



**HAL**  
open science

## Simulating Future Test and Redesign Considering Epistemic Model Uncertainty

Nathaniel B. Price, Mathieu Balesdent, Sébastien Defoort, Rodolphe Le Riche, Nam H. Kim, Raphael T. Haftka

► **To cite this version:**

Nathaniel B. Price, Mathieu Balesdent, Sébastien Defoort, Rodolphe Le Riche, Nam H. Kim, et al.. Simulating Future Test and Redesign Considering Epistemic Model Uncertainty. 18th AIAA Non-Deterministic Approaches Conference (AIAA SCITECH 2016), Jan 2016, SAN DIEGO, United States. 10.2514/6.2016-0950 . hal-01386664

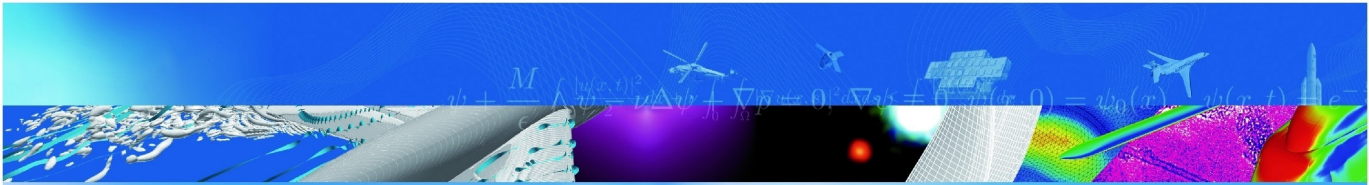
**HAL Id: hal-01386664**

**<https://hal.science/hal-01386664v1>**

Submitted on 14 Nov 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



COMMUNICATION A CONGRES

**Simulating future test and redesign  
considering epistemic model  
uncertainty**

N.-B. Price (ONERA), M. Balesdent (ONERA),  
S. Defoort (ONERA), R. Le Riche, N.-H. Kim,  
R.-T. Haftka

AIAA SCITECH 2016  
SAN DIEGO, U.S.A  
4-8 janvier 2016

TP 2016-601

**70** 2016  
ans

**ONERA**

THE FRENCH AEROSPACE LAB



# Simulating future test and redesign considering epistemic model uncertainty

Nathaniel B. Price\*, Mathieu Balesdent†, Sébastien Defoort‡,  
Rodolphe Le Riche§, Nam H. Kim¶, Raphael T. Haftka||

\*†‡ *ONERA - The French Aerospace Lab, Palaiseau, France*

\*¶|| *University of Florida, Gainesville, Florida, USA*

\*§ *École des Mines de Saint-Étienne, Saint-Étienne, France*

§ *CNRS LIMOS UMR 6158, Saint-Étienne, France*

At the initial design stage engineers often rely on low-fidelity models that have high epistemic uncertainty. Traditional safety-margin-based deterministic design resorts to testing to reduce epistemic uncertainty and achieve targeted levels of safety. Testing is used to calibrate models and prescribe redesign when tests are not passed. After calibration, reduced epistemic model uncertainty can be leveraged through redesign to restore safety or improve design performance; however, redesign may be associated with substantial costs or delays. In this paper, a methodology is described for optimizing the safety-margin-based design, testing, and redesign process to allow the designer to tradeoff between the risk of future redesign and the possible performance and reliability benefits. The proposed methodology represents the epistemic model uncertainty with a Kriging surrogate and is applicable in a wide range of design problems. The method is illustrated on a cantilever beam design problem where there is mixed epistemic model error and aleatory parameter uncertainty.

## Nomenclature

$\mathbf{x}$	Design variable vector
$\mathbf{U}$	Aleatory random variable vector
$e(\cdot, \cdot)$	Model error
$f(\cdot)$	Objective function
$g(\cdot, \cdot)$	Limit-state function
$n$	Safety margin
$q$	Redesign indicator function
$p_{re}$	Probability of redesign
$p_f$	Probability of failure
$E_{\Omega}[\cdot]$	Expected value with respect to epistemic uncertainty
$Pr_U[\cdot]$	Probability with respect to aleatory uncertainty

### Subscripts

$L$	Low-fidelity model
$H$	High-fidelity model
$det$	Deterministic value
$ini$	Initial design

---

\*Graduate Research Assistant, Mechanical & Aerospace Engineering, natprice@ufl.edu, AIAA Student Member

†Research Engineer, mathieu.balesdent@onera.fr

‡Research Engineer, sebastien.defoort@onera.fr

§CNRS Permanent Research Associate, leriche@emse.fr

¶Associate Professor, Mechanical & Aerospace Engineering, nkim@ufl.edu, AIAA Associated Fellow

||Distinguished Professor, Mechanical & Aerospace Engineering, haftka@ufl.edu, AIAA Fellow

<i>re</i>	Design after redesign
<i>final</i>	Final design after possible redesign
<i>lb</i>	Lower bound
<i>ub</i>	Upper bound

#### *Superscripts*

( <i>i</i> )	Realization of epistemic random variable or function
*	Target value in optimization

#### *Accents*

^	Epistemic random variables or functions
-	Mean prediction of Kriging model

## I. Introduction

At the initial design stage engineers often rely on low-fidelity models that have high uncertainty. This model uncertainty is reducible and is classified as epistemic uncertainty; uncertainty due to variability is irreducible and classified as aleatory uncertainty. Both forms of uncertainty can be implicitly compensated for using conservativeness such as conservative material properties, conservative limit loads, safety margins, and safety factors. However, if the design is too conservative then typically performance will suffer. Traditional safety-factor-based deterministic design has relied on testing in order to reduce epistemic uncertainty and achieve high levels of safety. Testing is used to calibrate models and prescribe redesign when tests are not passed. After calibration, reduced epistemic model uncertainty can be leveraged through redesign to restore safety or improve design performance; however, redesign may be associated with substantial costs or delays. Integrated optimization of the design, testing, and redesign process can allow the designer to tradeoff between the risk of future redesign and the possible performance and reliability benefits. Previous work has illustrated this tradeoff when there is only a fixed constant model bias.<sup>1-3</sup> This study builds on previous work by considering spatial correlation in the epistemic model uncertainty. A Kriging surrogate is used to provide a flexible representation of the epistemic model uncertainty that allows the method to be applicable to a wide range of engineering problems.

In this study, the epistemic model uncertainty is treated separately from the aleatory parameter uncertainty in the model inputs. This results in the challenging task of propagating aleatory uncertainty through an uncertain model. Furthermore, in order for the method to be applicable under current safety-factor-based design regulations,<sup>4</sup> a traditional deterministic safety-margin-based design approach is considered. Some studies have used the parallels between safety-factor-based design and reliability-based design optimization (RBDO) approaches to reduce computational cost of RBDO.<sup>5-7</sup> However, these studies have not considered epistemic model uncertainty. When there is only epistemic model uncertainty a safety margin balances the need for the final design to be feasible while at the same time not being so conservative that design performances suffer.<sup>8</sup> Few studies have considered the effects of both aleatory parameter uncertainty and epistemic model uncertainty. Mahadevan and Rebba have shown that failing to account for epistemic model uncertainty may lead to an overestimation of reliability and unsafe designs or underestimation of the reliability and designs that are more conservative than needed.<sup>9</sup> Studies that use surrogate models in RBDO also encounter a situation of mixed uncertainty. However, unlike this study where we are interested in epistemic model uncertainty as an inherent part of the low-fidelity model, these studies are usually motivated by a desire to reduce computational cost. Kim and Choi have shown that when using response surfaces in RBDO the epistemic model uncertainty results in uncertainty in the reliability index and additional sampling can be used to avoid being overly conservative.<sup>10</sup>

One of the important aspects of this study is the integration of the design and testing process: the effects of a future test and possible redesign are considered while optimizing the initial design. Since the test will be performed in the future, the test result is an epistemic random variable. Predicting possible test results requires a probabilistic formulation of the relationship between the low-fidelity model prediction, the true value, and the test result. In the context of calibrating computer models, Kennedy and O'Hagan proposed that the true model can be related to a computer model by multiplying by a constant scale parameter and adding a discrepancy function.<sup>11</sup> Similar formulations have subsequently been applied in many

other studies.<sup>12–17</sup> These formulations are similar in that they all relate the true model to the low-fidelity model by adding an uncertain discrepancy function. The formulations differ in the representation of the scale parameter. Methods range from omitting the scale parameter<sup>13,14</sup> to considering an uncertain scaling function.<sup>16</sup> In this study we consider only an uncertain discrepancy function to formulate the relationship between the high-fidelity model and the low-fidelity model. The uncertain discrepancy function is constructed in the joint design and aleatory input space in order to have epistemic model uncertainties that are correlated with respect to design and aleatory inputs.

