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A reduced-order modeling technique for the study of non-linear vibrations in dissipative systems

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Abstract

This work is dedicated to the presentation and testing of a new reduced-order modeling technique, devised in order to handle effectively vibrations of phenomenological models as well as large-scale industrial systems, in the presence of strongly activated nonlinearities. The method is quite flexible and does not require any restrictions concerning the nature of the nonlinear effects. Both smooth and non-smooth nonlinearities can be tackled, as well as dissipative phenomena. In this paper the main steps of this new reduction strategy are introduced, and the method is tested on a lumped-parameter model, representative of the bending behaviour of an industrial bladed-disk subjected to dry friction at the blade-disk joints. The method proves to efficiently capture and render the intricate dynamics of the model, while providing a significant improvement in terms of computational effort, allowing here to perform a short study of the amplification factor arising from mistuning in the presence of strong nonlinearities.

1 Introduction

Despite the growing performance of modern computing, the need to perform finite element analyses on ever more complex and large systems have made of component mode synthesis (CMS) methods [1–4] a key tool for structural dynamicists. However, to improve the design process or to further our understanding of complex systems, it can prove necessary to take into account nonlinear phenomena, which can severely impact the efficiency of these reduction techniques, usually requiring to retain all nonlinear degrees of freedom (DOF) as master coordinates to achieve sufficient accuracy. As a consequence, one way to improve classic CMS methods would be to build a reduction basis allowing to capture the nonlinear effects and thus not having to keep nonlinear DOF as master coordinates. With that in mind, recent works devoted to nonlinear complex modes [5, 6] make them appear as an interesting tool to tackle this problematic.

A well-known topic of research in the field of turbomachinery is what is nowadays commonly referred to as mistuning, denoting the loss of symmetry of a cyclic structure, such as a turbine or compressor bladed-disk. Addressed a few decades ago [7–10], the subject has led to a rich and topical scientific literature [11], reflecting the interest of both industrials and researchers in the matter. The phenomenon, resulting for instance from tolerancing limitations and in-operation wear, is indeed of primary importance in designing such components. It can, indeed, drastically disturb their dynamical behaviour, with a potential localization and amplification of the vibrations on a few blades. As a consequence, a great deal of reduced-order modeling methods dedicated to the analysis of mistuned structures can be found in the literature [12–15], but most of them are unfortunately confined to the study of linear systems, and are either inapplicable or far less efficient when confronted to nonlinearities.

The technique presented in this paper aims at providing a novel and efficient tool to study nonlinear and dissipative systems without requiring any symmetry assumptions, being hence directly applicable to strongly mistuned cyclic structures. The method is similar to a standard fixed-interface CMS, but furthers the reduction by not retaining the nonlinear DOFs as master coordinates, unlike the standard strategy. In order to allow this while avoiding any detrimental loss of information, the method makes use of the concept of nonlinear modes to capture the essence of the nonlinearities in the reduction basis. The fixed-interface linear modes of each substructure are thus replaced by fixed-interface nonlinear complex modes, which are still supplemented by classic static modeshapes to account for the displacement of the interface DOFs. The harmonic balance approach [5, 6, 16] used to compute them allows to benefit from the flexibility of numerical methods, compared to analytical approaches which can involve cumbersome algebra when dealing with large-scale systems and strong, non-smooth nonlinearities.

A few reminders concerning nonlinear complex modes and their computation are given in Section 2, before introducing the methodology used to build a nonlinear superelement from them. In Section 3, the method is applied on a phenomenological model representative of an industrial bladed-disk exhibiting friction at the blade-disk joints and subjected to structural mistuning, in order to appraise the performance of the proposed nonlinear CMS. As commonly tackled when it comes to mistuning, the evolution of the amplification factor with the mistuning magnitude is addressed, in the linear domain as well as at a strongly nonlinear level thanks to the capabilities of the proposed method.

2 Nonlinear reduced-order modeling

This section provides the theoretical background of the proposed reduced-order modeling strategy. The notion of nonlinear complex mode is first reminded and illustrated on an academic system, and the equations governing a nonlinear superelement are then derived. Since the method aims at being an extension of standard fixed-interface CMS, it is referred to as CNCMS throughout the paper, which stands for component nonlinear complex mode synthesis.

