} \frac{de}{d\beta} = -\text{sgn} \left\{ \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \left(\frac{v''_H}{v'_H} - \frac{v''_L}{v'_L} \right) \right\}$$

Proof. Differentiation of the first order conditions w.r.t. β yields:

$$\alpha u'_H \left(\frac{dt_H}{d\beta} - 1 \right) + (1 - \alpha)u'_L \left(\frac{dt_L}{d\beta} - 1 \right) = 0 \quad (41)$$

$$\alpha u''_H \left(\frac{dt_H}{d\beta} - 1 \right) + (1 - \alpha) u''_L \left(\frac{dt_L}{d\beta} - 1 \right) - (\alpha u''_H + (1 - \alpha) u''_L - \psi'') \frac{de}{d\beta} = 0 \quad (42)$$

$$-v''_H u'_L \frac{dt_H}{d\beta} + v'_H u''_L \left(\frac{dt_L}{d\beta} - 1 + \frac{de}{d\beta} \right) - u''_H v'_L \left(\frac{dt_H}{d\beta} - 1 + \frac{de}{d\beta} \right) + u'_H v''_L \frac{dt_L}{d\beta} = 0 \quad (43)$$

From (41) we have

$$\alpha u'_H \left(\frac{dt_H}{d\beta} - 1 \right) = -(1 - \alpha) u'_L \left(\frac{dt_L}{d\beta} - 1 \right)$$

Substituting this into (42) then yields

$$(1 - \alpha) u'_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \left(\frac{dt_L}{d\beta} - 1 \right) - (\alpha u''_H + (1 - \alpha) u''_L - \psi'') \frac{de}{d\beta} = 0 \quad (44)$$

We may rewrite (43) as

$$-v''_H u'_L \left(\frac{dt_H}{d\beta} - 1 \right) + v'_H u''_L \left(\frac{dt_L}{d\beta} - 1 + \frac{de}{d\beta} \right) - u''_H v'_L \left(\frac{dt_H}{d\beta} - 1 + \frac{de}{d\beta} \right) + u'_H v''_L \left(\frac{dt_L}{d\beta} - 1 \right) - v''_H u'_L + u'_H v''_L = 0$$

Using the relation $u'_L v'_H = v'_L u'_H$ this can be simplified to

$$-\left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \left(\frac{dt_H}{d\beta} - 1 \right) + \left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \right) \left(\frac{dt_L}{d\beta} - 1 \right) + \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \frac{de}{d\beta} + \left(\frac{v''_H}{v'_H} - \frac{v''_L}{v'_L} \right) = 0 \quad (45)$$

Substituting

$$\left(\frac{dt_H}{d\beta} - 1 \right) = -\frac{(1 - \alpha) u'_L}{\alpha u'_H} \left(\frac{dt_L}{d\beta} - 1 \right)$$

this becomes

$$\left\{ \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \frac{(1 - \alpha) u'_L}{\alpha u'_H} + \left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \right) \right\} \left(\frac{dt_L}{d\beta} - 1 \right) + \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \frac{de}{d\beta} + \left(\frac{v''_H}{v'_H} - \frac{v''_L}{v'_L} \right) = 0 \quad (46)$$

Multiplying this expression by

$$(1 - \alpha) u'_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right)$$

and using (44) then yields

$$0 = \left\{ \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \frac{(1 - \alpha) u'_L}{\alpha u'_H} + \left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \right) \right\} (\alpha u''_H + (1 - \alpha) u''_L - \psi'') \frac{de}{d\beta} \quad (47)$$

$$+ (1 - \alpha) u'_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right)^2 \frac{de}{d\beta} + (1 - \alpha) u'_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \left(\frac{v''_H}{v'_H} - \frac{v''_L}{v'_L} \right)$$

The coefficient of $\frac{de}{d\beta}$ in (47) is strictly positive, proving the desired result. ■

Proof of Proposition 4

Using the same argument as in the proof of Proposition 2, we may show that for any point $\{\underline{t}_L, \underline{t}_H, \bar{t}_L, \bar{t}_H, \underline{c}, \bar{c}\}$ belonging to the feasible set the constraint (7) is never binding, and that if $\underline{c} \leq \bar{c}$ and (5) holds with equality, then (6) is satisfied. We shall therefore drop the constraints (7) and (6), and maximize the objective (4) subject to the two remaining constraints (8) and (5).

Lemma 7 *Consider the program of maximizing (4) subject to the constraints (8) and (5). Then at the optimum, the constraints (8) and (5) are binding, and the optimality conditions (27), (26) and (28) hold.*

Proof. Let μ and γ denote the Lagrangean multipliers associated with the constraints (5) and (8), respectively. Maximizing the Lagrangean L w.r.t. the choice variables $\{\underline{t}_L, \underline{t}_H, \bar{t}_L, \bar{t}_H, \underline{c}, \bar{c}\}$ then yields the following first order conditions:

$$0 = -\pi v'(-\underline{t}_L) + (\pi + \mu)u'(\underline{t}_L - \underline{c} - \varepsilon_L);$$

$$0 = -\pi v'(-\underline{t}_H) + (\pi + \mu)u'(\underline{t}_H - \underline{c} - \varepsilon_H);$$

$$0 = -(1 - \pi)v'(-\bar{t}_L) + (1 - \pi + \gamma - \mu)u'(\bar{t}_L - \bar{c} - \varepsilon_L);$$

$$0 = -(1 - \pi)v'(-\bar{t}_H) + (1 - \pi + \gamma - \mu)u'(\bar{t}_H - \bar{c} - \varepsilon_H);$$

$$0 = \alpha u'(\underline{t}_H - \underline{c} - \varepsilon_H) + (1 - \alpha)u'(\underline{t}_L - \underline{c} - \varepsilon_L) - \psi'(\underline{\beta} - \underline{c});$$

$$0 = (1 - \pi + \gamma - \mu) [\alpha u'(\underline{t}_H - \bar{c} - \varepsilon_H) + (1 - \alpha)u'(\underline{t}_L - \bar{c} - \varepsilon_L) - \psi'(\underline{\beta} - \bar{c})] + \mu[\psi'(\underline{\beta} - \bar{c}) - \psi'(\bar{\beta} - \bar{c})].$$

The equality of the marginal rates of substitution (28) comes directly from the first four equations. The fifth equation yields the optimal effort condition in (27). Replacing μ and $1 - \pi + \gamma - \mu$ in the sixth equation by their values coming from the first and the fourth equation yield the optimal effort condition in (26).

We now show that the constraints (5) and (8) must be binding. Suppose first that (5) were not

binding. We would then have $\mu = 0$, implying

$$\begin{aligned}\frac{\partial L}{\partial \underline{t}_L} &= -\pi v'(-\underline{t}_L) + \pi u'(\underline{t}_L - \underline{c} - \varepsilon_L) < 0; \\ \frac{\partial L}{\partial \underline{t}_H} &= -\pi v'(-\underline{t}_H) + \pi u'(\underline{t}_H - \underline{c} - \varepsilon_H) < 0.\end{aligned}$$

Hence it would be optimal to set \underline{t}_L and \underline{t}_H as small as possible, i.e. $\underline{t}_L - \underline{c} - \varepsilon_L = 0$ and $\underline{t}_H - \underline{c} - \varepsilon_H = 0$. We would then have $\underline{U} = u(0) - \psi(\underline{c}) \leq 0$, contradicting the hypothesis that $\underline{U} > \Phi(\bar{c}) > 0$, and thereby proving that (5) is binding.

The proof that (8) must be binding proceeds in an entirely analogous way. ■

We now show that any solution to the equations (27),(26) and (28) must satisfy $\underline{c} < \bar{c}$, proving that at such a solution the neglected constraint (6) is satisfied.

Lemma 8 *Consider any solution to (27),(26) and (28). Then $\underline{c} < \bar{c}$ holds.*

Proof. Suppose to the contrary that at some solution to (27)-(28) we had $\underline{c} \geq \bar{c}$. Parametrize this solution by ε_L . Since at $\varepsilon_L = 0$, we have $\underline{c} < \bar{c}$, there would then exist some value of $\varepsilon_L \neq 0$ where $\underline{c} = \bar{c}$. We shall now show that this is impossible, proving the desired result.