In addition to the integration of design and testing, this study also seeks to integrate a redesign process. Redesign refers to changing the design variables conditional on the test result. Since the future test result is an epistemic random variable the design variable after redesign is also random variable. Villanueva et al. developed a method for simulating the effects of future tests and redesign when there is a constant but unknown model bias in the calculation and measurement.<sup>18</sup> In the context of constant model bias, Matsumura et al. compared RBDO considering future redesign to traditional RBDO.<sup>1</sup> Villanueva et al. also studied the tradeoff between future redesign and performance for an integrated thermal protection system.<sup>2</sup> Price et al. compared starting with a more conservative design and possibly redesigning to improve performance to starting with a less conservative design and possibly redesigning to improve safety.<sup>3</sup> These studies have demonstrated that integrated optimization of design, testing, and redesign can be used to manage redesign risk and tradeoff between the probability of future redesign and design performance. However, the assumption of constant model bias in these studies severely limits the types of problems where the method is applicable. In order to apply the method to a broader range of general engineering problems this study uses a Kriging model to represent model uncertainty whose conditional simulations allow uncertainty propagation.

In section II the general method of simulating a future test and possible redesign is described. In section III the demonstration example of a cantilever beam is described. In section IV the study is summarized and the implications of the method and results are discussed.

## II. Methods

The design, testing, and redesign process is formulated deterministically in terms of an initial safety margin  $n_{ini}$ , lower and upper bounds on acceptable safety margin  $n_{lb}$  and  $n_{ub}$ , and a redesign safety margin  $n_{re}$ . In section A the formulation of the optimization of the safety margins is presented. For each set of safety margins, a Monte Carlo simulation (MCS) of epistemic error realizations is performed as described in section B. A single sample in the MCS consists of a complete deterministic design / redesign process as described in section C. The results of the MCS are used to calculate the probability of redesign, expected probability of failure, and expected design cost as described in section D.

### A. Optimization of safety margins

The deterministic design process is controlled by a vector of safety margins  $\mathbf{n} = \{n_{ini}, n_{lb}, n_{ub}, n_{re}\}$ . The safety margins are optimized to minimize expected design cost while satisfying constraints on expected probability of failure and probability of redesign. The optimization of the safety margins is formulated as

$$\begin{aligned} \min_{\mathbf{n}} \quad & E_{\Omega} \left[ f(\hat{\mathbf{X}}_{final}) \right] \\ \text{s.t.} \quad & E_{\Omega} \left[ \hat{P}_{f,final} \right] \leq p_f^* \\ & p_{re} \leq p_{re}^* \end{aligned} \quad (1)$$

where  $E_{\Omega}[\cdot]$  is used to denote the expectation with respect to epistemic uncertainty,  $f(\cdot)$  is a cost function,  $\hat{\mathbf{X}}_{final}$  is a distribution of possible final designs,  $\hat{P}_{f,final}$  is a distribution of final probability of failure, and  $p_{re}$  is the probability of redesign. The final design  $\hat{\mathbf{X}}_{final}$  is an epistemic random variable because the design may be modified conditional on the future test result which is unknown at the initial design stage. The final probability of failure is an epistemic random variable because the final design is uncertain and because there is epistemic model uncertainty in the limit-state function  $g(\cdot, \cdot)$ . The tradeoff between expected cost and probability of redesign is captured by solving the single objective optimization problem for several values of the constraint  $p_{re}^*$ . The global optimization of the safety margins is performed using Covariance Matrix Adaptation Evolution Strategy (CMA-ES)<sup>19</sup> and a penalization strategy to handle the constraints. The optimizer calls a subfunction to perform a MCS of the deterministic design process as shown in figure 1. The

MCS of the deterministic design process is used to calculate the distribution of possible final designs, the distribution of final probabilities of failure, and the probability of redesign. To reduce the computational cost of optimizing the safety margins, surrogate models can be fit for the expected cost and expected probability of failure as a function of the safety margins (see appendix).

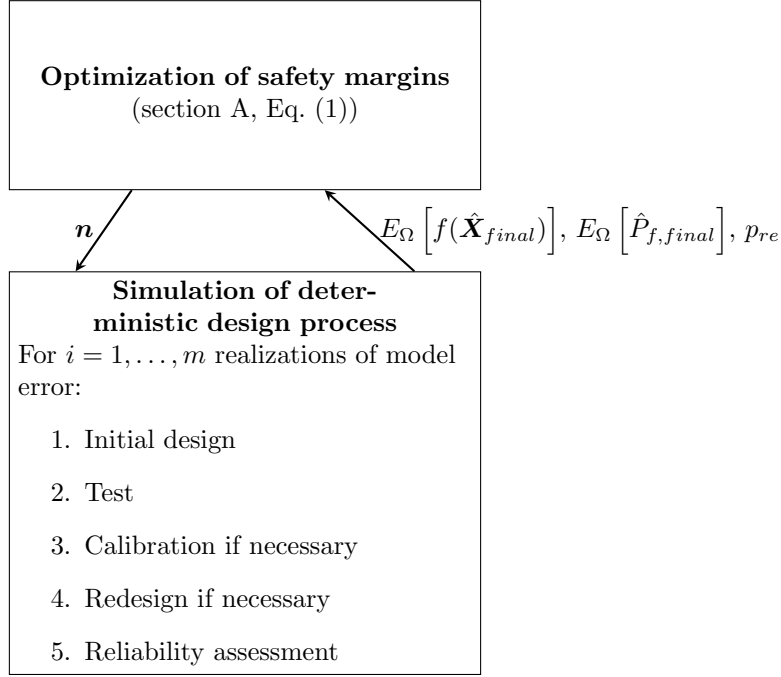


Figure 1: The optimization of the safety margins is based on a MCS of the deterministic design process

In this study, we define two different triggers for redesign. We will refer to redesign triggered by a low safety margin (less than  $n_{lb}$ ) as redesign for safety and redesign triggered by a high safety margin (greater than  $n_{ub}$ ) as redesign for performance. To force only redesign for safety the upperbound on acceptable safety margins can be removed from the optimization by setting  $n_{ub} = +\infty$ . To force only redesign for performance, the lowerbound on acceptable safety margins can be removed from the optimization by setting  $n_{lb} = -\infty$ . Considering only redesign for safety or only redesign for performance are special cases of the general formulation where all the safety margins  $\mathbf{n} = \{n_{ini}, n_{lb}, n_{ub}, n_{re}\}$  are optimized simultaneously.

## B. Monte-Carlo Simulation of Epistemic Model Error

The epistemic model uncertainty and aleatory parameter uncertainty are treated separately (see<sup>20-22</sup>). The true relationship between the models is assumed to be of the form

$$g_H(\mathbf{x}, \mathbf{u}) = g_L(\mathbf{x}, \mathbf{u}) + e(\mathbf{x}, \mathbf{u}) \quad (2)$$

where  $\mathbf{x} \in \mathbb{R}^d$  is a vector of design variables,  $\mathbf{U}$  is a vector of aleatory random variables with a realization  $\mathbf{u} \in \mathbb{R}^p$ ,  $g_H(\cdot, \cdot)$  is the high-fidelity model,  $g_L(\cdot, \cdot)$  is the low-fidelity model, and  $e(\cdot, \cdot)$  is the error between the low-fidelity and high-fidelity models. Typically, the error  $e(\cdot, \cdot)$  is unknown. The uncertainty in the model error is represented as a Kriging model  $\hat{E}(\cdot, \cdot)$ . The hat accent on the error is used to differentiate between the random distribution of possible error  $\hat{E}(\cdot, \cdot)$ , and the unknown, deterministic error  $e(\cdot, \cdot)$ . Based on the possible model errors the high-fidelity model is predicted as

$$\hat{G}_H(\mathbf{x}, \mathbf{u}) = g_L(\mathbf{x}, \mathbf{u}) + \hat{E}(\mathbf{x}, \mathbf{u}) \quad (3)$$

The Kriging model for the calculation error is constructed in the joint space of the aleatory variables,  $\mathbf{u}$ , and the design variables,  $\mathbf{x}$ . The uncertainty in  $\hat{G}_H(\mathbf{x}, \mathbf{u})$  in Eq. (3) is only due to epistemic model error  $\hat{E}(\cdot, \cdot)$ . Propagation of aleatory uncertainty  $\mathbf{U}$  through the uncertain model is discussed in section D. For simplicity

of notation, we will define the mean of the Kriging prediction for the error as  $\bar{e}(\cdot, \cdot)$  and the mean prediction of the high-fidelity model as

$$\bar{g}_H(\mathbf{x}, \mathbf{u}) = g_L(\mathbf{x}, \mathbf{u}) + \bar{e}(\mathbf{x}, \mathbf{u}) \quad (4)$$

