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**FRACTIONS IN THE *SUÀN SHÙ SHŪ*  
(CHINA, BEGINNING OF THE 2<sup>ND</sup> CENTURY BCE)**

**西漢出土文獻《算數書》分數表達**

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**ABSTRACT:** The *Suàn Shù Shū* contains 301 instances of regular expressions for fractions. They can be “mono-dimensional” (formed with one integer name only) for unit fractions, “bidimensional” (with two integer names) for both unit and non-unit fractions, or lexicalized only for 1/3, 1/2 and 2/3. The present paper gives a complete description of the diversity of these forms. Bidimensional expressions are predicative phrases: the name  $n fēn$  of a unit fraction  $1/n$  acts as subject and the numerator’s name as predicate; according to the syntactic context, the morpheme *zhī* can be used as an optional marker of this predicative relation.

**KEYWORDS:** Chinese historical syntax, Fractions, Numerical expressions, Separable semantic units, Measure words, *Suàn Shù Shū*.

**摘要:** 西漢出土文獻《算數書》中表達分數的短語有 301 例。本文全面描述該書中的那些短語。在文中，我把它們定義為“一維短語（即包括唯一一個整數名）”和“二維短語（即包括兩個整數名）”，前者表達單位分數，後者表達單位和非單位分數，除此之外還有三個詞彙化的短語專門用於表達 1/3、1/2 和 2/3。在二維短語中，單位分數  $1/n$  作為短語的主語表達分母，而表達分子的那個數名作為短語的謂語，根據語境，“之”字可作為短語標誌放在主謂語之間。

**主題詞:** 中文語法歷史，分數，表數短語，離合詞組，量詞，《算數書》。

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Abbreviations DECL: declarative; MW: measure word; NUM: numeral; OBJ: object of transitive verb; 3OBJ: third person object pronoun; { $n$ } (with a number  $n$  written in Arabic numerals): the mono-morphemic expression of the number  $n$  in a given language;  $A(B)$  and  $A(BC)$ : the character  $A$  is a rendition of the original character encountered in the Chinese corpus, the character  $B$  or the sequence  $BC$  in parenthesis is a modern form of what is understood for  $A$ . For example: 有(又), 廿(二十), 卅(三十), 卌(四十), 𠫪(七十), 泰(大).

## 1. FRACTIONS IN THE *SUÀN SHÙ SHŪ*

The *Suàn Shù Shū*<sup>1</sup> is a mathematical text written on 190 bamboo strips, consisting of approximately 7,000 characters. The manuscript was excavated during the winter of 1983–84 from a Han Dynasty tomb in Zhangjiashan where a calendar for the year 186 BCE was found, and so the tomb is thought to have been closed that very year: the book was probably written in the beginning of the 2<sup>nd</sup> century BCE. Peng Hao (2001: 4–6) states that the production and taxation standards mentioned in some passages prove that they were copied from originals written in the kingdom of Qin before the unification of China in 221 BCE, while other sections could only have been composed during the reign of the Western Han Dynasty which began in 206 BCE.

The text contains 301 occurrences (not all different) of regular expressions for fractions. This is much more than in Qin-Han manuscripts found in the same tomb<sup>2</sup>, or elsewhere but not specialized in mathematics. More Qin-Han texts of mathematics were discovered after the *Suàn Shù Shū*; they<sup>3</sup> shows no discrepancy concerning the expression of integers and fractions which are all *proper fractions*, i.e. smaller than 1, the denominator being larger than the numerator<sup>4</sup>. They are all written in the Chinese language; I use Arabic numerals in translations because they are more readable than numbers written in English, but there is no symbolic numerical notation in the original.

Expressions for fractions can be special lexicalized items, but clearly such a scheme is viable only for a few specific fractions; in Qin-Han texts this was limited to  $1/2$ ,  $1/3$  and  $2/3$ . On the other hand a *generic* linguistic pattern capable of communicating the fraction of any two integers has to account for both the numerator and the denominator, thus producing numerical expressions which I call *bidimensional*. Unit fractions  $1/n$  are a special case in the corpus since approximately 64% (exactly 83 out of 129) of them only state their denominator  $n$  and end up *monodimensional*.

English *two-fifths* can be inserted before nouns or measure words as the head of a noun phrase using the preposition *of*; for example *two-fifths of the*

<sup>1</sup> Authorised editions of the *Suàn Shù Shū* were successively published in [Wenwu 2000] and [Wenwu 2001]. An edition with commentary was published by Peng Hao (2001). Japanese translations were produced by Jochi S. (2001) and [Ōkawa et al. 2006]; the latter includes a translation into Contemporary Chinese by Ma Biao. Another Contemporary Chinese rendition was done by Hu Yitao (2006) who worked under the supervision of Zhang Xiancheng (Southwest University, Chongqing). Two independent English translations were successively produced by Cullen (2004) and Dauben (2008).

<sup>2</sup> Yang Lingrong (2008: 14–20) counts 328 fractions in the *Suàn Shù Shū* (including atypical expressions of fractions and expressions of proportions), but 58 fractions only in all the other corpora of Zhangjiashan.

<sup>3</sup> I checked *Shù* 數 (it belonged to the Yuèlù Academy, the text is now available in Xiao Can 2010; Xiao Can briefed me by email about integers and fractions in *Shù* before the formal publication), and *Suàn Shù* 算術 (which was excavated in Shuihudi in 2006; at the time of my research, only short excerpts were available in Xiong Beisheng et al. (2008), a joint publication by the Hubei Provincial Institute of Cultural Relics and Archaeology (Húběi Shěng Wénwù Kǎogǔ Yánjiūsuǒ) and the Yunmen Museum (Yúnmèng Xiàn Bówùguǎn) published in 2008, and in Chemla & Ma Biao (2011)).

<sup>4</sup> There is a wide range of values in the *Suàn Shù Shū* that depend on the calculations they are involved in. For example,  $1/50$  is on strip 4,  $12/18$  is on strip 55,  $47/98$  is on strip 92, and  $162/2016$  is on strip 20.

population or two-fifths of a liter. Contemporary Chinese expresses  $2/5$  as *wǔ fēn zhī èr*, i.e. {5} *fēn zhī* {2}, with the denominator's name given first, the compound *fēn zhī* coming next and the numerator's name in last position. The compound {5} *fēn zhī* {2} can in turn be inserted directly before a measure word or a noun to form the sequences “Fraction Name + MW” or “Fraction Name + Noun” respectively; placing the fraction name in the head position of a noun phrase “Noun (+ *de* 的) + Fraction Name” is also possible. Nothing can be introduced between the components of *two-fifths* or {5} *fēn zhī* {2}, therefore such compounds are *inseparable semantic units* or *inseparable fraction names*. But in Qin-Han manuscripts, the only inseparable fraction names were on one hand special lexicalized expressions of  $1/3$ ,  $1/2$  and  $2/3$ , and on the other hand the monodimensional expressions of unit fractions built according to the pattern “Denominator + *fēn*”; measure words were inserted after these expressions. Bidimensional expressions of fractions were built as predicative phrases with the name “Denominator's Name + *fēn*” of a unit fraction acting as subject and with the numerator's name acting as predicate. When a measure word was involved it was inserted right after “Denominator + *fēn*”. The morpheme *zhī* could be optionally added before the numerator's name as a marker of the predicative relation.

The present paper provides an exhaustive survey of the diversity of all the expressions for fractions in the *Suàn Shù Shū*<sup>5</sup>.

## 2. MEASURE WORDS, INTEGERS AND MIXED NUMBERS

Measure words can be found after the “Denominator + *fēn*” compounds. Measure words in the *Suàn Shù Shū* are mostly units of measurement<sup>6</sup>; they fit into the construction “NUM + MW” where the numeral can be the name of an integer, as well as lexicalized fraction names and monodimensional unit fraction expressions.

<sup>5</sup> Guo Shuchun (2002) and Yang Lingrong (2008) already presented the various patterns which can be encountered, but they failed to quantify their distribution and to relate them to their insertion contexts.

<sup>6</sup> Length units are *cùn* 寸, *chǐ* 尺, *bù* 步, *zhàng* 丈 and *lǐ* 里:  $1\text{ lǐ} = 180\text{ zhàng}$ ,  $1\text{ zhàng} = 10\text{ chǐ}$ ,  $1\text{ bù} = 6\text{ chǐ}$ ,  $1\text{ chǐ} = 10\text{ cùn}$ . There is also a specific unit *wéi* 韋(圍) used only for circumferences according to Peng Hao (2001):  $1\text{ wéi} = 3\text{ chǐ}$  (i.e.  $\approx \pi\text{ chǐ}$ , the circumference of a circle with diameter  $1\text{ chǐ}$ ). Surface area units can be *mǔ* 畝 and *qǐng* 頃 ( $1\text{ qǐng} = 100\text{ mǔ}$ ), or are derived from length units and contextually understood as referring to surfaces even though there is no special indication equivalent to English “square”. Capacity units are *shēng* 升, *dǒu* 斗 and *shí* 石 ( $1\text{ shí} = 10\text{ dǒu}$ ,  $1\text{ dǒu} = 10\text{ shēng}$ ). The only volume unit in the *Suàn Shù Shū* is *chǐ* 尺; it is derived from the length unit *chǐ* 尺 and occurs without any special indication equivalent to “cubic”. Weight units are *zhū* 朱(銖), *liǎng* 兩, *jīn* 斤, *jūn* 鈞 and *shí* 石:  $1\text{ shí} = 4\text{ jūn}$ ,  $1\text{ jūn} = 30\text{ jīn}$ ,  $1\text{ jīn} = 16\text{ liǎng}$ ,  $1\text{ liǎng} = 24\text{ zhū}$ . Note that *shí* 石 can designate both a capacity unit and a weight unit, and that the *Xiàndài Hànyǔ Guīfàn Cídiǎn* 现代汉语规范词典 [Dictionary of contemporary Chinese] (Beijing, 2010: 262) gives the pronunciation *dàn* for this character used as a measurement unit, but recommends *shí* when reading ancient texts.

Mass nouns *sù* 粟 [unhusked millet]<sup>7</sup>, *mǐ* 米 [husked millet], *bài* 粳 [milled millet]<sup>8</sup>, *shuǐ* 水 [water], *qī* 漆(漆) [lacquer], *jīn* 金 [gold], *guǎng* 廣 [width], *zòng* 縱 [length], etc. occur in “Noun + NUM + MW” sequences, whereas *rén* 人 [person] and nouns for countable items like *lútáng* 蘆唐 [bamboo tube], *jiǎn* 簡 [bamboo strip], *suàn* 筭(算) [string of coins]<sup>9</sup>, etc. all fit into the pattern “NUM + Noun” in the same manner as measure words<sup>10</sup>. The word *qián* 錢 is used as a currency unit; it occurs either in “NUM + MW” or in “Noun + NUM”. The numeral 1 is not always stated before a measure word; this is marked with  $\emptyset$  right before the measure word *chǐ* in (9), (42), and (43), and *wéi* in (69).

Fraction expressions are built from the names of the numerator and denominator, which are integers. Chinese names for integers belong to a decimal numeration. The digits are *yī* 一 {1}<sup>11</sup>, *èr* 二 {2}<sup>12</sup>, *sān* 三 {3}, *sì* 四 {4}, *wǔ* 五 {5}, *liù* 六 {6}, *qī* 七 {7}, *bā* 八 {8}, *jiǔ* 九 {9}. In the *Suàn Shù Shū*, the series of pivots is limited to *shí* 十 {10}, *bǎi* 百 {10<sup>2</sup>}, *qiān* 千 {10<sup>3</sup>} and *wàn* 萬 {10<sup>4</sup>}; the largest number being 10,000,000 expressed as *qiān wàn*, i.e. {10<sup>3</sup>}{10<sup>4</sup>}, on strip 11.