If we had $\underline{c} = \bar{c} = c$, then it follows from (5) that

$$\alpha u(\underline{t}_H - c - \varepsilon_H) + (1 - \alpha)u(\underline{t}_L - c - \varepsilon_L) = \alpha u(\bar{t}_H - c - \varepsilon_H) + (1 - \alpha)u(\bar{t}_L - c - \varepsilon_L).$$

Consider the iso-utility surface

$$\alpha u(t_H - c - \varepsilon_H) + (1 - \alpha)u(t_L - c - \varepsilon_L) = K.$$

Along this surface, we have

$$\frac{dt_H}{dt_L} = -\frac{(1 - \alpha)u'_L(t_L - c - \varepsilon_L)}{\alpha u'_H(t_H - c - \varepsilon_H)} < 0.$$

It follows that

$$\underline{t}_L \geq \bar{t}_L \Leftrightarrow \underline{t}_H \leq \bar{t}_H. \tag{48}$$

Next, consider the relation (28):

$$v'(-t_H)u'(t_L - c - \varepsilon_L) - u'(t_H - c - \varepsilon_H)v'(-t_L) = 0$$

which must hold at $(t_L, t_H) = (\underline{t}_L, \underline{t}_H)$ and $(t_L, t_H) = (\bar{t}_L, \bar{t}_H)$. Differentiating, we see that along this surface we have

$$-\left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H}\right) \frac{dt_H}{dt_L} + \left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L}\right) = 0,$$

i.e. along this surface we have

$$\frac{dt_H}{dt_L} = \frac{\left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L}\right)}{\left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H}\right)} > 0.$$

Hence (28) implies

$$\underline{t}_L \geq \bar{t}_L \Leftrightarrow \underline{t}_H \geq \bar{t}_H. \quad (49)$$

Combining (48) and (49), we see that $\underline{c} = \bar{c}$ would imply $\underline{t}_L = \bar{t}_L$ and $\underline{t}_H = \bar{t}_H$. But then (27) and (26) would imply

$$\psi'(\underline{e}) = Eu'(\underline{t} - c - \varepsilon) = Eu'(\bar{t} - c - \varepsilon) > \psi'(\bar{e}) = \psi'(\underline{e} + \Delta\beta),$$

which contradicts that ψ' is strictly increasing. ■

Our next result establishes that $\bar{e} < e(\bar{\beta})$ and $\underline{e} < e(\underline{\beta})$.

Lemma 9 *We have $\bar{e} < e(\bar{\beta})$ and $\underline{e} < e(\underline{\beta})$.*

Proof. The first order conditions for $\{\underline{t}_L, \underline{t}_H, \underline{e}\}$ may be rewritten as:

$$\begin{aligned} \alpha u(\underline{t}_H - \underline{e} - \varepsilon_H) + (1 - \alpha)u(\underline{t}_L - \underline{e} - \varepsilon_L) - \psi(\underline{e}) &= \Phi; \\ \alpha u'(\underline{t}_H - \underline{e} - \varepsilon_H) + (1 - \alpha)u'(\underline{t}_L - \underline{e} - \varepsilon_L) - \psi'(\underline{e}) &= 0; \\ v'(-\underline{t}_H)u'(\underline{t}_L - \underline{e} - \varepsilon_L) - u'(\underline{t}_H - \underline{e} - \varepsilon_H)v'(-\underline{t}_L) &= 0. \end{aligned}$$

Furthermore, the solution to this system at $\Phi = 0$ corresponds to the solution of (19) at $\beta = \underline{\beta}$. To prove that $\underline{e} < e(\underline{\beta})$, it therefore suffices to show that $\frac{d\underline{e}}{d\Phi} < 0$. Let us therefore consider the comparative statics of the above equation system w.r.t. the parameter Φ . Totally differentiating

yields:

$$\begin{bmatrix} \alpha u'_H & (1-\alpha)u'_L & 0 \\ -\left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H}\right) & \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L}\right) & \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H}\right) \\ \alpha u''_H & (1-\alpha)u''_L & \alpha u''_H + (1-\alpha)u''_L - \psi''(e) \end{bmatrix} \begin{bmatrix} \frac{dt_H}{d\Phi} \\ \frac{dt_L}{d\Phi} \\ \frac{de}{d\Phi} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Now the inverse of the 3×3 matrix

$$\begin{bmatrix} a & b & 0 \\ c & d & e \\ f & g & h \end{bmatrix}$$

equals

$$\frac{1}{bef - aeg - bch + adh} \begin{bmatrix} dh - eg & -bh & be \\ ef - ch & ah & -ae \\ cg - df & bf - ag & -bc + ad \end{bmatrix}.$$

Let $D = bef - aeg - bch + adh$. We then have

$$\frac{de}{d\Phi} = \frac{cg - df}{D}.$$

We now claim that $D > 0$. To see this, observe now that

$$\begin{aligned} bef - aeg &= \alpha(1-\alpha)u'_L u''_H \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H}\right) - \alpha(1-\alpha)u'_H u''_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H}\right) \\ &= -\alpha(1-\alpha)u'_L u'_H \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H}\right)^2, \end{aligned} \quad (50)$$

and

$$-bch + adh = \left(\alpha u'_H \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L}\right) + (1-\alpha)u'_L \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H}\right)\right) [\alpha u''_H + (1-\alpha)u''_L - \psi''(e)].$$

Hence we obtain

$$\begin{aligned} D &> -\alpha(1-\alpha)u'_L u'_H \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H}\right)^2 + [\alpha u''_H + (1-\alpha)u''_L] \left(\alpha u'_H \frac{u''_L}{u'_L} + (1-\alpha)u'_L \frac{u''_H}{u'_H}\right) \\ &= \left(\frac{u''_L}{u'_L} \frac{u''_H}{u'_H}\right) (\alpha u'_H + (1-\alpha)u'_L)^2 > 0. \end{aligned}$$

In addition, we may compute

$$cg - df = -(1 - \alpha)u_L'' \left(\frac{v_H''}{v_H'} + \frac{u_H''}{u_H'} \right) - \alpha u_H'' \left(\frac{u_L''}{u_L'} + \frac{v_L''}{v_L'} \right) < 0,$$

implying $\frac{d\bar{e}}{d\Phi} < 0$. We conclude that $\underline{e} < e(\underline{\beta})$.

Next, note that the first order conditions for $\{\bar{t}_L, \bar{t}_H, \bar{e}\}$ may be rewritten as:

$$\begin{aligned} \alpha u(\bar{t}_H - \bar{e} - \varepsilon_H) + (1 - \alpha)u(\bar{t}_L - \bar{e} - \varepsilon_L) - \psi(\bar{e}) &= 0; \\ v'(-\bar{t}_H)u'(\bar{t}_L - \bar{e} - \varepsilon_L) - u'((\bar{t}_H - \bar{e} - \varepsilon_H)v'(-\bar{t}_L)) &= 0; \\ \alpha u'(\bar{t}_H - \bar{e} - \varepsilon_H) + (1 - \alpha)u'(\bar{t}_L - \bar{e} - \varepsilon_L) - \psi'(\bar{e}) &= \Gamma. \end{aligned}$$

Furthermore, the solution to this system at $\Phi = 0$ corresponds to the solution of (19) at $\beta = \underline{\beta}$. To prove that $\bar{e} < e(\bar{\beta})$, it will therefore suffice to show that $\frac{d\bar{e}}{d\Gamma} < 0$. Totally differentiating yields:

$$\begin{bmatrix} \alpha u_H' & (1 - \alpha)u_L' & \Gamma \\ -\left(\frac{v_H''}{v_H'} + \frac{u_H''}{u_H'}\right) & \left(\frac{u_L''}{u_L'} + \frac{v_L''}{v_L'}\right) & \left(\frac{u_L''}{u_L'} - \frac{u_H''}{u_H'}\right) \\ \alpha u_H'' & (1 - \alpha)u_L'' & \alpha u_H'' + (1 - \alpha)u_L'' - \psi''(e) \end{bmatrix} \begin{bmatrix} \frac{d\bar{t}_H}{d\Phi} \\ \frac{d\bar{t}_L}{d\Phi} \\ \frac{d\bar{e}}{d\Phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Now the inverse of the 3×3 matrix

$$\begin{bmatrix} a & b & \Gamma \\ c & d & e \\ f & g & h \end{bmatrix}$$

equals

$$\frac{1}{\Delta} \begin{bmatrix} dh - eg & \Gamma g - bh & be - \Gamma d \\ ef - ch & ah - \Gamma f & \Gamma c - ae \\ cg - df & bf - ag & -bc + ad \end{bmatrix},$$

where $\Delta = bef - aeg - bch + adh + \Gamma cg - \Gamma df$. Hence

$$\frac{d\bar{e}}{d\Gamma} = \frac{ad - bc}{\Delta}.$$

We now claim that $\Delta > 0$. To see this, observe that

$$\Gamma(CG - df) > -(\alpha u'_H + (1 - \alpha)u'_L) \left((1 - \alpha)u''_L \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) + \alpha u''_H \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) \right)$$

and

$$adh - bch > \left(\alpha u'_H \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) + (1 - \alpha)u'_L \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \right) [\alpha u''_H + (1 - \alpha)u''_L].$$

Summing the right hand side of the two previous inequalities yields

$$\begin{aligned} & -\alpha(1 - \alpha)u'_H u''_L \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) - \alpha^2 u'_H u''_H \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) - (1 - \alpha)^2 u'_L u''_L \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \\ & - \alpha(1 - \alpha)u'_L u''_H \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) + \alpha^2 u'_H u''_H \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) + \alpha(1 - \alpha)u'_L u''_H \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \\ & + \alpha(1 - \alpha)u'_H u''_L \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) + (1 - \alpha)^2 u'_L u''_L \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \\ & = \alpha(1 - \alpha)u'_L u'_H \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right) \left\{ \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) - \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) \right\} \\ & = \alpha(1 - \alpha)u'_L u'_H \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right) \left\{ \left(\frac{v''_H}{v'_H} - \frac{v''_L}{v'_L} \right) + \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right) \right\}. \end{aligned}$$

Adding (50) then produces

$$\Delta > \alpha(1 - \alpha)u'_L u'_H \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right) \left(\frac{v''_H}{v'_H} - \frac{v''_L}{v'_L} \right) > 0.$$

We may compute

$$ad - bc = \alpha u'_H \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) + (1 - \alpha)u'_L \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) < 0,$$

implying $\frac{d\bar{e}}{d\Gamma} < 0$, and establishing that $\bar{e} < e(\bar{\beta})$. ■

We now study the effect of marginal variations in ε_L on the effort choices $\underline{e}(\varepsilon_L)$ and $\bar{e}(\varepsilon_L)$.