The epistemic random function  $\hat{E}(\cdot, \cdot)$  is used to represent the lack of knowledge regarding how well the low-fidelity model matches the high-fidelity model. Assuming initial test data is available, maximum likelihood estimation (MLE) will be used to estimate the parameters of the Kriging model. The prediction  $\hat{G}_H(\cdot, \cdot)$  is viewed as a distribution of possible functions. Samples or trajectories drawn from this distribution that are conditional on initial test data are referred to as conditional simulations. In the absence of test data these realizations are unconditional simulations. These simulations are spatially consistent Monte Carlo simulations. Let  $\hat{g}_H^{(i)}(\cdot, \cdot)$  denote the  $i$ -th realization of  $\hat{G}_H(\cdot, \cdot)$  based on a realization  $\hat{e}^{(i)}(\cdot, \cdot)$  of the Kriging model  $\hat{E}(\cdot, \cdot)$ . A variety of methods exist for generating these conditional simulations.<sup>23</sup> In this study, the conditional simulations are generated directly based on Cholesky factorization of the covariance matrix using the STK Matlab toolbox for Kriging<sup>24</sup> and by sequential conditioning.<sup>23</sup>

We can consider a Monte Carlo simulation of  $m$  conditional simulations  $i = 1, \dots, m$  corresponding to  $m$  possible futures. In practice, the sample size  $m$  is increased until the estimated coefficient of variation of the quantity of interest, such as expected probability of failure, is below a certain threshold. Let  $\Omega$  denote the epistemic uncertainty space of the model  $\hat{G}_H(\cdot, \cdot)$ . There is a realization,  $\exists \omega \in \Omega$ , such that the simulation,  $\hat{g}_H^{(\omega)}(\cdot, \cdot)$ , is arbitrarily close to the true model,  $g_H(\cdot, \cdot)$ . The design process conditional on one error realization is described in section C. By repeating the design process for many different error realizations (i.e. for different possible high-fidelity models through Eq. (3)) we can determine the distribution of possible final design outcomes. From the MCS, it is possible to estimate the risk of redesign and to predict how failing a test relates to final design performance or safety. This can in turn be used to optimize the safety margins that govern the deterministic design process.

### C. Deterministic Design Process

The deterministic design process is controlled by a vector of safety margins  $\mathbf{n}$ . There is an initial safety margin  $n_{ini}$ , lower and upper bounds on acceptable safety margin  $n_{lb}$  and  $n_{ub}$ , and a redesign safety margin  $n_{re}$ . First, an initial design is found based on deterministic optimization using the mean model prediction and a safety margin  $n_{ini}$ . Then, the optimum design is evaluated using the high-fidelity model to calculate the true safety margin with respect to  $g_H(\cdot, \cdot)$ . Based on the high-fidelity evaluation, the designer will consider the test passed and keep the initial design if the safety margin is greater than  $n_{lb}$  and less than  $n_{ub}$ . The lower bound  $n_{lb}$  is used to initiate redesign when the initial design is revealed to be unsafe. The upper bound  $n_{ub}$  is used to initiate redesign when the initial design is revealed to be so conservative that it is worthwhile to redesign to improve performance. If the test is failed, a calibration process is performed to update the model based on the test result. Finally, if redesign is performed a new design is found by performing deterministic optimization using the calibrated model and a safety margin  $n_{re}$ .

However, the future high-fidelity evaluation of the initial design (i.e. future test) is unknown and therefore modeled as an epistemic random variable. The redesign decision, calibration, and redesign optimum are conditional on a particular test result. In sections 1 to 3 the process is described conditional on the error realization  $\hat{E}(\cdot, \cdot) = \hat{e}^{(i)}(\cdot, \cdot)$ .

#### 1. Initial design

The design problem is formulated as a deterministic safety-margin-based optimization problem

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \bar{g}_H(\mathbf{x}, \mathbf{u}_{det}) - n_{ini} \geq 0 \end{aligned} \quad (5)$$

where  $\bar{g}_H(\cdot, \cdot)$  is the mean of the predicted high-fidelity model,  $n_{ini}$  is the initial safety margin,  $\mathbf{u}_{det}$  is a vector of conservative deterministic values used in place of aleatory random variables, and  $f(\mathbf{x})$  is a known deterministic objective function. We assume the limit-state function is formulated such that failure is defined as  $g(\cdot, \cdot) < 0$ . Let  $\mathbf{x}_{ini}$  denote the optimum design found from Eq. (5) using initial safety margin  $n_{ini}$ . There is no uncertainty in the initial design  $\mathbf{x}_{ini}$  because the optimization problem is defined using the mean of the model prediction and fixed conservative values,  $\mathbf{u}_{det}$ , are used in place of aleatory random variables.



## 2. Testing initial design and redesign decision

A possible high-fidelity evaluation,  $\hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det})$ , of the initial design  $\mathbf{x}_{ini}$  is simulated. The test will be passed if  $n_{lb} \leq \hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det}) \leq n_{ub}$ . If the measured safety margin is too low ( $\hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det}) < n_{lb}$ ) then the design is unsafe and redesign should be performed to restore safety. If the safety margin is too high ( $\hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det}) > n_{ub}$ ) then the design is too conservative and it may be worth redesigning to improve performance. Let  $\hat{q}^{(i)}$  denote an indicator function for the redesign decision that is 1 for redesign and 0 otherwise. We will refer to redesign triggered by a low safety margin as redesign for safety and redesign triggered by a high safety margin as redesign for performance. If the test is not passed then redesign should be performed to select a new design.

## 3. Calibration and redesign

To obtain the calibrated model, the test realization  $\hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det})$  corresponding to the error instance  $\hat{e}^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det})$  is treated as a new data point and the error instance is added to the design of experiment for the error model. The updated mean of the predicted high-fidelity model is

$$\bar{g}_{H,calib}^{(i)}(\mathbf{x}, \mathbf{u}) = E_{\Omega} \left[ \hat{G}_H(\mathbf{x}, \mathbf{u}) | \hat{G}_H(\mathbf{x}_{ini}, \mathbf{u}_{det}) = \hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det}) \right] \quad (6)$$

The redesign problem is formulated as a deterministic safety-margin-based optimization problem

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \bar{g}_{H,calib}^{(i)}(\mathbf{x}, \mathbf{u}_{det}) - n_{re} \geq 0 \end{aligned} \quad (7)$$

where the mean of the predicted high-fidelity model  $\bar{g}_{H,calib}^{(i)}(\cdot, \cdot)$  is calibrated conditional on the test result  $\hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{u}_{det})$  and  $n_{re}$  is a new safety margin that may be different than  $n_{ini}$ . Let  $\hat{\mathbf{x}}_{re}^{(i)}$  denote the optimum design after redesign found from Eq. (7) using the calibrated model and safety margin  $n_{re}$ .

Comparing the initial design problem in Eq. (5) to the redesign problem in Eq. (7), we see that there is a change in the feasible design space. One change is controlled by the safety margin  $n_{re}$ , but there is also a change based on the calibrated model used to calculate the safety margin. For example, if we choose  $n_{ini} = n_{re}$  then it is still possible for the feasible design space to increase or decrease based on the calibration. If the feasible design space increases then some high performance designs that were considered infeasible before the test may become feasible. Alternatively, the feasible design space may be reduced leading to worse design performance. This relationship between the possible change in feasible design space and the performance is precisely the change we are interested in modeling in order to select the safety margins.

## D. Probabilistic Evaluation

A vector of safety margins  $\mathbf{n}$  is associated with a probability of redesign  $p_{re}$  and a distribution of final designs  $\hat{\mathbf{X}}_{final}$  that translates into a distribution of probability of failure after possible redesign  $\hat{P}_{f,final}$ , and a distribution of design cost  $f(\hat{\mathbf{X}}_{final})$ . The distributions are approximated based on a Monte Carlo simulation of  $m$  error realizations  $i = 1, \dots, m$  as described in section B.

The probability of redesign is  $p_{re} = E_{\Omega} [\hat{Q}]$  where  $\hat{Q}$  is the indicator function for the redesign decision. The final design after possible redesign is

$$\hat{\mathbf{x}}_{final}^{(i)} = (1 - \hat{q}^{(i)}) \mathbf{x}_{ini} + \hat{q}^{(i)} \hat{\mathbf{x}}_{re}^{(i)} \quad (8)$$

Recall, that  $\hat{q}^{(i)} = 1$  corresponds to failing the test and performing redesign. The expected design cost after possible redesign is  $E_{\Omega} [f(\hat{\mathbf{X}}_{final})]$ . Since the redesign decision defines a partitioning of the epistemic outcome space, the law of total expectation allows the expectation to be written as

$$E_{\Omega} [f(\hat{\mathbf{X}}_{final})] = (1 - p_{re}) f(\mathbf{x}_{ini}) + p_{re} E_{\Omega} [f(\hat{\mathbf{X}}_{re})] \quad (9)$$

where  $f(\mathbf{x}_{ini})$  is the expected design cost conditional on the test being passed and the designer keeping the initial design and  $E_{\Omega} [f(\hat{\mathbf{X}}_{re})]$  is the expected design cost conditional on the test being failed and the designer performing redesign.