2.1 Nonlinear complex modes

Nonlinear modes and their applications have been thoroughly studied by structural dynamicists, and appear nowadays as a viable extension to linear modal analysis in the study of some industrial systems. A comprehensive state-of-the-art on the subject can be found in [17]. The concept of nonlinear complex modes was first introduced in [5] to provide an alternative to analytical approaches and handle efficiently strong and non-smooth nonlinearities thanks to a modified harmonic balance formulation. Recent papers [6, 16, 18] have been devoted to these modes and have proved their capability to deal with large-scale industrial systems and mistuned cyclic structures. The interested reader is referred to the aforementioned references for further information, as only the main steps of their computation are tackled below to introduce some notations.

In structural dynamics, the differential equations governing an autonomous system after spatial discretization can usually take the form given by Eq. (1),

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{f}_{nl}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{0} \quad (1)$$

where the terms \mathbf{M} , \mathbf{C} , and \mathbf{K} refer to the mass, viscous damping, and linear stiffness matrices, respectively, and \mathbf{x} is the vector of DOFs. The nonlinear restoring and dissipative forces acting on the system are gathered in the column vector \mathbf{f}_{nl} . The n -th nonlinear complex mode of the system is sought as a multi-harmonic pseudo-oscillation,

$$\mathbf{x}_n(t) = \frac{1}{2} \left\{ \sum_{k=1}^{n_h} \hat{\mathbf{x}}_{n,k} e^{kt\lambda_n} + c.c. \right\} \quad (2)$$

where $\lambda_n = -\beta_n + i\omega_n$ is the eigenvalue of the mode, with ω_n the fundamental natural frequency and β_n the modal damping coefficient, and where *c.c.* refers to the complex conjugate terms. The substitution of $\mathbf{x}_n(t)$ in Eq. (1) yields a set of residual equations, which can be orthogonalized to the subspace spanned by the basis functions $\{e_k = e^{ikt\omega_n}\}, \forall k \in \llbracket 1, n_h \rrbracket$ with respect to the bilinear form (3), where the overline denotes the complex conjugate operator.

$$\langle f|g \rangle = \frac{2}{T} \int_0^T f(t)\overline{g(t)}dt, \text{ with } T = \frac{2\pi}{\omega_n} \quad (3)$$

As suggested in former papers [5, 6, 16, 18], it is assumed that the solution is undamped over the period T used in Eq. (3), which allows to take advantage of the orthogonality properties of the complex exponentials with respect to the bilinear form. The procedure results in a nonlinear algebraic system (4) governing the mode,

$$\forall k \in \llbracket 1, n_h \rrbracket : \quad [(k\lambda_n)^2\mathbf{M} + (k\lambda_n)\mathbf{C} + \mathbf{K}] \hat{\mathbf{x}}_{n,k} + \langle \mathbf{f}_{nl}|e_k \rangle = \mathbf{0} \quad (4)$$

in which $\langle \mathbf{f}_{nl}|e_k \rangle$ can couple the whole set of equations. Unlike a standard harmonic balance procedure, the system is here underdetermined due to the unknowns ω_n and β_n , and requires two additional equations to be solved. These equations are usually taking the form of a continuation condition and a phase constraint [18], which will not be detailed in this paper. The evaluation of $\langle \mathbf{f}_{nl}|e_k \rangle$ at each iteration of the solver is achieved by an alternating frequency-time (AFT) procedure [19], which allows to deal readily with non-smooth nonlinear forces such as contact and dry friction.

In order to illustrate the modal phenomenology of friction-damped systems, the first nonlinear complex mode of the 2-DOFs system shown on Fig. (1) is computed. Table 1 lists the value taken by each parameter. The evolution of the natural frequency ω_n and modal damping β_n with respect to the amplitude of vibration $|x_1|$ is plotted on Fig. (2). These curves are classically referred to as backbones of the nonlinear mode, and are well described and explained in former papers [5, 6, 18]. Let us just point out that the modal parameters are controlled by the activation of the nonlinearity, here embodied by the amplitude $|x_1|$, and that dry friction is a softening nonlinearity that induces a reduction of the natural frequency and a potential maximum in modal damping.