I use the notation {*number*} with a number written in Arabic numerals between braces to represent the numerical morpheme which expresses the bracketed number in a particular language. For instance the same notation {10} can represent the word *ten* in English and the morpheme *shí* in Chinese. The notation {10<sup>4</sup>} represents *wàn* in Chinese, but would not occur for English *ten thousand*, which is represented as {10}{10<sup>3</sup>} and stands for the succession of the mono-morphemic items *ten* {10} and *thousand* {10<sup>3</sup>}.

In the *Suàn Shù Shū*, the morpheme {1} is used before all pivots in a number name but the highest one. This is visualized with a shaded *yī* 一 {1} in examples (1), (2), (4), (6), and a shaded  $\emptyset$  in (3), (5), (6), (7). The sequences {*digit*}{*pivot*} and {*smaller pivot*}{*larger pivot*} express products, they are concatenated directly. These concatenations express sums.

The conjunction *yòu* was used in the Western Zhou inscriptions on bronze vessels to join the tens and units places, and sometimes also the hundreds and tens, but in the *Suàn Shù Shū* it occurs only in expressions of mixed numbers to

<sup>7</sup> I refer to Dauben (2008: 169-170) for the English names of crops and their byproducts in the *Suàn Shù Shū*.

<sup>8</sup> In the *Suàn Shù Shū* as in the *Nine Chapters* [*Jiǔ Zhāng Suàn Shù* 九章算術], *bài* 粳 refers to milled millet, not to a particular variety of millet.

<sup>9</sup> The word *suàn* 筭(算) refers either to a string of coins (Cullen 2004: 29) or to a Han dynasty unit of taxation (Chemla & Guo Shuchun 2004: 989).

<sup>10</sup> In Chinese the distinction between mass and countable nouns is semantic, not grammatical.

<sup>11</sup> According to the rules of *pīnyīn* transcription, the digit {1} is always romanized as *yī* with a first tone mark regardless of the actual tone in Contemporary Chinese. This depends on the tone of the following syllable; for example {1}{10<sup>4</sup>} is actually pronounced *yí wàn*, but this is only noted in publications concerned with pronunciation.

<sup>12</sup> The Contemporary Chinese variant *liǎng* 兩 was hardly used in exact number names before the 20th century CE and the digit *èr* 二 is the only numeral for 2 in the *Suàn Shù Shū*.

link an integer and a fraction as in (7) and it is free pattern<sup>13</sup>. The term *líng* 零 is not encountered anywhere in the corpus and was not used in integer names before the 12<sup>th</sup> century CE<sup>14</sup>.

Additionally, to write tens, the copyists of the *Suàn Shù Shū* could use ligatures instead of the corresponding two-character forms. The numbers 50, 60, 80 and 90 were either written with two separate characters, or as ligatures of the characters for 5, 6, 8 or 9 written in a reduced size above the character for 10; I always transcribe these with the two-character forms *wǔ shí* 五十, *liù shí* 六十, *bā shí* 八十 and *jiǔ shí* 九十 respectively. But 20, 30, 40 and 70 were always written 廿, 卅, 卌 and 𠄎<sup>15</sup> respectively. Conforming to the current scholarly usage, I transcribe them with their actual written forms followed by the two-character forms in parentheses: *èr shí* 廿(二十), *sān shí* 卅(三十), *sì shí* 卌(四十) and *qī shí* 𠄎(七十) respectively; the *pīnyīn* romanization and glosses are those of the disyllabic compounds<sup>16</sup>; see for example the expressions with 20 in (4) and (6).

(1)	二百	一十
in <i>Suàn Shù Shū</i>	<i>èr bǎi</i>	<i>yī shí</i>
strip 172	{2}{10 <sup>2</sup> }	{1}{10}
	'210'	

(2)	二千	一十	六
in <i>Suàn Shù Shū</i>	<i>èr qiān</i>	<i>yī shí</i>	<i>liù</i>
strip 20	{2}{10 <sup>3</sup> }	{1}{10}	{6}
	'2016'		

(3)	錢	百	五十
in <i>Suàn Shù Shū</i>	<i>qián</i>	<i>bǎi</i>	<i>wǔ shí</i>
strip 76	<i>qián</i>	{10 <sup>2</sup> }	{5}{10}
	'150 <i>qián</i> '		

(4)	七千	一百	廿(二十)	九
in <i>Suàn Shù Shū</i>	<i>qī qiān</i>	<i>yī bǎi</i>	<i>èr shí</i>	<i>jiǔ</i>
strip 176	{7}{10 <sup>3</sup> }	{1}{10 <sup>2</sup> }	{2}{10}	{9}
	'7129'			

<sup>13</sup> When a measure word separates the integer and the fraction, the item *yòu* can be present as in (7) or absent as in (18). When there is no measure word, the item *yòu* is sometimes not used as in (27) or used as in (119).

<sup>14</sup> Readers can find more details on Chinese integer names in Anicotte (2015 a).

<sup>15</sup> The ligature 𠄎 for 70 is also found in the Qin-Han manuscripts *Shù* and *Suàn Shù* mentioned above, but not in dictionaries of Middle Chinese.

<sup>16</sup> Modern dictionaries give the pronunciations *niàn* for 廿, *sà* for 卅, and *xì* for 卌, but other ligatures for tens are never mentioned; this justifies the present usage of glossing *all* of these ligatures as disyllabic compounds. I presume nothing about the actual pronunciation of tens during the Qin-Han period; this matter is beyond the scope of the present paper.

- (5)                    千      八十      九  
 in *Suàn Shù Shū*     $\emptyset$ qiān    bā shí    jiǔ  
 strip 172             $\emptyset\{10^3\}$     {8}{10}    {9}  
                          '1089'
- (6)                    萬      一千      五百      廿(二十) 銖  
 in *Suàn Shù Shū*     $\emptyset$ wàn    yī qiān    wǔ bǎi    èr shí    zhū  
 strip 47             $\emptyset\{10^4\}$     {1}{10^3}    {5}{10^2}    {2}{10}    zhū  
                          '11520 zhū'
- (7)                    十六      尺 有(又) 十八分      尺  
 in *Suàn Shù Shū*     $\emptyset$ shí liù    chǐ yòu    shí bā fēn    chǐ  
 strip 55             $\emptyset\{10\}\{6\}$     chǐ and    {10}{8} fēn    chǐ  
                          '16 chǐ and 12/18 chǐ',<sup>17</sup>
- 十二  
                           $\emptyset$ shí èr  
                           $\emptyset\{10\}\{2\}$

The mixed number 16 12/18 in (7) is inserted with the measure word *chǐ*. The integer and the fraction are dealt with as two independent quantification phrases which are concatenated with *yòu* here or juxtaposed in other instances. The measure word *chǐ* occurs first with the integer and is inserted again within the expression for the fraction 12/18. This is a regular pattern in the *Suàn Shù Shū*<sup>18</sup>, even the nouns *jiǎn* 簡 [bamboo strip] and *suàn* 算(算) [string of coins], in (55) and (57) respectively, appear first after the name of the integer and then again with the proper fraction after *fēn*. There are only a few exceptions: 3 can be seen in (28)–(30) with a measure word stated only after the integer, and there is also (76) with the noun *lútáng* 盧唐 [bamboo tube] placed in the measure word position in the integer expression but not repeated after *fēn*. On the other hand, (31) is a unique case of over-repetition, with the measure word *bù* also found after the denominator's name and therefore appearing three times.

### 3. GENERIC UNIT FRACTION EXPRESSIONS

#### 3–1. Monodimensional unit fraction expressions

In the *Suàn Shù Shū* as in other Qin-Han texts of mathematics, the canonical names for unit fractions  $1/n$  were  $n$  fēn. They stated only the denominator  $n$  but not the numerator 1 and were therefore monodimensional numerical expressions. Nothing could be inserted between the two constituents  $n$  and *fēn*; that is, the

<sup>17</sup> English would simply write 16 12/18 *chǐ* and say “sixteen and twelve-eighteenths of a *chǐ*”. My translations are meant to highlight the repetition of the measure words and the presence of linking terms in the original.

<sup>18</sup> This repetition shows that the integer and the fraction were dealt with independently; this has nothing to do with “echo” constructions which occur in *one* quantification phrase “Noun + Num + Noun” (e.g. *qiāng bǎi qiāng* 羌百羌 [Qiang hundred Qiang] i.e. ‘one hundred Qiang people’ on bone inscription H32042).

$n fēn$  compounds were inseparable semantic units. Measure words followed  $n fēn$  just as they followed integers. In other words, these  $n fēn$  unit fraction designations were numerals in their own right.

The item  $fēn$  in  $n fēn$  can be considered syntactically neutralized and fossilized through its involvement in a word-formation process. It was used on occasion in the corpus as a noun for *part* or *fraction*, in which cases it could be read  $fèn$  with a falling tone regardless of the modern reading  $fēn$ . It was also used as a verb meaning *to share* (which should be read as  $fēn$ ); but considering  $fēn$  in the compounds  $n fēn$  synchronically, it seems pointless to try and interpret it as a noun or as a verb. From a semantic point of view, the question would be whether  $fēn$  referred more to the action of partitioning or more to the result (the parts) of this action. This might be interesting in an attempt to reconstruct the *emergence* of the expressions  $n fēn$ , but again this seems irrelevant synchronically.

In the *Suàn Shù Shū*, there are 83 instances (not all different) of monodimensional expressions of  $1/n$  unit fractions. Among them, 76 do not involve a measure word, while 8 instances do. The 76 instances without measure words are:

- $sān fēn$  for  $1/3$  (16 instances, on strips 3, 5, 6, 24, 27, 139, 168, 169, 170, 171, 172, 174, 176 and 179);
- $sì fēn$  for  $1/4$  (15 instances, on strips 3, 5, 6, 9, 14, 27, 169, 171, 172, 174, 176 and 179);
- $wǔ fēn$  for  $1/5$  (14 instances, on strips 5, 6, 8, 9, 14, 170, 171, 172, 174, 176 and 179);
- $liù fēn$  for  $1/6$  (10 instances, on strips 9, 10, 171, 172, 174, 176 and 179);
- $qī fēn$  for  $1/7$  (9 instances, on strips 5, 9, 10, 172, 174, 176 and 179);
- $bā fēn$  for  $1/8$  (5 instances, on strips 10, 174, 176 and 179);
- $jiǔ fēn$  for  $1/9$  (2 instances, on strips 176 and 177);
- $shí fēn$  for  $1/10$  (2 instances, on strips 179 and 180);
- $bǎi fēn$  for  $1/100$  (2 instances, on strips 14 and 16).

The 8 instances of expressions for unit fractions followed by a measure word are:

- $sān fēn cùn$  for  $1/3 cùn$  on strip 2;
- $bā fēn cùn$  for  $1/8 cùn$  on strip 2;
- $sì fēn cùn$  for  $1/4 cùn$  on strip 4;
- $wǔ fēn cùn$  for  $1/5 cùn$  on strip 4;
- $liù fēn cùn$  for  $1/6 cùn$  on strip 4;
- $liù shí fēn chǐ$  for  $1/60 chǐ$  on strip 4;
- $jiǔ fēn zhū$  for  $1/9 zhū$  on strip 29.

