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Robust adaptive magnetic control of satellites with uncertain parameters

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Abstract

Recently developed results for direct adaptive control design in the context of uncertain systems in descriptor form are discussed and applied to a satellite attitude control problem. Thanks to descriptor model handling the methodology allows to address with convexity arguments uncertainties on the inertia matrix. The design of the adaptive law is done with semi-definite programming tools (LMI solvers) and is applicable with the sole knowledge of a controller stabilizing the nominal system (no passivity assumptions). The tackled satellite model is linear (assuming small pointing errors), full 3D rotations are considered (axes are coupled), and mixed reaction wheel and magnetotorquer actuation is adopted. Compared to the provided initial LTI control law, adaptive control increases robustness margins by a factor 262%.

1 Introduction

Attitude control of satellites has been a rising issue for years and keeps being an important topic, both in research ([5], [15], [23]) and in engineering ([24], [19], [17]). A wide range of satellites, including those of CNES Myriade series ([16]), are actuated by reaction wheels, which allow a high pointing accuracy but which can saturate, evolving the uncontrollability of the spacecraft. To prevent reaction wheel saturation and counter external disturbances, satellites are also

equipped with magnetotorquers ([14], [1], [22]).

Such devices are low cost and reliable but they suffer from the unavoidable constraint to generate a torque lying in the plane orthogonal to the local direction of the geomagnetic field. Since this last evolves along the satellite orbit, the obtained system is time varying, making it hard to control with classical techniques. Recently, some interesting solutions have been proposed to tackle this issue ([26], [4], [20]), but none of them takes into account parametric uncertainties in the model. In most papers on attitude control, the inertia of a satellite is either assumed known ([12], [25], [21], [23]), or not well known but with an uncertainty never more than 30% around its nominal value ([5], [3]). Since adaptive control theory has been proved to deal well with systems with high parametric uncertainties ([7], [8], [11]), the controller proposed in this paper is adaptive.

In this paper, we aim at stabilizing a 3 axes satellite with several actuators for the widest range of inertia. A multi input/multi output synthesis is then required. Moreover, since the considered uncertainties are all scalar (on the diagonal components of the matrix of inertia of the satellite), the method to be employed must guarantee parametric robustness with respect to structured uncertainties ([6]). In [16], an optimal controller has already been computed and widely used in the case when the

reaction wheels are far from saturation. This one will be used as a baseline for the adaptive controller synthesis.

Theoretical and practical results have been published in [10] and [11]. They satisfy all these criteria, excepted the taking into account of the data yielded by the magnetotorquers. They use LMI-based methods ([2]) with S-variables, making their resolution effective. Neither passivity assumption, nor parameter estimation is required, allowing direct adaptation and therefore simple implementation.

The paper is organized as follows: The problem is formulated in section 2. In section 3, theoretical results to build a robust direct adaptive controller are given. Based on these results, an adaptive attitude controller for a CNES satellite is designed, with no worse robustness than a corresponding static output feedback in section 4, and with improved robustness in section 5. Conclusion is given in section 6.

Notation. I stands for the identity matrix. $\{1; V\}$ is the set of all the integers between 1 and V . A^T is the transpose of the matrix A . A^S stands for the symmetric matrix $A + A^T$. $A(\preceq) \prec B$ is the matrix inequality stating that $A - B$ is negative (semi-)definite.

2 Problem statement

Performance and robustness of the attitude control loop are two fundamental requirements to have a high attitude pointing accuracy level. In the context of small deviations, the satellite model is linearized around a target attitude. Without loss of generality, this one is assumed equal to zero for easier calculations. The considered satellite has three reaction wheels whose angular momentum is controlled by three magneto-torquers. When the scientific mission must be achieved, the classical attitude control loop has the modelisation of Figure 1.

