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Adaptation of Kron's Tensorial Analysis of Network for the EMC Design and Analysis of Systems*

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Gabriel Kron developed tensorial analysis of networks (TAN) in 1939 to deal with electrical machines. This formalism has been extended to applications in the Electromagnetic Compatibility (EMC) domain. Using such a topological approach to deal with complex systems is particularly relevant in the design phases compared to numerical approaches and allows a problem to be studied in an abstract way. In this paper the robust mathematical approach is synthesized. Then, after explaining how cords are added to the TAN method, we apply the formalism to three typical examples.

I. INTRODUCTION

Nowadays, there is an increasing number of electronic systems that are more and more complex. As systems integrate different functions, it becomes essential to prevent electromagnetic disturbances between the different parts. As a result, the design phase is crucial and must be carried out with precision to ensure of the correct functioning when manufacturing the system. One way to attain this objective is to use numerical simulation. For this, different numerical methods have given rise to numerous electromagnetic commercial codes. We can cite Finite Element, Finite Difference or Moment Methods that are very popular in the EMC domain. But when dealing with the design phase, the whole system has to be simulated to take all the interactions into account and to prevent strong parasitic coupling. As a result, it becomes difficult to detect the origin of disturbances responsible for malfunctions by only trying different configurations. This approach is time consuming, and supposes that the numerical tools are adapted to the problem. One alternative is to use the engineer knowledge to have a more complete vision of the system. In fact, as mentioned above, a system is generally consists of subsystems. Thus it is more efficient to have a macro-model vision of each subsystem and to analyse the different connections between them. Such a topological approach provides a metric giving the value for each connection and allows the comparison between the different propagation path of disturbances. To follow such an approach, Gabriel Kron, developed an engineering approach based on Tensorial Analysis of Networks. This approach first developed for electrical machines, has recently been extended to EMC problems [1] to take into account not only conducting disturbances but also radiation couplings and even non linear ones. Furthermore, this approach allows EMC problems to be studied in an abstract way. In section II, the Einstein notation used in this paper will be reviewed. In

section III the principle of the classic Kron's method will be explained in details. Section IV presents three typical examples that show different kinds of models that can be integrated into Kron's approach. Then we will conclude on the interest of the method in section V.

II. NOTATIONS

In all of this paper we use Einstein's mute index notation. It means that each time an index is repeated, it implies a summation operation on the values of this index, i.e.:

$$\sum_{\alpha} a_{\alpha} u^{\alpha} \rightarrow a_{\alpha} u^{\alpha} \quad (1)$$

Vectors projected in a base \vec{b}_k are identified by their coordinates: $\vec{u} = u^k \vec{b}_k \rightarrow u^k$, with the index up. The k component of a vector appears as a superscript. Covectors are noted $\alpha (\vec{u} \cdot \vec{b}_k) = \omega_k$. The k component of a covector appears as a subscript.

III. KRON'S METHOD

Tensorial Analysis of Networks (TAN) as created by Kron[2] starts from a graph that represents the engineer's thinking. From this graph, a system of non linear integro-differential equations is deduced. This system is then studied theoretically. According to Kron, once equations are obtained, the circuit and its associated graph are no longer useful. The whole analysis should be conducted theoretically. Currents in the nodes pair space or the mesh space constitute the orthogonal axes of a multi-dimensional space that can be studied using differential geometry tools[4]. The concept of topology in Kron's approach is a mathematical one, and is not directly linked to the physical appearance of the system. It calls for a higher degree of theoretical thinking. However it is not constrained by any good shielding approximation [5] or

* EMC: electromagnetic compatibility

any other specific approximations, for example[6]. The topological side of Kron's method is in related to mathematical algebraic topology and its operators [7][8].

A. Graphs

In graphs nodes, branches and meshes are easily identifiable. These elements are part of a cellular topology [7]. Each branch is linked to a real electronic object and not at all to its geometrical aspect. Electronic objects are characterized by equations and intervals. A branch is associated with an equation that gives the relationship between an electromotive force e_u applied to the object and the current i^u flowing through the object under applied electromotive force $e_u = z_{uu}(i^u)$. When engineers connect branches together to make a graph, they construct a real system represented by a drawing. Intervals specify minimum and maximum values of the parameters. A single graph is made of various connected networks (space R , where any node is connected to two branches). Each network has branches (space B) connected together by nodes (space N), where a connected group of branches makes a mesh (space M) [9]. Euler-Poincaré's topological equation implies (see figure 1):

$$M - B + N - R = 0 \quad (2)$$

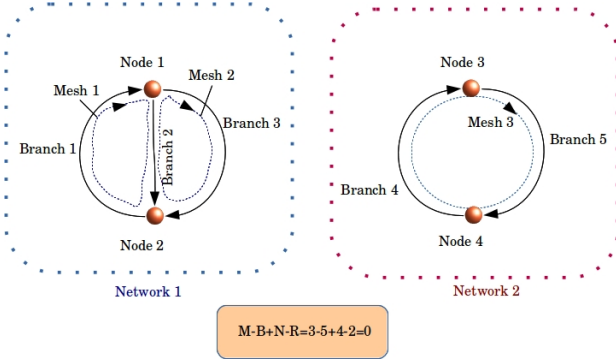


FIG. 1. Euler-Poincaré's relation

B. Circuits as dynamical systems with n degrees of freedom - currents in mesh space

Let us consider the case of a circuit made only with passive elements i.e. resistors, inductances and capacitances. In order to characterize an equivalent circuit associated with a branch, one connects an electromotive force across a branch and measures the current. Kron called this a primitive circuit. Using the Laplace operator, branch characterization reduces to simple expressions: $u_a = z_{aa}i^a$ (figure 2). On a branch, a difference in potential exists between the two ends of the branch but, making a mesh forces the difference in potential to

be null. Connecting two branches to the same mesh (figure 2b.), the two differences in potential are equal (figure 2a.) and therefore, their difference all around the mesh is zero.