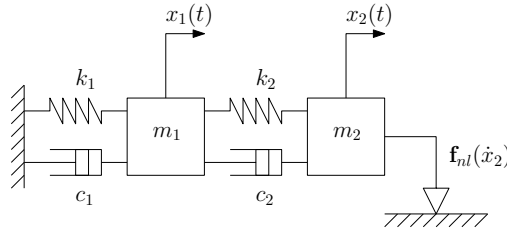


Figure 1: 2-DOFs dry-friction damped model

– (unit)	m (kg)	c (Ns/m)	k (N/m)
DOF1	1	0.5	640
DOF2	0.02	0.5	40

Table 1: Values of the lumped-parameters for the 2-DOFs system

2.2 Nonlinear superelement equations

Assuming that the structure has been divided into a given set of substructures, this section derives the equations governing one superelement. Since only one substructure is considered throughout the section, the non-

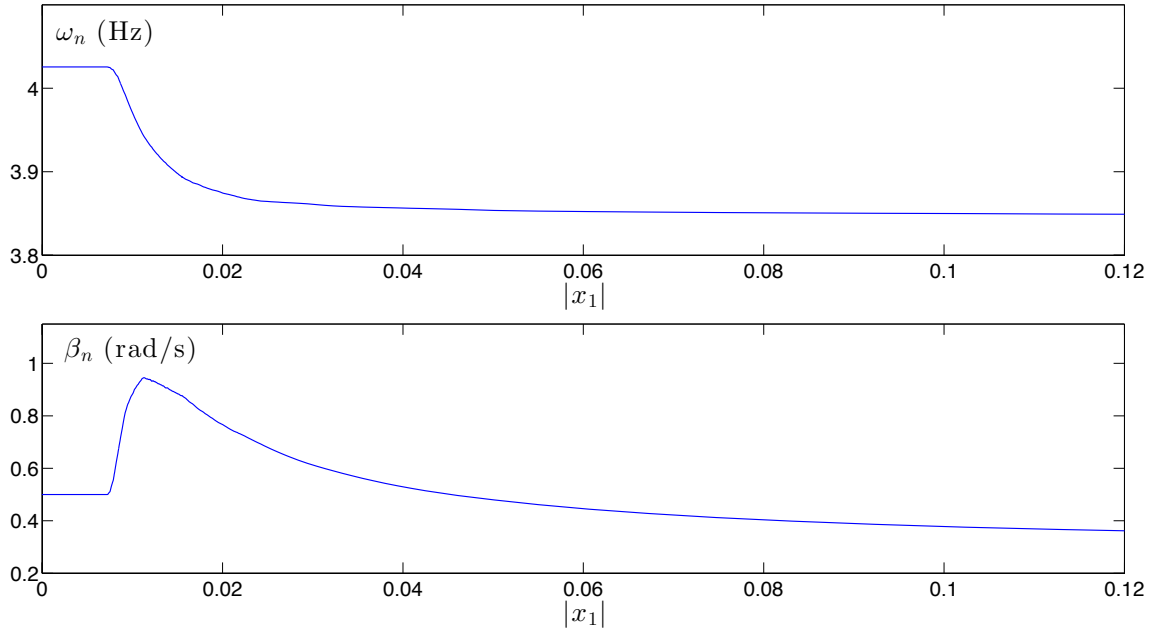


Figure 2: Nonlinear natural frequency and modal damping of the 2-DOFs model

linear superelement can perfectly be coupled to linear superelements computed by any other fixed-interface CMS method, such as the Craig-Bampton method. It is assumed that at least one nonlinear complex mode of the substructure has been computed by means of the methodology presented above for fixed boundary conditions, and that the excitation is a multi-harmonic forcing of fundamental frequency Ω , which can be written in the form,

$$\mathbf{f}_e(t) = \frac{1}{2} \left(\sum_k \langle \mathbf{f}_e | e_k \rangle e^{ik\Omega t} + c.c. \right) \quad (5)$$

using the same notations than in Eq. (2).