For control design, we consider the following linearised satellite dynamics:

$$J\ddot{\Theta} = T, \quad \dot{\Theta} = \Omega \quad (1)$$

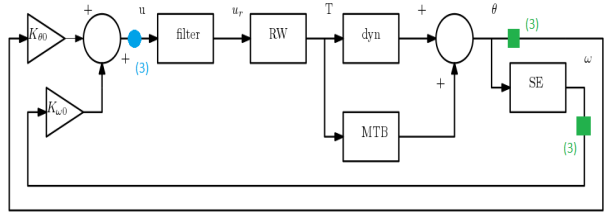


Figure 1: Classical attitude control scheme in closed-loop

where $\Theta \in \mathbb{R}^3$ is the vector of attitude angles, $\Omega \in \mathbb{R}^3$ is the vector of angular rates and $J \in \mathbb{R}^{3 \times 3}$ is the inertia of the satellite, whose diagonal coefficients are assumed uncertain.

The aim of the study is to build a controller which stabilizes the system for the highest range of inertia. In all the following, these notations will be used:

$$J = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{bmatrix}$$

$$J_{ii} \in [(1 - q_i)J_{ii,nom}; (1 + q_i)J_{ii,nom}]; \quad q_i \in [0 \quad \bar{q}]$$

For all $i \in \{x, y, z\}$, $J_{ii,nom}$ is known: $J_{xx,nom} = 31.38kg.m^2$, $J_{yy,nom} = 21.19kg.m^2$ and $J_{zz,nom} = 35.70kg.m^2$. For $j \neq i$, J_{ij} is known: $J_{xy} = J_{yx} = -1.114kg.m^2$, $J_{xz} = J_{zx} = -0.260kg.m^2$ and $J_{yz} = J_{zy} = -0.778kg.m^2$. Notice that the uncertainty range \bar{q} is the same on all the three axes. \bar{q} is to be maximized.

The "RW" block in Figure 1 is the model for the reaction wheel actuators. These are represented in the local linearization frame as three decoupled second order lowpass filters. In simulations the reaction wheels angular rates are saturated at $\pm 293rad/s$.

The three magneto-torquers controlling the angular momentum of the three reaction wheels are presented in Figure 2. Assuming that the target angular momentum is zero, the green square on the left of Figure 2 corresponds to the error between the real reaction wheel and the target kinetic momenta. A PI controller is then applied to this error, with a saturation

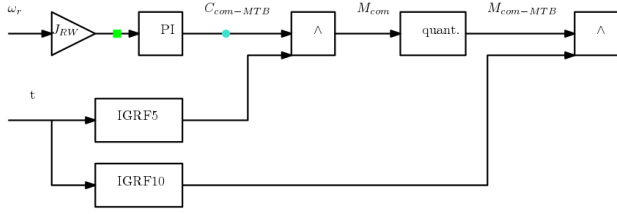


Figure 2: Momentum controller model

of the integral term. This gives a torque $C_{com-MTB}$. Apart from that, the local magnetic field B_{sat} is simulated at any time. The magnetic field model used is IGRF5. Then, the component of $C_{com-MTB}$ which is parallel to the local magnetic field B_{sat} is removed, so that it just remains the magnetic momentum M_{com} needed in order to control the magnetotorquers. The quantifier block aims at modeling the effect of the modulator, which is converting the momentum input into a number of 1/16s time slots activation. Due to the modulator, the magnetic momentum applied to the satellite is quantified with a $M_{max}/16$ step. This creates, together with the local magnetic field vector B_{sat} , but this time using IGRF10 model (which is more accurate but involves more calculations), a torque on the satellite $C_{MTB} = M_{com} \wedge B_{sat}$. This model reflects the reality and will be used for simulations. For the synthesis of the controllers (see section 3 and further), a linearized model provided by CNES is used.

Attitude control of the nominal satellite with inertia equal to J_{nom} is achieved by the combination of three identical speed estimators

$$\omega_i = \frac{s}{1 + \tau s} \theta_i$$

where the time constant τ is equal to 0.5s; three stabilizing filters with coefficients chosen by multi-objective H_2/H_∞ synthesis with pole placement in a LMI region ([18]); and a known static output feedback $u = K_{\Theta 0} \Theta + K_{\Omega 0} \Omega$.