Lemma 10 *For all ε_L , there exist $A > 0$ and $B > 0$ such that*

$$\text{sgn} \left(A \frac{d\underline{e}}{d\varepsilon_L} + B \frac{d\bar{e}}{d\varepsilon_L} \right) = \text{sgn} \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right).$$

Proof. Consider the three optimality equations for type $\underline{\beta}$:

$$\underline{U} = \psi(\bar{\beta} - \bar{c}) - \psi(\bar{\beta} - \underline{c}) = \Phi(\bar{e}); \quad (51)$$

$$\alpha u'(\underline{t}_H - \underline{c} + \varepsilon_H) + (1 - \alpha)u'(\underline{t}_L - \underline{c} - \varepsilon_L) - \psi'(\underline{\beta} - \underline{c}) = 0; \quad (52)$$

$$\frac{v'(-\underline{t}_H)}{v'(-\underline{t}_L)} = \frac{u'(\underline{t}_H - \underline{c} - \varepsilon_H)}{u'(\underline{t}_L - \underline{c} - \varepsilon_L)}. \quad (53)$$

Totally differentiating (51), we obtain:

$$\alpha \underline{u}'_H \left(\frac{d\underline{t}_H}{d\varepsilon_L} + \frac{d\underline{e}}{d\varepsilon_L} + \frac{1 - \alpha}{\alpha} \right) + (1 - \alpha) \underline{u}'_L \left(\frac{d\underline{t}_L}{d\varepsilon_L} + \frac{d\underline{e}}{d\varepsilon_L} - 1 \right) - \psi'(\underline{e}) \frac{d\underline{e}}{d\varepsilon_L} = \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}.$$

Using (52), this simplifies to

$$\alpha \underline{u}'_H \left(\frac{d\underline{t}_H}{d\varepsilon_L} + \frac{1 - \alpha}{\alpha} \right) + (1 - \alpha) \underline{u}'_L \left(\frac{d\underline{t}_L}{d\varepsilon_L} - 1 \right) = \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}. \quad (54)$$

Totally differentiating (52) yields:

$$(\alpha \underline{u}''_H + (1 - \alpha) \underline{u}''_L - \psi''(\underline{e})) \frac{d\underline{e}}{d\varepsilon_L} + \alpha \underline{u}''_H \left(\frac{d\underline{t}_H}{d\varepsilon_L} + \frac{1 - \alpha}{\alpha} \right) + (1 - \alpha) \underline{u}''_L \left(\frac{d\underline{t}_L}{d\varepsilon_L} - 1 \right) = 0.$$

From (54) we have:

$$\alpha \left(\frac{d\underline{t}_H}{d\varepsilon_L} + \frac{1 - \alpha}{\alpha} \right) = -(1 - \alpha) \frac{\underline{u}'_L}{\underline{u}'_H} \left(\frac{d\underline{t}_L}{d\varepsilon_L} - 1 \right) + \frac{\Phi'(\bar{e})}{\underline{u}'_H} \frac{d\bar{e}}{d\varepsilon_L} = 0.$$

Substituting into the previous equation yields:

$$(\alpha \underline{u}''_H + (1 - \alpha) \underline{u}''_L - \psi''(\underline{e})) \frac{d\underline{e}}{d\varepsilon_L} + (1 - \alpha) \left(\frac{d\underline{t}_L}{d\varepsilon_L} - 1 \right) \underline{u}'_L \left(\frac{\underline{u}''_L}{\underline{u}'_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \right) + \frac{\underline{u}''_H}{\underline{u}'_H} \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L} = 0. \quad (55)$$

Totally differentiating (53) yields:

$$-\frac{\underline{v}''_H \underline{u}'_L}{\underline{v}'_H \underline{u}'_L} \frac{d\underline{t}_H}{d\varepsilon_L} + \frac{\underline{v}'_H \underline{u}''_L}{\underline{v}'_H \underline{u}'_L} \left(\frac{d\underline{t}_L}{d\varepsilon_L} + \frac{d\underline{e}}{d\varepsilon_L} - 1 \right) + \frac{\underline{v}''_L \underline{u}'_H}{\underline{v}'_L \underline{u}'_H} \frac{d\underline{t}_L}{d\varepsilon_L} - \frac{\underline{v}'_L \underline{u}''_H}{\underline{v}'_L \underline{u}'_H} \left(\frac{d\underline{t}_H}{d\varepsilon_L} + \frac{d\underline{e}}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) = 0.$$

Using the fact that $\underline{v}'_H \underline{u}'_L = \underline{u}'_H \underline{v}'_L$ holds, we may rewrite the latter equation as

$$0 = \left(-\frac{\underline{v}''_H}{\underline{v}'_H} \frac{d\underline{t}_H}{d\varepsilon_L} + \frac{\underline{u}''_L}{\underline{u}'_L} \left(\frac{d\underline{t}_L}{d\varepsilon_L} + \frac{d\underline{e}}{d\varepsilon_L} - 1 \right) \right) + \left(\frac{\underline{v}''_L}{\underline{v}'_L} \frac{d\underline{t}_L}{d\varepsilon_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \left(\frac{d\underline{t}_H}{d\varepsilon_L} + \frac{d\underline{e}}{d\varepsilon_L} - \frac{d\varepsilon_H}{d\varepsilon_L} \right) \right). \quad (56)$$

We will now solve system (54)-(56). From (54) we have:

$$\frac{d\underline{t}_H}{d\varepsilon_L} + \frac{1-\alpha}{\alpha} = -\frac{(1-\alpha)}{\alpha} \frac{\underline{u}'_L}{\underline{u}'_H} \left(\frac{d\underline{t}_L}{d\varepsilon_L} - 1 \right) + \frac{1}{\alpha} \frac{\Phi'(\bar{e})}{\underline{u}'_H} \frac{d\bar{e}}{d\varepsilon_L}.$$

Substituting into (56) produces:

$$\begin{aligned} 0 = & \left(\frac{(1-\alpha)}{\alpha} \frac{\underline{u}'_L}{\underline{u}'_H} \left(\frac{\underline{v}''_H}{\underline{v}'_H} + \frac{\underline{u}''_H}{\underline{u}'_H} \right) + \left(\frac{\underline{v}''_L}{\underline{v}'_L} + \frac{\underline{u}''_L}{\underline{u}'_L} \right) \right) \left(\frac{d\underline{t}_L}{d\varepsilon_L} - 1 \right) \\ & + \left(\frac{\underline{v}''_L}{\underline{v}'_L} - \frac{\underline{v}''_H}{\underline{v}'_H} \frac{d\varepsilon_H}{d\varepsilon_L} \right) - \left(\frac{\underline{u}''_H}{\underline{u}'_H} - \frac{\underline{u}''_L}{\underline{u}'_L} \right) \frac{d\underline{e}}{d\varepsilon_L} - \left(\frac{\underline{v}''_H}{\underline{v}'_H} + \frac{\underline{u}''_H}{\underline{u}'_H} \right) \frac{1}{\alpha} \frac{\Phi'(\bar{e})}{\underline{u}'_H} \frac{d\bar{e}}{d\varepsilon_L}. \end{aligned} \quad (57)$$

From (55) we obtain

$$(1-\alpha) \left(\frac{d\underline{t}_L}{d\varepsilon_L} - 1 \right) \underline{u}'_L \left(\frac{\underline{u}''_L}{\underline{u}'_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \right) = -(\alpha \underline{u}''_H + (1-\alpha) \underline{u}''_L - \psi''(\underline{e})) \frac{d\underline{e}}{d\varepsilon_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}.$$