Similarly to a classic fixed-interface CMS, the displacement field is approximated by a multiharmonic superposition of internal eigenvectors $\varphi_{n,k}$, corresponding to the nonlinear modes computed for fixed boundaries, and a set of static modeshapes $\psi_{s,k}$,

$$\mathbf{x}(t) = \frac{1}{2} \left(\sum_n q_n \sum_k \varphi_{n,k} (|q_n|) e^{ik\Omega t} + \sum_s p_s \sum_k \psi_{s,k} e^{ik\Omega t} + c.c. \right) \quad (6)$$

where q_n and p_s are the generalized coordinates of the nonlinear modes and static modeshapes, respectively. The vectors $\varphi_{n,k}$ are derived from the vectors $\hat{\mathbf{x}}_{n,k}$ of the previous section by a standard normalization,

$$\hat{\mathbf{x}}_{n,k} = q_n \varphi_{n,k} \quad (7)$$

The static modeshapes are here those of a classic Craig-Bampton CMS, regardless of the harmonic considered,

$$\forall k \in \llbracket 1, n_h \rrbracket :$$

$$\psi_{s,k} = -\mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} \quad \text{with} \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \quad (8)$$

where subscript i refers to internal DOFs and subscript b refers to boundary DOFs. It should be pointed out that if different vectors $\psi_{s,k}$ were to be used, the method could still be applied without any modifications.

For a given mode, the quantity driving the activation of the nonlinearities, and thus the modal parameters, is the amplitude of its coordinate $|q_n|$, formerly embodied by $|x_1|$ for the 2-DOFs system of the previous section. Even if not explicitly written to improve the legibility, $\varphi_{n,k}$ and λ_n are still functions of $|q_n|$ in the rest of the paper.

Adding the excitation vector \mathbf{f}_e of Eq. (5) on the right-hand-side of Eq. (1) yields the nonlinear system of differential equations to be solved. The substitution of Eq. (6) into this non-autonomous system yields a set of residual equations, which can be orthogonalized to the subspace spanned by the basis functions $\{e_k = e^{ik\Omega t}\}$ by means of the bilinear form defined in Eq. (3). The procedure leads to

$$\forall k \in \llbracket 1, n_h \rrbracket : \quad [(ik\Omega)^2 \mathbf{M} + (ik\Omega) \mathbf{C} + \mathbf{K}] \left\{ \sum_n q_n \varphi_{n,k} + \sum_s p_s \psi_{s,k} \right\} + \langle \mathbf{f}_{nl} | e_k \rangle = \langle \mathbf{f}_e | e_k \rangle \quad (9)$$

In order to drop the projection of the nonlinear forces $\langle \mathbf{f}_{nl} | e_k \rangle$, the sum of the corresponding terms in the eigenproblems (4) is used to express it as a function of the nonlinear eigenvalues λ_n , leading to

$$\begin{aligned} \forall k \in \llbracket 1, n_h \rrbracket : \\ & \sum_n [(ik\Omega)^2 \mathbf{M} + (ik\Omega) \mathbf{C} + \mathbf{K}] q_n \varphi_{n,k} \\ & - \sum_n [(k\lambda_n)^2 \mathbf{M} + (k\lambda_n) \mathbf{C} + \tilde{\mathbf{K}}] q_n \varphi_{n,k} \\ & + \sum_s [(ik\Omega)^2 \mathbf{M} + (ik\Omega) \mathbf{C} + \mathbf{K}] p_s \psi_{s,k} \\ & - \langle \mathbf{f}_e | e_k \rangle \approx \mathbf{0} \end{aligned} \quad (10)$$

where $\langle \mathbf{f}_{nl} | e_k \rangle$ has been replaced by the terms on the second line. Since the nonlinear modes were computed for fixed-boundary conditions, the stiffness matrix $\tilde{\mathbf{K}}$ is not strictly equal to \mathbf{K} . The last step of the procedure is to orthogonalize the residual equations (10) to the subspace spanned by the nonlinear eigenvectors $\varphi_{n,k}$ and the static modeshapes $\psi_{s,k}$ by means of the standard Hermitian form of \mathbb{C}^n , yielding eventually the governing equations of a nonlinear superelement,

$$\forall k \in \llbracket 1, n_h \rrbracket : \quad \rho_k = \begin{bmatrix} \Phi_k^\dagger (\mathbf{Z}_k \Phi_k - \xi_k) & \Phi_k^\dagger \mathbf{Z}_k \Psi_k \\ \Psi_k^\dagger (\mathbf{Z}_k \Phi_k - \xi_k) & \Psi_k^\dagger \mathbf{Z}_k \Psi_k \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \mathbf{p} \end{Bmatrix} - \begin{Bmatrix} \Phi_k^\dagger \langle \mathbf{f}_e | e_k \rangle \\ \Psi_k^\dagger \langle \mathbf{f}_e | e_k \rangle \end{Bmatrix} = \mathbf{0} \quad (11)$$