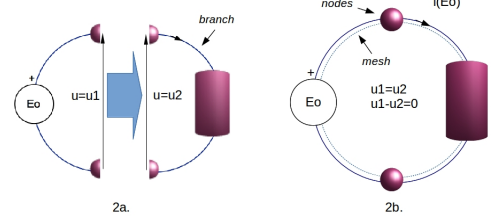


FIG. 2. Simple primitive circuit

1. Lagrange's equation

Under our hypothesis and using the Laplace operator p , circuit laws become: $E_0 = Ri$, $E_0 = Lpi$, $E_0 = 1/(Cp)i$ for a resistance, an inductance or a capacitance respectively. We now introduce how to couple these primitive circuits. If M, B, N, R are the topological characteristics of one circuit, we represent by $i^k, k = 1, 2, \dots, M$ the mesh currents associated with M meshes chosen in the circuit. $e_k, k = 1, 2, \dots, M$ are the electromotive forces of the meshes. We associate mesh loads q^k with variables i^k such as $i^k = \dot{q}^k$. figure 3 shows two coupled meshes where inductances, elastances and resistances are associated with branches or meshes, mesh currents and loads. A *cord* (defined in more detail below) represents a mutual inductance between meshes.

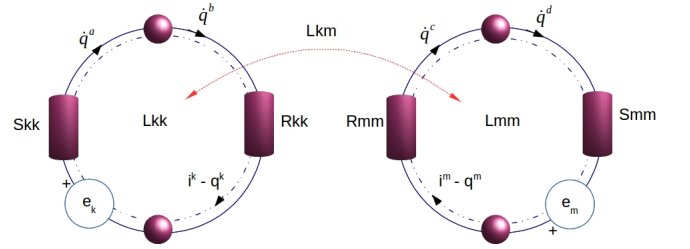


FIG. 3. Example of network for Lagrange's equation

In the whole circuit, kinetic energy is given by:

$$T = \frac{1}{2} \mathcal{L}_{km} \dot{q}^k \dot{q}^m \quad (3)$$

For the potential energy we have:

$$\mathcal{U} = \frac{1}{2} S_{km} q^k q^m \quad (4)$$

where S_{km} are elastance values (inverse of capacitance). And finally for dissipations:

$$\mathcal{W} = \frac{1}{2} R_{km} \dot{q}^k \dot{q}^m \quad (5)$$

Lagrange's equations for the circuit are given by:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}^k} \right) + \frac{\partial \mathcal{U}}{\partial q^k} + \frac{\partial \mathcal{W}}{\partial \dot{q}^k} = e_k \quad (6)$$

with (3),(4),(5) and using Laplace's operator p and transformations on i and e , with i_0^k initial conditions in current and q_0^k initial conditions in loads, Lagrange's equation becomes:

$$\mathcal{L}_{km} p(i^m - i_0^m) + \mathcal{R}_{km} i^m + \mathcal{S}_{km} \left(\frac{i^m}{p} + q_0^m \right) = e_k \quad (7)$$

Defining: $Z_{km} = \mathcal{L}_{km} p + \frac{\mathcal{S}_{km}}{p} + \mathcal{R}_{km}$, this is equivalent to: $Z_{km} i^m - \mathcal{L}_{km} i_0^m + \mathcal{S}_{km} q_0^m = e_k$. Usually, \mathcal{L}_{kk} are the self inductances of meshes, $\mathcal{L}_{km}, k \neq m$ the mutual inductances between meshes. \mathcal{S}_{kk} the self elastances of meshes and $\mathcal{S}_{km}, k \neq m$ common elastances between meshes. \mathcal{R}_{kk} are the self dissipations of meshes and $\mathcal{R}_{km}, k \neq m$ common dissipations between meshes. Note that for null initial conditions, Lagrange's equation can be written:

$$\left(\mathcal{L}_{km} p + \frac{\mathcal{S}_{km}}{p} + \mathcal{R}_{km} \right) i^m = e_k \quad (8)$$

2. Invariant

The fundamental concept of tensorial analysis is the existence of an invariant. In the case of Kron's proposal, this invariant is:

$$\omega = e_k i^k = f_k h^k \quad (9)$$

for two different referentials (e_k, i^k) and (f_k, h^k) . Suppose a change of referential to use another mesh base defined by:

$$i^a = C_a^k h^k \quad (10)$$

h^k being new mesh currents. Using (10) in (9) gives $f_k = e_a C_a^k$. Now using (10) in (8) leads to:

$$Z_{km} C_a^m h^a = e_k \quad (11)$$

Multiplying the two terms by C_n^i gives (with the relation on f_k):

$$C_n^k Z_{km} C_a^m h^a = C_n^k e_k = f_n \quad (12)$$

As C_n^k are constant, in general $C_n^k Z_{km} C_a^m = Z_{na}$, which transforms (12) into:

$$Z_{na} h^a = f_n \quad (13)$$

The equation is unchanged under any choice of mesh base. It means that the mesh space, under our hypothesis, can be considered as a Riemannian space with a Euclidian metric as all the Z_{km} are constant and Z_{km} is its fundamental tensor [10][11].