The true probability of failure of the final design is unknown since there is epistemic uncertainty in the model  $\hat{G}_H(\cdot, \cdot)$ . A realization of the probability of failure is calculated conditional on an error realization  $\hat{E}(\cdot, \cdot) = \hat{e}^{(i)}(\cdot, \cdot)$ . A realization of the probability of failure for the initial design is

$$\hat{p}_{f,ini}^{(i)} = Pr_U \left[ \hat{g}_H^{(i)}(\mathbf{x}_{ini}, \mathbf{U}) < 0 \right] \quad (10)$$

where  $Pr_U[\cdot]$  denotes the probability with respect to aleatory uncertainty. Note that the epistemic model uncertainty is treated separately from the aleatory uncertainty to distinguish between the quantity of interest, the probability of failure with respect to the high-fidelity model and aleatory uncertainty, and the lack of knowledge regarding this quantity. The error in the low-fidelity model  $\hat{E}(\cdot, \cdot)$  has no impact on the reliability with respect to the high-fidelity model  $g_H(\cdot, \cdot)$ . However, since the high-fidelity model is unknown, the probability of failure calculation is repeated many times conditional on all possible realizations of the high-fidelity model  $\hat{g}_H^{(i)}(\cdot, \cdot)$  as shown in Eq. (10). A realization of the final probability of failure after possible redesign is

$$\hat{p}_{f,re}^{(i)} = Pr_U \left[ \hat{g}_H^{(i)}(\hat{\mathbf{x}}_{re}^{(i)}, \mathbf{U}) \leq 0 \right] \quad (11)$$

After redesign, the design variable  $\hat{\mathbf{x}}_{re}^{(i)}$  is also an epistemic random variable in addition to the limit state function  $\hat{g}_H^{(i)}(\cdot, \cdot)$ . Many different methods are available for calculating the probability of failure. In this study, first order reliability method (FORM) is used to calculate the probability of failure for each epistemic realization. The final probability of failure after possible redesign is

$$\hat{p}_{f,final}^{(i)} = \left( 1 - \hat{q}^{(i)} \right) \hat{p}_{f,ini}^{(i)} + \hat{q}^{(i)} \hat{p}_{f,re}^{(i)} \quad (12)$$

Note that the redesign decision  $\hat{q}^{(i)}$  shapes the final probability of failure distribution because we will have the opportunity in the future to correct the initial design if it fails the deterministic test. The expected probability of failure after possible redesign is  $E_\Omega \left[ \hat{P}_{f,final} \right]$ . As above, the expectation can be written as

$$E_\Omega \left[ \hat{P}_{f,final} \right] = (1 - p_{re}) E_\Omega \left[ \hat{P}_{f,ini} | \hat{Q} = 0 \right] + p_{re} E_\Omega \left[ \hat{P}_{f,re} | \hat{Q} = 1 \right] \quad (13)$$

where the  $E_\Omega \left[ \hat{P}_{f,ini} | \hat{Q} = 0 \right]$  is the expected probability of failure conditional on the test being passed and the designer keeping the initial design and  $E_\Omega \left[ \hat{P}_{f,re} | \hat{Q} = 1 \right]$  is the expected probability of failure conditional on the test being failed and the designer performing redesign.

### III. Demonstration Example

#### A. Overview

The demonstration problem is adapted from an example by Wu et al.<sup>6</sup> The example is the design of a cantilever beam to minimize mass subject to a constraint on tip displacement. The original problem involved the design of a long slender beam and therefore used Euler-Bernoulli beam theory. In this example, the length of the beam is reduced such that shear stress effects become important and Timoshenko beam theory is more accurate. The low-fidelity model of the tip displacement is formulated based on Euler-Bernoulli beam theory and the high-fidelity model is formulated based on Timoshenko beam theory. The design optimization (Eqs. (5) and (7)) is performed using sequential quadratic programming (SQP).

The low-fidelity model of the limit state function is

$$g_L(\mathbf{x}, \mathbf{U}) = d^* - \frac{4l^3}{ewt} \sqrt{\left( \frac{F_Y}{t^2} \right)^2 + \left( \frac{F_X}{w^2} \right)^2} \quad (14)$$

where  $\mathbf{x} = \{w, t\}$  are the design variables and  $\mathbf{U} = \{F_X, F_Y\}$  are the aleatory variables. The high-fidelity model of the limit state function is

$$g_H(\mathbf{x}, \mathbf{U}) = d^* - \sqrt{(d_x(\mathbf{x}, \mathbf{U}))^2 + (d_y(\mathbf{x}, \mathbf{U}))^2} \quad (15)$$

where  $d_x$  and  $d_y$  are given by Eqs. (16) and (17). The problem parameters are described in table 1.

$$d_x(\mathbf{x}, \mathbf{U}) = \left( \frac{3lF_X}{2gwt} + \frac{4l^3F_X}{ewt^3} \right) \quad (16)$$

$$d_y(\mathbf{x}, \mathbf{U}) = \left( \frac{3lF_Y}{2gwt} + \frac{4l^3F_Y}{ewt^3} \right) \quad (17)$$

The objective function is the cross-sectional area of the beam

$$f(\mathbf{x}) = wt \quad (18)$$

Table 1: Parameters for cantilever beam example

	Parameter	Notation	Value
Design variables, $\mathbf{x}$	Width of cross section	$w$	$2.5 \leq w \leq 5.5$ in
	Thickness of cross section	$t$	$1.5 \leq t \leq 4.5$ in
Aleatory variables, $\mathbf{U}$	Horizontal load	$F_X$	$F_X \sim N(500, 100^2)$ lbs
	Vertical load	$F_Y$	$F_Y \sim N(1000, 100^2)$ lbs
Constants	Elastic modulus	$e$	$29 \times 10^6$ psi
	Shear modulus	$g$	$11.2 \times 10^6$ psi
	Length of beam	$l$	10 in
	Allowable tip displacement	$d^*$	$2.25 \times 10^{-3}$ in
	Conservative aleatory values	$\mathbf{u}_{det}$	{664.5, 1164.5} lbs
	Target mean probability of failure	$p_f^*$	$1.35 \times 10^{-3}$

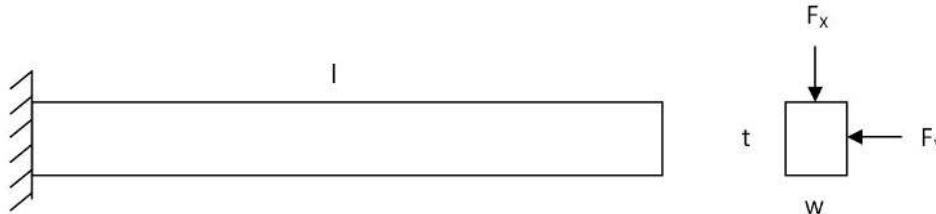


Figure 2: The beam is subject to horizontal and vertical tip loads

## B. Error model

It is assumed that some preliminary test data is available for constructing the surrogate model  $\hat{E}(\mathbf{x}, \mathbf{U})$ . In this example, the preliminary test data corresponds to evaluations of  $g_H(\cdot, \cdot)$  at the 16 corner points of the joint design-aleatory space. The corner points were selected for illustration purposes in order to ensure there is reasonably high epistemic model uncertainty for points inside the design domain. In practice, other designs of experiments (DoE) could be used or any available test data could be used to construct the error model. The design space is defined according to the bounds on  $\mathbf{x}$  in table 1 and bounds on  $\mathbf{U}$  corresponding to  $-2\sigma$  to  $+7\sigma$ . Based on this DoE the parameters for the Kriging error model are estimated using maximum likelihood estimation (MLE). A Gaussian covariance function was selected for the Kriging model. The error model is constructed in the joint space of design variables,  $\mathbf{x}$ , and aleatory variables,  $\mathbf{U}$ . Recall, the design optimization problem is formulated using fixed conservative values,  $\mathbf{U} = \mathbf{u}_{det}$ . In figure 3a, the design optimization problem is shown along with the 95% confidence interval of model uncertainty. In figure 3b, the reliability analysis is shown for the optimum design found using  $n_{ini} = 0$  along with the 95% confidence interval of model uncertainty. Selecting a different design by using a different safety margin will alter the plot shown in figure 3b. However, the plot is provided as an example to show how the model uncertainty

results in a wide confidence interval in the aleatory space. The wide confidence intervals in aleatory space will result in high uncertainty in the probability of failure. The mean and variance of the model error vary with design variables,  $\mathbf{x}$ , and aleatory variables,  $\mathbf{U}$  as shown in figure 4. The variance is zero at the corners of the design space since these points correspond to sample locations in the DoE. Although the absolute values of the error appear small, the error is significant relative to the model predictions of tip displacement. For example, the allowable tip displacement in this example is  $d^* = 2.25 \times 10^{-3}$  inches.