### 3–2. Bidimensional expressions of unit fractions

The numerator's name {1} is not compulsory and is usually omitted when the fraction comes as a factor in a multiplication: check  $\emptyset$  right after  $fēn$  in (8) with no measure word, and after the measure word  $cùn$  in (9).



(8) 四分 $\emptyset$  乘 四分 $\emptyset$  十六分 一  
 in *Suàn Shù Shū* *sì fēn*  $\emptyset$  *chéng* *sì fēn*  $\emptyset$  *shí liù fēn* *yī*  
 strip 9 {4} *fēn*  $\emptyset$  multiply {4} *fēn*  $\emptyset$  {10}{6} *fēn* {1}  
 ‘1/4 times 1/4 are 1/16’

(9) 五分 寸 $\emptyset$  乘  $\emptyset$ 尺  
 in *Suàn Shù Shū* *wǔ fēn* *cùn*  $\emptyset$  *chéng*  $\emptyset$  *chǐ*  
 strip 4 {5} *fēn* *cùn*  $\emptyset$  multiply  $\emptyset$  *chǐ*  
 ‘1/5 *cùn* times [one] *chǐ* are

五十分 尺 一 也  
*wǔ shí fēn* *chǐ* *yī* *yě*  
 {5}{10} *fēn* *chǐ* {1} DECL  
 1/50 [square] *chǐ*<sup>19</sup>

But {1} is not omitted (yielding bidimensional numerical expressions) when stating the result of multiplications involving *n fēn* unit fraction names or lexicalized names for 1/2 and 1/3. Examples can be seen in (8) and (9) above for the former situation, and in (117) and (122) from Sect. 5 for the latter one. We can analyze the combination of *n fēn* and {1} as a predicative clause with the monodimensional name of the unit fraction acting as the subject and the number name {1} acting as the predicate. There are 46 instances (not all different) of these bidimensional expressions of unit fractions in the *Suàn Shù Shū*. They are distributed among the following patterns (the category  $c_1$  includes no instances, but is known to exist in at least one other Qin-Han text):

(a<sub>1</sub>): “Denominator + *fēn* + {1}”: 24 instances presented in Sect 3–2–1

(b<sub>1</sub>): “Denominator + *fēn* + MW + {1}”: 11 instances in Sect 3–2–2.

(c<sub>1</sub>): “Denominator + *fēn* + *zhī* + {1}”: no instances (Sect 3–2–3).

(d<sub>1</sub>): “Denominator + *fēn* + MW + *zhī* + {1}”: 11 instances in Sect 3–2–4.

[Wenwu 2001] – and then Peng Hao (2001), [Ōkawa et al. 2006], Hu Yitao (2006) – considered that a MW was omitted between *fēn* and the numerator in some instances of the category (a<sub>1</sub>) and added the MW which was implied by the context. Yang Lingrong (2008: 17–19) already argued that such additions were unnecessary; I can only emphasize that they must be rejected in the study of the actual expressions of fractions in the corpus.

### 3–2–1. “Denominator + *fēn* + {1}”

The 24 instances are:

→ *sān fēn yī* for 1/3 (1 instance, on strip 3);

<sup>19</sup>This is the calculation of a surface. It also gives the conversion 1/5 *cùn* = 1/50 *chǐ* for length units (given that 1 *cùn* = 1/10 *chǐ*); the conversion is expressed as a product.

- *sì fēn yī* for 1/4 (3 instances, on strips 3, 4 and 8);  
 → *wú fēn yī* for 1/5 (1 instance, on strip 5);  
 → *liù fēn yī* for 1/6 (2 instances, on strips 3 and 8);  
 → *bā fēn yī* for 1/8 (1 instance, on strip 5);  
 → *jiǔ fēn yī* for 1/9 (2 instances, on strips 3 and 8);  
 → *shí fēn yī* for 1/10 (1 instance, on strip 5);  
 → *shí èr fēn yī* for 1/12 (1 instance, on strip 5);  
 → *shí wǔ fēn yī* for 1/15 (1 instance, on strip 6);  
 → *shí liù fēn yī* for 1/16 (2 instances, on strip 5 and 9);  
 → *èr shí fēn yī* for 1/20 (2 instances, on strip 6 and 9);  
 → *èr shí wǔ fēn yī* for 1/25 (2 instances, on strips 6 and 8–9);  
 → *sān shí fēn yī* for 1/30 (1 instance, on strip 9);  
 → *sān shí liù fēn yī* for 1/36 (1 instance, on strip 9);  
 → *sì shí èr fēn yī* for 1/42 (1 instance, on strip 10);  
 → *sì shí jiǔ fēn yī* for 1/49 (1 instance, on strip 9);  
 → *wǔ shí fēn yī* for 1/50 (1 instance, on strip 10).

### 3–2–2. “Denominator + *fēn* + MW + {1}”

The 11 instances are given in (10)–(20):

- (10)                    十分      尺    一  
 in *Suàn Shù Shū*    *shí fēn      chǐ    yī*  
 strip 1                {10} *fēn      chǐ    {1}*  
                              ‘1/10 *chǐ*’
- (11)                    廿(二十)分    尺    一  
 in *Suàn Shù Shū*    *èr shí fēn      chǐ    yī*  
 strip 1                {2}{10} *fēn      chǐ    {1}*  
                              ‘1/20 *chǐ*’
- (12)                    卅(三十)分    尺    一  
 in *Suàn Shù Shū*    *sān shí fēn      chǐ    yī*  
 strip 2                {3}{10} *fēn      chǐ    {1}*  
                              ‘1/30 *chǐ*’
- (13)                    八十分        尺    一  
 in *Suàn Shù Shū*    *bā shí fēn      chǐ    yī*  
 strip 2                {8}{10} *fēn      chǐ    {1}*  
                              ‘1/80 *chǐ*’
- (14)                    八分        尺    一  
 in *Suàn Shù Shū*    *bā fēn      chǐ    yī*  
 strip 4                {8} *fēn      chǐ    {1}*  
                              ‘1/8 *chǐ*’

10

(15) 冊(四十)分 尺 一  
 in *Suàn Shù Shū* *sì shí fēn chǐ yī*  
 strip 4 {4}{10}fēn chǐ {1}  
 ‘1/40 chǐ’

(16) 五十分 尺 一  
 in *Suàn Shù Shū* *wǔ shí fēn chǐ yī*  
 strip 4 {5}{10}fēn chǐ {1}  
 ‘1/50 chǐ’

(17) 四分 步 一  
 in *Suàn Shù Shū* *sì fēn bù yī*  
 strips 86–87 {4}fēn bù {1}  
 ‘1/4 bù’

The following instances are inserted in mixed numbers.

(18) 一 錢 五分 錢 一  
 in *Suàn Shù Shū* *yī qián wǔ fēn qián yī*  
 strip 33 {1} qián {5}fēn qián {1}  
 ‘1 qián 1/5 qián’

(19) 十 斤 十二 兩 十九 朱(銖)  
 in *Suàn Shù Shū* *shí jīn shí èr liǎng shí jiǔ zhū*  
 strip 79 {10} jīn {10}{2} liǎng {10}{9} zhū  
 ‘10 jīn 12 liǎng 19 zhū’

五分 朱(銖) 一  
*wǔ fēn zhū yī*  
 {5}fēn zhū {1}  
 1/5 zhū’

(20) 七 斗 三分 升 一  
 in *Suàn Shù Shū* *qī dòu sān fēn shēng yī*  
 strip 119 {7} dòu {3}fēn shēng {1}  
 ‘7 dòu 1/3 shēng’

### 3–2–3. “Denominator + fēn + zhī + {1}”

In the whole *Suàn Shù Shū*, there are no instances of the sequence *fēn zhī* uninterrupted by a measure word before {1}. This absence does not prove the pattern to be impossible; actually there is an instance of it with the expression *sān shí fēn zhī yī* 卅(三十)分之一 {3}{10} fēn zhī {1} on strip 0778 of *Shù* (Xiao Can 2010: 51).

### 3–2–4. “Denominator + *fēn* + MW + *zhī* + {1}”

The 11 instances of this pattern are given in (21)–(26). All are quantification phrases starting with a noun.

(21) 粳 五分 升 之 一  
 in *Suàn Shù Shū* *bài* *wú fēn shēng zhī yī*  
 strip 100 milled millet {5} *fēn shēng zhī* {1}  
 ‘1/5 *shēng* of milled millet’

(22) 粳米 四分 升 之 一  
 in *Suàn Shù Shū* *bài mǐ sì fēn shēng zhī yī*  
 strips 101–102 milled millet {4} *fēn shēng zhī* {1}  
 4 instances ‘1/4 *shēng* of milled millet’

(23) 毀(穀)米 四分 升 之 一  
 in *Suàn Shù Shū* *huǐ mǐ sì fēn shēng zhī yī*  
 strips 102, 104 polished millet {4} *fēn shēng zhī* {1}  
 2 instances ‘1/4 *shēng* of polished millet’

(24) 毀(穀) 四分 升 之 一  
 in *Suàn Shù Shū* *huǐ sì fēn shēng zhī yī*  
 strip 103 polished millet {4} *fēn shēng zhī* {1}  
 2 instances ‘1/4 *shēng* of polished millet’

Beware that the Chinese name for ‘polished millet’ is *huǐ mǐ* in (23) and *huǐ* in (24).

(25) 米 六 升 四分 升 之 一  
 in *Suàn Shù Shū* *mǐ liù shēng sì fēn shēng zhī yī*  
 strip 121 husked millet {6} *shēng* {4} *fēn shēng zhī* {1}  
 ‘6 *shēng* 1/4 *shēng* of husked millet’

(26) 從(縱) 一 步 六分 步 之 一  
 in *Suàn Shù Shū* *zòng yī bù liù fēn bù zhī yī*  
 strip 121 length {1} *bù* {6} *fēn bù zhī* {1}  
 ‘a length of 1 *bù* 1/6 *bù*’

## 4. GENERIC EXPRESSIONS OF NON-UNIT FRACTIONS

There are 97 instances (not all different) of expressions of non-unit fractions stating both a numerator and a denominator in the *Suàn Shù Shū*. These bidimensional expressions of fractions are distributed among the following patterns:

(a<sub>2</sub>): “Denominator + *fēn* + Numerator”: 11 instances, in Sect. 4–1.

(b<sub>2</sub>): “Denominator + *fēn* + MW + Numerator”: 43 instances, in Sect. 4–2.

(c<sub>2</sub>): “Denominator + *fēn* + *zhī* + Numerator”: 7 instances, in Sect. 4–3.

(d<sub>2</sub>): “Denominator + *fēn* + MW + *zhī* + Numerator”: 36 instance, in Sect. 4–4.

As above with category (a<sub>1</sub>), I reject the additions of a MW by [Wenwu 2001] in some instances of category (a<sub>2</sub>).

#### 4–1. “Denominator + *fēn* + Numerator” for non-unit fractions

The 11 instances are: *jiǔ fēn èr* for 2/9 on strip 8, *jiǔ fēn qī* for 7/9 on strip 30, on strips 22–23 there are *wǔ fēn èr* for 2/5, *liù fēn sān* for 3/6, *sān fēn èr* for 2/3, *shí fēn bā* for 8/10, *shí èr fēn qī* for 7/12, and finally 4 instances, all of them inserted in mixed numbers, given in (27)–(30); among them the 2 instances in (28) and (29) come in predicative position after a mass noun.