3. Application of Kron's method to EMC: Generalized interactions through cords

Each time a current i^o creates a field, this field induces an electromotive force e_k at the considered part of the network. This field is coupled after a transport process associated to any adequately defined Green's function. As for branches, these functions can be anything as long as their mathematical expression is defined. Starting from this proposal, the generic name of cord [12][13] was submitted to represent the coupling functions between branches or meshes. Green's functions can be those of free space, inside cavities between wires and field modes, and many other interactions. The coupling can be associated with the source covector e_k or represented using an impedance coupling function obtained from the ratio $e_k/i^m = z_{km}$. From this general approach of coupling functions, several models were developed, such as [14]:

- i Branin's model for transmission lines
- ii field to wire interactions
- iii coupling to objects within cavities

These physical processes are the major ones for EMC work. But also unusual EMC problems can be addressed, such as the interaction between particles and power supply on electronics [15] or the emissions of printed circuit board tracks [16]. Any near field or far field interaction can be easily incorporated in Kron's formalism using cords. Many cases were tested with good results providing a very elegant formulation of the problem under the mesh space. One particular example to illustrate this is the near field interaction between antennas achieved in [17].

A good illustration for general use of cords can be the coupling between two matched antennas in free space. The first mesh consists of the source generator, source impedance and radiation resistance. The second mesh is similar to the first but without a generator and a matched load. The cord is defined by Friis's equation:

$$\frac{P_2}{P_1} = G_1 G_2 \left(\frac{\lambda}{4\pi R_{12}} \right)^2 e^{-\frac{R_{12}}{c} p} \quad (14)$$

where P_2 is the received electromagnetic power on the second antenna, P_1 the transmitted power to the first antenna, G_1 the first antenna gain, G_2 the second one, R_{12} the distance between the two antennas, λ the wavelength, c the speed of light and p the Laplace operator. To define the cord z_{21} we define e_2 as a received electromotive force and i^1 as a transmitted current. Since $P_1 = Z_0 (i^1)^2$ and $P_2 = (e_2)^2 / Z_0$, Z_0 being the radiation resistance and source impedance, we easily obtain:

$$\frac{e_2}{i^1} = z_{21} = Z_0 \left(\frac{\lambda}{4\pi R_{12}} \right) \sqrt{G_1 G_2} e^{-\frac{R_{12}}{c} p} \quad (15)$$

figure 4 illustrates this example.

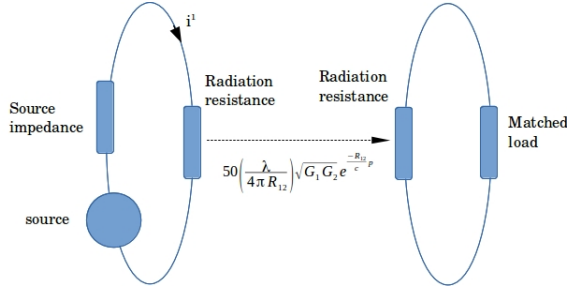


FIG. 4. cord illustration

For antennas, the coupling is symmetric. The fundamental tensor linked with this system is:

$$Z = \begin{bmatrix} Z_0 & z_{21} \\ z_{21} & Z_0 \end{bmatrix} \quad (16)$$

C. Intrinsic mesh properties

The advantage of having an additional space dimension compared to nodal techniques is to have a direct correspondence with surface flux through Hodge's operator [18]. There is no need for partial inductances: one inductance of a loop can be directly associated with this loop in the mesh space. A proposal to show the inductance in the center of the mesh in the graph was made [1]. Mathematically, once the impedance matrix Z is defined in the branch space, its transformation by the matrix C to the mesh space is carried out using the bilinear product $\zeta = [C]^T Z C$. Now if some properties belong to the mesh space, a μ matrix can be defined where self inductance of meshes, mutual inductances, etc., are computed and in the mesh space, the equation becomes:

$$e_a = (\zeta_{ab} + \mu_{ab}) i^b \quad (17)$$

D. Building a mesh set, additional branches and virtual meshes

On any graph G including nodes, branches, meshes and networks, one spanning tree ST can be defined for each network. It is a sequence of branches that go one and one time only through all the nodes of the network. If we impose to choose of a spanning tree made of successive branches, then if b_u is an unused branch: $\exists (n_1, n_2) \in \{\partial ST\} / (n_1, n_2) \in b_u$ (∂ being the boundary operator). Then if A is the incidence matrix so that $n_1 = A_1^k b_k$, $n_2 = A_2^q b_q$ and $(b_k, b_q) \in ST^2$. If $\{b_k, \dots, b_q\}$ is the set of branches included between b_k and b_q on ST , then $\{b_k, \dots, b_q, b_u\}$ makes a mesh M_u using what is called a closing branch b_u . This technique provides a guide to choose meshes in any network. It leads to more adequate matrix conditioning except when two meshes are coupled through a filter structure. figure 5 shows the

two cases of a shared branch or a spanning tree process to define mesh-to-branch connectivity.

