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# Fan-Beam Reconstruction Under Motion and Data Truncation: Mapping Analytic and Iterative Approaches

Jan Hoskovec, Fabien Momey, Rolf Clackdoyle, Laurent Desbat and Simon Rit

**Abstract**—In this paper we propose comparisons and correlations between analytic and iterative fan-beam reconstruction approaches when object’s rigid motion and data truncation occur during a circular scan. Based on our recent work presenting an exact analytic reconstruction method, we are able to predict the field of theoretically reconstructible points and transform the problem from a dynamic to a static point a view where the source trajectory is virtually modified taking into account the known rigid motion. We implement the iterative reconstruction as the convex minimization of a data-fidelity term under non-negativity constraint and regularization to solve this virtually static inverse problem. We compare the reconstructed field of view by the two methods on 2D fan-beam Shepp-Logan phantom simulations. Our results show that both methods validate the predicted reconstructible zone and are in good correlation in terms of reconstruction quality. The iterative reconstruction also demonstrates that in certain cases it is possible to recover structures beyond the analytic strict frontier of reconstructibility.

**Index Terms**—Tomography, Region-Of-Interest Tomography, Dynamic Tomography.

## I. INTRODUCTION

In [2] we reported a method for performing exact analytic 2D fan-beam reconstruction when the object of interest has undergone a perfectly known rigid translation during the circular scan, also involving data truncation. Such an object’s perturbation transforms the circular source trajectory into a virtual one which also involve data truncation. The method proposes to take advantage of data redundancy from the  $2\pi$  source trajectory to extend the field of reconstructible points by cleverly selecting the needed lines of response to be able to perform Differentiated Back-Projection with Hilbert filtering (DBP-H) approach [6]. The algorithm has been tested in a proof-of-concept study on Shepp-Logan phantom simulations with several motion cases and detector sizes.

In this paper, we propose to map the results given by our analytic resolution with an iterative resolution of the inverse problem, particularly in terms of the predicted field of reconstructible points. Our results from Shepp-Logan phantom simulations show a very good match regarding the almost perfectly reconstructed zone of the phantom, and highlight possibilities for the iterative method to reconstruct beyond the predicted field of view.

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## II. MATERIALS AND METHODS

### A. Geometry

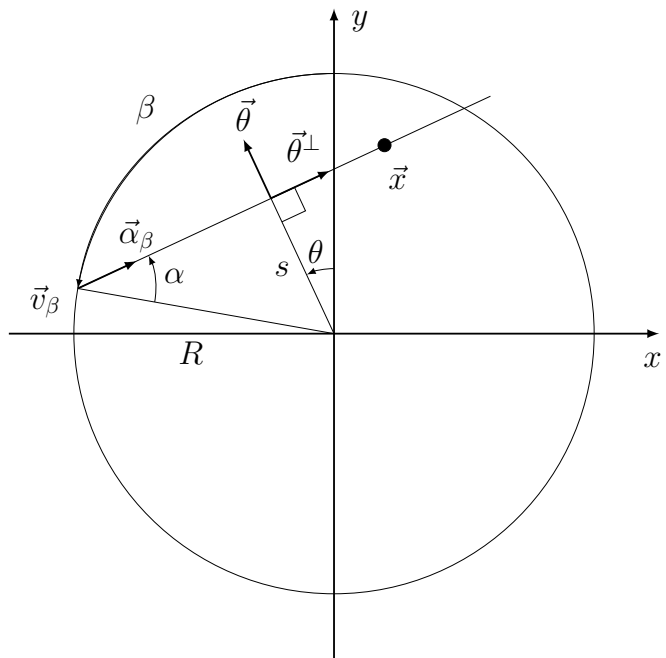


Fig. 1. The fan-beam projection geometry. The source is at  $\vec{v}_\beta$  and the trajectory radius is  $R$ . A measured ray in the fan-beam geometry is parametrized by  $(\alpha, \beta)$ . Also shown are the equivalent parameters  $(s, \theta)$  expressing the same ray in the parallel projection geometry.