Multiplying (57) by

$$(1-\alpha) \underline{u}'_L \left(\frac{\underline{u}''_L}{\underline{u}'_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \right),$$

and substituting the previous equation yields:

$$\begin{aligned} 0 = & \left(\frac{(1-\alpha)}{\alpha} \frac{\underline{u}'_L}{\underline{u}'_H} \left(\frac{\underline{v}''_H}{\underline{v}'_H} + \frac{\underline{u}''_H}{\underline{u}'_H} \right) + \left(\frac{\underline{v}''_L}{\underline{v}'_L} + \frac{\underline{u}''_L}{\underline{u}'_L} \right) \right) \\ & \left[-(\alpha \underline{u}''_H + (1-\alpha) \underline{u}''_L - \psi''(\underline{e})) \frac{d\underline{e}}{d\varepsilon_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L} \right] \\ & + (1-\alpha) \underline{u}'_L \left(\frac{\underline{u}''_L}{\underline{u}'_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \right) \left(\frac{\underline{v}''_L}{\underline{v}'_L} - \frac{\underline{v}''_H}{\underline{v}'_H} \frac{d\varepsilon_H}{d\varepsilon_L} \right) \\ & + (1-\alpha) \underline{u}'_L \left(\frac{\underline{u}''_H}{\underline{u}'_H} - \frac{\underline{u}''_L}{\underline{u}'_L} \right)^2 \frac{d\underline{e}}{d\varepsilon_L} \\ & - \frac{(1-\alpha)}{\alpha} \frac{\underline{u}'_L}{\underline{u}'_H} \left(\frac{\underline{u}''_L}{\underline{u}'_L} - \frac{\underline{u}''_H}{\underline{u}'_H} \right) \left(\frac{\underline{v}''_H}{\underline{v}'_H} + \frac{\underline{u}''_H}{\underline{u}'_H} \right) \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}. \end{aligned} \quad (58)$$

Let

$$A = - \left(\frac{(1-\alpha) u'_L}{\alpha u'_H} \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) + \left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \right) \right) (\alpha u''_H + (1-\alpha) u''_L - \psi''(\underline{\varepsilon})) \\ + (1-\alpha) u'_L \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right)^2,$$

$$B = \left(\frac{(1-\alpha) u'_L}{\alpha u'_H} \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) + \left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \right) \right) \left(-\frac{u''_H}{u'_H} \right) \Phi'(\bar{\varepsilon}) \\ - \frac{(1-\alpha) u'_L}{\alpha u'_H} \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \Phi'(\bar{\varepsilon}),$$

$$C = (1-\alpha) u'_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \left(\frac{v''_L}{v'_L} + \frac{1-\alpha}{\alpha} \frac{v''_H}{v'_H} \right).$$

Then (58) can be rewritten as

$$A \frac{d\underline{\varepsilon}}{d\varepsilon_L} + B \frac{d\bar{\varepsilon}}{d\varepsilon_L} + C = 0.$$

As in Lemma 5 we may show that $A < 0$. Furthermore, we may calculate:

$$B = - \left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \right) \frac{u''_H}{u'_H} \Phi'(\bar{\varepsilon}) - \frac{(1-\alpha) u'_L}{\alpha u'_H} \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) \frac{u''_L}{u'_L} \Phi'(\bar{\varepsilon}) < 0.$$

Since

$$\text{sgn}C = \text{sgn} \left(\frac{u''_H}{u'_H} - \frac{u''_L}{u'_L} \right),$$

this implies the desired result. ■

We conclude that if u is of the DARA type, then a marginal mean preserving spread must increase either the effort of the low type, or the effort of the high type. Furthermore, if u is of the CARA type, then $\frac{d\underline{\varepsilon}}{d\varepsilon_L}$ and $\frac{d\bar{\varepsilon}}{d\varepsilon_L}$ must have opposite sign.

Proof of Proposition 5

Consider the three optimality equations for type $\bar{\beta}$:

$$\alpha u(\bar{t}_H - \bar{c} - \varepsilon_H) + (1-\alpha) u(\bar{t}_L - \bar{c} - \varepsilon_L) - \psi(\bar{\varepsilon}) = 0; \quad (59)$$

$$\alpha u'(\bar{t}_H - \bar{c} - \varepsilon_H) + (1 - \alpha) u'(\bar{t}_L - \bar{c} - \varepsilon_L) - \psi'(\bar{e}) = \frac{\pi}{1 - \pi} \left(\frac{v'(-\underline{t}_L)}{u'(\underline{t}_L - \underline{c} - \varepsilon_L)} - 1 \right) \frac{u'(\bar{t}_L - \bar{c} - \varepsilon_L)}{v'(-\bar{t}_L)} \Phi'(\bar{e}); \quad (60)$$

$$v'(-\bar{t}_H) u'(\bar{t}_L - \bar{c} - \varepsilon_L) - u'(\bar{t}_H - \bar{c} - \varepsilon_H) v'(S - \bar{t}_L) = 0. \quad (61)$$

From (59) we have

$$\alpha \bar{u}'_H \left(\frac{d\bar{t}_H}{d\varepsilon_L} + \frac{1 - \alpha}{\alpha} \right) + (1 - \alpha) \bar{u}'_L \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) + (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})) \frac{d\bar{e}}{d\varepsilon_L} = 0. \quad (62)$$

Analogously to (37), from (61) we may derive

$$0 = \left(-\frac{\bar{v}''_H}{\bar{v}'_H} \frac{d\bar{t}_H}{d\varepsilon_L} + \frac{\bar{u}''_L}{\bar{u}'_L} \left(\frac{d\bar{t}_L}{d\varepsilon_L} + \frac{d\bar{e}}{d\varepsilon_L} - 1 \right) \right) + \left(\frac{\bar{v}''_L}{\bar{v}'_L} \frac{d\bar{t}_L}{d\varepsilon_L} - \frac{\bar{u}''_H}{\bar{u}'_H} \left(\frac{d\bar{t}_H}{d\varepsilon_L} + \frac{d\bar{e}}{d\varepsilon_L} + \frac{1 - \alpha}{\alpha} \right) \right). \quad (63)$$

Grouping terms, and using (62) we obtain:

$$\begin{aligned} 0 &= \left(\frac{(1 - \alpha)}{\alpha} \frac{\bar{u}'_L}{\bar{u}'_H} \left(\frac{\bar{v}''_H}{\bar{v}'_H} + \frac{\bar{u}''_H}{\bar{u}'_H} \right) + \left(\frac{\bar{v}''_L}{\bar{v}'_L} + \frac{\bar{u}''_L}{\bar{u}'_L} \right) \right) \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) \\ &+ \left(\frac{\bar{v}''_L}{\bar{v}'_L} + \frac{\bar{v}''_H}{\bar{v}'_H} \frac{1 + \alpha}{\alpha} \right) - \left(\frac{\bar{u}''_H}{\bar{u}'_H} - \frac{\bar{u}''_L}{\bar{u}'_L} \right) \frac{d\bar{e}}{d\varepsilon_L} + \left(\frac{\bar{v}''_H}{\bar{v}'_H} + \frac{\bar{u}''_H}{\bar{u}'_H} \right) \frac{\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})}{\alpha \bar{u}'_H} \frac{d\bar{e}}{d\varepsilon_L}. \end{aligned} \quad (64)$$

Next, differentiating (60)

$$\begin{aligned} 0 &= (\alpha \bar{u}''_H + (1 - \alpha) \bar{u}''_L - \psi''(\bar{e})) \frac{d\bar{e}}{d\varepsilon_L} + \alpha \bar{u}''_H \left(\frac{d\bar{t}_H}{d\varepsilon_L} + \frac{1 - \alpha}{\alpha} \right) + (1 - \alpha) \bar{u}''_L \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) \\ &- \frac{\pi}{1 - \pi} \left(-\frac{v''_L}{u'_L} \frac{d\underline{t}_L}{d\varepsilon_L} - \frac{v'_L u''_L}{(u'_L)^2} \left(\frac{d\underline{t}_L}{d\varepsilon_L} + \frac{d\underline{e}}{d\varepsilon_L} - 1 \right) \right) \frac{\bar{u}'_L}{\bar{v}'_L} \Phi'(\bar{e}) \\ &- \frac{\pi}{1 - \pi} \left(\frac{v'_L}{u'_L} - 1 \right) \left(\frac{\bar{u}''_L}{\bar{v}'_L} \left(\frac{d\bar{t}_L}{d\varepsilon_L} + \frac{d\bar{e}}{d\varepsilon_L} - 1 \right) + \frac{\bar{u}'_L \bar{v}''_L}{(\bar{v}'_L)^2} \frac{d\bar{t}_L}{d\varepsilon_L} \right) \Phi'(\bar{e}) \\ &- \frac{\pi}{1 - \pi} \left(\frac{v'_L}{u'_L} - 1 \right) \frac{\bar{u}'_L}{\bar{v}'_L} \Phi''(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}. \end{aligned}$$