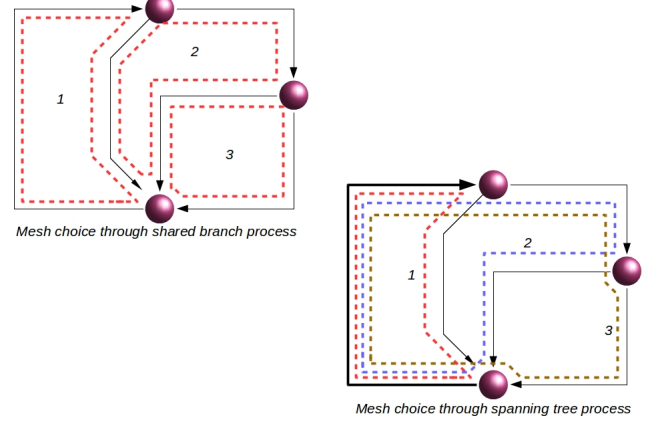


FIG. 5. Mesh definitions

In the case of the filter structure, it is clear that the boundary made by the filter must be the region where currents are shared and distributed depending on frequency and filter characteristics. We consider two branches of branch currents i^1 and i^2 , connected through a common mesh (they can be the boundaries of two larger networks). We can always add a branch of current i^a connected to the same two nodes of branches i^1 and i^2 to make two meshes: the first one of currents i^1, i^a and the second one of currents $-i^a, i^2$. The impedance of this added branch appears as an extra-diagonal component of the impedance matrix of the two mesh networks. In many cases, this added branch has a capacitor as its impedance, which is the classical first order filter in EMC (figure 6).

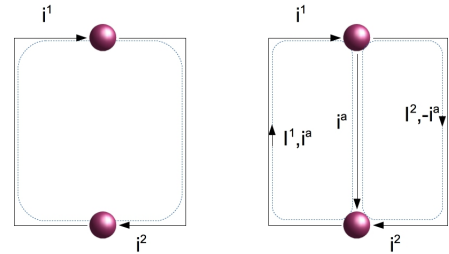


FIG. 6. Shared branch detail

Another kind of mesh can be attached to the branch of ST : virtual meshes. These are meshes of known mesh currents J^v [19]. The general equation becomes: $e_a = z_{ab} i^b + z_{av} J^v$.

E. Kron's approach

In Kron's approach, each component of a system is seen as a primitive element involved in the system construction [20]. The set of all primitive networks constitutes

the diagonal part of Kron's fundamental tensor. All the electromagnetic interactions between the primitive elements remain to be added. Kron's fundamental tensor is a mathematical image of the EMC interaction matrix[21]. Kron's method can be broken down into six steps:

1. build up the set of primitive circuits z_{mn} involved in the system as defined in the branch space (figure 7);
2. carry out the direct summation of all these components of impedance tensor z_{mm} : $z_{complete} = \oplus_m z_{mm}$ (figure 7);
3. add coupling between branches A_{mn} (figure 8);
4. define connectivity C , choose a spanning tree to identify meshes then transform to the mesh space $Z_{\mu\nu} = C_{\mu}^m C_{\nu}^n (z_{mm}|_{m=n} + A_{mn}|_{m \neq n})$ (figure 9). Capital letters are now used to identify objects that belong to the mesh space;
5. add self mesh properties $S_{\mu\nu}$, interactions between meshes and current sources: $(Z_{\mu\nu} + S_{\mu\nu}) = G_{\mu\nu}$, to obtain

$$E_{\sigma} = G_{\sigma\beta} v^{\beta} + G_{\mu\alpha} J^{\alpha} \quad (18)$$

(figure 10). ;

6. solve the system.

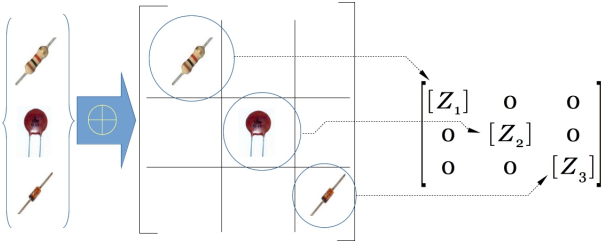


FIG. 7. A set of primitive circuits in the mesh space (1st step of Kron's model)

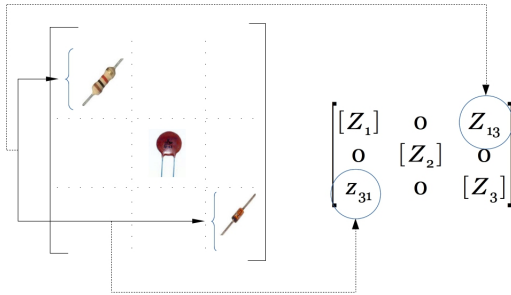


FIG. 8. Addition of interactions (coupling) between branches (3rd step in Kron's model)