Replacing the static output feedback by an adaptive output feedback has been the topic of [13], but the control of the reaction wheel angular momentum was not taken into account in the design process. Adap-

tion was only on attitude angle Θ and angular rate Ω . The particularity of this study lies in the fact that in addition to the six adaptive gains (three for the attitude and three for the angular rates), we also consider that the proportional gains of the magnetotorquers PI controllers (one per axis) in Figure 2 are time varying. Consequently, the considered system has six inputs (blue circles on Figures 1 and 2) and nine outputs (green squares on Figures 1 and 2). The corresponding static output feedback has the following form:

$$K_0 = \begin{bmatrix} K_{\Theta 0} & 0 & 0 & K_{\Omega 0} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{\Theta 0} & 0 & 0 & K_{\Omega 0} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{\Theta 0} & 0 & 0 & K_{\Omega 0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{p0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{p0} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{p0} \end{bmatrix} \quad (2)$$

3 Robust adaptive controller design

This part is mainly a recall of theoretical results detailed in [10] and [11], but they are essential for the understanding of their application to satellite attitude control. Let the following linear state-space model:

$$\begin{aligned} E_{xx}(q) \dot{x}(t) + E_{x\pi}(q) \pi(t) &= A(q)x(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (3)$$

where $x \in \mathbb{R}^{n_x}$ is the state of the plant, $u \in \mathbb{R}^{n_u}$ is the control input, $\pi \in \mathbb{R}^{n_\pi}$ is an auxiliary signal, $y \in \mathbb{R}^{n_y}$ is the output signal. $E_{xx}(q) \in \mathbb{R}^{n_x \times n_x}$, $E_{x\pi}(q) \in \mathbb{R}^{n_x \times n_\pi}$ and $A(q) \in \mathbb{R}^{n_x \times n_x}$ must be affine functions of the uncertain vector q , whose components are the uncertain parameters (here $q \in \mathbb{R}^3$ is composed by the diagonal coefficients of the inertia J). $B \in \mathbb{R}^{n_x \times n_u}$ and $C \in \mathbb{R}^{n_y \times n_x}$. Without loss of generality, the q -dependent matrices can be rewritten into a polytopic form: $E_{xx}(q) = E_{xx}(\delta) = \sum_{v=1}^V \delta_v E_{xx}^{[v]}$, $E_{x\pi}(q) = E_{x\pi}(\delta) = \sum_{v=1}^V \delta_v E_{x\pi}^{[v]}$ and

$A(q) = A(\delta) = \sum_{v=1}^V \delta_v A^{[v]}$. The polytope is defined by its V vertices $\delta_1 \dots \delta_V$, which are the $V (= 2^3 \text{ here})$ extremal combinations of the components of q . Every descriptor model with matrices rational with respect to the components of the uncertain vector can be rewritten into such a form. A proof of this result and general techniques to manage it in practice are given in [6].

The main idea of this study is to replace the given static output feedback by the following adaptive law:

$$u(t) = (K_0 + LK(t)R)y(t) \quad (4)$$

where $K(t) = \text{diag}(K_1(t), K_2(t), \dots)$, $L = [L_1 \ L_2 \ \dots]$, $R^T = [R_1^T \ R_2^T \ \dots]$. Low rank matrices L_k and R_k allow $LK(t)R$ to have the same form as K_0 in (2). The time-varying gains $K_k(t)$ evolve according to:

$$\begin{aligned} \dot{K}_k(t) &= \mathcal{I}_{D_k}(K_k(t), W_k(t)) \\ W_k(t) &= \gamma_k(-G_k y(t)(R_k y(t))^T - \sigma_k K_k(t)) \end{aligned} \quad (5)$$

where G_k and D_k are computed by solving LMIs ([13],[9]). The value of G_k gives the direction of the adaptation. D_k enters in the definition of the saturated integrator \mathcal{I}_{D_k} , which pushes $K_k(t)$ inside a set \mathcal{E}_k when it is at its border, so that the adaptive gains are bounded. The set \mathcal{E}_k is defined by:

$$K_k \in \mathcal{E}_k \Leftrightarrow \text{Tr}(K_k^T D_k K_k) \leq 1. \quad (6)$$

The forgetting factor σ_k drives $K_k(t)$ to zero when the output y is zero. The value of γ_k gives the variation speed of $K_k(t)$.