Our work takes place in the context of 2D fan-beam reconstruction from a circular scan around the object. The geometry is illustrated in Fig.1. Fan-beam projections can be written as:

$$p(\alpha, \beta) = \int_{-\infty}^{\infty} f(\vec{v}_\beta - t\vec{\alpha}_\beta) dt \quad (1)$$

with  $\beta$  the polar angle of the source from the vertical axis,  $\vec{v}_\beta = R\vec{\beta} = R(-\sin \beta, \cos \beta)^T$  the source position and  $\vec{\alpha}_\beta = (-\sin \alpha + \beta, \cos \alpha + \beta)^T$ ,  $\alpha$  being the fan angle. The angular conventions taken here are illustrated Fig. 1.

$p(\alpha, \beta)$  can be related to equivalent parallel-beam projections  $\bar{p}(s, \theta)$  via the following change of variables:

$$\theta = \alpha + \beta - \frac{\pi}{2} \quad (2)$$

$$s = R \sin \alpha, \quad (3)$$

therefore

$$p(\alpha, \beta) = \bar{p}(R \sin \alpha, \alpha + \beta - \frac{\pi}{2}). \quad (4)$$

### B. DBP-H Formula

We summarize here our analytical reconstruction method, which is clearly presented in [2]. This method belongs to the DBP-H family, sometimes called simply DBP, or BPF, for Backprojection-Filtration. More specifically, the DBP-H algorithm used here is of the “backproject first” approach [10], which begins with performing two backprojections of the unprocessed sinogram data onto the target pixel grid. Then, via a simple sum of partial derivatives of each weighted backprojection, we can obtain the same Hilbert image of the object of interest as if we had performed a differentiation along the flat detector before backprojecting.

The general DBP-H reconstruction formula is given by

$$H_\phi f(\vec{x}) = \frac{-1}{2\pi} b_\phi(\vec{x}) = \frac{-1}{2\pi} \int_\phi^{\phi+\pi} \frac{\partial}{\partial s} \bar{p}(s, \theta) \Big|_{s=\vec{x} \cdot \vec{\theta}} d\theta. \quad (5)$$

where  $H_\phi f$  denotes a 1D Hilbert transform along the vector  $\vec{\phi}^\perp = (\cos \phi, \sin \phi)^T$ . With the “backproject first” approach, we obtain  $b_\phi(\vec{x})$  by the following relation (see [2] or [10] for full derivation):

$$b_\phi(\vec{x}) = \frac{\partial}{\partial x} \int_\phi^{\phi+\pi} \bar{p}(\vec{x} \cdot \vec{\theta}, \theta) (-\sin \theta) d\theta + \frac{\partial}{\partial y} \int_\phi^{\phi+\pi} \bar{p}(\vec{x} \cdot \vec{\theta}, \theta) (\cos \theta) d\theta \quad (6)$$

1) *Handling motion and truncation for DPB-H reconstruction:* The “backproject first” DBP-H algorithm is useful in the context of motion-compensated reconstruction, since all motion corrections can be included *before* the sinogram data is processed in any manner. Our algorithm from [2] does just that when it rearranges motion contaminated full-scan fan-beam data into an equivalent, static, parallel-beam geometry.

When the object undergoes a rigid translation, the sinogram data can become truncated (a part of the object “leaves” the scanner’s field-of-view during a part of the scan). Since a rigid displacement of the object (described by a vector  $\vec{d}_\beta$  parametrized by the gantry angle) is equivalent to a deformation of the X-Ray source trajectory by subtracting the same vector from its regular path, static truncation (due to a reduced width of the detector) and dynamic truncation (induced by object motion) can be handled as the same problem.