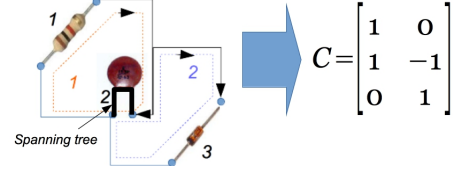


FIG. 9. Transformation to the mesh space through the connectivity matrix C (4th step in Kron's model)

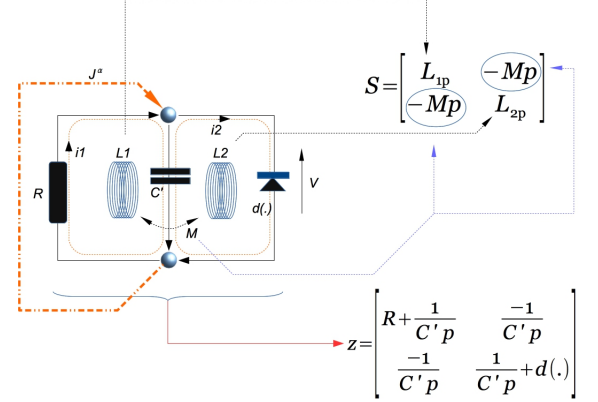


FIG. 10. Addition of mesh self properties and current sources in the mesh space (5th step in Kron's model)

The result of using this method is this system of equations. Kron's method is a rigorous mathematical basis to theoretically study the behavior of electronic systems in EMC. Once the system of equations is available, it can be solved using numerical techniques.

In Kron's fundamental tensor $G_{\sigma\beta}$, components coming from various techniques may be included: Z matrix for moment method, macro-models derived from measurements, and so on. Due to its mathematical origin, Kron's method may be applied to many different problems[22][23].

IV. THREE EXAMPLES OF KRON'S METHOD USE FOR EMC

In order to illustrate the use of Kron's method for EMC, we have chosen three examples. The first one focuses on a shielded cable problem. Internal pigtailed still pose a difficult problem that cannot be solved using 3D codes or SPICE-like software. Kron's method is an efficient way to approach the problem, to write its mathematical expression rigorously and to compute the order of magnitudes. The second example focuses on cavities. Numerical software doesn't always take all losses into account inside cavities. Even if techniques are now available, it remains difficult to analyse the result without the order of magnitudes. Once again Kron's method gives the means to make this analysis. And finally the third case concerns abstract study. It shows how Kron's formalism leads to the rigorous analytical equations of

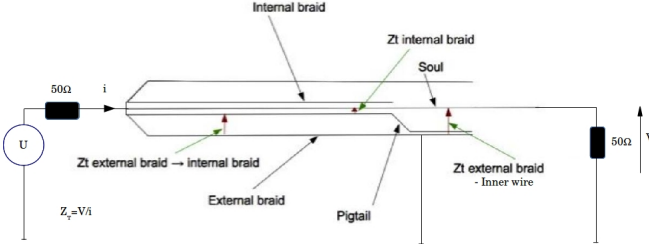


FIG. 11. Shielded cable with internal pigtailed

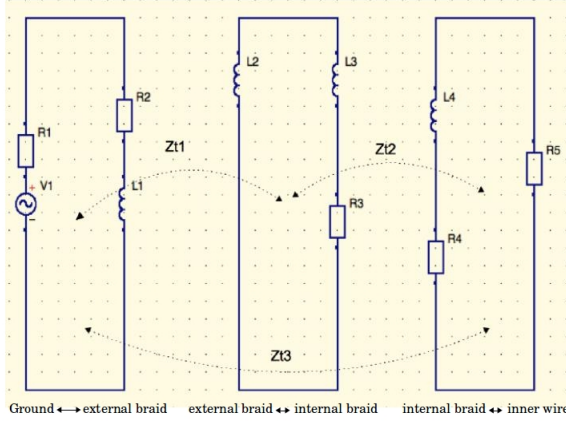


FIG. 12. Graph for multi-braid cable

coupling systems in relative acceleration.

A. A multi-braid cable with pigtail

Working in the mesh space is an efficient technique to obtain equations in the case of cable transfer impedances. The technique was applied in a difficult case, where the internal braid was connected to the external one through a pigtail [24]. As long as the wavelength is greater than the cable length, an external domain including the external braid, ground plane and shield short circuits can be reduced to one mesh. The same representation can be used for any other intermediate domain. figure 11 shows the cable configuration and figure 12 how the case was managed with two braids linked through pigtailed.