Theorem 3.1 ([11]) yields a mean to find appropriate values for G_k and D_k such that system (3) is robustly stable with controller (4):

Theorem 3.1 *Considering system (3), if there exist matrices $\hat{P}^{[v]} = \hat{P}^{[v]T}$ and \hat{S} such that the following condition holds for all $v \in \{1; V\}$:*

$$\begin{bmatrix} 0 & 0 & \hat{P}^{[v]} \\ 0 & 0 & 0 \\ \hat{P}^{[v]} & 0 & 0 \end{bmatrix} + \left\{ \hat{S} \begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} & -A_c^{[v]} \end{bmatrix} \right\}^S \prec 0 \quad (7)$$

where $A_c^{[v]} = A^{[v]} + BK_0C$, then:

- (i) *The closed-loop is robustly stable*
- (ii) *There exist matrices $P^{[v]}$, S , $G^T = [G_1^T, \dots, G_{\bar{k}}^T]$, $D = \text{diag}(D_1, \dots, D_{\bar{k}})$ and $\epsilon > 0$ such that the following equation holds $\forall v \in \{1; V\}$:*

$$\begin{bmatrix} 0 & 0 & P^{[v]} & 0 \\ 0 & 0 & 0 & 0 \\ P^{[v]} & 0 & \epsilon I + 2C^T R^T R C & -C^T G^T \\ 0 & 0 & -GC & -2D \end{bmatrix} + \left\{ S \begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} & -A_c^{[v]} & -BL \end{bmatrix} \right\}^S \prec 0. \quad (8)$$

Besides, the solution is such that the adaptive control (4) stabilizes the plant whatever positive values of σ_k , γ_k and for all $\delta \in \Delta^V := \{\delta \in \mathbb{R}^V : \delta \geq 0, \mathbf{1}^T \delta = 1\}$.

Conditions (7) imply that the static output feedback $u(t) = K_0 y(t)$ stabilizes the system for every value of the uncertain vector in the polytope Δ^V ([6]). An important remark is that checking the feasibility of LMIs (8) on the vertices of the plant with a common S is sufficient to prove the stability of the system with controller (4) for every value of the uncertain vector. This is possible thanks to the fact that the matrices of the system are affine functions of the uncertainty. A detailed proof of Theorem 3.1 is given in [10].

Theorem 3.1 only ensures that the designed adaptive controller is no less robust than the corresponding static output feedback. Theorem 3.2 proves that the adaptive controller is actually more robust than the static output feedback.

Theorem 3.2 *Consider the following matrix inequalities with $\check{P}^{[v]} \succ 0$ and $\check{\epsilon} > 0$ for all $v \in \{1; V\}$:*

$$\begin{bmatrix} 0 & 0 & \check{P}^{[v]} & 0 \\ 0 & 0 & 0 & 0 \\ \check{P}^{[v]} & 0 & \check{\epsilon} I + 2C^T R^T R C + \{C^T R^T F^{[v]} G C\}^S & -C^T G^T \\ 0 & 0 & -GC & -2D \end{bmatrix} + \left\{ S \begin{bmatrix} E_{xx}^{[v]} & E_{x\pi}^{[v]} & -A_{c\Delta}^{[v]} & -BL \end{bmatrix} \right\}^S \prec 0 \quad (9)$$

and

$$\begin{bmatrix} T_k & F_k^{[v]T} D_k \\ D_k F_k^{[v]} & D_k \end{bmatrix} \succeq 0, \text{Tr}(T_k) \leq 1 \quad \forall k \in \{1; \bar{k}\} \quad (10)$$

where $A_{c\Delta}^{[v]} = A^{[v]} + B(K_0 + LF^{[v]}R)C$.

These constraints are such that:

- (i) For fixed K_0 , G , S and $D = \text{diag}(D_1, \dots, D_{\bar{k}})$ the constraints are LMIs in $\check{P}^{[v]}$, $\check{\xi}$ and $F^{[v]}$.
- (ii) For K_0 , G , S and D solution to constraints in Theorem 3.1, LMIs (9) and (10) are feasible.
- (iii) If the constraints are feasible, then for all $k \in \{1; \bar{k}\}$, $F(\delta)$ is such that $\text{Tr}(F_k(\delta)D_kF_k(\delta)) \leq 1$ and $u(t) = (K_0 + LF(\delta)R)y(t)$ stabilizes the plant (3) for any value of the uncertainty δ , where $F(\delta) = \sum_{v=1}^V \delta_v F^{[v]}$.
- (iv) If the constraints are feasible, then whatever positive γ_k the adaptive control (4) robustly stabilizes the set of the states x such that $x = 0$ when all $\sigma_k = 0$ and robustly stabilizes a neighborhood of this same set when at least one $\sigma_k > 0$.