Observing the virtual trajectory  $\vec{v}_\beta - \vec{d}_\beta$ , we recall that with the DBP-H methods, a point’s Hilbert image can be recovered if the point is observable during a portion of the scan long enough for the trajectory to contain two endpoints of a segment on which the point lays [6][7][9]. (Such a point becomes theoretically reconstructible via (5).) We refer to this type of point as a Hilbert point.

Taking advantage of the data redundancy inherent to a fan-beam full-scan, our algorithm can also recover points for which such a segment of the virtual trajectory is not available, but where data from the opposite side of the scan can fill in the gap. See [2] for details.

In practice, to reconstruct the object, we need to be able to invert the Hilbert transform on the Hilbert points we obtain. This is not trivial because the Hilbert transform is an infinite support operation. However, with a small *a priori* about the image, we can invert Hilbert points using the finite-support Hilbert transform inversion formula from [4] on all segments of Hilbert points which cross the entire object support.

### C. Iterative reconstruction method for solving the virtually static inverse problem

Our iterative reconstruction takes the form of an inverse problem, the goal of which is to seek the static image  $\mathbf{f}$  which minimizes the least square criterion - data-fidelity term - under non-negativity constraint, with a regularization term:

$$\mathbf{f}^+ = \arg \min_{\mathbf{f} > 0} \left\{ \left\| \mathbf{R}^{\bar{\beta}} \cdot \mathbf{f} - \mathbf{p}^{\bar{\beta}} \right\|_2^2 + \mu \mathcal{J}_{\text{prior}}(\mathbf{f}) \right\}, \quad (7)$$

where  $\mathbf{p}^{\bar{\beta}} = \{\mathbf{p}^{\bar{\beta}_k} | k = 1 \dots N_\beta\}$  stands for the set of  $N_\beta$  fan-beam projections, and  $\mathbf{R}^{\bar{\beta}}$  is the projection model approximating the Radon transform, calibrated by the virtual (perturbed) source trajectory at the virtual angular positions  $\{\bar{\beta}_k | k = 1 \dots N_\beta\}$ .

The data-fidelity term ensures consistency of the model with the data. A non-negativity constraint is also handled as the object to be reconstructed is known to have positive values. The term  $\mathcal{J}_{\text{prior}}$  accounts for prior information. The constraint and the regularizer are necessary for the reconstruction algorithm to effectively converge to a relevant solution by avoiding artifacts amplifications and noise. The hyperparameter  $\mu$  controls the tradeoff between data fitting and regularity.

We chose an edge-preserving smoothness regularizer expressed as a relaxed total variation (TV) prior [8]:

$$\mathcal{J}_{\text{prior}}(\mathbf{f}) = \sum_i \sqrt{\|\nabla_i \cdot \mathbf{f}\|_2^2 + \epsilon^2}, \quad (8)$$

with  $\epsilon > 0$  the relaxation parameter and  $\nabla_i$  a finite difference operator approximating the spatial gradient at position  $i$ .

The minimization of (7) is carried out by the VMLM algorithm [5], a limited memory quasi-Newton method, for which we have added the handling of the non-negativity constraint.

## III. SIMULATIONS AND RECONSTRUCTIONS

We simulated two cases for getting the projection data of an off-centered slice of the 3D Shepp-Logan phantom [3]. Each case corresponds to a given rigid translation of the phantom during a circular scan of radius  $R = 360$  mm with a flat detector at 480 mm from the source with 0.5 mm pixel spacing. We will refer to these two cases as *motion 1* and *motion 2*. The rigid motions, as well as the equivalent source trajectories if the object remained static, are illustrated Fig.2.