The problem concerns three domains: the external domain with the external braid and the ground plane, the intermediate domain with the internal braid and pigtailed and the internal domain with the internal braid and the wire. Each domain includes the braid resistance and self inductance and for the internal wire, the same elements but given for a wire inside a braid. Couplings are transfer impedances, computed using equations given in [25] and refer to equations (19) to (22). For the direct coupling between the external braid and the wire through the internal pigtailed, a special equation was developed in [24] and it shows how the method easily accepts any kind of newly developed equation. The transfer impedance is

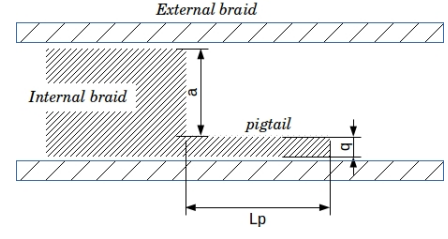


FIG. 13. Pigtail dimensions

given by:

$$Z_{pigtail} = p \frac{\mu L_p}{2\pi} \text{Log} \left(\frac{a}{q} \right) \quad (19)$$

L_p being the pigtail length, a the supposed constant distance between the straight pigtail and the height reference q , located at the surface of the pigtail (see figure 13). The internal braid has a transfer impedance given by:

$$Z'_{t2} = (R_\alpha + \xi p) x \quad (20)$$

where x is the cable length, R_α the DC resistance of the braid and ξ its transfer inductance value. However, since the internal braid is terminated by the internal pigtail, the total transfer impedance between this braid and the inner wire is given by $Z_{t2} = Z'_{t2} + Z_{pigtail}$; so:

$$Z_{t2} = R_\alpha (x - L_p) + p \left(\xi (x - L_p) + \frac{\mu L_p}{2\pi} \text{Log} \left(\frac{a}{q} \right) \right) \quad (21)$$

The external braid is coupled with the internal braid over its whole length and is coupled with the inner wire only along the pigtail length. Transfer impedance of braids is given, for braid angles around 45 degrees (ψ) and a diffusion process, by:

$$Z_t = \frac{4}{\pi d^2 N C \sigma \cos(\psi)} \frac{(1+j)^{\frac{d}{\delta}}}{sh \left[(1+j)^{\frac{d}{\delta}} \right]} \quad (22)$$

where σ is the braid conductivity, ψ the braid angle, C the number of conductor groups in the braid, d the conductor diameter and δ the skin depth given by:

$$\delta = \left(\frac{2}{\omega \mu_0 \mu_r \sigma} \right)^{\frac{1}{2}} \quad (23)$$

To the diffusion process must be added the diffraction process, represented by a transfer inductance. It is given, for angles around 45 degrees by:

$$L_t = \nu \frac{\alpha_m \mu_0}{\pi^2 D^2} \quad (24)$$

where D is the braid diameter, ν the number of holes per meter and α_m the magnetic polarisability of each hole. α_m is approximated by:

$$\alpha_m \approx \frac{L_g^3}{6} \quad (25)$$

L_g being the hole length. A correction term ct is given by:

$$\begin{cases} Kp = \frac{1,16}{NCd} \text{atan}\left(\frac{N}{3}\right) \sin\left(\frac{\pi}{2} - 2\psi\right) \sqrt{\frac{\mu_0}{\sigma}} \\ ct = Kpe^{-j\frac{\pi}{4}}\omega^{0,7} \end{cases} \quad (26)$$

and Z_{t1} is given by:

$$Z_{t1} = (Z_t + ct + L_tp)(x - L_p) \quad (27)$$

while Z_{t3} is given by $Z_{t3} = (Z_t + ct + L_tp)L_p$. These equations provides the means to compute both Z_{t1} , Z'_{t2} and Z_{t3} values. Z_{t3} is applied on the pigtail length L_p . The global impedance tensor of the problem is:

$$Z = \begin{bmatrix} R_1 + R_2 + L_1p & 0 & 0 \\ -Z_{t1}(x - L_p) & L_2p + R_3 + L_3p & 0 \\ -Z_{t3}L_p & -Z_{t2} & R_4 + R_5 + L_4p \end{bmatrix} \quad (28)$$

R_k with $k = 1, 2, 3, 4, 5$ are the loads. For the internal braid, the only significant resistance is the pigtail connection to the end of the cable. L_3 is the pigtail inductance (approximated by $\mu\pi r/2$, with r radius of the equivalent circle made by the pigtail and its distance to the inner wire).

The measurement was made in a triaxial test bench for transfer impedances. Inductances L_k for $k = 1, 2, 4$ were computed using the fundamental coaxial laws. The first one gives the cable characteristic impedance:

$$Z_c = 60 \ln\left(\frac{r_e}{r_i}\right) \quad (29)$$

r_e and r_i are the internal and external radii of the braid and the internal wire. Then, we can obtain the inductance using:

$$L = \frac{Z_c}{v}x \quad (30)$$

where v is the propagation speed in the cable. The measurement was made on a 10 cm diameter and 30 cm long triaxial test bench, which can make measurements up to 100 MHz [24]. figure 14 shows the measurements and computation curves. The average difference between measurement m_i and computation c_i ($|m_i - c_i|/m_i$) is 33% for a standard deviation of 26% ($\psi = 45.2$, $\sigma = 10^8$, $N = 16$, $C = 120$, $d = 112.5\mu m$, $D = 15mm$).

Note that if we had used an approach by topological decomposition of domains, it would have been difficult to take into account the Z_{t3} interaction. This interaction is of major significance. By not taking it into account the computation differs enormously from the measurement as shown in figure 15.