Here again, a detailed proof of Theorem 3.2 is given in [11] and follows the same lines as the one of Theorem 3.1.

The most important point is that the robust stability of the system with the static output feedback is not required anymore. This strong improvement will be used in the next section.

4 Application to attitude control with magneto-torquers - no worse robustness

4.1 Design of the controller

The first step is to write the system modelised in Figure 1 into the form of (3). We get 2^3 systems (one for each extremal value of the components of J) that we call (S). In this subsection only, the maximal uncertainty around every component of J ,

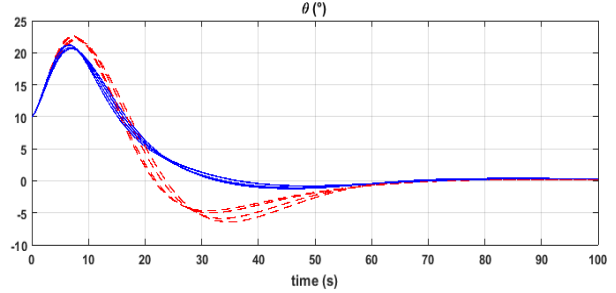


Figure 3: Attitude angle θ_z . Dashed lines: with SOF. Solid lines: with adaptive controller

\bar{q} , is fixed and equal to 30%, which is a classical assumption. The main objective of the study will be reached in the next subsection.

The only requirement to apply Theorem 3.1 is the knowledge of a static output feedback which satisfies (7). K_0 in (2), whose numerical value is known, is used in all the following. We check that (7) is satisfied with a common matrix \hat{S} for the 8 systems of (S). Then, we apply Theorem 3.1 and obtain numerical values for parameters G_k , D_k , σ_k and γ_k . LMIs (8) contain 5845 variables, 552 rows and are solved in 77 sec.

4.2 Simulations results

Time variations of attitude angle θ_z and of reaction wheel speed ω_{rz} along z-axis are given in Figures 3 and 4. The same results are observable for the x and y axes. Dashed blue curves are with the static output feedback (2) and solid red curves are with adaptive controller (4). Each curve corresponds to one random value of the inertia in the polytope Δ^V . The same random inertias have been used to plot the blue and the red curves. Initial conditions are the same for all the simulations: $\Theta_0 = [10 \ 10 \ 10]^\circ$; $\Omega_0 = [-0.01 \ -0.01 \ -0.01]$ and $\Omega_{r0} = [290 \ 290 \ 290] \text{rad/s}$, that is, very close to the saturation.

Time variations of the attitude and reaction wheel speed are smoother with the adaptive controller (4) than with the static output feedback (2). More-

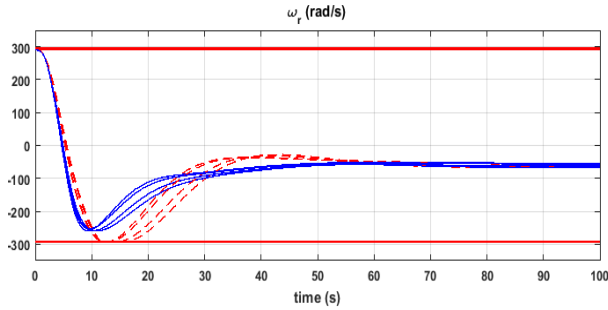


Figure 4: Reaction wheel speed ω_{rz} . Dashed lines: with SOF. Solid lines: with adaptive controller