where \bullet^\dagger is the conjugate transpose operator, $\mathbf{Z}_k = [(ik\Omega)^2 \mathbf{M} + (ik\Omega) \mathbf{C} + \mathbf{K}]$ is the dynamic stiffness matrix of the k -th harmonic, $\xi_k = [\mathbf{M} \Phi_k \Lambda^2 k^2 + \mathbf{C} \Phi_k \Lambda k + \tilde{\mathbf{K}} \Phi_k]$ is the term accounting for the substitution of $\langle \mathbf{f}_{nl} | e_k \rangle$, Φ_k is the matrix of the vectors $\varphi_{n,k}$, Ψ_k is the matrix made of the static modeshapes $\psi_{s,k}$, and Λ is the diagonal spectral matrix gathering the eigenvalues λ_n . Finally, the vectors \mathbf{q} and \mathbf{p} list the unknowns of the system *i.e.* the coordinates q_n et p_s , respectively.

Once a superelement has been built for every substructure, the reduced-order model can be obtained by coupling the superelements through the generalized coordinates p_s , as in a classic fixed-interface CMS. It should be pointed out that if the maximum harmonic order n_h retained is larger than 1, the method results in an overdetermined system. However, neglecting the equations corresponding to $\langle \mathbf{f}_e | e_k \rangle = \mathbf{0}$ has proved to have a negligible influence on the solution. As a consequence, for a mono-harmonic excitation, the system to be solved possesses as many unknowns as equations. Due to $\varphi_{n,k}$ and λ_n being functions of the variables $|q_n|$, the final algebraic system is nonlinear and requires an iterative algorithm to be solved numerically. Since a given nonlinear complex mode is computed over a discrete range of $|q_n|$, it is necessary to perform an interpolation at each iteration of the solver to get an approximation of $\varphi_{n,k}$ and λ_n for the ongoing value of $|q_n|$.

3 Numerical applications

In this section, the method is tested on a phenomenological model of mistuned bladed disk subjected to dry friction nonlinearities. First, the forced response synthesised by CNCMS is compared to that obtained with a standard harmonic balance method (HBM), and the computational performance of both are appraised. Then, a short study on the amplification factor arising from the mistuning is presented, in order to compare the behaviour of the nonlinear system and that of the underlying linear structure.

3.1 Phenomenological model

The phenomenological model used in the following sections is made of 24 substructures, or sectors, representing the bending behaviour of a bladed-disk. The fundamental sector of the ideal, perfectly tuned structure, is shown on Fig. 3, with the structural lumped-parameters taking their values in Tab. 2. The damping coefficients of the dashpots are tuned so as to enforce a modal damping ratio of 0.1 for the first mode. The nonlinear nature of the model arises from the dry friction element linking the DOFs representative of the root of the blades and the disk, the nonlinear force being modeled by a hyperbolic tangent function [18]. The maximum number of harmonics n_h retained for both nonlinear complex mode and HBM calculations is taken equal to 5.

– (unit)	Tip	Middle	Root	Disk	Ground
m (kg)	0.2	0.3	0.4	1.2	–
c (Ns/m)	1.3	0.7	26.7	33.3	0.4
k ($10^6 \cdot \text{N/m}$)	2	1	40	50	0.6