(27)                    十二        𠄎(七十)二分    十一  
 in *Suàn Shù Shū*    *shí èr*        *qī shí èr fēn*        *shí yī*  
 strip 36                {10}{2}    {7}{10}{2} *fēn*    {10}{1}  
                               ‘12 11/72’

(28)                    糲                七    斗    五分    三  
 in *Suàn Shù Shū*    *bài*                *qī*    *dǒu*    *wǔ fēn*    *sān*  
 strip 135                milled millet {7}    *dǒu*    {5} *fēn*    {3}  
                               ‘7 3/5 *dǒu* of milled millet’

(29)                    糲                二    斗    五分    二  
 in *Suàn Shù Shū*    *lì*                *èr*    *dǒu*    *wǔ fēn*    *èr*  
 strip 136                husked millet {2}    *dǒu*    {5} *fēn*    {2}  
                               ‘2 2/5 *dǒu* of husked millet’

(30)                    四    韋(圍)    二    寸    廿(二十)五分    十四  
 in *Suàn Shù Shū*    *sì*    *wéi*        *èr*    *cùn*    *èr shí wǔ fēn*    *shí sì*  
 strip 154                {4}    *wéi*        {2}    *cùn*    {2}{10}{5} *fēn*    {10}{4}  
                               ‘4 *wéi* 2 14/25 *cùn*’

#### 4–2. “Denominator + *fēn* + MW + Numerator” for non-unit fractions

The 43 instances are given in (31)–(72) along with the preceding integer when there is one. Of these 2 are identical and 11 follow a noun.

(31)                    十一        步    有(又)  
 in *Suàn Shù Shū*    *shí yī*        *bù*    *yòu*  
 strip 84                {10}{1}    *bù*    and  
                               ‘11 *bù* and

- 九十七分 步 卅(七十)九 步<sup>20</sup>  
*jiǔ shí qī fēn bù qī shí jiǔ bù*  
 {9}{10}{7} fēn bù {7}{10}{9} bù  
 79/97 bù'
- (32) 二 錢 六十分 錢 五十七  
 in *Suàn Shù Shū* èr qián liù shí fēn qián wǔ shí qī  
 strip 23 {2} qián {6}{10} fēn qián {5}{10}{7}  
 '2 qián 57/60 qián'
- (33) 一 錢 卅(三十)分 錢 十七  
 in *Suàn Shù Shū* yī qián sān shí fēn qián shí qī  
 strips 23–24 {1} qián {3}{10} fēn qián {10}{7}  
 '1 qián 17/30 qián'
- (34) 金 三 朱(銖) 九分 朱(銖) 五  
 in *Suàn Shù Shū* jīn sān zhū jiǔ fēn zhū wǔ  
 strip 28 gold {3} zhū {9} fēn zhū {5}  
 '3 zhū 5/9 zhū of gold'
- (35) 七分 朱(銖) 六  
 in *Suàn Shù Shū* qī fēn zhū liù  
 strip 28 {7} fēn zhū {6}  
 '6/7 zhū'
- (36) 金 二 朱(銖)  
 in *Suàn Shù Shū* jīn èr zhū  
 strip 28 gold {2} zhū  
 '2 zhū'
- 六十三分 朱(銖) 卅(四十)四  
*liùshí sān fēn zhū sì shí sì*  
 {6}{10}{3} fēn zhū {4}{10}{4}  
 44/63 zhū of gold'
- (37) 六十三分 朱(銖) 廿(二十)二  
 in *Suàn Shù Shū* liù shí sān fēn zhū èr shí èr  
 strip 30 {6}{10}{3} fēn zhū {2}{10}{2}  
 '22/63 zhū'

<sup>20</sup> The third occurrence of *bù* is superfluous and likely a copyist's mistake; this changes nothing about the classification of this fraction.

- (38) 五分 錢 四  
 in *Suàn Shù Shū* *wǔ fēn qiàn sì*  
 strip 33 {5} *fēn* {4} *qiàn*  
 ‘4/5 *qiàn*’
- (39) 一 寸 六十二分 寸 卅(三十)八  
 in *Suàn Shù Shū* *yī cùn liù shí èr fēn cùn sān shí bā*  
 strip 40 {1} *cùn* {6}{10}{2} *fēn* *cùn* {3}{10}{8}  
 ‘1 *cùn* 38/62 *cùn*’
- (40) 三 寸 六十二分 寸 十四  
 in *Suàn Shù Shū* *sān cùn liù shí èr fēn cùn shí sì*  
 strips 40–41 {3} *cùn* {6}{10}{2} *fēn* *cùn* {10}{4}  
 ‘3 *cùn* 14/62 *cùn*’
- (41) 六 寸 六十二分 寸 廿(二十)八  
 in *Suàn Shù Shū* *liù cùn liù shí èr fēn cùn èr shí bā*  
 strip 41 {6} *cùn* {6}{10}{2} *fēn* *cùn* {2}{10}{8}  
 ‘6 *cùn* 28/62 *cùn*’
- (42) ∅尺 二 寸 六十二分 寸 五十六  
 in *Suàn Shù Shū* *∅ chǐ èr cùn liù shí èr fēn cùn wǔ shí liù*  
 strip 41 *∅ chǐ* {2} *cùn* {6}{10}{2} *fēn* *cùn* {5}{10}{6}  
 ‘[one] *chǐ* 2 *cùn* 56/62 *cùn*’
- (43) ∅尺 五 寸 六十二分 寸 五十  
 in *Suàn Shù Shū* *∅ chǐ wǔ cùn liù shí èr fēn cùn wǔ shí*  
 strip 41 *∅ chǐ* {5} *cùn* {6}{10}{2} *fēn* *cùn* {5}{10}  
 ‘[one] *chǐ* 5 *cùn* 50/62 *cùn*’
- (44) 二 斗 三 升 十一 分 升 八  
 in *Suàn Shù Shū* *èr dòu sān shēng shí yī fēn shēng bā*  
 strip 48 {2} *dǒu* {3} *shēng* {10}{1} *fēn* *shēng* {8}  
 ‘2 *dǒu* 3 *shēng* 8/11 *shēng*’
- (45) 一 兩 十 朱(銖)  
 in *Suàn Shù Shū* *yī liǎng shí zhū*  
 strip 50 {1} *liǎng* {10} *zhū*  
 ‘1 *liǎng* 10 *zhū*’
- 百卅(四十)四分 朱(銖) 九十二  
*bǎi sì shí sì fēn zhū jiǔ shí èr*  
 {10<sup>2</sup>}{4}{10}{4} *fēn* *zhū* {9}{10}{2}  
 92/144 *zhū*’

- (46) 一 錢 百一十四分 錢 卅(七十)一  
 in *Suàn Shù Shū* yī qián bǎi yī shí sì fēn qián qī shí yī  
 strip 57 {1} qián {10<sup>2</sup>}{1}{10}{4} fēn qián {7}{10}{1}  
 ‘1 qián 71/114 qián’
- (47) 卅(四十)分 斗 五  
 in *Suàn Shù Shū* sì shí fēn dòu wǔ  
 strip 59 {4}{10} fēn dòu {5}  
 ‘5/40 dòu’
- (48) 四 錢 八分 錢 三  
 in *Suàn Shù Shū* sì qián bā fēn qián sān  
 strip 59 {4} qián {8} fēn qián {3}  
 ‘4 qián 3/8 qián’
- (49) 八 寸 十一分 寸 二  
 in *Suàn Shù Shū* bā cùn shí yī fēn cùn èr  
 strips 61–62 {8} cùn {10}{1} fēn cùn {2}  
 ‘8 cùn 2/11 cùn’
- (50) 十八 錢 十一分 錢 九  
 in *Suàn Shù Shū* shí bā qián shí yī fēn qián jiǔ  
 strip 62 {10}{8} qián {10}{1} fēn qián {9}  
 ‘18 qián 9/11 qián’
- (51) 廿(二十)五分 錢 廿(二十)四  
 in *Suàn Shù Shū* èr shí wǔ fēn qián èr shí sì  
 strip 64 {2}{10}{5} fēn qián {2}{10}{4}  
 ‘24/25 qián’
- (52) 漆(漆) 卅(三十)七分 升 卅(三十)  
 in *Suàn Shù Shū* qī sǎn shí qī fēn shēng sān shí  
 strip 66 lacquer {3}{10}{7} fēn shēng {3}{10}  
 ‘30/37 shēng of lacquer’
- (53) 水 二 升 卅(三十)七分 升 七  
 in *Suàn Shù Shū* shuǐ èr shēng sān shí qī fēn shēng qī  
 strips 66–67 water {2} shēng {3}{10}{7} fēn shēng {7}  
 ‘2 shēng 7/37 shēng of water’
- (54) 七 步 卅(三十)七分 步 廿(二十)三  
 in *Suàn Shù Shū* qī bù sān shí qī fēn bù èr shí sān  
 strip 68 {7} bù {3}{10}{7} fēn bù {2}{10}{3}  
 ‘7 bù 23/37 bù’



The nouns *jiǎn* 簡 [*bamboo strip*] and *suàn* 算(算) [*string of coins*] occur after *fēn* in (55)–(57), and also after the name of the integer in (55) and (57). They behave in the same manner as measure words.

- (55)                    二百五            簡    八分    簡    七  
in *Suàn Shù Shū*    èr bǎi w ŭ      jiǎn   bā fēn   jiǎn   qī  
strip 70                {2}{10<sup>2</sup>}{5} strip {8} *fēn* strip {7}  
‘205 strips and 7/8 of a strip’
- (56)                    八分    簡    一  
in *Suàn Shù Shū*    bā fēn   jiǎn   yī  
strip 71                {8} *fēn* strip {1}  
‘1/8 of 1 strip’
- (57)                    十七    算(算)  
in *Suàn Shù Shū*    shí qī   suàn  
strip 73                {10}{7} string of coins  
‘17 strings of coins’
- 二百六十九分      算(算)            十一  
                          èr bǎi liù shí jiǔ fēn   suàn                shí yī  
                          {6}{10}{9} *fēn*      string of coins    {10}{1}  
                          11/69 of a string of coins’
- (58)                    百三    錢    四百卅(三十)分      錢    九十  
in *Suàn Shù Shū*    bǎi sān   qián   sì bǎi sān shí fēn      qián   jiǔ shí  
strip 76                {10<sup>2</sup>}{3}   qián   {4}{10<sup>2</sup>}{3}{10} *fēn*      qián   {9}{10}  
‘103 *qián* 90/430 *qián*’
- (59)                    水    三    斗<sup>21</sup>    四分    升    三  
in *Suàn Shù Shū*    shuǐ   sān   dòu   sì fēn   shēng   sān  
strips 80–81        water {3}   dòu   {4} *fēn*   shēng   {3}  
‘3 *dòu* 3/4 *shēng* of water’
- (60)                    十一    步    有(又)  
in *Suàn Shù Shū*    shí yī   bù   yòu  
strip 84                {10}{1}   bù   and  
‘11 *bù*

<sup>21</sup> Peng Hao (2001: 76 note 10), Hu Yitao (2006: 41) and [Ōkawa et al. 2006: 93] consider, for the sake of computational coherence, that *dòu* 斗 as found in the text is actually a copyist’s error for *shēng* 升 assuming the text means ‘3 and 3/4 *shēng* of water’; this correction changes nothing for our classification.