Using (62), (60), and simplifying, we obtain:

$$\begin{aligned}
0 = & - \left(\psi''(\bar{e}) - \frac{\bar{u}''_H}{\bar{u}'_H} \psi'(\bar{e}) \right) \frac{d\bar{e}}{d\varepsilon_L} + (1 - \alpha) \bar{u}'_L \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) \left(\frac{\bar{u}''_L}{\bar{u}'_L} - \frac{\bar{u}''_H}{\bar{u}'_H} \right) \\
& + \frac{\pi}{1 - \pi} \left(\left(\frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \right) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) + \frac{v''_L}{v'_L} + \frac{u''_L}{u'_L} \frac{d\varepsilon}{d\varepsilon_L} \right) \frac{v'_L}{u'_L} \frac{\bar{u}'_L}{\bar{v}'_L} \Phi'(\bar{e}) \\
& - \frac{\pi}{1 - \pi} \left(\frac{v'_L}{u'_L} - 1 \right) \left(\left(\frac{\bar{u}''_L}{\bar{u}'_L} + \frac{\bar{v}''_L}{\bar{v}'_L} \right) \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) + \frac{\bar{v}''_L}{\bar{v}'_L} + \frac{\bar{u}''_L}{\bar{u}'_L} \frac{d\bar{e}}{d\varepsilon_L} \right) \frac{\bar{u}'_L}{\bar{v}'_L} \Phi'(\bar{e}) \\
& - \frac{\pi}{1 - \pi} \left(\frac{v'_L}{u'_L} - 1 \right) \frac{\bar{u}'_L}{\bar{v}'_L} \Phi''(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}.
\end{aligned} \tag{65}$$

Using (60), and the fact that u and v are CARA, (65) can be rewritten as:

$$\begin{aligned}
0 = & - (\psi''(\bar{e}) + \rho_u \psi'(\bar{e})) \frac{d\bar{e}}{d\varepsilon_L} \\
& - (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})) \left(\frac{v'_L}{u'_L} - 1 \right)^{-1} \frac{v'_L}{u'_L} \left((\rho_u + \rho_v) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) + \rho_v + \rho_u \frac{d\varepsilon}{d\varepsilon_L} \right) \\
& + (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})) \left((\rho_u + \rho_v) \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) + \rho_v + \rho_u \frac{d\bar{e}}{d\varepsilon_L} \right) \\
& - (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})) \frac{\Phi''(\bar{e})}{\Phi'(\bar{e})} \frac{d\bar{e}}{d\varepsilon_L}.
\end{aligned} \tag{66}$$

Furthermore, (64) becomes:

$$0 = (\rho_u + \rho_v) [\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L] \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) + \rho_v \bar{u}'_H + (\rho_u + \rho_v) [\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})] \frac{d\bar{e}}{d\varepsilon_L},$$

and so

$$\begin{aligned}
& (\rho_u + \rho_v) \left(\frac{d\bar{t}_L}{d\varepsilon_L} - 1 \right) + \rho_v + \rho_u \frac{d\bar{e}}{d\varepsilon_L} \\
& = (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L)^{-1} \left\{ (1 - \alpha) (\bar{u}'_L - \bar{u}'_H) \rho_v + [\rho_u \psi'(\bar{e}) - \rho_v (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e}))] \frac{d\bar{e}}{d\varepsilon_L} \right\}.
\end{aligned} \tag{67}$$

From (57) we obtain

$$0 = -(\rho_u + \rho_v) (\alpha \underline{u}'_H + (1 - \alpha) \underline{u}'_L) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) - \rho_v \underline{u}'_H + (\rho_u + \rho_v) \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L},$$

and so

$$\begin{aligned}
& (\rho_u + \rho_v) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) + \rho_v + \rho_u \frac{d\underline{e}}{d\varepsilon_L} \\
&= (\alpha \underline{u}'_H + (1 - \alpha) \underline{u}'_L)^{-1} \left((1 - \alpha) (\underline{u}'_L - \underline{u}'_H) \rho_v + (\rho_u + \rho_v) \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L} \right) + \rho_u \frac{d\underline{e}}{d\varepsilon_L}.
\end{aligned} \tag{68}$$

Furthermore, (58) becomes

$$(\alpha \rho_u \underline{u}'_H + (1 - \alpha) \rho_u \underline{u}'_L + \psi''(\underline{e})) \frac{d\underline{e}}{d\varepsilon_L} = -\rho_u \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}.$$

Using (52) this can be rewritten as

$$(\rho_u \psi'(\underline{e}) + \psi''(\underline{e})) \frac{d\underline{e}}{d\varepsilon_L} = -\rho_u \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L}.$$

Substituting $\frac{d\underline{e}}{d\varepsilon_L}$ into (68), and using (52) yields:

$$\begin{aligned}
& (\rho_u + \rho_v) \left(\frac{dt_L}{d\varepsilon_L} - 1 \right) + \rho_v + \rho_u \frac{d\underline{e}}{d\varepsilon_L} \\
&= (\alpha \underline{u}'_H + (1 - \alpha) \underline{u}'_L)^{-1} \left((1 - \alpha) (\underline{u}'_L - \underline{u}'_H) \rho_v + \left(\rho_u \frac{\psi''(\underline{e})}{\rho_u \psi'(\underline{e}) + \psi''(\underline{e})} + \rho_v \right) \Phi'(\bar{e}) \frac{d\bar{e}}{d\varepsilon_L} \right).
\end{aligned} \tag{69}$$

Substituting (67) and (69) into (66), and collecting terms, results in an expression of the form

$$C \frac{d\bar{e}}{d\varepsilon_L} + (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})) D = 0.$$

It follows from the assumption that $\psi''' \geq 0$ that $\Phi''(e) = \psi''(e) - \psi''(e - \Delta\beta) \geq 0$. Hence we have

$$\begin{aligned}
C &\leq -(\psi''(\bar{e}) + \rho_u \psi'(\bar{e})) + (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})) \rho_u \psi'(\bar{e}) (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L)^{-1} \\
&= -\psi''(\bar{e}) - \rho_u \psi'(\bar{e})^2 (\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L)^{-1} < 0.
\end{aligned}$$

If, π is small, then (60) implies that $\alpha \bar{u}'_H + (1 - \alpha) \bar{u}'_L - \psi'(\bar{e})$ is small, so the sign of C will equal the sign of $-(\psi''(\bar{e}) + \rho_u \psi'(\bar{e})) < 0$.

To show that $\frac{d\bar{\varepsilon}}{d\varepsilon_L} < 0$, it therefore suffices to show that $D < 0$. Since

$$\left(\frac{v'_L}{u'_L} - 1\right)^{-1} \frac{v'_L}{u'_L} > 1$$

we have

$$D < (1 - \alpha)\rho_v \left(-(\alpha u'_H + (1 - \alpha)u'_L)^{-1} (u'_L - u'_H) + (\alpha \bar{u}'_H + (1 - \alpha)\bar{u}'_L)^{-1} (\bar{u}'_L - \bar{u}'_H) \right). \quad (70)$$

The sign of the right hand side is the same as the sign of

$$(\alpha \bar{u}'_H + (1 - \alpha)\bar{u}'_L) (u'_L - u'_H) - (\alpha u'_H + (1 - \alpha)u'_L) (\bar{u}'_L - \bar{u}'_H) = u'_L \bar{u}'_H - \bar{u}'_L u'_H = 0. \quad (71)$$

To establish the final equality in (71), we will show that

$$\frac{\bar{u}'_H}{\bar{u}'_L} = \frac{u'_H}{u'_L}.$$

Using the CARA property, this is in turn equivalent to showing that

$$\bar{y}_H - \bar{y}_L = y_H - y_L.$$

For CARA case the optimal risk sharing conditions (28) become:

$$\frac{e^{-\rho_v(Y-t_H)}}{e^{-\rho_v(Y-t_L)}} = \frac{e^{-\rho_u(t_H-c-\varepsilon_H)}}{e^{-\rho_u(t_L-c-\varepsilon_L)}},$$

which implies

$$\rho_v(t_H - t_L) = \rho_u(t_L - t_H + \varepsilon_H - \varepsilon_L).$$

Hence

$$(t_H - t_L) = \frac{\rho_u}{\rho_u + \rho_v} (\varepsilon_H - \varepsilon_L). \quad (72)$$

It follows from (72) that

$$y_H - y_L = (t_H - t_L) - \varepsilon_H + \varepsilon_L = -\frac{\rho_v}{\rho_u + \rho_v} (\varepsilon_H - \varepsilon_L).$$

Hence we have $\bar{y}_H - \bar{y}_L = \underline{y}_H - \underline{y}_L$, proving the desired result.