In this study the motion is perfectly known, so is the corresponding virtual trajectory which is used to calibrate the iterative reprojection model  $\mathbf{R}^{\bar{\beta}}$ . For both methods, we have reconstructed an image of  $510 \times 510$  pixels large with a sampling rate of 1 mm in both directions. For the iterative

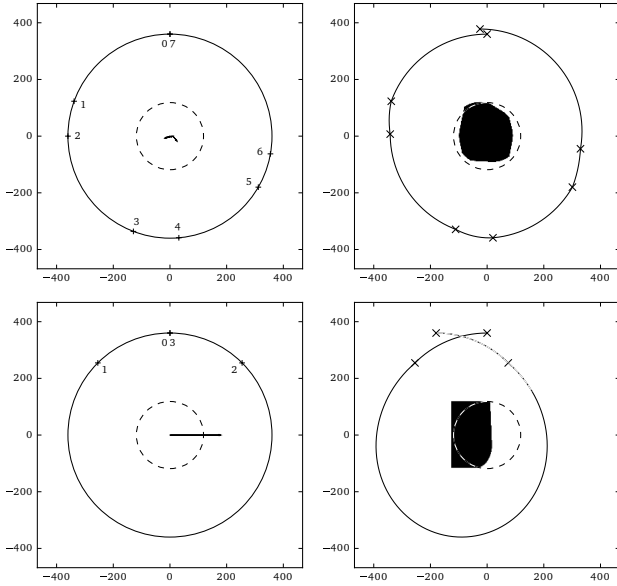


Fig. 2. Top left: Representation of motion 1. Bottom left: Representation of motion 2. Top right and bottom right: virtual trajectory, static FOV and regions of Hilbert points (in black) obtained respectively with the motions 1 and 2.

reconstruction, the value of  $\epsilon$  was chosen to be  $10^{-3}$ , *i.e.* 1/10 of the minimum contrast value of the Shepp-Logan phantom. Therefore structures in the image involving contrasts superior to this value will be preserved in the image, and smoothed otherwise. The hyperparameter  $\mu$  was carefully tuned “by hand” until a satisfactory reconstruction quality was reached. The typical value found is  $\mu = 10^3$ . The quality of the iterative reconstruction hardly depends on the injected amount of regularization. A weak value of  $\mu$  or no regularization will cause errors due to reprojection model approximations or noise to be amplified. Thus it is absolutely mandatory to regularize the solution, and preliminary results tend to verify this claim (*cf.* Fig. 3). We also observed in our reconstructions that the non-negativity constraint was also mandatory, and that it acts quite as a support constraint to force the algorithm to “put the information” in the right zones *cf.* Fig. 3(c-d).

Fig. 4 shows the reconstructions obtained with both methods. Fig. 5 displays horizontal profiles taken across different parts the phantom. We can see that the reconstruction is accurate in the predicted field of view. For the iterative reconstruction, we can notice that the efficiency of the reconstruction seems to go beyond this strict frontier between reconstructible and not reconstructible points, even if the error is higher in the zone of uncertainties (*cf.* Fig. 4(g-h) and Fig. 5). This is probably due to the fact that the points located in the uncertainty zone are partially covered by the scanning geometry, although not well enough to allow for an analytic solution. Therefore a sufficient information can be brought to the iterative algorithm to retrieve some structures, also helped by the regularization. Obviously the quality of such extrapolation is hardly object-dependent, and some lines of response can bring more information than others if the structures are oriented in suitable directions. We also notice