- 九十七分 步 卅(七十)九  
*jiǔ shí qī fēn bù qī shí jiǔ*  
 {9}{10}{7} fēn bù {7}{10}{9}  
 79/97 bù'
- (61) 十一分 步 五  
 in *Suàn Shù Shū* *shí yī fēn bù wǔ*  
 strip 85 {10}{1} fēn bù {5}  
 '5/11 bù'
- (62) 十一 步 有(又)  
 in *Suàn Shù Shū* *shí yī bù yòu*  
 strip 92 {10}{1} bù and  
 '11 bù'
- 九十八分 步 卅(四十)七  
*jiǔ shí bā fēn bù sì shí qī*  
 {9}{10}{8} fēn bù {4}{10}{7}  
 47/98 bù'
- (63) 九 步 五分 步 三  
 in *Suàn Shù Shū* *jiǔ bù wǔ fēn bù sān*  
 strip 96 {9} bù {5} fēn bù {3}  
 '9 bù 3/5 bù'
- (64) 糲 卅(三十)二分 升 九  
 in *Suàn Shù Shū* *bài sān shí èr fēn shēng jiǔ*  
 strip 103 milled millet {3}{10}{2} fēn shēng {9}  
 '9/32 shēng of milled millet'
- (65) 米 七分 升 六  
 in *Suàn Shù Shū* *mǐ qī fēn shēng liù*  
 strip 113 husked millet {7} fēn shēng {6}  
 '6/7 shēng of husked millet'
- (66) 粟 七分 升 六  
 in *Suàn Shù Shū* *sù qī fēn shēng liù*  
 strip 115 unhusked millet {7} fēn shēng {6}  
 '6/7 shēng of unhusked millet'
- (67) 米 一 升 七分 升 三  
 in *Suàn Shù Shū* *mǐ yī shēng qī fēn shēng sān*  
 strip 115 husked millet {1} shēng {7} fēn shēng {3}  
 '1 shēng 3/7 shēng of husked millet'

- (68) 二千五十五 尺  
 in *Suàn Shù Shū* èr qiān wǔ shí wǔ chǐ  
 strip 149 {2}{10<sup>3</sup>}{5}{10}{5} chǐ  
 ‘2055 chǐ’
- 卅(三十)六分 尺 廿(二十)  
 sān shí liù fēn chǐ èr shí  
 {3}{10}{6} fēn chǐ {2}{10}  
 20/36 chǐ’
- (69) ∅ 韋(圍) 二 寸 廿(二十)五分 寸 十四  
 in *Suàn Shù Shū* ∅ wéi èr cùn èr shí wú fēn cùn shí sì  
 strip 153 ∅ wéi {2} cùn {2}{10}{5} fēn cùn {10}{4}  
 ‘[one] wéi 2 14/25 cùn’<sup>22</sup>
- (70) 七 寸 五分 寸 三  
 in *Suàn Shù Shū* qī cùn wǔ fēn cùn sān  
 strips 153, 154 {7} cùn {5} fēn cùn {3}  
 2 instances ‘7 cùn 3/5 cùn’
- (71) 從(縱) 九十七 步 有(又)  
 in *Suàn Shù Shū* zòng jiǔ shí qī bù yòu  
 strips 171–173<sup>23</sup> length {9}{10}{7} bù and  
 ‘a length of 97 bù and 141/147 bù’
- 百卅(四十)七分 步  $\boxed{?}^{24}$ 百卅(四十)一  
 bǎi sì shí qī fēn bù  $\boxed{?}$  bǎi sì shí  
 {10<sup>2</sup>}{4}{10}{7} fēn bù  $\boxed{?}$  {10<sup>2</sup>}{4}{10}{1}

<sup>22</sup> The term *wéi* is a unit of length used for circumferences; it also appeared in (30). The shaded ∅ signals the absence of the number name {1} before the measure word *wéi*, this also occurs in (9), (42) and (43) before *chǐ*.

<sup>23</sup> Strip 171 is followed by strip 173. [Wenwu 2001] and Peng Hao (2001: 117) initially had the sequence 171-172-173, but Peng Hao had changed this to 171-173-172 in the last release of the digital corpus he sent me in 2010. Hu Yitao (2006: 80, 82 note 14) and [Ōkawa et al. 2006: 1, 7 note 12] all have 171-173.

<sup>24</sup> I write a question mark  $\boxed{?}$  where the original shows an unclear written mark which looks like *wǔ* 五 {5}. But this would give a numerator equal to 541, which seems unlikely since it would yield an improper fraction. For the sake of coherence, the numerator should be 141. Peng Hao (2001: 121 note 39) says the digit *wǔ* 五 {5} should be corrected to *yī* 一 {1}, while Hu Yitao (2006: 82 note 14) and [Ōkawa et al. 2006: 93] say it is superfluous. This later formulation is in my view more acceptable since {1} was never used before the highest pivot of a number name in the *Suàn Shù Shū*. In any case this instance is undoubtedly an example of the pattern “Denominator’s name *fēn* + MW + Numerator’s name” whatever the value of the numerator.

- (72) 方 十五 步 卅(三十)一分 步 十五  
 in *Suàn Shù Shū* *fāng shí wǔ bù sān shí yī fēn bù shí wǔ*  
 strip 185 side {10}{5} *bù* {3}{10}{1} *fēn* *bù* {10}{5}  
 ‘a side of 15 *bù* 15/31 *bù*’ (the length of the side)

#### 4–3. “Denominator + *fēn* + *zhī* + Numerator” for non-unit fractions

The 7 instances are given in (73)–(79). Among them, 4 follow a noun and 1 follows a “Verb + OBJ” sequence. The numerical expressions in (73) and (75) are 2 of the only 4 instances of expressions of fractions with *zhī* which are not in predicative position after a noun or a phrase, the other 2 instances being those in (81) and (82) of the next section.

- (73) 二千一十六分 之 百六十二  
 in *Suàn Shù Shū* *èr qiān yī shí liù fēn zhī bǎi liù shí èr*  
 strip 20 {2}{10<sup>3</sup>}{1}{10}{6} *fēn* *zhī* {10<sup>2</sup>}{6}{10}{2}  
 ‘162/2016’

In (74), the expression for the fraction occurs after the pronoun object *zhī* of the verb *yuē*. The fraction is in the position of predicate with regard to the phrase *yuē zhī*; it expresses the result of a reduction.

- (74) 約 之 百一十二分 之 九  
 in *Suàn Shù Shū* *yuē zhī bǎi yī shí èr fēn zhī jiǔ*  
 strip 20 reduce 3OBJ {10<sup>2</sup>}{1}{10}{2} *fēn* *zhī* {9}  
 ‘reduce it [referring to 162/2016], [it is] 9/112’

In (75) the fraction is the object of a verb.

- (75) 各 受 卅(三十)分 之 廿(二十)三  
 in *Suàn Shù Shū* *gè shòu sān shí fēn zhī èr shí sān*  
 strip 26 each get {3}{10} *fēn* *zhī* {2}{10}{3}  
 ‘each gets 23/30’

- (76) 十三 盧唐 四分 之 三  
 in *Suàn Shù Shū* *shí sān lútáng sì fēn zhī sān*  
 strips 129–130 {10}{3} bamboo tube {4} *fēn* *zhī* {3}  
 ‘13 bamboo tubes 3/4’

- (77) 盾(膾) 九分 之 五  
 in *Suàn Shù Shū* *tú jiǔ fēn zhī wǔ*  
 strip 82 lard {9} *fēn* *zhī* {5}  
 ‘5/9 of lard’<sup>25</sup>

<sup>25</sup> The weigh unit *jīn* which can be deduced from the context is omitted after *fēn*. For our classification we only need to acknowledge this absence.

(78) 田 七<sup>26</sup>分 之 四  
 in *Suàn Shù Shū* *tián qī fēn zhī sì*  
 strip 162 field {7} *fēn* *zhī* {4}  
 ‘a field of 4/7’<sup>27</sup>

(79) 從(縱) 廿(二十)一分 之 十六  
 in *Suàn Shù Shū* *zòng èr shí yī fēn zhī shí liù*  
 strip 162 length {2}{10}{1} *fēn* *zhī* {10}{6}  
 ‘a length is 16/21’<sup>28</sup>

#### 4-4. “Denominator + *fēn* + MW + *zhī* + Numerator” for non-unit fractions

The 36 instances are given in (80)–(115) together with the preceding integer when the fraction is inserted in a mixed number. Note that in (81) and (82), the numerical expressions are objects of a verb, which was already the case with (75) in the previous section. The status of (115) in this matter is unclear because some characters are illegible. In all of the other 33 instances the numerical expression is inserted in a predicative clause, and the subject is a mass noun.

(80) 金 七分 朱(銖) 之 三  
 in *Suàn Shù Shū* *jīn qī fēn zhū zhī sān*  
 strip 30 gold {7} *fēn* *zhū* *zhī* {3}  
 ‘3/7 *zhū* of gold’

(81)<sup>29</sup> 長者 受 十六 尺 有(又)  
 in *Suàn Shù Shū* *zhǎngzhě shòu shí liù chǐ yòu*  
 strip 55 elder get {10}{6} *chǐ* and  
 ‘the elder gets 16 *chǐ* and

十八分 尺 之 十二  
*shí bā fēn chǐ zhī shí èr*  
 {10}{8} *fēn* *chǐ* *zhī* {10}{2}  
 12/18 *chǐ*’

<sup>26</sup> The character on the strip looks more like *yī* 一 {1} than *qī* 七 {7}; but linguistic and conceptual coherence require us to choose *qī* 七 {7} to have a proper fraction. Peng Hao (2001: 115 note 5), Hu Yitao (2006: 79) and [Ōkawa et al. 2006: 20 note 4] all discussed this point. Anyway, it is of no consequence for our classification.

<sup>27</sup> This takes place in the calculation of one side of a rectangle when the other side and the surface are known. The textual coherence implies that a surface in square *bù* should be understood, even though the measure word is not stated. We only need to acknowledge this absence for our classification.

<sup>28</sup> As for the instance in (78), we only need to acknowledge the absence of a measure word to classify the expression.

<sup>29</sup> The fraction in (81) was already stated in (7) withdrawn from its insertion context.