Proof of Proposition 6

For this exercise, the low cost firm remains equally efficient as in the complete information scenario, i.e. we have $\underline{\beta} = \beta$. The high cost firm's efficiency declines, i.e. we have $\bar{\beta} = \beta + \Delta\beta$. We examine the comparative statics of the solution $\{\underline{e}_L, \underline{e}_H, \bar{e}_L, \bar{e}_H, \underline{t}_L, \underline{t}_L, \bar{t}_L, \bar{t}_H\}$ with respect to $\Delta\beta$, in a neighborhood of $\Delta\beta = 0$. Recall that

$$\bar{U} = \alpha u(\bar{t}_H - \bar{\beta} + \bar{e} - \varepsilon_H) + (1 - \alpha)u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L) - \psi(\bar{e}) = 0; \quad (73)$$

$$\begin{aligned} \psi'(\bar{e}) &= \alpha u'(\bar{t}_H - \bar{\beta} + \bar{e} - \varepsilon_H) + (1 - \alpha)u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L) \\ &\quad - \frac{\pi}{1 - \pi} \left(\frac{v'(-\underline{t}_L)}{u'(\underline{t}_L - \underline{\beta} + \underline{e} - \varepsilon_L)} - 1 \right) \frac{u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L)}{v'(-\bar{t}_L)} \Phi'(\bar{e}); \end{aligned} \quad (74)$$

and

$$\frac{v'(-\bar{t}_H)}{v'(-\bar{t}_L)} = \frac{u'(\bar{t}_H - \bar{\beta} + \bar{e} - \varepsilon_H)}{u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L)}, \quad (75)$$

where

$$\Phi'(\bar{e}) = \psi'(\bar{e}) - \psi'(\bar{e} - \Delta\beta).$$

Define

$$\Omega = \frac{\pi}{1 - \pi} \left(\frac{v'(-\underline{t}_L)}{u'(\underline{t}_L - \underline{\beta} + \underline{e} - \varepsilon_L)} - 1 \right) \frac{u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L)}{v'(-\bar{t}_L)}.$$

Then we can simplify (74) as

$$\alpha u'(\bar{t}_H - \bar{\beta} + \bar{e} - \varepsilon_H) + (1 - \alpha)u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L) - \Omega \Phi'(\bar{e}) = \psi'(\bar{e}). \quad (76)$$

Differentiating (73) yields:

$$\alpha u'_H \left(\frac{d\bar{t}_H}{d\Delta\beta} - 1 + \frac{d\bar{e}}{d\Delta\beta} \right) + (1 - \alpha)u'_L \left(\frac{d\bar{t}_L}{d\Delta\beta} - 1 + \frac{d\bar{e}}{d\Delta\beta} \right) - \psi'(\bar{e}) \frac{d\bar{e}}{d\Delta\beta} = 0.$$

Collecting terms then produces:

$$\alpha u'_H \left(\frac{d\bar{t}_H}{d\Delta\beta} - 1 \right) + (1 - \alpha)u'_L \left(\frac{d\bar{t}_L}{d\Delta\beta} - 1 \right) + (\alpha u'_H + (1 - \alpha)u'_L - \psi'(\bar{e})) \frac{d\bar{e}}{d\Delta\beta} = 0.$$

Since $\Phi'(\bar{e}) = 0$ at $\Delta\beta = 0$, we have from (76) that

$$\alpha u'_H + (1 - \alpha)u'_L - \psi'(\bar{e}) = 0.$$

Substituting this into the previous expression, we then obtain

$$\alpha u'_H \frac{d\bar{t}_H}{d\Delta\beta} + (1 - \alpha)u'_L \frac{d\bar{t}_L}{d\Delta\beta} = \psi'(\bar{e}). \quad (77)$$

Next, let us re-write (75) as follows:

$$v'(-\bar{t}_H)u'(\bar{t}_L - \bar{\beta} + \bar{e} - \varepsilon_L) - v'(-\bar{t}_L)u'(\bar{t}_H - \bar{\beta} + \bar{e} - \varepsilon_H) = 0.$$

Differentiating this expression produces:

$$-v''_H u'_L \frac{d\bar{t}_H}{d\Delta\beta} + v'_H u''_L \left(\frac{d\bar{t}_L}{d\Delta\beta} - 1 + \frac{d\bar{e}}{d\Delta\beta} \right) + v''_L u'_H \frac{d\bar{t}_L}{d\Delta\beta} - v'_L u''_H \left(\frac{d\bar{t}_H}{d\Delta\beta} - 1 + \frac{d\bar{e}}{d\Delta\beta} \right) = 0.$$

Collecting terms reduces this to:

$$-(v''_H u'_L + v'_L u''_H) \frac{d\bar{t}_H}{d\Delta\beta} + (v'_H u''_L + v''_L u'_H) \frac{d\bar{t}_L}{d\Delta\beta} + (v'_H u''_L - v'_L u''_H) \frac{d\bar{e}}{d\Delta\beta} - (v'_H u''_L - v'_L u''_H) = 0.$$

Using (75) this becomes:

$$-\left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_L} \right) \frac{d\bar{t}_H}{d\Delta\beta} + \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) \frac{d\bar{t}_L}{d\Delta\beta} + \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \frac{d\bar{e}}{d\Delta\beta} - \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) = 0. \quad (78)$$

Finally, let us differentiate (76):

$$\alpha u''_H \left(\frac{d\bar{t}_H}{d\Delta\beta} - 1 + \frac{d\bar{e}}{d\Delta\beta} \right) + (1 - \alpha)u''_L \left(\frac{d\bar{t}_L}{d\Delta\beta} - 1 + \frac{d\bar{e}}{d\Delta\beta} \right) - \psi''(\bar{e}) \frac{d\bar{e}}{d\Delta\beta} - \frac{d\Omega}{d\Delta\beta} \Phi'(\bar{e}) - \Omega \frac{d\Phi'}{d\Delta\beta} = 0.$$

Now we may calculate:

$$\frac{d}{d\Delta\beta} \Phi'(\bar{e}) = \frac{d}{d\Delta\beta} [\psi'(\bar{e}) - \psi'(\bar{e} - \Delta\beta)] = [\psi''(\bar{e}) - \psi''(\bar{e} - \Delta\beta)] \frac{d\bar{e}}{d\Delta\beta} + \psi''(\bar{e} - \Delta\beta).$$

Evaluating at $\Delta\beta = 0$ produces:

$$\frac{d}{d\Delta\beta}\Phi'(\bar{e}) = \psi''(\bar{e}).$$

Since we also have $\Phi'(\bar{e}) = 0$ at $\Delta\beta = 0$, we then obtain:

$$\alpha u''_H \frac{d\bar{t}_H}{d\Delta\beta} + (1-\alpha)u''_L \frac{d\bar{t}_L}{d\Delta\beta} + (\alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e})) \frac{d\bar{e}}{d\Delta\beta} - (\alpha u''_H + (1-\alpha)u''_L + \Omega\psi''(\bar{e})) = 0. \quad (79)$$

The three equations (77)-(79) can be written in matrix form, as follows:

$$\begin{bmatrix} \alpha u'_H & (1-\alpha)u'_L & 0 \\ -\left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_L}\right) & \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L}\right) & \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H}\right) \\ \alpha u''_H & (1-\alpha)u''_L & \alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e}) \end{bmatrix} \begin{bmatrix} \frac{d\bar{t}_H}{d\Delta\beta} \\ \frac{d\bar{t}_L}{d\Delta\beta} \\ \frac{d\bar{e}}{d\Delta\beta} \end{bmatrix} = \begin{bmatrix} \psi'(\bar{e}) \\ \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H}\right) \\ \alpha u''_H + (1-\alpha)u''_L + \Omega\psi''(\bar{e}) \end{bmatrix}.$$

The inverse of the 3×3 matrix

$$\begin{bmatrix} a & b & 0 \\ c & d & e \\ f & g & h \end{bmatrix}$$

equals

$$\frac{1}{bef - aeg - bch + adh} \begin{bmatrix} dh - eg & -bh & be \\ ef - ch & ah & -ae \\ cg - df & bf - ag & -bc + ad \end{bmatrix}.$$

Since the 3×3 matrix is identical to the one in the proof of Lemma 9, we know that

$$D = bef - aeg - bch + adh > 0.$$

We then have:

$$\frac{d\bar{t}_H}{d\Delta\beta} = \frac{1}{D} \left[(dh - eg) \psi'(\bar{e}) - bh \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) + be [\alpha u''_H + (1-\alpha)u''_L + \Omega\psi''(\bar{e})] \right].$$

The term in square brackets may be computed as

$$\begin{aligned} & \left[\left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) [\alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e})] - (1-\alpha)u''_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \right] \psi'(\bar{e}) \\ & - (1-\alpha)u'_L [\alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e})] \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \\ & + (1-\alpha)u'_L \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) [\alpha u''_H + (1-\alpha)u''_L + \Omega \psi''(\bar{e})]. \end{aligned}$$

Collecting terms, we then obtain:

$$\begin{aligned} \frac{d\bar{t}_H}{d\Delta\beta} &= \frac{1}{D} \times \\ & \left\{ \left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) [\alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e})] \psi'(\bar{e}) + (1-\alpha)u'_L \left(-\frac{u''_L}{u'_L} \psi'(\bar{e}) + \psi''(\bar{e}) [1 + \Omega] \right) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \right\}. \end{aligned}$$

Next, let us compute $\frac{d\bar{t}_L}{d\Delta\beta}$. We have

$$\frac{d\bar{t}_L}{d\Delta\beta} = \frac{1}{D} \left[(ef - ch) \psi'(\bar{e}) + ah \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) - ae [\alpha u''_H + (1-\alpha)u''_L + \Omega \psi''(\bar{e})] \right].$$

The term in square brackets may be computed as

$$\begin{aligned} & \left[\alpha u''_H \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) + \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_L} \right) [\alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e})] \right] \psi'(\bar{e}) \\ & + \alpha u'_H [\alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e})] \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \\ & - \alpha u'_H [\alpha u''_H + (1-\alpha)u''_L + \Omega \psi''(\bar{e})] \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right). \end{aligned}$$