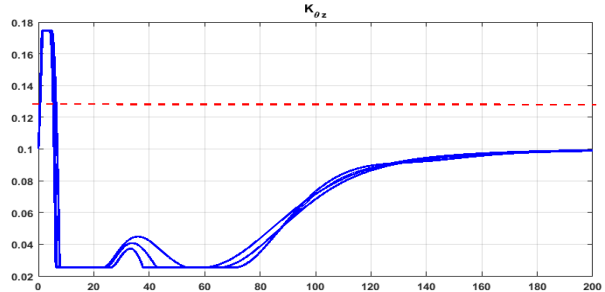


Figure 5: Attitude gain $K_{\theta z}$. Dashed lines: with SOF. Solid lines: with adaptive controller

over, there are less differences between the solid red curves than between the dashed blue curves: Adaptive controller (4) seems more robust than the corresponding static output feedback (2). The most important remark is that the reaction wheel speed saturates very quickly with the static output feedback, whatever the real value of the inertia. In opposite, it never saturates with the adaptive controller. The adaptive controller designed in [13], with adaptation only on attitude angle and angular speed, could not avoid the saturation of the reaction wheels. This improvement is due to the fact that we made the proportional gains of the PI controllers of the magneto-torquers time-varying and then evolving depending on the measurements in real time. The fact that the reaction wheel rate w_r does not converge to zero is due to the presence of external disturbances in the simulated model. Besides, the torque $C_{com-MTB}$ never reaches its saturation limit. Finally, it can be mentioned that the initial position of the satellite along its orbit does not affect the efficiency of the adaptive controller.

Time variations of the adaptive gains along z-axis are given in Figures 5, 6 and 7. Blue curves, corresponding to the static output feedback, are logically constant whereas those standing for the adaptive controller are time-varying. It is clear that all the adaptive gains are bounded.

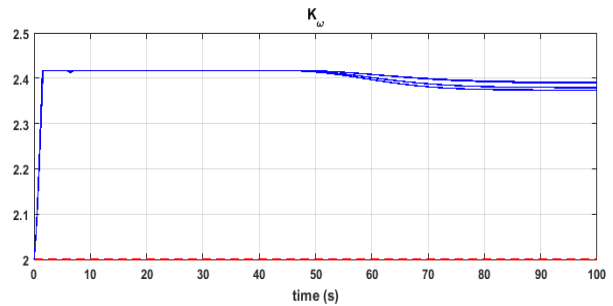


Figure 6: Angular speed gain $K_{\omega z}$. Dashed lines: with SOF. Solid lines: with adaptive controller

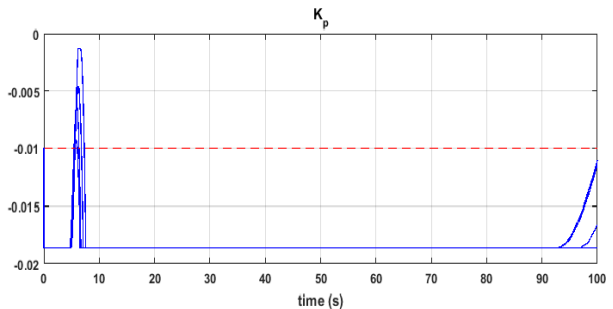


Figure 7: Magneto-torquer gain K_{pz} . Dashed lines: with SOF. Solid lines: with adaptive controller

5 Application to attitude control with magneto-torquers - improved robustness

In this section, the value of the maximal uncertainty \bar{q} on the components of the inertia of the satellite is not fixed anymore (it was of 30% in section 4). Nevertheless, it is still assumed that the range is the same along the three axes.

As said in section 3, the strength of Theorem 3.2 lies in the fact that the static output feedback does not need to robustly stabilize the system anymore. But rather than modifying the value of the static output feedback, the following method is applied to system (S), with static output feedback (2):

- The values of parameters G_k , D_k and S are fixed and equal to the ones computed in section 4, when solving LMIs (8).
- Using (i) in Theorem 3.2, we get that inequalities (9) are LMIs.
- We find \bar{q} , the maximal value for which LMIs (9) are feasible for system (S) with inertia $J_{ii} \in [(1 - \bar{q})J_{ii,nom}; (1 + \bar{q})J_{ii,nom}]$.

Based on (ii) in Theorem 3.2 and application of Theorem 3.1 in section 4, we know that $\bar{q} \geq 30\%$. LMIs (7) with static output feedback (2) become unfeasible when the diagonal components of J have 34% of uncertainty. Yet, as LMIs (7) are only sufficient conditions, their unfeasibility is not enough to conclude about the instability of the closed-loop. However, system (3) without uncertainty, with diagonal components of J increased by 34% from their nominal value is unstable. Consequently, 34% of uncertainty is the limit for the robust stability of system (3) with static output feedback (2).