Table 2: Values of the parameters for the fundamental sector

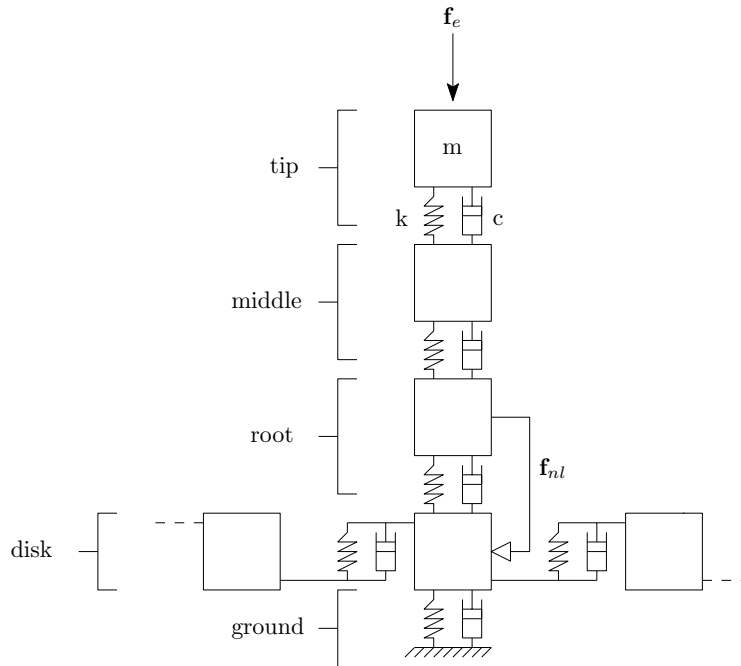


Figure 3: Fundamental sector of the cyclic model

In the next section, the structural mistuning is introduced by defining a second fundamental sector with different stiffness coefficients for the blade middle and tip DOFs, as reported in Tab. 3. This new set of structural parameters induces a 5% shift of the first natural frequency of the sector. These two fundamental sectors are randomly distributed to form the bladed-disk, according to the pattern given in Tab. 4, where A refers to the initial fundamental sector and B refers to the second one.

– (unit)	Tip	Middle	Root	Disk	Ground
m (kg)	0.2	0.3	0.4	1.2	–
k ($10^6 \cdot \text{N/m}$)	1.8	0.9	40	50	0.6

Table 3: Values of the parameters of sector B

Sector	1	2	3	4	5	6	7	8	9	10	11	12
Type	A	B	A	A	B	B	B	A	A	B	A	B
Sector	13	14	15	16	17	18	19	20	21	22	23	24
Type	A	A	B	B	A	A	A	A	A	A	B	B

Table 4: Mistuning pattern

3.2 Forced response synthesis and computational performance

The first nonlinear complex mode of each fundamental sector is computed by means of the methodology reminded in Sec.2.1 for fixed boundary conditions, and the corresponding superelements are then built from Eq. (11), and finally assembled through the generalized coordinates p_s . Figure 4 compares the backbones of the nonlinear mode of the two sectors, where the 5% frequency shift mentioned above can be clearly observed on the upper plot.

The reduced-order model of this mistuned system is then subjected to a traveling wave excitation, as commonly encountered in bladed-disk dynamics [18], with an arbitrary number of 6 nodal diameters. The intensity of the excitation is set high enough to bring the system close to the vibration level yielding the maximum nonlinear damping *i.e.* in the vicinity of 1mm (see Fig. 4). The response of all 24 blades, monitored at the tip DOFs, is plotted on Fig. 5 in blue solid lines, and compared to a reference in black dashed lines computed by HBM. The impact of the mistuning can be clearly observed as the responses of the blades significantly differ from one another. The accuracy of the CNCMS proves here very satisfactory.

Figure 6 reports the ratio of the computation time of the HBM with respect to the computation time of the CNCMS to compute the response of the system, as a function of the excitation level F , so as to compare the methods for different nonlinear levels. Throughout the excitation range, the CNCMS proves to be significantly more time efficient than the HBM, outperforming the latter by a ratio of about 55 close to the maximum nonlinear damping point ($F=5\text{N}$) *i.e.* to compute the response on Fig. 5. Viewing the performance of the method, a quantitative analysis of the amplification factor, in the nonlinear domain, appears to be feasible at reasonable computational cost, which is the topic of the next section.

3.3 Mistuning amplification factor in the presence of dry-friction

As mentioned in the introduction, a direct consequence of mistuning is a potential amplification of the vibration level on a few blades of the structure. A lot of effort has been dedicated to the understanding and