Hence we have

$$\begin{aligned} \frac{d\bar{t}_L}{d\Delta\beta} &= \frac{1}{D} \times \\ & \left\{ \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_L} \right) [\alpha u''_H + (1-\alpha)u''_L - \psi''(\bar{e})] \psi'(\bar{e}) - \alpha u'_H \left[-\frac{u''_H}{u'_H} \psi'(\bar{e}) + \psi''(\bar{e}) (1 + \Omega) \right] \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \right\}. \end{aligned}$$

It follows that

$$\begin{aligned} D \frac{d(\bar{t}_H - \bar{t}_L)}{d\Delta\beta} &= [\alpha u''_H + (1 - \alpha)u''_L - \psi''(\bar{e})] \psi'(\bar{e}) \left\{ \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) + \left(\frac{v''_L}{v'_L} - \frac{v''_H}{v'_H} \right) \right\} \\ &\quad + (1 - \alpha)u'_L \left(-\frac{u''_L}{u'_L} \psi'(\bar{e}) + \psi''(\bar{e}) [1 + \Omega] \right) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \\ &\quad + \alpha u'_H \left[-\frac{u''_H}{u'_H} \psi'(\bar{e}) + \psi''(\bar{e}) (1 + \Omega) \right] \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right). \end{aligned}$$

Further simplifying yields:

$$\begin{aligned} D \frac{d(\bar{t}_H - \bar{t}_L)}{d\Delta\beta} &= [\alpha u''_H + (1 - \alpha)u''_L - \psi''(\bar{e})] \psi'(\bar{e}) \left\{ \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) + \left(\frac{v''_L}{v'_L} - \frac{v''_H}{v'_H} \right) \right\} \\ &\quad - [(1 - \alpha)u'_L + \alpha u'_H] \psi'(\bar{e}) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \\ &\quad + [\alpha u'_H + (1 - \alpha)u'_L] \psi''(\bar{e}) (1 + \Omega) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right). \end{aligned}$$

Using $\alpha u'_H + (1 - \alpha)u'_L = \psi'(\bar{e})$ and simplifying a third time gives:

$$\frac{D}{\psi'(\bar{e})} \frac{d(\bar{t}_H - \bar{t}_L)}{d\Delta\beta} = [\alpha u''_H + (1 - \alpha)u''_L - \psi''(\bar{e})] \left(\frac{v''_L}{v'_L} - \frac{v''_H}{v'_H} \right) + \psi''(\bar{e}) \Omega \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right).$$

On the other hand, following the procedure from Lemma 9, we may compute:

$$\begin{aligned} \Delta \underline{t}_H &= \frac{\psi'(\bar{e})\Delta\beta}{D} \left(\left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) [\alpha u''_H + (1 - \alpha)u''_L - \psi''(e)] - \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) (1 - \alpha)u''_L \right); \\ \Delta \underline{t}_L &= \frac{\psi'(\bar{e})\Delta\beta}{D} \left(\alpha u''_H \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) + \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) [\alpha u''_H + (1 - \alpha)u''_L - \psi''(e)] \right). \end{aligned}$$

and so

$$\frac{D}{\psi'(\bar{e})} \frac{d(\underline{t}_H - \underline{t}_L)}{d\Delta\beta} = [\alpha u''_H + (1 - \alpha)u''_L - \psi''(e)] \left(\frac{v''_L}{v'_L} - \frac{v''_H}{v'_H} \right) - \psi''(e) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right).$$

We therefore may conclude that if the consumer has constant absolute risk aversion over the range

of exogenous uncertainty, then

$$\begin{aligned}\frac{d(\bar{t}_H - \bar{t}_L)}{d\Delta\beta} &= \frac{\psi'(\bar{e})}{D} \psi''(\bar{e}) \Omega \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) \\ \frac{d(\underline{t}_H - \underline{t}_L)}{d\Delta\beta} &= -\frac{\psi'(\bar{e})}{D} \psi''(e) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right)\end{aligned}$$

Thus the effect of a small amount of asymmetric information will then be opposite on the low and high efficiency firm. Because at $\Delta\beta = 0$ we have

$$\Pi_L > \Pi_H$$

if the firm has decreasing absolute risk aversion, then

$$-\frac{u''_L}{u'_L} < -\frac{u''_H}{u'_H},$$

and:

$$\begin{aligned}\Delta \underline{t}_H &= \frac{\psi'(\bar{e})\Delta\beta}{D} \left(\left(\frac{u''_L}{u'_L} + \frac{v''_L}{v'_L} \right) [\alpha u''_H + (1-\alpha)u''_L - \psi''(e)] - \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) (1-\alpha)u''_L \right); \\ \Delta \underline{t}_L &= \frac{\psi'(\bar{e})\Delta\beta}{D} \left(\alpha u''_H \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right) + \left(\frac{v''_H}{v'_H} + \frac{u''_H}{u'_H} \right) [\alpha u''_H + (1-\alpha)u''_L - \psi''(e)] \right).\end{aligned}$$

and so

$$\frac{D}{\psi'(\bar{e})} \frac{d(\underline{t}_H - \underline{t}_L)}{d\Delta\beta} = [\alpha u''_H + (1-\alpha)u''_L - \psi''(e)] \left(\frac{v''_L}{v'_L} - \frac{v''_H}{v'_H} \right) - \psi''(e) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right).$$

Thus we have

$$\frac{d(\bar{t}_H - \bar{t}_L)}{d\Delta\beta} > 0 \text{ and } \frac{d(\underline{t}_H - \underline{t}_L)}{d\Delta\beta} < 0.$$

We have shown that when $\Delta\beta = 0$, then $t_H > t_L$ and $\Pi_H < \Pi_L$. Hence if $(t_H - t_L)$ increases, then the consumer bears more risk, and the firm bears less risk. The converse is true when $(t_H - t_L)$ decreases.

Without any assumptions on consumers' absolute risk aversion, we may derive the following:

$$\frac{d(\bar{t}_H - \bar{t}_L)}{d\Delta\beta} - \frac{d(\underline{t}_H - \underline{t}_L)}{d\Delta\beta} = \frac{\psi'(\bar{e})\psi''(\bar{e})}{D}(1 + \Omega) \left(\frac{u''_L}{u'_L} - \frac{u''_H}{u'_H} \right).$$

Thus if the firm exhibits DARA, then we have

$$\frac{d(\bar{t}_H - \bar{t}_L)}{d\Delta\beta} > \frac{d(\underline{t}_H - \underline{t}_L)}{d\Delta\beta},$$

proving the desired result.

FIGURE 1

COMPARISON OF THE FIRST BEST EFFORT LEVEL e^* AND \underline{e} , THE EQUILIBRIUM EFFORT LEVEL OF THE EFFICIENT FIRM IN THE ABSENCE OF EXOGENOUS RISK