- (82) 少者 受 八 尺 有(又)  
 in *Suàn Shù Shū* *shàozhě shòu bā chǐ yòu*  
 strip 55 younger get {8} *chǐ* and  
 ‘the younger gets 8 *chǐ* and  
 十八分 尺 之 六  
*shí bā fēn chǐ zhī liù*  
 {10}{8} *fēn chǐ zhī* {6}  
 6/18 *chǐ*’
- (83) 糲 十分 升 之 三  
 in *Suàn Shù Shū* *bài shí fēn shēng zhī sān*  
 strip 98 milled millet {10} *fēn shēng zhī* {3}  
 ‘3/10 *shēng* of milled millet’
- (84) 米 十五分 升 之 四  
 in *Suàn Shù Shū* *mǐ shí wǔ fēn shēng zhī sì*  
 strip 98 husked millet {10}{5} *fēn shēng zhī* {4}  
 ‘4/15 *shēng* of husked millet’
- (85) 粟 廿(二十)七分 升 之 十  
 in *Suàn Shù Shū* *sù èr shí qī fēn shēng zhī shí*  
 strip 99 unhusked millet {2}{10}{7} *fēn shēng zhī* {10}  
 ‘10/27 *shēng* of unhusked millet’
- (86) 米 九分 升 之 二  
 in *Suàn Shù Shū* *mǐ jiǔ fēn shēng zhī èr*  
 strip 99 husked millet {9} *fēn shēng zhī* {2}  
 ‘2/9 *shēng* of husked millet’
- (87) 毀(穀) 卅(四十)五分 升 之 八  
 in *Suàn Shù Shū* *huǐ sà shí wǔ fēn shēng zhī bā*  
 strip 100 polished millet {4}{10}{5} *fēn shēng zhī* {8}  
 ‘8/45 *shēng* of polished millet’
- (88) 粟  
 in *Suàn Shù Shū* *sù*  
 strip 101 unhusked millet  
 ‘25/54 *shēng* of unhusked millet’  
 五十四分 升 之 廿(二十)五  
*wǔ shí sì fēn shēng zhī èr shí wǔ*  
 {5}{10}{4} *fēn shēng zhī* {2}{10}{5}

- (89) 米 十八分 升 之 五  
 in *Suàn Shù Shū* *mǐ* *shí bā fēn* *shēng zhī wǔ*  
 strip 101 husked millet {10}{8} *fēn shēng zhī* {5}  
 ‘5/18 *shēng* of husked millet’
- (90) 毀(穀)米 九分 升 之 二  
 in *Suàn Shù Shū* *huǐ mǐ* *jiǔ fēn* *shēng zhī èr*  
 strip 102 polished millet {9} *fēn shēng zhī* {2}  
 ‘2/9 *shēng* of polished millet’
- (91) 麥 十二分 升 之 五  
 in *Suàn Shù Shū* *mài* *shí èr fēn* *shēng zhī wǔ*  
 strip 102 wheat {10}{2} *fēn shēng zhī* {5}  
 ‘5/12 *shēng* of wheat’
- (92) 米 十六分 升 之 五  
 in *Suàn Shù Shū* *mǐ* *shí liù fēn* *shēng zhī wǔ*  
 strips 102–103 husked millet {10}{6} *fēn shēng zhī* {5}  
 ‘5/16 *shēng* of husked millet’
- (93) 麥 卅(三十)二分 升 之 十五  
 in *Suàn Shù Shū* *mài* *sān shí èr fēn* *shēng zhī shí wǔ*  
 strip 103 wheat {3}{10}{2} *fēn shēng zhī* {10}{5}  
 ‘15/32 *shēng* of wheat’
- (94) 粟  
 in *Suàn Shù Shū* *sù*  
 strip 104 unhusked millet  
 ‘25/48 *shēng* of unhusked millet’
- 卅(四十)八分 升 之 廿(二十)五  
*sì shí bā fēn* *shēng zhī èr shí wǔ*  
 {4}{10}{8} *fēn shēng zhī* {2}{10}{5}
- (95) 粟  
 in *Suàn Shù Shū* *sù*  
 strip 105 unhusked millet  
 ‘500/789 *shēng* of unhusked millet’
- 七百八十九分 升 之 五百  
*qī bǎi bā shí jiǔ fēn* *shēng zhī wǔ bǎi*  
 {7}{10<sup>2</sup>}{8}{10}{9} *fēn shēng zhī* {5}{10<sup>2</sup>}

- (96) 粟 一 升  
 in *Suàn Shù Shū* *sù* *yī shēng*  
 strips 105–106 unhusked millet {1} *shēng*  
 ‘1 *shēng*
- 二百六十三分 之 二百卅(三十)七  
*èr bǎi liù shí sān fēn zhī èrbǎi sān shí qī*  
 $\{2\}\{10^2\}\{6\}\{10\}\{3\}fēn$  *zhī*  $\{2\}\{10^2\}\{3\}\{10\}\{7\}$   
 237/263 *shēng* of unhusked millet’
- (97) 粟 一 斗 九 升  
 in *Suàn Shù Shū* *sù* *yī dǒu jiǔ shēng*  
 strip 106 unhusked millet {1} *dǒu* {9} *shēng*  
 ‘1 *dǒu* 9 *shēng*
- 有(又) 二百六十三分 升 之 三  
*yòu èr bǎi liù shí sān fēn shēng zhī sān*  
 and  $\{2\}\{10^2\}\{6\}\{10\}\{3\}fēn$  *shēng zhī* {3}  
 and 3/263 *shēng* of unhusked millet’
- (98) 粟 十九 斗 有(又)  
 in *Suàn Shù Shū* *sù* *shí jiǔ dǒu yòu*  
 strip 106 unhusked millet {10}{9} *dǒu* and  
 ‘19 *dǒu* and
- 二百六十三分 升 之 卅(三十)  
*èr bǎi liù shí sān fēn shēng zhī sān shí*  
 $\{2\}\{10^2\}\{6\}\{10\}\{3\}fēn$  *shēng zhī*  $\{3\}\{10\}$   
 30/263 *shēng* of unhusked millet’
- (99) 粟  
 in *Suàn Shù Shū* *sù*  
 strip 107 unhusked millet  
 ‘100/171 *shēng* of unhusked millet’
- 百~~十~~(七十)一分 升 之 百  
*bǎi qī shí yī fēn shēng zhī bǎi*  
 $\{10^2\}\{7\}\{10\}\{1\}fēn$  *shēng zhī*  $\{10^2\}$
- (100) 粟 一 升 有(又)  
 in *Suàn Shù Shū* *sù* *yī shēng yòu*  
 strip 107 unhusked millet {1} *shēng* and  
 ‘1 *shēng* and



二百八十五分 升 之 二百~~十~~(七十)五  
*èr bǎi bā shí wǔ fēn shēng zhī èr bǎi qī shí wǔ*  
 $\{2\}\{10^2\}\{8\}\{10\}\{5\}fēn shēng zhī \{2\}\{10^2\}\{7\}\{10\}\{5\}$   
 275/285 *shēng* of unhusked millet'

(101) 粟 十七 升 有(又)  
 in *Suàn Shù Shū* *sù shí qī shēng yòu*  
 strip 108 unhusked millet {10}{7} *shēng* and  
 '17 *shēng* and

二百八十五分 升 之 百五十  
*èr bǎi bā shí wǔ fēn shēng zhī bǎi wǔ shí*  
 $\{2\}\{10^2\}\{8\}\{10\}\{5\}fēn shēng zhī \{10^2\}\{5\}\{10\}$   
 150/285 *shēng* of unhusked millet'

(102) 粟 十七 斗 五 升  
 in *Suàn Shù Shū* *sù shí qī dòu wǔ shēng*  
 strip 106 unhusked millet {10}{7} *dǒu* {5} *shēng*  
 '17 *dǒu* 5 *shēng*

有(又) 二百八十五分 升 之 百廿(二十)五  
*yòu èr bǎi bā shí wǔ fēn shēng zhī bǎi èr shí wǔ*  
 and  $\{2\}\{10^2\}\{8\}\{10\}\{5\}fēn shēng zhī \{10^2\}\{2\}\{10\}\{5\}$   
 and 125/285 *shēng* of unhusked millet'

(103) 米 卅(四十)六 石  
 in *Suàn Shù Shū* *mǐ sì shí liù shí*  
 strip 146 husked millet {4}{10}{6} *shí*  
 '46 *shí*

廿(二十)七分 石 之 八  
*èr shí qī fēn shí zhī bā*  
 $\{2\}\{10\}\{7\}fēn shí zhī \{8\}$   
 8/27 *shí* of husked millet'

(104) 廣 八分 步 之 六  
 in *Suàn Shù Shū* *guǎng bā fēn bù zhī liù*  
 strip 162 width {8} *fēn* *bù* *zhī* {6}  
 'a width of 6/8 *bù*'

(105) 廣 七分 步 之 三  
 in *Suàn Shù Shū* *guǎng qī fēn bù zhī sān*  
 strip 162 width {7} *fēn* *bù* *zhī* {3}  
 'a width of 3/7 *bù*'

- (106) 田 四分 步 之 二  
 in *Suàn Shù Shū* *tián sì fēn bù zhī èr*  
 strip 162 field {4} *fēn* *bù zhī* {2}  
 ‘a field of 2/4 [square] *bù*’
- (107) 從(縱) 百卅(三十) 步 有(又)  
 in *Suàn Shù Shū* *zòng bǎi sān shí bù yòu*  
 strip 168 length {10<sup>2</sup>}{3}{10} *bù* and  
 ‘a length of 130 *bù* and
- 十一分 步 之 十  
*shí yī fēn bù zhī shí*  
 {10}{1} *fēn* *bù zhī* {10}  
 10/11 *bù*’
- (108) 從(縱) 百一十五 步 有(又)  
 in *Suàn Shù Shū* *zòng bǎi yī shí wǔ bù yòu*  
 strip 169 length {10<sup>2</sup>}{1}{10}{5} *bù* and  
 ‘a length of 115 *bù* and
- 廿(二十)五分 步 之 五  
*èr shí wǔ fēn bù zhī wǔ*  
 {2}{10}{5} *fēn* *bù zhī* {5}  
 5/25 *bù*’
- (109) 從(縱) 百五 步 有(又)  
 in *Suàn Shù Shū* *zòng bǎi wǔ bù yòu*  
 strip 170 length {10<sup>2</sup>}{5} *bù* and  
 ‘a length of 105 *bù* and
- 百卅(三十)七分 步 之 十五  
*bǎi sān shí qī fēn bù zhī shí wǔ*  
 {10<sup>2</sup>}{3}{10}{7} *fēn* *bù zhī* {10}{5}  
 15/137 *bù*’
- (110) 從(縱) 九十二 步 有(又)  
 in *Suàn Shù Shū* *zòng jiǔ shí èr bù yòu*  
 strips 172–183<sup>30</sup> length {9}{10}{2} *bù* and  
 ‘a length of 92 *bù* and

<sup>30</sup> Strip 172 is followed by strip 183, see Hu Yitao (2006: 82).