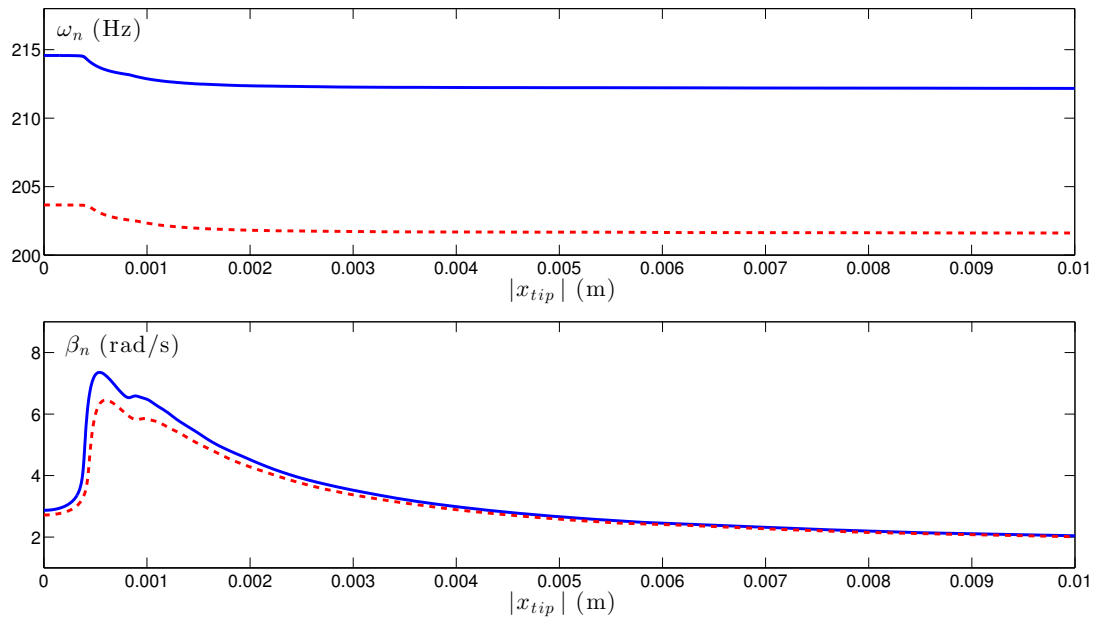


Figure 4: Backbones of the first mode of the two types of sectors with fixed boundary conditions (sector A- and sector B - -)

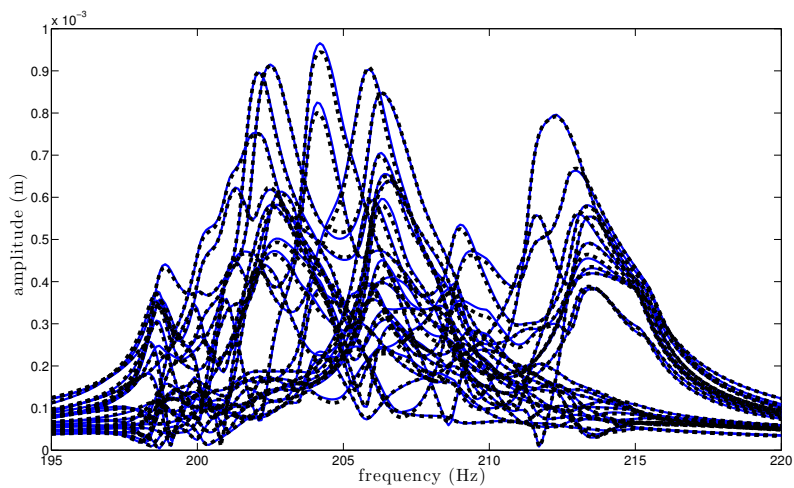


Figure 5: Response of all the blades of the mistuned system for a travelling wave excitation with 6 nodal diameters (CNCMS -, HBM - -)