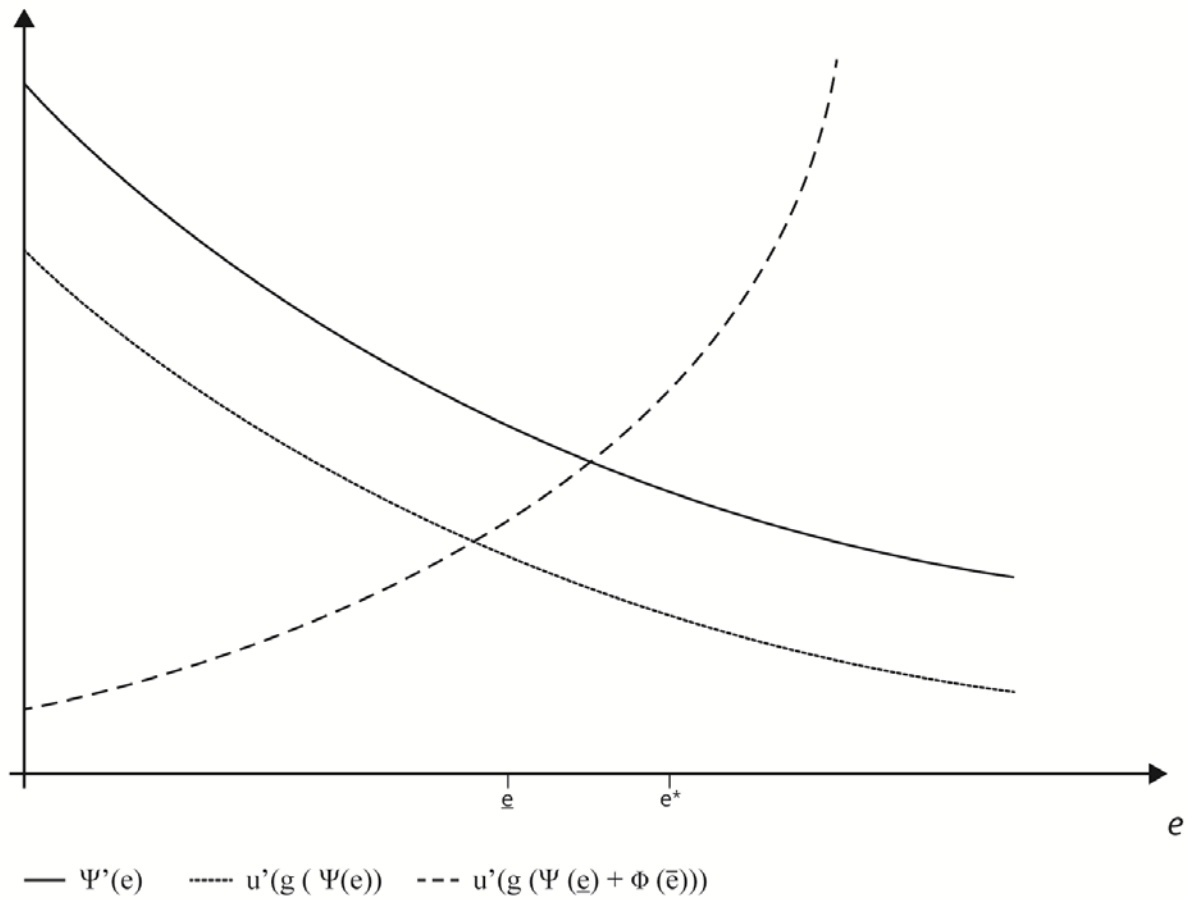


FIGURE 2

COMPARISON OF THE EFFICIENT FIRM'S EQUILIBRIUM EFFORT LEVEL, $\underline{e}(\pi)$, TO THE INEFFICIENT FIRM'S EQUILIBRIUM EFFORT LEVEL, $\bar{e}(\pi)$

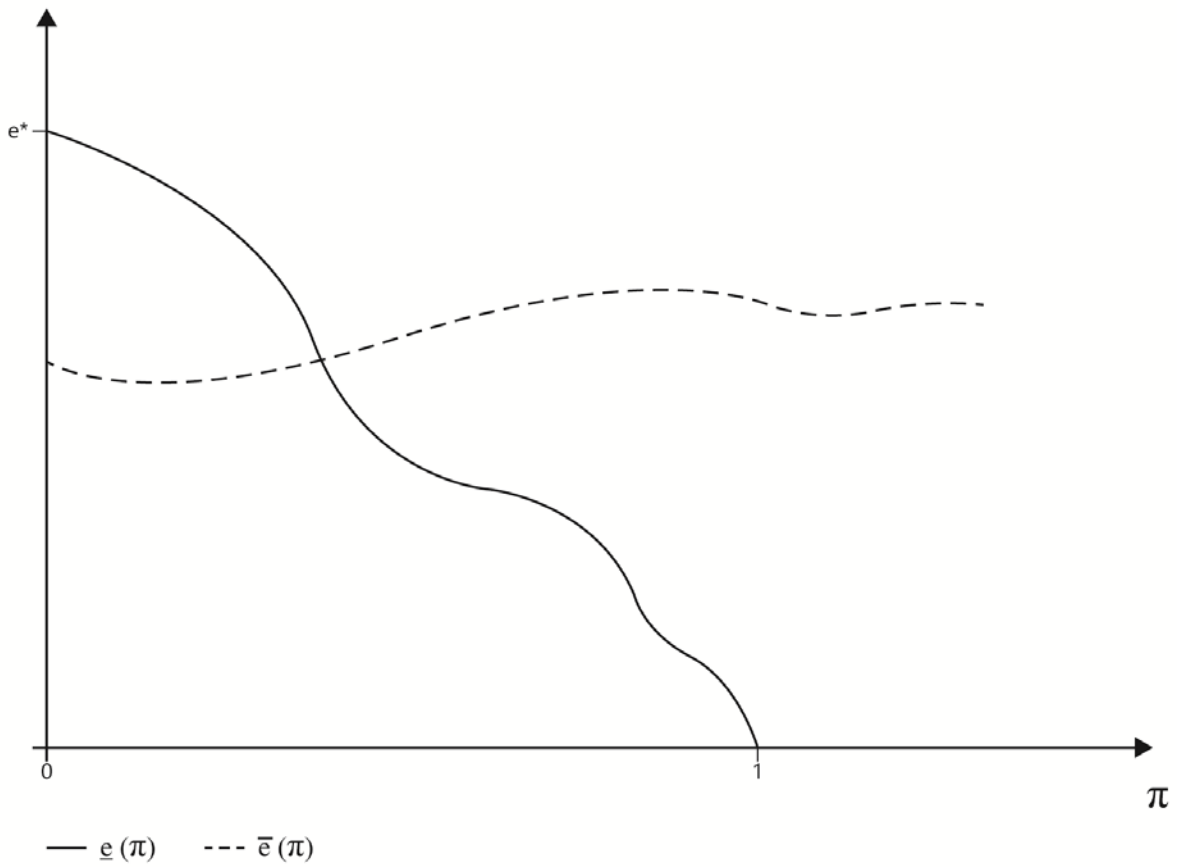


FIGURE 3

EFFECT OF LOWERING EFFORT WHILE MAINTAINING PRODUCTIVE EFFICIENCY AND KEEPING THE FIRM AT ITS RESERVATION UTILITY LEVEL WHEN u IS OF THE DARA TYPE

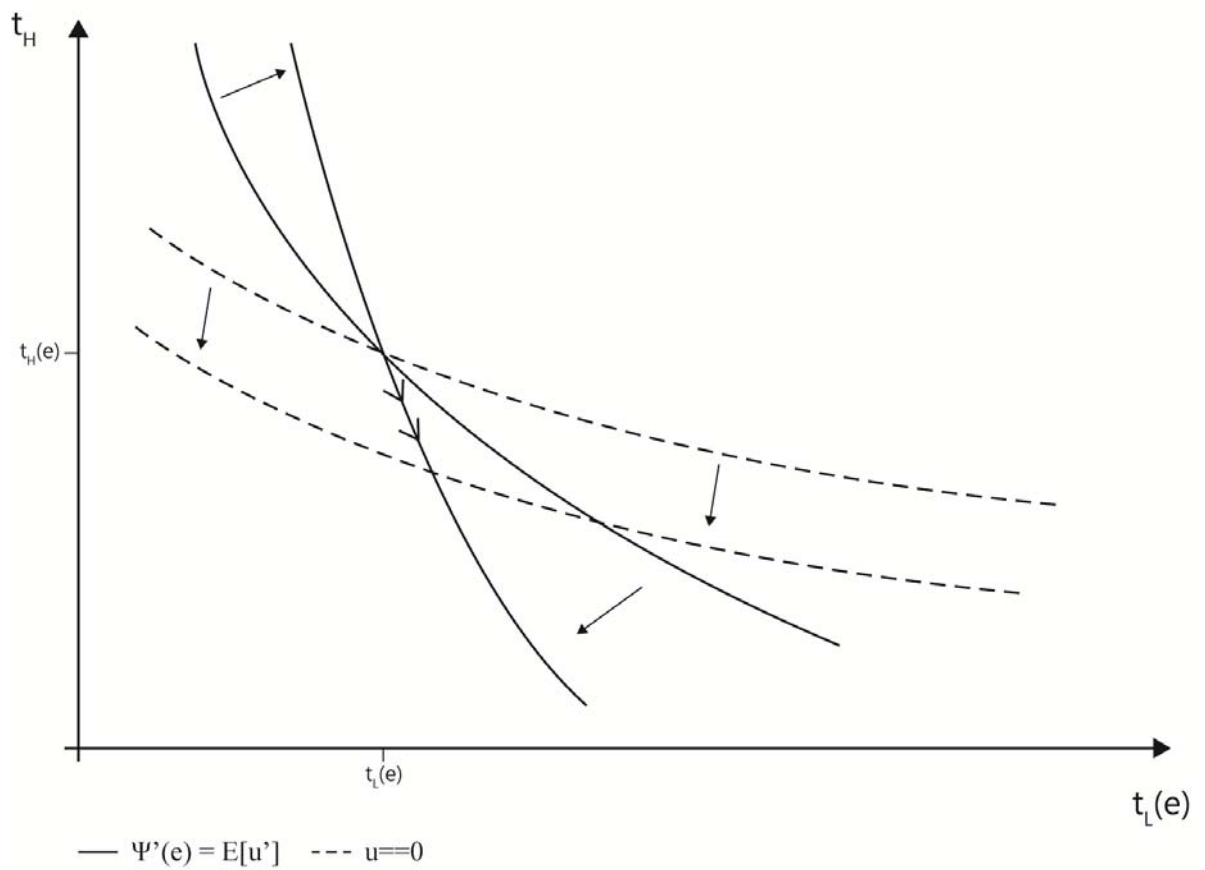


FIGURE 4

EFFECT OF A MEAN PRESERVING SPREAD IN EXOGENOUS UNCERTAINTY, WHILE MAINTAINING OPTIMAL RISK SHARING AND PRODUCTIVE EFFICIENCY, AND KEEPING EFFORT LEVEL \underline{e} FIXED