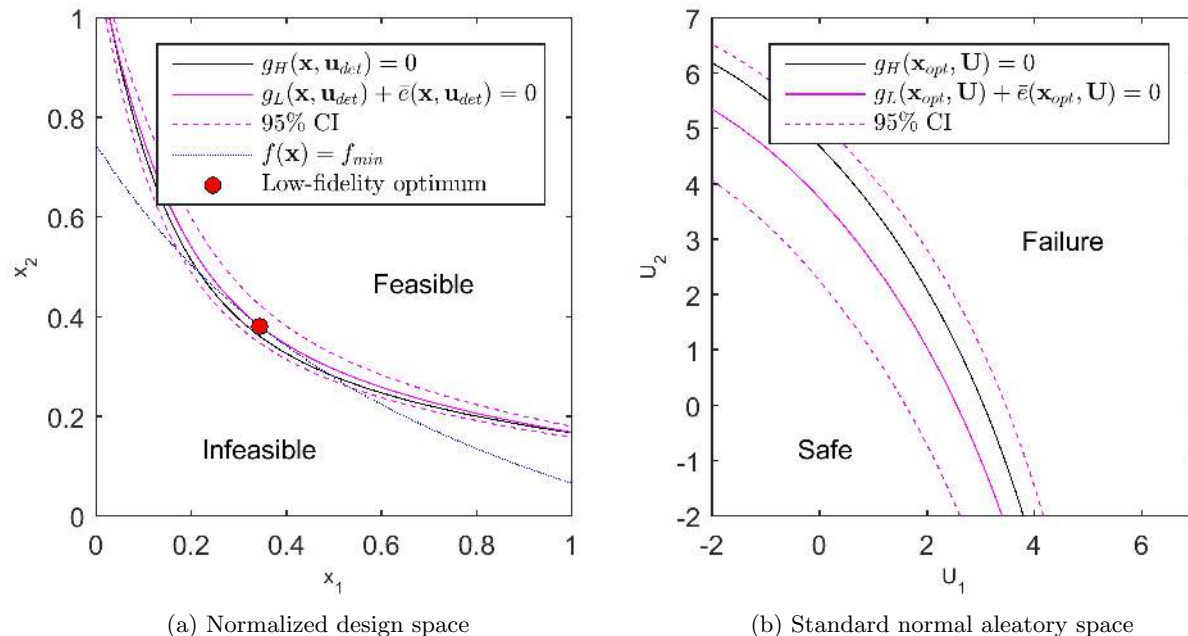


Figure 3: The figure on the left shows the design optimization when using a safety margin  $n_{ini} = 0$  and fixed conservative values  $\mathbf{u}_{det}$  in place of aleatory variables  $\mathbf{U}$ . The figure on the right shows the reliability of the optimum design found on the left by plotting the limit-state function in standard normal space.

### C. Results

To reduce the computational cost, the optimization of the safety margins was performed using surrogate models for expected cost and expected probability of failure (Eq. (1)). Details on the surrogate models can be found in the appendix. Tradeoff curves for expected cost versus probability of redesign are shown in figure 5. For zero probability of redesign, the problem reduces to finding an initial safety margin,  $n_{ini}$ , that minimizes the mass of the initial design,  $f(\mathbf{x}_{ini})$ , while ensuring that the mean probability of failure for the initial design,  $E_{\Omega} [\hat{P}_{f,ini}]$ , satisfies the reliability constraint. With increasing probability of redesign, redesign can be used to improve safety if the initial design is revealed to be dangerous or improve performance if the initial design is revealed to be too conservative. Redesign for safety allows for a lighter initial design because the initial design will be corrected if the tip displacement is later revealed to be too high (i.e. unsafe). If redesign is required, the final beam will become heavier during redesign because making the beam stiffer (i.e. safer) results in a mass increase. Redesign for performance starts with a heavier initial design and redesign will be performed if the tip displacement of the initial design is later revealed to be too low (i.e. very conservative or safe). If redesign is required, the final design can be made lighter during redesign because the beam can be made less stiff. It is observed that redesign for safety results in a lower mean mass than redesign for performance. It is also observed that a mixed redesign strategy offers a slight improvement over redesign for safety. However, as indicated by the error bars it is not clear if this difference is a result of bias in the surrogate models used when optimizing the safety margins.

The simulation results can be explored in more detail by looking at a single point on the tradeoff curve. The safety margins corresponding to 20% probability of redesign were selected for more detailed investigation.

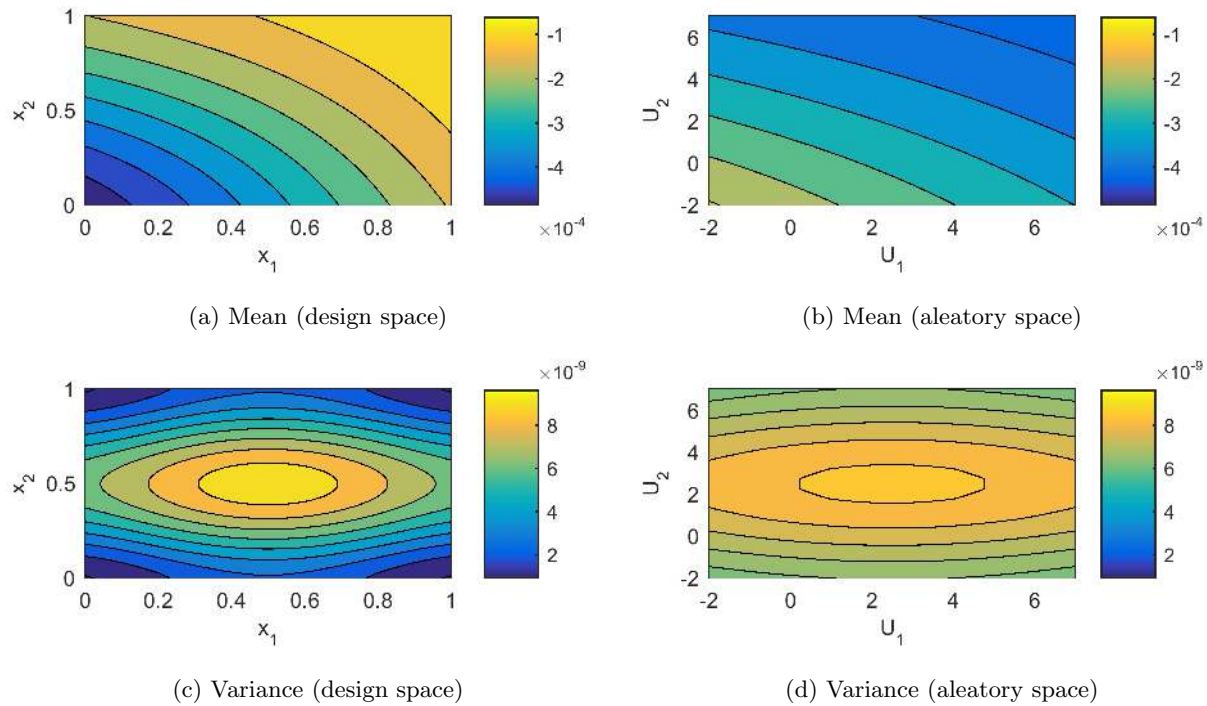


Figure 4: On the left, the mean and variance of the error are plotted in a normalized design space with fixed conservative values  $\mathbf{u}_{det}$  in place of aleatory variables  $\mathbf{U}$ . On the right, the mean and variance of the error are plotted in standard normal aleatory space for optimum design found using  $n_{ini} = 0$ . The error is in inches.

Figure 6 shows the distribution of possible high-fidelity safety margins for the initial design that are predicted based on the model error. Both distributions capture the true safety margin if we were to evaluate the initial design using the high-fidelity model. In the case of redesign for performance, we see that the true safety margin is greater than  $n_{ub}$  and therefore redesign would be required. If we calibrate using the true high-fidelity evaluation and perform redesign the true safety margin is now very close to  $n_{re}$  which agrees with the predicted change in the safety margin. Figure 7 shows the joint distributions of the design variables corresponding to the width and thickness of the beam cross section. The peak in the distributions corresponds to the initial design,  $\mathbf{x}_{ini}$ . The safety margins have been optimized such that there is an 80% probability that the design will not require any changes after the future test. The other designs in the figure correspond to failing the future test and performing redesign. Figure 8 shows the distributions of cross-sectional area corresponding to the designs in figure 7. The mass is reduced if redesign for performance is required and the mass is increased if redesign for safety is required. We can see in the distribution of cross sectional area for redesign for performance that the predicted mass reduction after redesign is close to the true value. Comparing the safety margin distributions in figure 6 to the reliability index distributions in figure 9 we observe similar distribution shapes. Both distributions capture the true reliability index of the initial design as calculated with respect the high-fidelity model. After redesign for performance the true reliability index is reduced in order to reduce the mass of the beam. The true reliability index after redesign falls within the predicted distribution of possible final reliability indexes. Histograms of the most-probable point (MPP) are shown in figure 10. The fixed deterministic values we selected  $\mathbf{u}_{det}$  are slightly outside the distribution of possible MPP's. However, the values are not totally unreasonable since they are much closer to the center of the distribution than, for example, the mean of the distributions which is located at  $[0, 0]$ .