By simple dichotomy method, we get $\bar{q} = 89\%$, meaning that adaptive control (4) can stabilize system (S) when considering that the inertia has 89% of uncertainty. One application of Theorem 3.2 lasts 36 sec on average. LMIs of Theorem 3.2 contain 3731 variables and 579 rows.

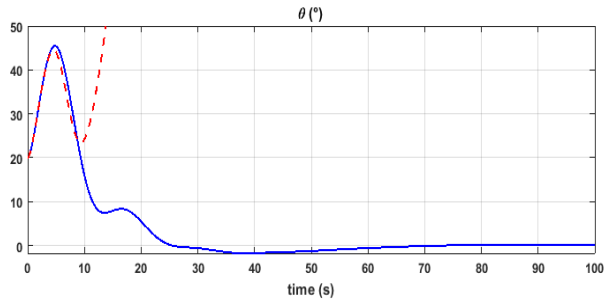


Figure 8: Attitude angle θ_z . Dashed lines: with SOF. Solid lines: with adaptive controller

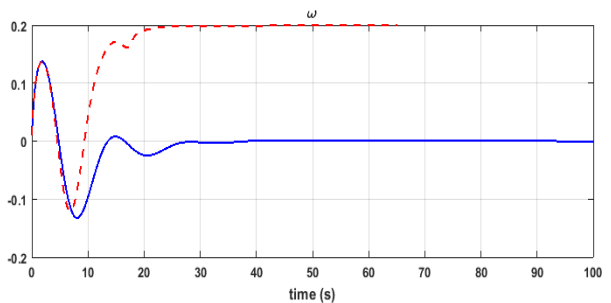


Figure 9: Angular rate ω_z . Dashed lines: with SOF. Solid lines: with adaptive controller

In order to illustrate this result, time variations of the states of the system are plotted in Figures 8 (attitude angle θ), 9 (angular speed ω) and 10 (reaction wheel speed ω_r). Simulation parameters are the same as in section 4 and here again, similar results have been obtained for x and y axes.

On one hand, curves in dashed red lines clearly show that the static output feedback (2) does not robustly stabilize system (S) for $q_z = \bar{q}$. Attitude angle θ diverges and angular and reaction wheel rates saturate after 20 seconds of simulation. This is not surprising since the robust stabilization of the system with the static output feedback is not a requirement to apply Theorem 3.2. By the way, LMIs (7) are not satisfied for $q_z = \bar{q}$. On the other hand, curves in solid blue lines attest to the fact that with adaptive control (4) with parameters of Theorem 3.2, the

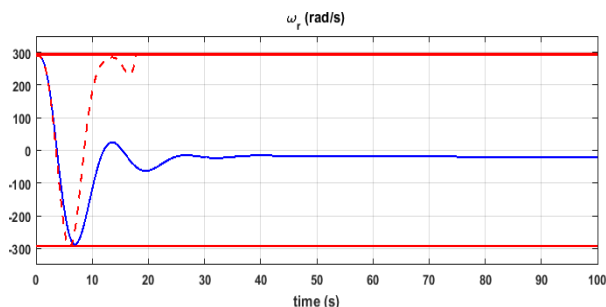


Figure 10: Reaction wheel rate ω_{rz} . Dashed lines: with SOF. Solid lines: with adaptive controller

states of system (S) converge at least to a neighborhood of the origin. Notice nonetheless that in that case, overshoots on θ , ω and ω_r are not avoided.

6 Conclusion

A robust adaptive attitude controller has been designed for satellite with magnetotorquers. Information yielded by the magnetotorquers is fully used, adaptation being made not only on attitude angle and angular rate, but also on reaction wheel torque. The adaptive controller turns out to be very robust with respect to parametric uncertainties. It allows to stabilize the satellite with a considered uncertainty range on its inertia of $\pm 89\%$. The case when the cross coefficients of the inertia are uncertain will be considered in a future work. From a practical point of view, this can take place in the context of a shock between the satellite and another spacecraft or debris, or during the deployment/folding of the panels of the satellite. In such cases, the inertia of the satellite can be subject to important and sudden changes, making it unknown at much more than classical $\pm 30\%$.

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