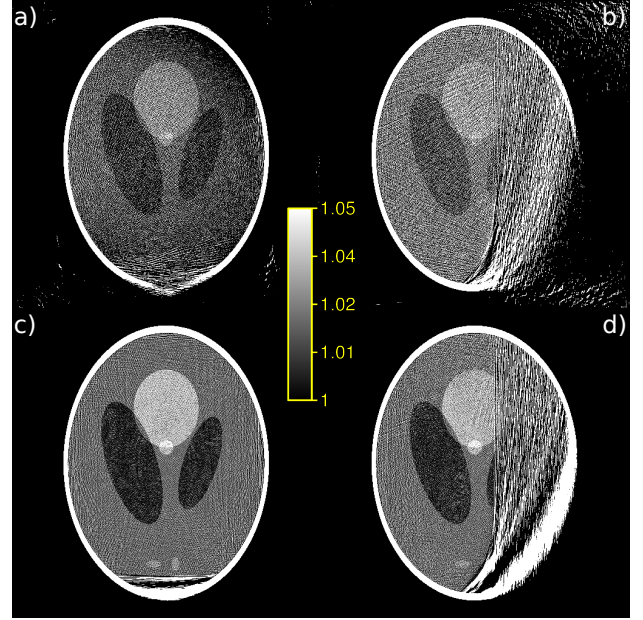


Fig. 3. Reconstructions of simulated cases *motion 1* (first column) and *motion2* (second column) with the iterative reconstruction method. (a-b) Reconstruction without non-negativity constraint and without regularization. (c-d) Reconstruction with non-negativity constraint and without regularization.

that injecting regularization causes some bias in the recovered values of the finer structures regarding the ground truth. This is a typical known behaviour which can be avoided by decreasing the value of the hyperparameter  $\mu$ , but at the cost of a dramatic increase of the variance of errors as we can see in Fig. 3. Hence regularizing can be seen as nothing but a trade-off between bias and variance on the final solution. However we can observe that the unregularized reconstructions in Fig. 3(c-d) make appear two different regions with two different levels of errors: one which remains almost stable regarding the ground truth and one which seems to completely diverge. The zone with the weaker error matches perfectly the predicted field of reconstructible points. This observation tend to show that regions of uniqueness and stability of the solution can be identified by the iterative reconstruction, if we put aside the amplifications of modelization errors. A finer model would probably have yielded a finer solution in these regions, and would have allowed to reduce the amount of regularization needed to erase these artifacts, reducing at the same time the bias.

#### IV. DISCUSSION AND CONCLUSION

The results of our simulations show good coherence before the region-of-interest predicted by the analytic method and the part of the image where the reconstruction by the iterative method is quantitatively successful. The iterative method, however, manages to “make up” certain features outside that region in a way which is still readable.

Our results also show that a priori information injected in the inverse problem - regularization, non-negativity constraint - is absolutely mandatory for the iterative reconstruction to be in good correlation, meaning a good tradeoff between bias and variance of the error.