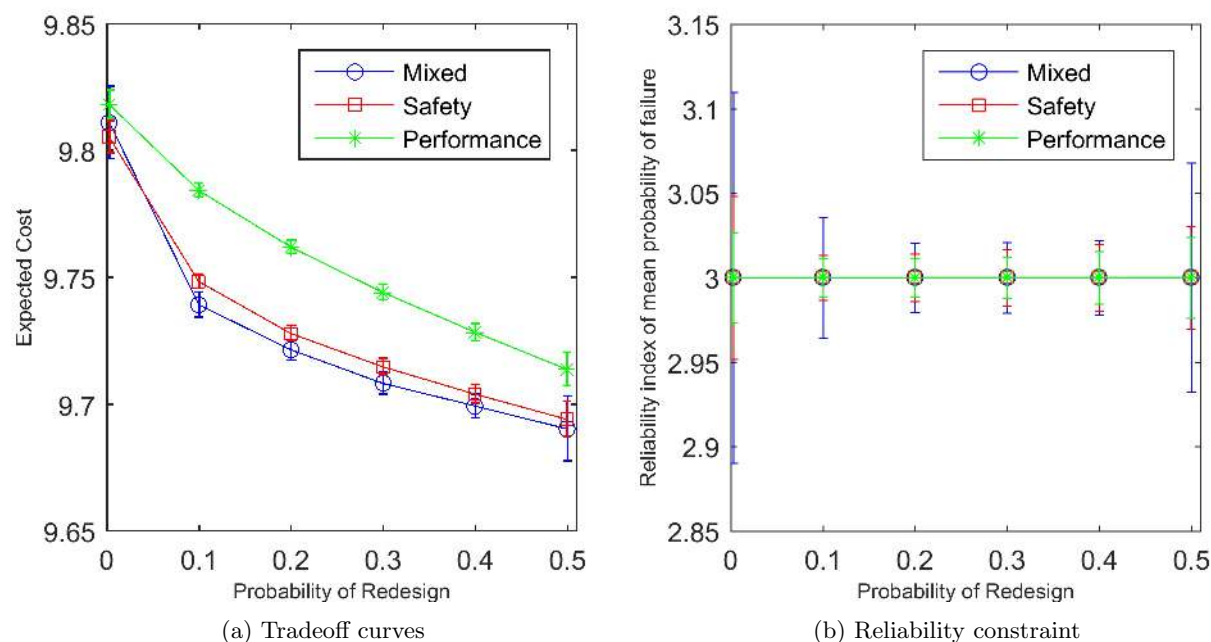


Figure 5: Tradeoff curves for expected cost (cross sectional area in square inches) as a function of probability of redesign. The curve labeled “mixed” corresponds to simultaneous optimization of  $\mathbf{n} = \{n_{ini}, n_{lb}, n_{ub}, n_{re}\}$ . The curve labeled “safety” corresponds to optimizing  $\{n_{ini}, n_{lb}, n_{re}\}$  with  $n_{ub} = +\infty$ . The curve labeled “performance” corresponds to optimizing  $\{n_{ini}, n_{ub}, n_{re}\}$  with  $n_{lb} = -\infty$ . Error bars are based on surrogate models used during optimization.

## IV. Discussion & Conclusions

In this study we described a method for the optimization of a safety-margin-based design process that allows the designer to tradeoff between the expected design performance and probability of redesign. Previous

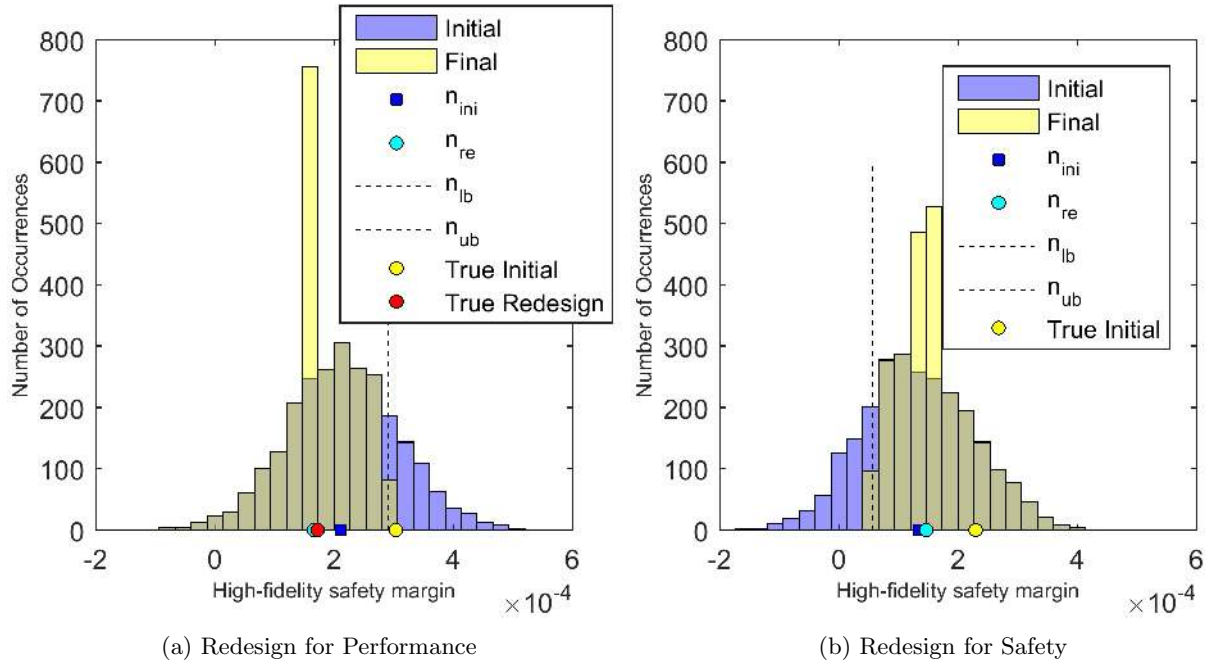


Figure 6: Histograms of possible safety margin distributions for 20% probability of redesign. Plots show overlapping transparent histograms.

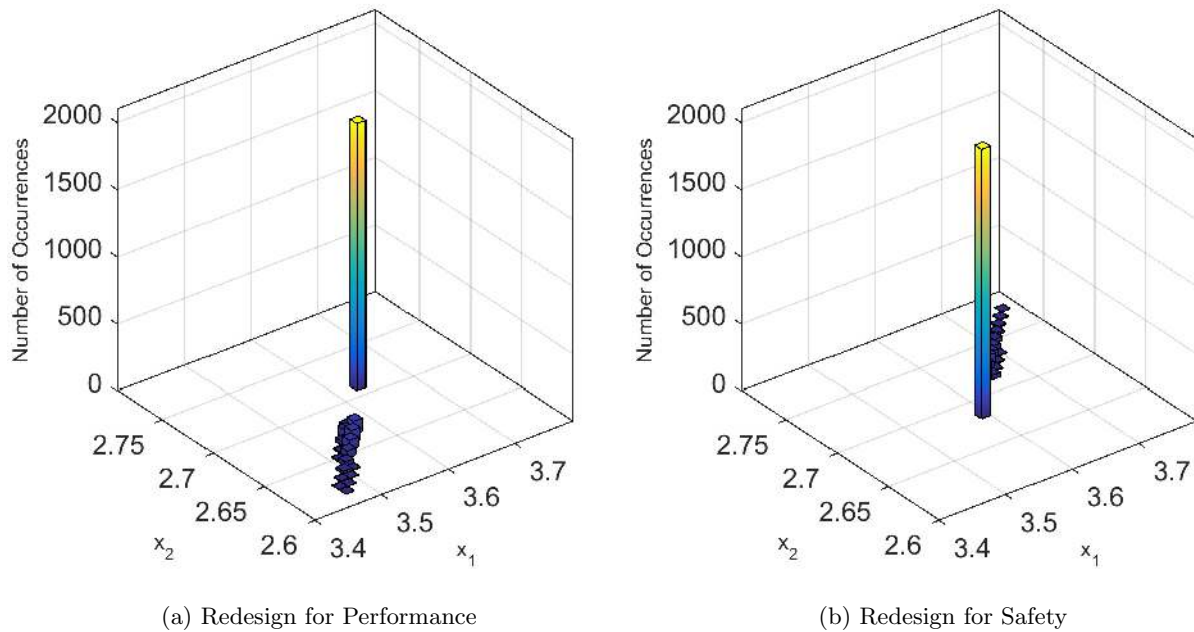


Figure 7: Joint distribution of design variables for possible final designs for 20% probability of redesign. Peak is located at initial design.