B. Coupling inside cavities

Various models were developed to compute problems in cavities (without including those that use 3D codes)

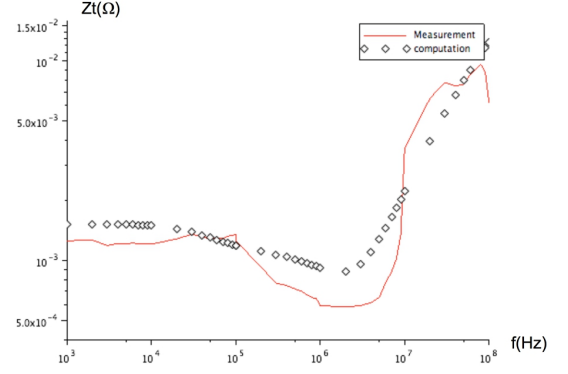


FIG. 14. Comparison computation - measurements

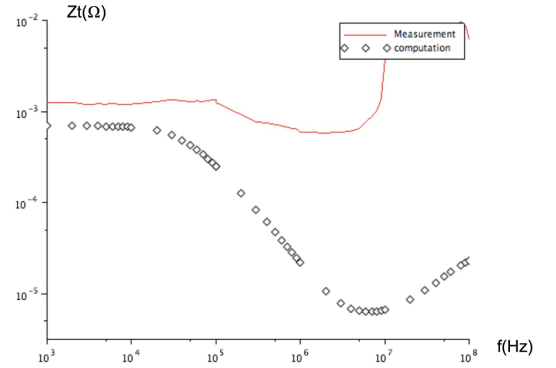


FIG. 15. Comparison computation - measurements without internal pigtail

[14][26][27][28][29]. To model confined fields many techniques may be incorporated in Kron's formalism such as chain matrices, Branin's model for guides, RLC circuit networks or transfer functions that behave like random couplings. One of these techniques may be more efficient than the others depending on the problem being investigated. Modelling the cavity itself is not difficult since an EMC engineer naturally looks mainly for order of magnitudes. Estimating the coupling between wires and field inside cavities is much more difficult. Using Kron's method, once the coupling impedances between the field modes and wires are obtained, the problem is entirely solved. For simple geometries, it is quite easy to estimate the analytical formulation in order to compute the coupling functions, and various works show this capability [30]. For more complex cavities, one technique consists in following Collins [31] to approach cavity geometry using several guided waves. With more than one guided wave and various interfaces, many geometries can be approached sufficiently well to give good results compared to measurements [32].

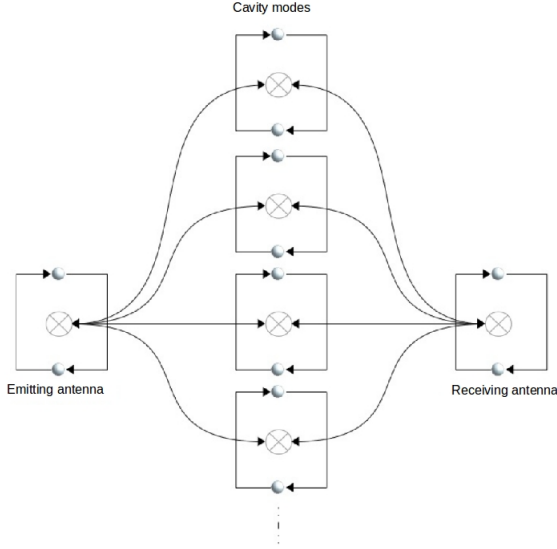


FIG. 16. Interaction inside a cavity: graph

1. Cavity parameters for resonators

We describe here the "RLC" approach (resonators) which is very efficient, giving accuracy of the same order as when using 3D codes. It requires a great effort to determine the circuit components [33]. But it is possible to benefit from all the works done previously in high frequency modelling involving cavities (see for example [34] [35]). figure 16 shows our problem made of two small antennas inside a cavity. Each field mode is represented by an "RLC" oscillator. Various elements inside the cavity interact with these modes, and if coupling exists between modes, these oscillators can be coupled with each other.

Each time a coupling occurs, a cord represents it on the graph making an electromagnetic link between the coupled elements. The emitting antenna is a simple loop. This kind of primitive circuit is directly given in the mesh space with an impedance equal to:

$$z_e = R_e + L_e p + \frac{1}{C_e p} \quad (31)$$

p being the Laplace operator. The same equation is used for the receiving antenna with index "r". Under the hypothesis of weak losses, the resonance frequencies for a rectangular cavity of length L , height H and width w are given by:

$$f_{mnq} = \frac{c}{2} \sqrt{\left(\frac{m}{L}\right)^2 + \left(\frac{n}{H}\right)^2 + \left(\frac{q}{w}\right)^2} \quad (32)$$

Under the hypothesis that only one polarisation exists in one dimension H , we can obtain the capacitor value for one resonator at one field mode making the equivalence between the electric field energy stored in the volume of the cavity and $1/2CV^2$ formulation, V being the field

work along the dimension H and C the capacitor we want to calculate. It means that:

$$\begin{aligned} \epsilon_0 \int_0^w \int_0^L \int_0^H dx dy dz \left[\sin\left(\frac{m\pi x}{w}\right) \sin\left(\frac{q\pi y}{L}\right) \right]^2 \dots \\ \dots = C \left(\int_0^H dz \right)^2 \end{aligned} \quad (33)$$