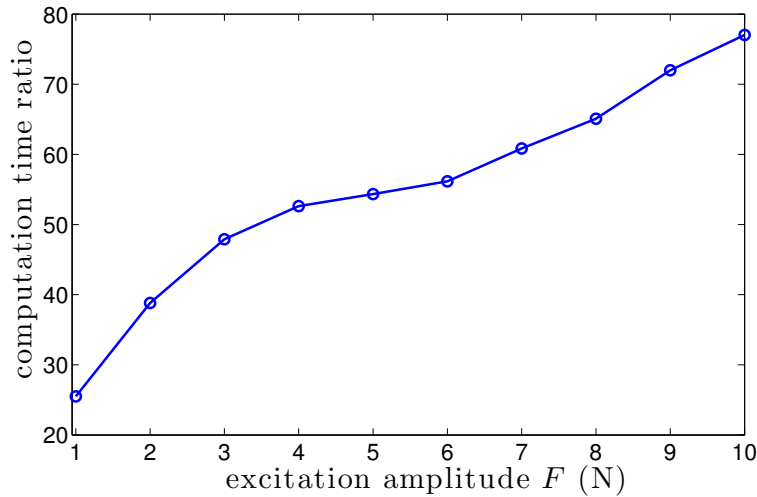


Figure 6: Computation time ratio between HBM and CNCMS

prediction of this amplification phenomenon [11]. However, most of the time the analysis is restricted to linear bladed-disks, such studies typically requiring thousands of computations, which can prove very costly for nonlinear systems. In light of the performance of the CNCMS presented in the previous section, such studies could actually be led at a reasonable cost with this new method. To illustrate the promising capabilities of the CNCMS to tackle this problematic, a few series of simulations were performed on the phenomenological model of Sec. 3.1 in order to highlight the differences in terms of amplification between a dry-friction damped bladed-disk and the underlying linear structure, as a function of the mistuning intensity. For these simulations, the mistuning was applied separately to each sector, by addition of a random perturbation shifting the backbone of the natural frequency ω_n corresponding to the nonlinear mode used to build the superelement. The standard deviation of this perturbation with respect to the nominal natural frequency is the parameter controlling the mistuning level in the following results.

Figure 7 shows the evolution of the amplification factor after 12000 random draws, which proved sufficient to ensure the convergence of the simulations. Similarly to the previous section, the forced responses are computed for a travelling wave excitation with 6 nodal diameters. The nonlinear behaviour is obtained by setting the excitation intensity close to the maximum nonlinear damping point ($F=5N$), and compared to a linear analysis. It can be observed that the linear behaviour is perfectly standard [11], with a high sensitivity range at low level of mistuning, a maximum amplification value, followed by a reduction of the amplification factor. In the nonlinear domain however, the evolution of the amplification factor differs significantly from that of the underlying linear system. The range of high sensitivity and the maximum in amplification are still observed, although the mistuning level yielding the maximum slightly differs from the linear case. The reduction in amplification is, on the other hand, nearly insignificant. Since intentional mistuning appears as an interesting alternative to improve the robustness of the design [11], this result could be of great interest if extended to industrial models, as the curves on Fig 7 show that the nonlinear effects can significantly affect the way a given mistuning level amplifies the response.

4 Conclusion

This paper presented a novel reduced-order modeling technique to study nonlinear vibrations in dissipative systems, the CNCMS. The method can be seen as an extension of the well-known fixed-interface CMS method, making use of nonlinear complex modes instead of classic linear modes in the reduction basis. It

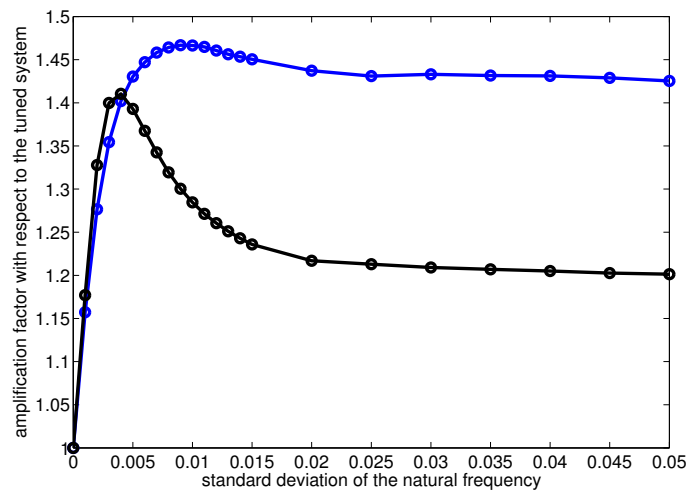


Figure 7: Amplification factor arising from the mistuning (linear domain -, nonlinear domain -)

has proved very promising in the study mistuned cyclic structures such as turbines and compressors bladed-disks, enabling a drastic reduction of the computation time compared to a standard HBM procedure. Since the method is quite general and does not require any symmetry assumption to derive the expression of a super-element, it could be directly used in the study of non-cyclic structures. As a fixed-interface CMS method, it could be used in combination with other linear fixed-interface CMS methods, as long as the same interface DOFs are retained in the super-elements to be assembled.

Thanks to the flexibility of nonlinear complex modes, a great variety of nonlinearities could be addressed, such as large displacement and large deformation, without any loss in order reduction. The method could also be adapted to other families of substructuring methods, such as free-interface or hybrid CMS. The authors are currently working on the implementation of the method on a 3D finite element model, in order to appraise its performance on full scale industrial components.

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