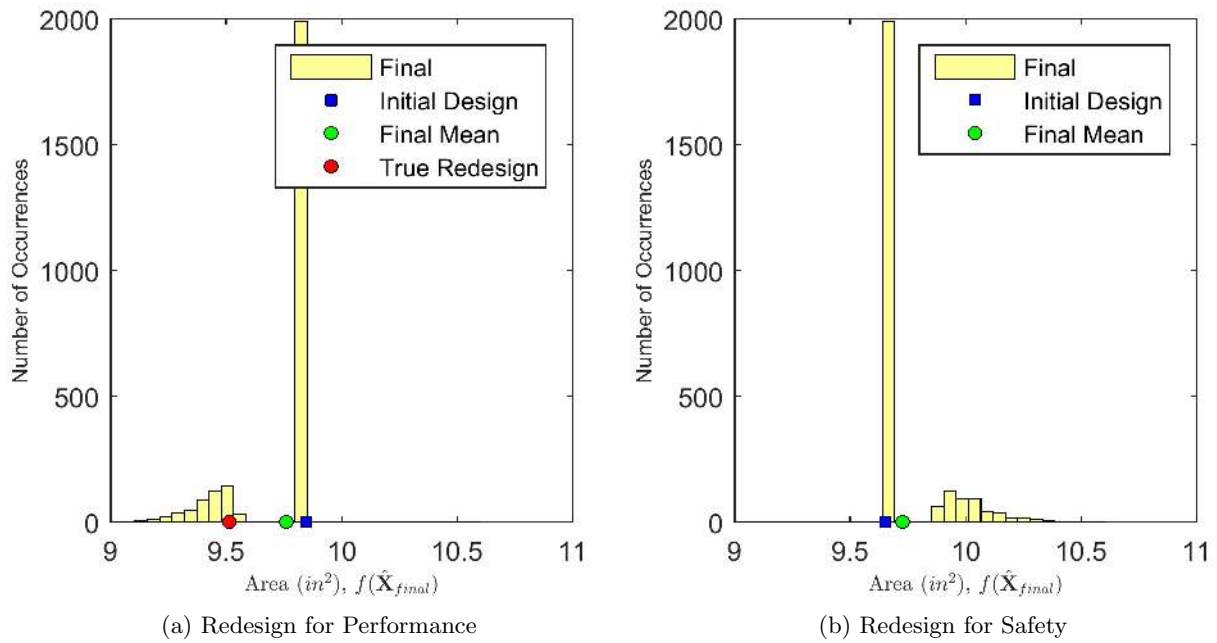


Figure 8: Histograms of cross-sectional area distributions for 20% probability of redesign.

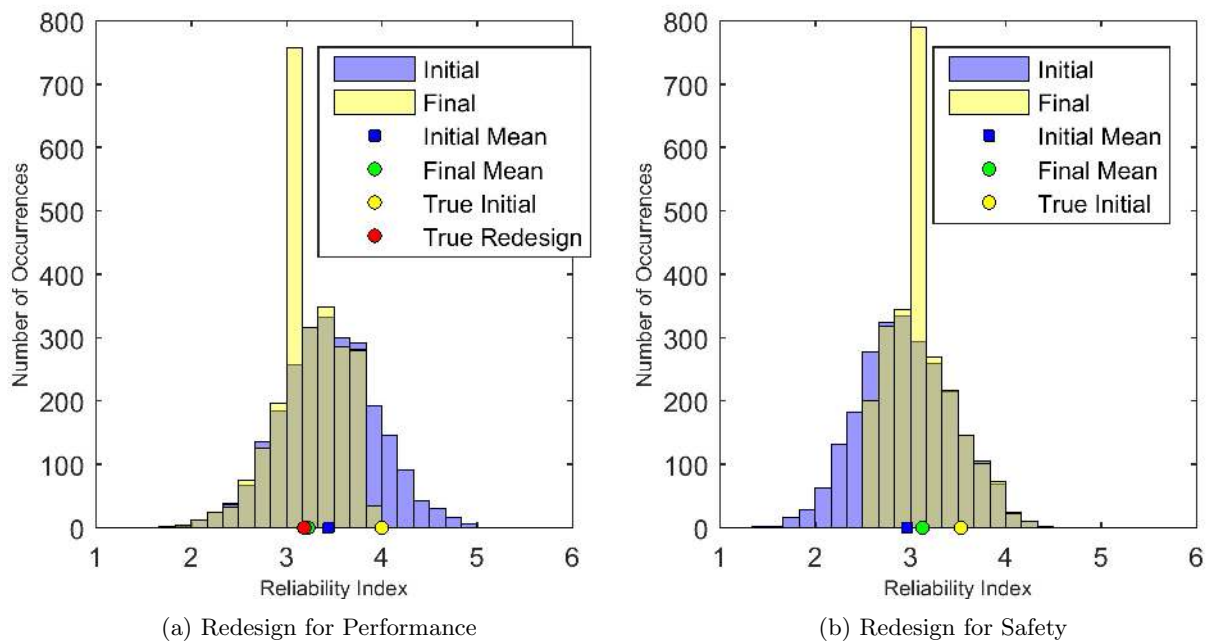


Figure 9: Histograms of reliability index distributions for 20% probability of redesign. Plots show overlapping transparent histograms.



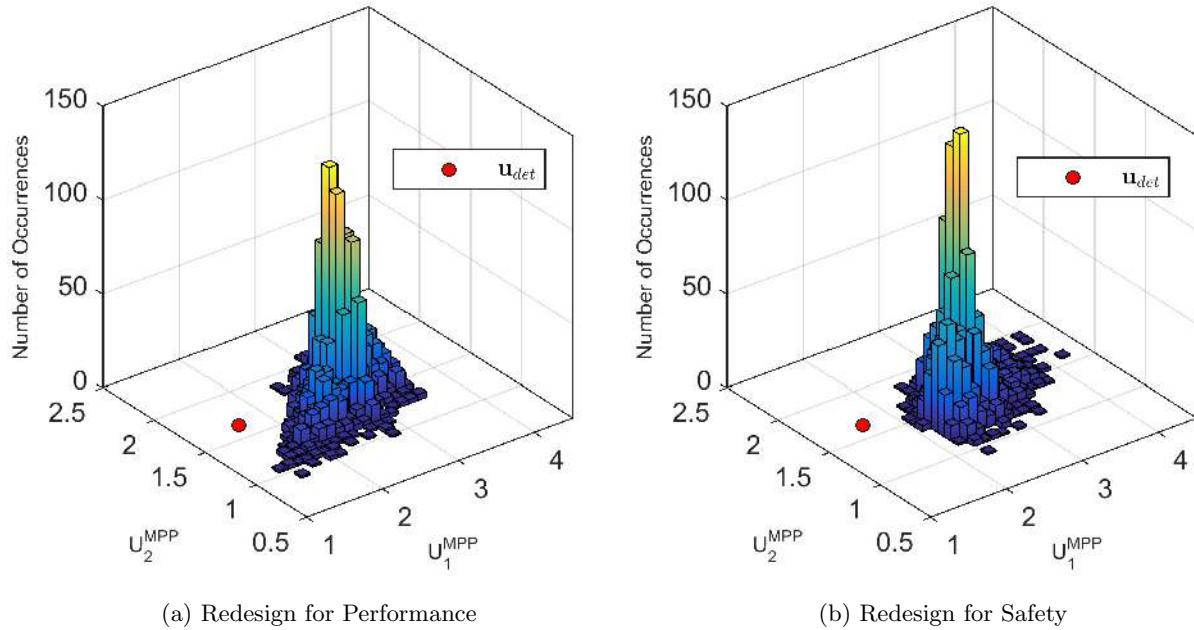


Figure 10: Joint distribution of possible most probable points (MPP's) for 20% probability of redesign.

studies on the optimization of safety margins when considering future redesign required an assumption of constant model bias.<sup>1-3</sup> However, in engineering design problems the model bias may vary with the design variables as well as the aleatory variables, such as in the case of the cantilever beam example. This study improves on previous work by introducing a Kriging model as a more general model of the epistemic uncertainty in the low-fidelity model. The Kriging model offers several practical benefits over the previous method. In particular, the Kriging model easily allows for the incorporation of preliminary high-fidelity data and a simple calibration of the model when new high-fidelity data becomes available. The Kriging error model captures the intuitive idea that the variance of the model error is greatest in unexplored regions and a minimum at existing data points. The Kriging model also provides several theoretical improvements of the method. One benefit is that it is likely that there exists a realization taken from the Kriging model that is arbitrarily close to the actual error between the low and high-fidelity models. Therefore, it is likely that there also exists a realization of the probability of failure that is close to the true probability of failure with respect to the high-fidelity model. In addition, it is likely the Kriging model will converge to the true error as more high-fidelity evaluations (or tests) are performed. If the Kriging model converges to the true error, then the distribution of probability of failure will also converge to the true probability of failure with respect to the high-fidelity model. Previous work was not capable of modeling the convergence of the model error because under the assumption of constant model bias only a single high-fidelity evaluation was necessary to remove all epistemic uncertainty.

