B. Marchand[1], L. Chamoin[1], C. Rey[2]

{marchand,chamoin}@lmt.ens-cachan.fr, christian.rey@safran.fr

[1] LMT Cachan/ENS Cachan/CNRS/Paris-Saclay University
[2] Safran Tech

Coupling Modified Constitutive Relation Error, Model Reduction and Kalman filtering algorithms for Real-Time Parameters Identification

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Context

Dynamic Data Driven Application Systems (DDDAS) paradigm [4] is based on a continuous exchange between physical system and its numerical model in order to :

(i) Use observations data to identify and update model parameters

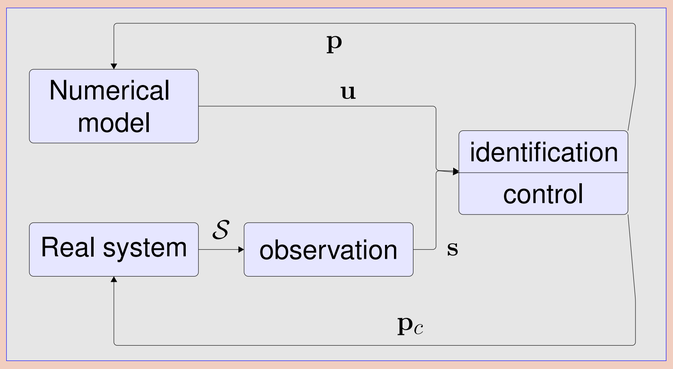
Inverse problem

(ii) Control system evolution according to criteria

Optimal control problem

Application example :

Damage evolution identification and control in composite structures [8].



Mathematical representation

Dynamical system :

$$\begin{cases} \mathbf{u}^{(k+1)} = \mathcal{M}(\mathbf{u}^{(k)}) + \mathbf{e}_M \\ \mathbf{s}^{(k)} = \mathcal{H}(\mathbf{u}^{(k)}) + \mathbf{e}_S \end{cases}$$

$\mathbf{u}^{(k)} \in \mathbb{R}^n$ state vector at time $t^{(k)}$
$\mathbf{s}^{(k)} \in \mathbb{R}^m$ observation vector at time $t^{(k)}$
$\mathbf{e}_M \in \mathbb{R}^n$ model error assumed to be Gaussian white noise
$\mathbf{e}_S \in \mathbb{R}^m$ observation error assumed to be Gaussian white noise
$\mathcal{M} : \mathbb{R}^n \to \mathbb{R}^n$ model operator
$\mathcal{H} : \mathbb{R}^n \to \mathbb{R}^m$ observation operator

Kalman filter based identification

Inverse problem resolution based on Kalman filtering [6] using non-linear formulation, here the Unscented Kalman Filter [5]. The idea is to define an expanded state vector $\bar{\mathbf{u}}$ and associated dynamical system :

$$\begin{cases} \bar{\mathbf{u}}^{(k+1)} = \mathcal{M}(\bar{\mathbf{u}}^{(k)}) + \mathbf{e}_M \\ \mathbf{s}^{(k)} = \mathcal{H}(\bar{\mathbf{u}}^{(k)}) + \mathbf{e}_S \end{cases} \quad \text{with} \quad \bar{\mathbf{u}}^{(k)} = \begin{bmatrix} \mathbf{u}^{(k)} \\ \mathbf{p}^{(k)} \end{bmatrix}$$

UKF idea is based on Unscented Transform which allows to reconstruct mean and covariance with sample of $2 \cdot \dim(\bar{\mathbf{u}}) + 1$ points. But complexity is $O\left(\dim(\bar{\mathbf{u}})^3\right)$

Modified CRE

Modified Constitutive Relation Error [7] is based on energy functionnal (transient thermal case)

$$\mathcal{E}_m^2(u, \mathbf{q}; \mathbf{s}, \mathbf{p}) = \frac{1}{2}\int_T \int_\Omega (\mathbf{q} - \kappa\nabla u)\,\kappa^{-1}\,(\mathbf{q} - \kappa\nabla u) + \frac{1}{2}\frac{r}{1-r}\int_T \|\Pi u - \mathbf{s}\|^2$$

The identification problem can be formulated as, find $\mathbf{p} \in \mathcal{P}$ such that;

$$\mathbf{p} = \underset{\xi\in\mathcal{P}}{\operatorname{argmin}} \min_{(u,\mathbf{q})\in\mathcal{U}\times\mathcal{S}} \mathcal{E}_m^2(u, \mathbf{q}; \mathbf{s}, \xi)$$

Resolution using alternating minimization + fixed point algorithm

- Minimization according to $\mathcal{U} \times \mathcal{S} \longrightarrow (u_{ad}, \mathbf{q}_{ad})$ (coupled forward/backward problem in time)
- Minimization according to $\mathcal{P} \longrightarrow \mathbf{p}$ (gradient method)