千八十九分                      步 之 六百一十二  
*qiān bā shí jiǔ fēn      bù zhī liù bǎi yī shí èr*  
 $\{10^3\}\{8\}\{10\}\{9\} fēn      bù zhī \{6\}\{10^2\}\{1\}\{10\}\{2\}$   
 612/1089 *bù'*

(111)                      從(縱) 八十八                      步 有(又)  
 in *Suàn Shù Shū*                      *zòng bā shí bā      bù yòu*  
 strip 175                      length  $\{8\}\{10\}\{8\}$  *bù* and  
    'a length of 88 *bù* and

二千二百八十三分                      步 之 六百九十六  
*èr qiān èr bǎi bā shí sān fēn      bù zhī liù bǎi jiǔ shí liù*  
 $\{2\}\{10^3\}\{2\}\{10^2\}\{8\}\{10\}\{3\}      bù zhī \{6\}\{10^2\}\{9\}\{10\}\{6\}$   
 696/2283 *bù'*

(112)                      從(縱) 八十四                      步 有(又)  
 in *Suàn Shù Shū*                      *zòng bā shí sì      bù yòu*  
 strips 177–178                      length  $\{8\}\{10\}\{4\}$  *bù* and  
    'a length of 84 *bù* and

七千一百廿(二十)九分                      步  
*qī qiān yī bǎi èr shí jiǔ fēn      bù*  
 $\{7\}\{10^3\}\{1\}\{10^2\}\{2\}\{10\}\{9\} fēn      bù$   
 5764/7129 *bù'*

之 五千七百六十四  
*zhī wǔ qiān qī bǎi liù shí sì*  
*zhī \{5\}\{10^3\}\{7\}\{10^2\}\{6\}\{10\}\{4\}*

(113)                      從(縱) 八十一                      步 有(又)  
 in *Suàn Shù Shū*                      *zòng bā shí yī      bù yòu*  
 strips 180–181                      length  $\{8\}\{10\}\{1\}$  *bù* and  
    'a length of 81 *bù* and

七千三百八十一分                      步  
*qī qiān sān bǎi bā shí yī fēn      bù*  
 $\{7\}\{10^3\}\{3\}\{10^2\}\{8\}\{10\}\{1\} fēn      bù$   
 68??/7381 *bù'*

之 六千八百 [illegible]<sup>31</sup>  
*zhī liù qiān bā bǎi* [illegible]  
*zhī* {6}{10<sup>3</sup>}{8}{10<sup>2</sup>} [illegible]

(114) 廣  
 in *Suàn Shù Shū* *guǎng*  
 strip 183 width  
 ‘a width of

七 步 冊(四十)九分 步 之 [illegible]<sup>32</sup>  
*qī bù sī shí jiǔ fēn* *bù zhī* [illegible]  
 {7} *bù* {4}{10}{9} *fēn* *bù zhī* [illegible]  
 7 and [illegible]/9 *bù*’

(115) [illegible]<sup>33</sup> 六十四 步 有(又)  
 in *Suàn Shù Shū* [illegible] *liù shí sì* *bù yòu*  
 strip 183 [illegible] {6}{10}{4} *bù* and  
 ‘[illegible] 64 *bù* and

三百冊(四十)三分 步 之 二百~~十~~(七十)三  
*sān bǎi sī shí sān fēn bù* *bù zhī èrbǎi qī shí sān*  
 {3}{10<sup>2</sup>}{4}{10}{3} *fēn* *bù zhī* {2}{10<sup>2</sup>}{7}{10}{3}  
 273/343 *bù*’

## 5. LEXICALIZED EXPRESSIONS FOR 1/2, 1/3 AND 2/3

In the *Suàn Shù Shū*, the terms *bàn* 半 [half], *shǎobàn* 少半 [the smaller half] and *tàibàn* 大半 [the larger half]<sup>34</sup> are used as exact number names in calculations. They are lexicalized expressions of the fractions 1/2, 1/3 and 2/3; these values are revealed in (116)–(118). They can appear in expressions of mixed numbers in the order *bàn shǎobàn*, i.e. {1/2} {1/3} for 1/2+1/3 on strip 26 in (119), or in the order *shǎobàn bàn*, i.e. {1/3} {1/2} for 1/3+1/2 on strip 23. There are no other lexicalized forms for fractions in the whole *Suàn Shù Shū*<sup>35</sup>.

The instances in (120) and (121) show that these numerals can work as verbs in “NUM + OBJ” constructions with the meaning of multiplying the value of the object by the numeral; this capability is shared by the names of integers.

<sup>31</sup> The tens and units digits are illegible and transcribed as ?? in the English translation. This does not change anything to the classification of the expression.

<sup>32</sup> The numerator is illegible and transcribed as [illegible] in the English translation.

<sup>33</sup> A noun or the upper rank digits of the integer are illegible and transcribed as [illegible] in the English translation.

<sup>34</sup> Only the instance of *tàibàn* on strip 8 is written 大半, the other three instances are written 泰半. I chose the reading *tàibàn* which fits the two written forms. The reading *dàibàn* would be possible for 大半 but hardly for 泰半.

<sup>35</sup> In Contemporary Chinese only *yī bàn* 一半 for 1/2 remains. The terms *tàibàn* (written 泰半 or 太半) or *dàibàn* 大半 are still used on occasion today but only as approximate numbers meaning *most*, no longer as exact numbers.

- (116) 一半 乘 一 半 也  
 in *Suàn Shù Shū* *yī bàn chéng yī bàn yě*  
 strip 3 {1}{1/2} multiply {1} {1/2} DECL  
 ‘1/2 times 1 is 1/2’
- 乘 半 四分 一 也  
*chéng bàn sì fēn yī yě*  
 multiply {1/2} {4} *fēn* {1} DECL  
 ‘times 1/2 is 1/4’
- (117) 少半 乘 少半  
 in *Suàn Shù Shū* *shǎobàn chéng shǎobàn*  
 strip 8 {1/3} multiply {1/3}  
 ‘1/3 times 1/3’
- 九分 一 也  
*jiǔ fēn yī yě*  
 {9} *fēn* {1} DECL  
 ‘is 1/9’
- (118) 少半 乘 大半 九分 二 也  
 in *Suàn Shù Shū* *shǎobàn chéng tài bàn jiǔ fēn èr yě*  
 strip 8 {1/3} multiply {2/3} {9} *fēn* {2} DECL  
 ‘1/3 times 2/3 is 2/9’
- (119) 五 人 分 三 有(又)  
 in *Suàn Shù Shū* *wǔ rén fēn sān yòu*  
 strip 26 {5} person share {3} and  
 ‘Five people share 3 and’
- 半 少半 各 受 卅(三十)分 之 廿(二十)三  
*bàn shǎobàn gè shòu sān shí fēn zhī èr shí sān*  
 {1/2} {1/3} each get {3}{10} *fēn* *zhī* {2}{10}{3}  
 1/2 1/3 [a sum of three terms], each gets 23/30’  
 [i.e. the result of  $(3+1/2+1/3)÷5$ ].
- (120) 可 半 半 之  
 in *Suàn Shù Shū* *kě bàn bàn zhī*  
 strip 17 can {1/2} {1/2} 3OBJ  
 ‘If it can be multiplied by 1/2 [i.e. is divisible by 2],  
 then multiply it by 1/2.’<sup>36</sup>

<sup>36</sup> This comes from a passage about the reduction of fractions on strips 17-20.

- (121) 半 母 亦 半 子  
 in *Suàn Shù Shū* *bàn mǔ yì bàn zǐ*  
 strip 19 {1/2} denominator also {1/2} numerator  
 ‘Multiply the denominator by 1/2, and multiply the numerator by 1/2.’<sup>37</sup>
- (122) 半 步 乘 半 步 四分一  
 in *Suàn Shù Shū* *bàn bù chéng bàn bù sì fēn yī*  
 strip 8 {1/2} *bù* multiply {1/2} *bù* {4} *fēn* {1}  
 ‘1/2 *bù* times 1/2 *bù* is 1/4’
- (123) 二 斗 泰(大)半 斗  
 in *Suàn Shù Shū* *èr dǒu tài bàn dǒu*  
 strip 52 {2} *dǒu* {2/3} *dǒu*  
 ‘2 *dǒu* 2/3 *dǒu*’
- (124) 粟 十六 斗 泰(大)半 斗  
 in *Suàn Shù Shū* *sù shí liù dǒu tài bàn dǒu*  
 strip 88 unhusked millet {10}{6} *dǒu* {2/3} *dǒu*  
 ‘16 *dǒu* 2/3 *dǒu* of unhusked millet’
- (125) 米 六 斗 泰(大)半 斗  
 in *Suàn Shù Shū* *mǐ liù dǒu tài bàn dǒu*  
 strip 89 husked millet {6} *dǒu* {2/3} *dǒu*  
 ‘6 *dǒu* 2/3 *dǒu* of husked millet’
- (126) 三分 而 乘 一  
 in *Suàn Shù Shū* *sān fēn ér chéng yī*  
 strip 3 {3} *fēn* and then multiply {1}  
 ‘1/3 times 1’
- 三分 一 也  
*sān fēn yī yě*  
 {3} *fēn* {1} DECL  
 is 1/3’
- (127) 七 斗 三分 升 一  
 in *Suàn Shù Shū* *qī dǒu sān fēn shēng yī*  
 strip 119 {7} *dǒu* {3} *fēn* *shēng* {1}  
 ‘7 *dǒu* 1/3 *shēng*’

The only expressions for 1/2 in the corpus are 1 instance of {1} *bàn*, not followed by any measure word, in (116), and 46 instances of ∅ *bàn*, 12 of which are followed by a measure word, see for example (122); 33 are not, and a last

<sup>37</sup> This is from the same passage on strips 17-20.

instance on strip 1 is uncertain since the following characters are illegible. The regular compound “{2} *fēn*” is nowhere to be found in the text<sup>38</sup>.

To express 1/3 there are 24 instances of the lexicalized *shǎobàn* (15 with a measure word, 9 without), and to express 2/3 there are 4 instances of *tàibàn*, one without any measure word on strip 8, see (118); and 3 followed by the measure word *dǒu* on strips 52, 88 and 89; see (118), (123)–(125) respectively.

The regular compound *sān fēn*, i.e. {3} *fēn*, however, is found twice in the sequence {3} *fēn* {1} to express 1/3 on strips 3 and 119: see (126) without a measure word and (127) with the measure word *shēng* inserted between *fēn* and the numerator’s name {1}. It is also found 17 times (already mentioned in Sect. 3–1) in monodimensional expressions of 1/3 with the numerator 1 not stated. There are three instances in expressions of 2/3: one in the sequence {3} *fēn* {2} on strip 23 (without measure word), and two on strips 138–139 with the measure word *qián* inserted between *fēn* and the numerator {2}. Therefore, in the corpus, among the 50 instances of expressions for 1/3 or 2/3, there is a choice between the lexicalized forms (28 instances) and the regular forms built with {3} *fēn* (22 instances). Any of these forms can be used to denote dimensioned quantities (weight, length, surface, etc.) and can be followed by measure words in data, calculations or results. They can also denote dimensionless coefficients in some calculations or in presentations of arithmetical procedures.

**Tab. 1: Distribution of the expressions for 1/3 and 2/3 in the *Suàn Shù Shū***

	Lexicalized names <i>shǎobàn</i> and <i>tàibàn</i>	Regular forms with <i>sān fēn</i>	
MW +	18	4	22
MW –	10	18	28
	28	22	50

The distribution given in Tab. 1 shows that there is no grammatical obligation concerning the choice between lexicalized or regular items. Nevertheless, when no measure word is present, there is some preference for the unlexicalized form with {3} *fēn* since the occurrence rate of such configurations is  $18/28 \times 100 \approx 64\%$ . Conversely, there is preference for the lexicalized numerical items in adjectival position before a measure word since the occurrence rate of such configurations is  $18/22 \times 100 \approx 82\%$ ; this may be because the lexicalized items yielding noun phrases are more economical than bidimensional expressions which produce predicative clauses.

## 6. CONTEXTUAL OMISSION OF THE DENOMINATOR OF A NON-UNIT FRACTION

In four passages there are series of fractions which have the same denominator, as unambiguously shown by the *context*, but this denominator is stated only in

<sup>38</sup> In Contemporary Chinese the lexicalized fraction name *yī bàn* for 1/2 can usually be replaced by the regular form {2} *fēn zhī* {1}, but not, for example, in the time expression *bā diǎn bàn* for 8:30.

the expression of the first fraction and is understood thereafter. Abbreviations with *fēn* not preceded by the numerator's name can be construed as free reinterpretations of the item *fēn* as a noun meaning *parts* in a given partitioning.

In (128), taken from a passage about the taxation of pelts, the denominator 7 is stated only once. The following occurrences of *fēn* are contextually understood to designate sevenths.