The method was applied to a simple cantilever beam design problem of minimizing the mass, or equivalently cross-sectional area, subject to a constraint on tip-displacement. Only a few high-fidelity evaluations were needed to construct the Kriging model that was used to provide the distribution of model uncertainty. A distribution of probability of failure was obtained through the combination of FORM and a MCS of error realizations (i.e. conditional simulations). It was shown that the distribution of possible reliability indexes captured the true reliability index of the initial design with respect to the high-fidelity model. Furthermore, it was shown that the predicted change in reliability after redesign agreed with the actual redesign outcome when the high-fidelity model was evaluated and redesign was performed. The safety margins governing a deterministic design process were optimized to tradeoff between the probability of redesign and the expected mass of the final design. It was shown that the predicted mass reduction (i.e. performance improvement) agreed with the actual change in performance after evaluating the high-fidelity model and performing redesign. For this example, it was found that it was better to start with a less conservative, lighter design and

implement a test and redesign process that would restore safety if the initial design was later revealed by the high-fidelity model to be unsafe. This process was contrasted with starting with a more conservative, heavier design and implementing a test and redesign process that would improve design performance if the initial design was later revealed by the high-fidelity model be too conservative. A mixed design strategy where redesign would restore safety or improve performance conditional on the results of the high-fidelity evaluation was found to be comparable to the redesign for safety approach. It is hypothesized that the best redesign strategy is problem dependent. In general, there is no need to specify a redesign for safety or redesign for performance a priori because when allowed to control all the safety margins the optimizer will converge to the best redesign strategy.

## Appendix

The optimization problem in Eq. (1) may be prohibitively expensive if a MCS is performed for each evaluation of the objective and constraint equations. Surrogate models were used to reduce the computational cost of the optimization of the safety margins. Kriging models of the mean final probability of failure  $E_{\Omega} [\hat{P}_{f,final}]$  and mean final design cost  $E_{\Omega} [f(\hat{X}_{final})]$  were fit as a function of the safety margins  $\mathbf{n} = \{n_{ini}, n_{lb}, n_{ub}, n_{re}\}$ . The mean probability of failure was transformed to a reliability index before fitting the surrogate models. The Kriging models were fit based on a DoE consisting of 400 points generated using Latin hypercube sampling (LHS) and the corner points in the design space. Each point in the DoE required a MCS of epistemic model uncertainty. The sample size of the MCS (i.e. number of conditional simulations) was adapted to reach a target coefficient of variation on the expected final probability of failure of 5% with a maximum sample size of  $m = 5000$ . Kriging with nugget was used in an effort to filter out some of the noise introduced by MCS. A Gaussian covariance function was used and parameters were estimated based on MLE. Three different sets of surrogate models were constructed corresponding to a mixed redesign strategy, redesign for performance, and redesign for safety. The redesign for performance and redesign for safety surrogate models were 3-dimensional surrogate models while the mixed redesign strategy required 4-dimensional surrogates. The error in the surrogate models was estimated based on leave-one-out cross validation (LOOCV). It should be noted that LOOCV may overestimate the error due to the noise filtering effect of Kriging with nugget. Error estimates for the surrogate models are listed in table 2.

Table 2: 95% confidence interval for relative error of surrogate models based on LOOCV

	Mixed	Performance	Safety
Expected probability of failure	[-33, 25]%	[-17, 13]%	[-17, 14]%
Expected cross sectional area	[-0.12, 0.12]%	[-0.05, 0.07]%	[-0.08, 0.06]%

## Acknowledgments

Funding for this research was provided by Onera - The French Aerospace Lab.

## References

- <sup>1</sup>Matsumura, T. and Haftka, R. T., "Reliability Based Design Optimization Modeling Future Redesign With Different Epistemic Uncertainty Treatments," *Journal of Mechanical Design*, Vol. 135, No. 9, July 2013, pp. 091006–091006.
- <sup>2</sup>Villanueva, D., Haftka, R. T., and Sankar, B. V., "Accounting for future redesign to balance performance and development costs," *Reliability Engineering & System Safety*, Vol. 124, April 2014, pp. 56–67.
- <sup>3</sup>Price, N. B., Matsumura, T., Haftka, R. T., and Kim, N. H., "Deciding How Conservative A Designer Should Be: Simulating Future Tests and Redesign," *16th AIAA Non-Deterministic Approaches Conference*, American Institute of Aeronautics and Astronautics, National Harbor, Maryland, Jan. 2014.
- <sup>4</sup>"§25.303 Factor of safety," *Federal Aviation Regulations*, Federal Aviation Administration, 2015.
- <sup>5</sup>Wu, Y. T. and Wang, W., "Efficient probabilistic design by converting reliability constraints to approximately equivalent deterministic constraints," *J. Integr. Des. Process Sci.*, Vol. 2, No. 4, 1998, pp. 13–21.
- <sup>6</sup>Wu, Y. T., Shin, Y., Sues, R., and Cesare, M., "Safety-factor based approach for probability-based design optimization," *Proc. 42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, number AIAA-2001-1522, Seattle, WA*, Vol. 196, 2001, pp. 199–342.

- <sup>7</sup>Du, X. and Chen, W., “Sequential optimization and reliability assessment method for efficient probabilistic design,” *Journal of Mechanical Design*, Vol. 126, No. 2, 2004, pp. 225–233.
- <sup>8</sup>Zhu, P., Pan, F., Chen, W., and Viana, F. A. C., “Lightweight design of vehicle parameters under crashworthiness using conservative surrogates,” *Computers in Industry*, Vol. 64, No. 3, April 2013, pp. 280–289.
- <sup>9</sup>Mahadevan, S. and Rebba, R., “Inclusion of Model Errors in Reliability-Based Optimization,” *Journal of Mechanical Design*, Vol. 128, No. 4, Jan. 2006, pp. 936–944.
- <sup>10</sup>Kim, C. and Choi, K. K., “Reliability-Based Design Optimization Using Response Surface Method With Prediction Interval Estimation,” *Journal of Mechanical Design*, Vol. 130, No. 12, Oct. 2008, pp. 121401–121401.
- <sup>11</sup>Kennedy, M. C. and O’Hagan, A., “Bayesian calibration of computer models,” *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, Vol. 63, No. 3, Jan. 2001, pp. 425–464.
- <sup>12</sup>Forrester, A. I., Sobester, A., and Keane, A. J., “Multi-fidelity optimization via surrogate modelling,” *Proceedings of the royal society A: mathematical, physical and engineering science*, Vol. 463, No. 2088, 2007, pp. 3251–3269.
- <sup>13</sup>Chen, W., Xiong, Y., Tsui, K.-L., and Wang, S., “A design-driven validation approach using Bayesian prediction models,” *Journal of Mechanical Design*, Vol. 130, No. 2, 2008, pp. 021101.
- <sup>14</sup>Huang, D., Allen, T. T., Notz, W. I., and Miller, R. A., “Sequential kriging optimization using multiple-fidelity evaluations,” *Structural and Multidisciplinary Optimization*, Vol. 32, No. 5, 2006, pp. 369–382.
- <sup>15</sup>Qian, Z., Seepersad, C. C., Joseph, V. R., Allen, J. K., and Wu, C. J., “Building surrogate models based on detailed and approximate simulations,” *Journal of Mechanical Design*, Vol. 128, No. 4, 2006, pp. 668–677.
- <sup>16</sup>Qian, P. Z. and Wu, C. J., “Bayesian hierarchical modeling for integrating low-accuracy and high-accuracy experiments,” *Technometrics*, Vol. 50, No. 2, 2008, pp. 192–204.
- <sup>17</sup>Xiong, Y., Chen, W., and Tsui, K.-L., “A New Variable-Fidelity Optimization Framework Based on Model Fusion and Objective-Oriented Sequential Sampling,” *Journal of Mechanical Design*, Vol. 130, No. 11, Oct. 2008, pp. 111401–111401.
- <sup>18</sup>Villanueva, D., Haftka, R. T., and Sankar, B. V., “Including the Effect of a Future Test and Redesign in Reliability Calculations,” *AIAA Journal*, Vol. 49, No. 12, 2011, pp. 2760–2769.
- <sup>19</sup>Hansen, N., “The CMA evolution strategy: a comparing review,” *Towards a new evolutionary computation*, Springer, 2006, pp. 75–102.
- <sup>20</sup>Hoffman, F. O. and Hammonds, J. S., “Propagation of Uncertainty in Risk Assessments: The Need to Distinguish Between Uncertainty Due to Lack of Knowledge and Uncertainty Due to Variability,” *Risk Analysis*, Vol. 14, No. 5, Oct. 1994, pp. 707–712.
- <sup>21</sup>Paté-Cornell, M. E., “Uncertainties in risk analysis: Six levels of treatment,” *Reliability Engineering & System Safety*, Vol. 54, No. 2–3, Nov. 1996, pp. 95–111.
- <sup>22</sup>Helton, J. C., “Treatment of uncertainty in performance assessments for complex systems,” *Risk analysis*, Vol. 14, No. 4, 1994, pp. 483–511.
- <sup>23</sup>Chilès, J.-P. and Delfiner, P., “Chapter 7 - Conditional Simulations,” *Geostatistics: Modeling Spatial Uncertainty, Second Edition*, John Wiley & Sons, Inc., 2012, pp. 478–628.
- <sup>24</sup>Bect, J. and Vazquez, E., “STK a Small (Matlab/Octave) Toolbox for Kriging,” .