Modified Kalman Filter

The approach we propose, **Modified Kalman Filter** (MKF), is slightly similar to the approach proposed in [1]

$$\begin{cases} \mathbf{p}^{(k+1)} = \mathbf{p}^{(k)} + \mathbf{e}_P \\ \mathbf{s}^{(k)} = \mathcal{H}_m(\mathbf{p}^{(k)}, \mathbf{s}^{(k-1:k)}) + \mathbf{e}_S \end{cases}$$

with observation operator defined as follow :

$$\mathcal{H}_m\left(\mathbf{p}^{(k)}, \mathbf{s}^{(k-1:k)}\right) = \mathcal{H} \circ \text{mCRE}_1\left(\mathbf{p}^{(k)}, \mathbf{s}^{(k-1:k)}\right)$$

$\mathcal{H}$: projector
mCRE$_1$: Admissible fields computation

Advantages :

1. Implementation, since the method is based on sampling points
2. Robustness, since the system state is computed as an admissible field according to mCRE

Drawbacks :

Computation time associated to the coupled forward/backward problem.

Proper Generalized Decomposition

Proper Generalized Decomposition [3] based on :

(i) Variable separation
(ii) Parametric solution
(iii) *offline* modes computation
(iv) *online* solution evaluation

The incremental approach we consider involves that PGD decomposition must be on the following form :

$$\mathbf{u}^{(k)} \simeq \mathbf{u}_M\left(\mathbf{x}, t^{(k)}, \mathbf{p}, \mathbf{s}^{(k-1)}, \mathbf{s}^{(k)}, \mathbf{u}_0^{(k)}\right)$$

$\mathbf{u}_0^{(k)}$ initial condition on $I_t^{(k)} = [t^{(k-1)}, t^{(k)}]$.

Numerical results

Problem setting

- parabolic PDE (transient thermal problem)
- Time-dependant Neumann Boundary Condition identification (example 1)
- Time-independant conductivity identification in multi-layer structure (example 2)

Simulated observations

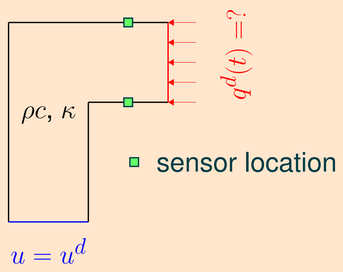
(i) Solve direct problem with fine time discretization
(ii) Extract values according to sensors placement and aquisition times (coarser than for resolution)
(iii) Add Gaussian white noise

$$s_i^{(k)} = s_{i,exact}^{(k)}\,(1 + \mathcal{N}(0, \Sigma)) \quad \forall i = 1, .., m \; ; \; \forall k$$

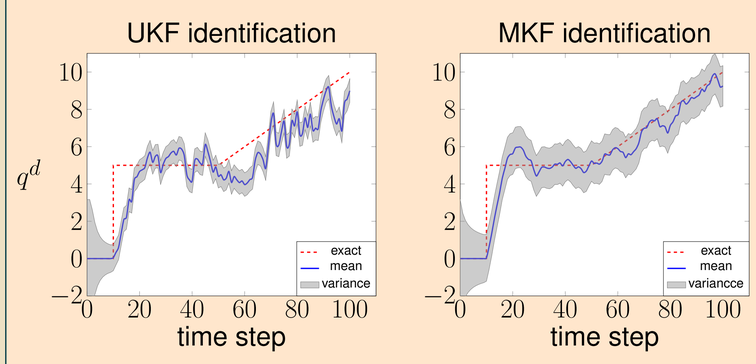
Global identification strategy

Offline – compute PGD parametric solution
Online – identification process using MKF approach

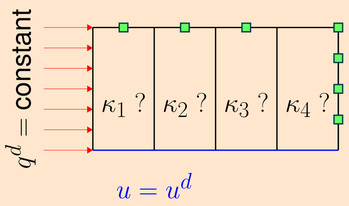
Example 1



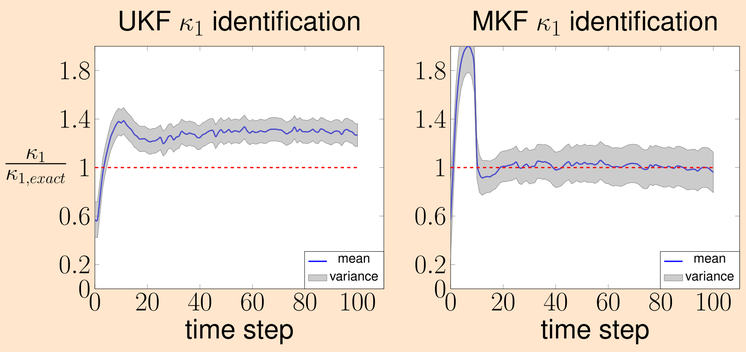
space : 980 TRI3
time : 100 time steps
noise : $\Sigma = 0.2$



Example 2



space : 148 TRI3
time : 100 time steps
noise : $\Sigma = 0.1$



Conclusions

The strategy we propose, based on coupling between Unscented Kalman filter, modified Constitutive Relation Error and PGD reduced model offers :

(i) more robustness to highly corrupted data
(ii) a significant computation time-saving

Future works

(i) Extend Modified Kalman Filter approach coupled with PGD to field parameters identification
  (a) adaptive mesh approach
  (b) multiscale method
(ii) Address full-fields measurements

References

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