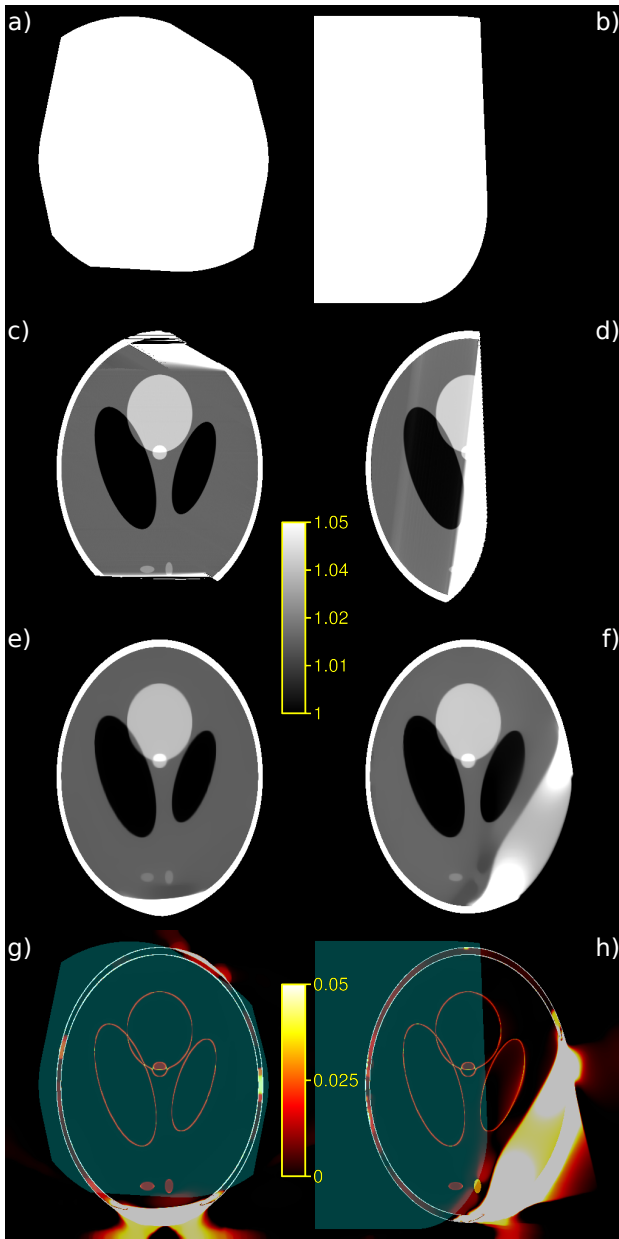


Fig. 4. Reconstructions of simulated cases *motion 1* (first column) and *motion 2* (second column) with both the analytic and iterative reconstruction methods. (a-b) Predicted FOV of reconstructible points. (c-d) Reconstruction with the analytic DBP-H method. (e-f) Iterative reconstruction. (g-h) Absolute value of the difference's map between ground truth image and iterative reconstruction, superimposed with the predicted FOV (in cyan).

The analytic method's potential is constrained by the need to have Hilbert points aligned on a segment crossing the object support entirely in order to be able to recover that part of the image. Implementing an iterative one-sided Hilbert transform inversion method alongside the analytic backprojection could lead to a reconstruction method where the whole reconstructible region is recovered, while the uncertain regions are discarded.

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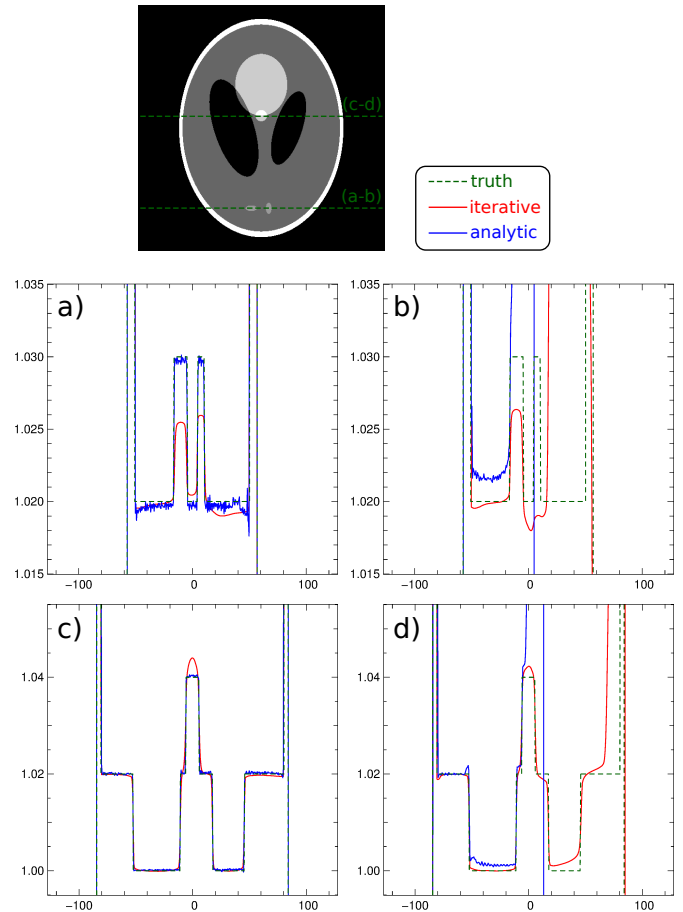


Fig. 5. Profiles taken in the reconstructions of Fig. 4. (a-c) *motion 1*. (b-d) *motion 2*.

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