(128) 犬 出 十五 錢 七分 六  
 in *Suàn Shù Shū* *quǎn chū shí wǔ qián qī fēn liù*  
 strips 34–35 *dog exit {10}{5} qián {7} fēn {6}*  
 ‘dog pelt is taxed at 15 and  $6/7$  *qián* [each]

狸 出 卅(三十)一 錢 五分 五  
*lí chū sān shí yī qián wū fēn wǔ*  
 wild cat exit {3}{10}{1} *qián wū fēn {5}*  
 wild cat pelt is taxed at 31 and  $5/7$  *qián* [each]

狐 出 六十三 錢 三分 三  
*hú chū liù shí sān qián sān fēn sān*  
 fox exit {6}{10}{3} *qián sān fēn {3}*  
 fox pelt is taxed at 63 and  $3/7$  *qián* [each]’

In (129), from another passage about the taxation of pelts, the denominator 72 is stated only once and the following occurrences of *fēn* are understood to refer to the same partitioning.

(129) 狐 出 十二 十一(七十)二分 十一  
 in *Suàn Shù Shū* *hú chū shí èr qī shí èr fēn shí yī*  
 strips 36–37 *fox pay {10}{2} {7}{10}{2} fēn {10}{1}*  
 ‘fox pelt is taxed at 12  $11/72$  [each]

狸 出 八 卅(四十)九 分  
*lí chū bā sì shí jiǔ fēn*  
 wild cat pay {8} *fēn {4}{10}{9}*  
 wild cat pelt is taxed at 8  $49/72$  [each]

犬 出 四 十二(七十)二分  
*quǎn chū sì shí èr fēn*  
 dog pay {4} *fēn {10}{2}*  
 dog pelt is taxed at 4  $12/72$  [each]’

In (130), a passage about the taxation of crops, the denominator 47 is stated only once and the following occurrences of *fēn* are understood to refer to the same partitioning.



(130) 禾 租 四 斗 卅(四十)七分 十二  
 in *Suàn Shù Shū* *hé zū sì dǒu sì shí qī fēn shí èr*  
 strips 43–44 millet tax {4} *dǒu* {4}{10}{7} *fēn* {10}{2}  
 ‘the tax for millet amounts to 4 *dǒu* 12/47 *dǒu*’

麥 租 三 斗 分 九  
*mài zū sān dǒu fēn jiǔ*  
 wheat tax {3} *dǒu* *fēn* {9}  
 the tax for wheat amounts to 3 *dǒu* 9/[47] *dǒu*

荅 租 二 斗 分 廿(二十)六  
*dá zū èr dǒu fēn èr shí liù*  
 beans tax {2} *dǒu* *fēn* {2}{10}{6}  
 the tax for beans amounts to 2 *dǒu* 26/[47] *dǒu*’

In (131), the number 36 is first announced as a divisor in the calculation of a volume and then understood as the denominator of the fraction in the result.

(131) 卅(三十)六 成<sup>39</sup> 今  
 in *Suàn Shù Shū* *sān shí liù chéng jīn*  
 strip 150 {3}{10}{6} divide now  
 ‘36 divides, now [we get]’

二千五十五 尺 分 廿(二十)  
*èr qiān wǔ shí wǔ chǐ fēn èr shí*  
 {2}{10<sup>3</sup>}{5}{10}{5} *chǐ* *fēn* {2}{10}  
 2055 [cubic] *chǐ* 20/[36] [cubic] *chǐ*’

## 7. CONTEXTUAL USE OF AN INTEGER NAME TO EXPRESS A DENOMINATOR

On two occasions in the corpus, the name of an integer is used to mean a fraction: see shaded  $\emptyset$  in (132) and (133). The integers {7} in (132) and {4} in (133) can be understood as fractions only because the text gives the result of the calculations.

(132) 六分 乘 七 $\emptyset$  卅(四十)二分 一  
 in *Suàn Shù Shū* *liù fēn chéng qī $\emptyset$  sì shí èr fēn yī*  
 strips 9–10 {6} *fēn* multiply {7} $\emptyset$  {4}{10}{2} *fēn* {1}  
 ‘1/6 times [1/7] is 1/42’

<sup>39</sup> The phrase “*n chéng*” (*n* 成) is an abbreviation of “*n chéng yī*” (*n* 成一) which expresses a division by *n* (Peng Hao 2001: 108 note 3), [Ōkawa et al. 2006: 29].

(133)	四	乘	五分	廿(二十)分	一
in <i>Suàn Shù Shū</i>	sì	chéng	wǔ fēn	èr shí fēn	yī
strip 9	{4}	multiply	{5} fēn	{2}{10} fēn	{1}
	‘[1/4] times 1/5 is 1/20’				

In the corpus, the sequence “NUM<sub>1</sub> + MW + *chéng* 乘 + NUM<sub>2</sub> (+MW)” can express the product of two lengths yielding to a surface; the unit of measurement can be dropped when it is the same for the two numbers. The instance given in (132) parallels this pattern if we consider the item *fēn* to be freely reinterpreted as a noun fitting into the measure word slot. The instance in (133) is similar except for the permutation of the two numerical expressions.

Hu Changqing (1996) cites other instances of these abbreviations in other corpora.

## 8. CASES OF ISOLATED NUMERATORS WITH SPECIAL MARKING

Two integer names preceded by *xiǎo* 小 [small] are found: *xiǎo* {5} on strip 29 and *xiǎo* {10} on strip 166. According to Peng Hao (2001: 50), the former makes sense from the context only if we interpret it as the numerator of the fraction 5/9 which is stated before in the text. As for the latter, Peng Hao (2001: 119) deduces by analogy that it must also be the numerator of a fraction which is not actually otherwise specified. Peng Hao’s interpretation is quite convincing for *xiǎo* {5} but not for *xiǎo* {10} given the respective contexts.

## 9. SUMMARY OF PROMINENT FEATURES

In the *Suàn Shù Shū*, the only inseparable fraction names were on one hand the special lexicalized expressions of 1/3, 1/2 and 2/3, and on the other hand the monodimensional expressions of unit fractions built according to the pattern “Denominator’s name + *fēn*” (83 instances). There were numerals which could be inserted before measure words in the same way as names for integers.

There were lexicalized forms only for 1/2, 1/3 and 2/3. Only the lexicalized form was used for 1/2 (47 instances). But to express 1/3 or 2/3, there was a choice between the lexicalized forms (28 instances) and the regular forms built with “{3} *fēn*” (22 instances). Any of these forms could be used with or without a measure word and there was no definite grammatical obligation, but a preference for the lexicalized items when a measure word was present.

Bidimensional expressions of fractions were built as predicative phrases with the unit fraction name “Denominator’s Name + *fēn*” acting as subject and with the numerator’s name acting as predicate. The resulting expressions were not inseparable and when a measure word was involved it was inserted right after “Denominator + *fēn*”. The morpheme *zhī* was used optionally as a marker of the predicative relation. The form of the bidimensional expression of a fraction belonged to one of the four patterns defined by whether the item *zhī* was used and whether a measure word was involved. Adding the 46 instances (not all different) of bidimensional expressions of unit fractions and the 97 instances (not all different) of bidimensional expressions of non-unit fractions we get the following distribution for the total of 143 instances:

- (a): “Denominator +  $f\bar{e}n$  + Numerator”: 35 instances.  
 (b): “Denominator +  $f\bar{e}n$  + MW + Numerator”: 54 instances.  
 (c): “Denominator +  $f\bar{e}n$  +  $zh\bar{i}$  + Numerator”: 7 instances.  
 (d): “Denominator +  $f\bar{e}n$  + MW +  $zh\bar{i}$  + Numerator”: 47 instances.

The item  $zh\bar{i}$  occurred only in bidimensional expressions followed by the numerator’s name and was therefore never used with the *mono*-dimensional expressions of unit fractions. Moreover the use of  $zh\bar{i}$  was correlated with the insertion of the fraction either as the predicate in a quantification clause or as the object of a verb; Tab. 2 provides a mapping of the situation.

**Tab. 2: Bidimensional expressions of fractions in the *Suàn Shù Shū*:  
The item  $zh\bar{i}$  and the insertion of fractions**

	$zh\bar{i}$ –	$zh\bar{i}$ +	
Inserted –	76	2	78
Inserted +	13	51	64
	89	53	142

The characters placed before the fraction in (115) are illegible, so I do not count it here and the grand total in Tab. 2 is only 142 and not 143.

The fraction is not inserted when it occurs as the results of the calculation as in the examples (132) and (133).

The occurrence rate of  $zh\bar{i}$  when the fraction is inserted as a predicate or an object amounts to  $51/64 \times 100 \approx 80\%$ . Conversely the occurrence rate of configurations without  $zh\bar{i}$  is  $76/89 \times 100 \approx 85\%$  when the fraction is not inserted. This allows us to state that the use of  $zh\bar{i}$  inside bidimensional fraction expressions was directly correlated with the syntactical insertion of these expressions as a dependent clause used as the predicate of a quantification phrase or as the object of a verb. Readers can refer to Anicotte (2015 b) for a detailed discussion on the use of  $zh\bar{i}$  in the expressions for fractions in Chinese.

## 10. ADDENDUM: BIDIMENSIONAL EXPRESSIONS OF PROPORTIONS

When we talk about a *fraction* of a given quantity, we assume the fraction to be one numerical item defined by a numerator and a denominator; for instance in the statement “ $2/3$  of 9 is 6”, the fraction  $2/3$  is one individualized number formed with the integers 2 and 3. However a numerical *proportion* between two things, or two kinds of items, can be expressed with two separate numbers.

For example, the sequence “Noun<sub>1</sub> + NUM<sub>1</sub> + Noun<sub>2</sub> + NUM<sub>2</sub>” reproduced in (134) is built with two numerals both in predicative positions.

- (134) 米 一  
 in *Suàn Shù Shū* *mǐ* *yī*  
 strip 119 husked millet {1}  
 ‘one [part of] husked millet,
- 粟 二  
*sù* *èr*  
 unhusked millet {2}  
 two [parts of] unhusked millet,
- 凡 十 斗  
*fán shí dǒu*  
 all {10} *dǒu*  
 altogether 10 *dǒu*’

This expresses a proportion of one part of *husked millet* [*mǐ* 米] for two parts of *unhusked millet* [*sù* 粟] making a total volume of 10 *dǒu*. From the proportion, we can deduce that the total amount is composed of 1/3 husked millet and 2/3 unhusked millet, however these fractions 1/3 or 2/3 are not stated and their denominator 3 does not appear at all; therefore the phrase “Noun<sub>1</sub> + NUM<sub>1</sub> + Noun<sub>2</sub> + NUM<sub>2</sub>” is not the expression of a fraction, but the bidimensional expressions of a proportion; therefore they should not be included in a study on fraction names.

Yang Lingrong (2008: 15–16) lists 14 examples of such expressions of proportions in the *Suàn Shù Shū* including our example (134). To these 14 examples, we can as well add this instance on strip 52:

- (135) 芻 藁 二 石 今  
 in *Suàn Shù Shū* *chú gǎo èr shí jīn*  
 strip 52 hay straw {2} *shí* now  
 ‘2 *shí* of hay and straw, now [there are]
- 芻 三 而 藁 二  
*chú sān ér gǎo èr*  
 hay {3} and straw {2}  
 three [parts of] hay and two [parts of] straw’

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