After integration we obtain for an mnq mode:

$$\begin{aligned} C_{mnq} = \frac{\epsilon_0}{2H} \left\{ wL - \frac{wL}{2mq\pi^2} (\cos(2q\pi) \cos(2m\pi) \dots \right. \\ \left. \dots - \cos(q\pi) - \cos(m\pi) + 2) \right\} \end{aligned} \quad (34)$$

From C_{mnq} we can compute L_{mnq} using:

$$L_{mnq} \approx \frac{1}{4\pi^2 f_{mnq}^2 C_{mnq}} \quad (35)$$

Losses are not taken into account in this equation, always under the hypothesis that they are sufficiently weak. These losses have to be calculated, to obtain the resistance values for the resonators. We start from the composite quality coefficient Q . It involves different loss mechanisms such as losses in the walls (Q_W) and losses associated with antennas (Q_a). Q_W accounts for the contribution of overlapping orders of modes given by:

$$Q_W = \frac{3v}{2\delta S \mu_r} \left[1 + \frac{3\lambda}{16} \left(\frac{1}{L} + \frac{1}{w} + \frac{1}{H} \right) \right]^{-1} \quad (36)$$

where v is the cavity volume, δ the skin depth, λ the wavelength and S the wall surface. For the receiving and emitting antennas, the quality coefficient is:

$$Q_a = \frac{L_a \omega}{R_a} \quad (37)$$

The whole quality coefficient for our problem is obtained through:

$$\frac{1}{Q_r} = \frac{1}{Q_W} + \frac{2}{Q_a} \quad (38)$$

This gives the resistance for each resonator. Now, the resistance associated with each mode is given by:

$$R_{mnq} = \frac{L_{mnq} 2\pi f_{mnq}}{Q_r} \quad (39)$$

Once the cavity model is determined (we suppose that models for the antennas are available and given by z_a), the coupling coefficient between all the components involved in the problem remains to be obtained. At this level, the impedance tensor results from the direct summation of all the primitive elements involved in the problem. It means that:

$$z = \oplus_k z_k = \begin{bmatrix} z_a & 0 & \dots & 0 \\ 0 & R_{mnq} + L_{mnq}p + \frac{1}{C_{mnq}p} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & z_a \end{bmatrix} \quad (40)$$

2. Coupling function calculation

In general, if we consider a coupled system, it is identified by a fundamental impedance tensor given by:

$$g = \begin{bmatrix} a & \beta \\ \beta & b \end{bmatrix} \quad (41)$$

The β function represents the coupling phenomenon. In the mesh space of mesh currents J^k , the system of equations for one source on the first mesh is given by:

$$\begin{cases} e_1 = aj^1 + \beta j^2 \\ 0 = \beta j^1 + bj^2 \end{cases} \quad (42)$$

To identify the condition that leads to the maximum energy on the receiving part, we look for:

$$\frac{\partial j^2}{\partial \beta} = 0 \quad (43)$$

This gives $\beta = \sqrt{ab}$. For magnetic energy, the coefficient is in general $-\alpha\sqrt{L_1 L_2 p}$. The coefficient α accounts for losses in the coupling process. So in general we have $\beta = \alpha\sqrt{ab}$. Determining the coupling function is reduced to the identification of this coefficient α . In the case of a closed circuit of normal \vec{n} , we consider the coupling with the magnetic field. The coupling coefficient is in this case given by:

$$\alpha = \iint dr d\theta \vec{n} \cdot \vec{B}^1(x, y, z) \quad (44)$$

\vec{B}^1 is a normalized field (here in $T \cdot m^{-3}$) in order to have α value without dimensions reaching 1 maximum. The magnetic field modes \vec{b} are defined by:

$$\begin{aligned} \vec{b} &= \cos\left(\frac{q\pi y}{L}\right) \sin\left(\frac{m\pi x}{w(t)}\right) \vec{u}_x + \dots \\ &\dots + \sin\left(\frac{q\pi y}{L}\right) \cos\left(\frac{m\pi x}{w(t)}\right) \vec{u}_y \end{aligned} \quad (45)$$

For example, locating a piece of wire as the emitting antenna at the center of the cavity with its normal following y on x_0 point for a length χ leads to the next coefficient $\alpha = \alpha_e$:

$$\alpha_e = \frac{1}{\chi} \int_0^\chi dx \cos\left(\frac{m\pi x}{w(t)}\right) \sin\left(\frac{\pi(x - x_0)}{R_b}\right) \quad (46)$$

The same calculation has to be made for the receiving antenna. Adding these coupling functions to the impedance tensor gives the complete matrix of this tensor and solves the complete coupled system. The fundamental tensor looks like:

$$z = \begin{bmatrix} z_a & \alpha_e & \dots & \alpha_{aa} \\ \alpha_e & R_{mnq} + L_{mnq}p + \frac{1}{C_{mnq}p} & \dots & \alpha_{ca} \\ \dots & \dots & \dots & \dots \\ \alpha_{aa} & \alpha_{ca} & \dots & z_a \end{bmatrix} \quad (47)$$

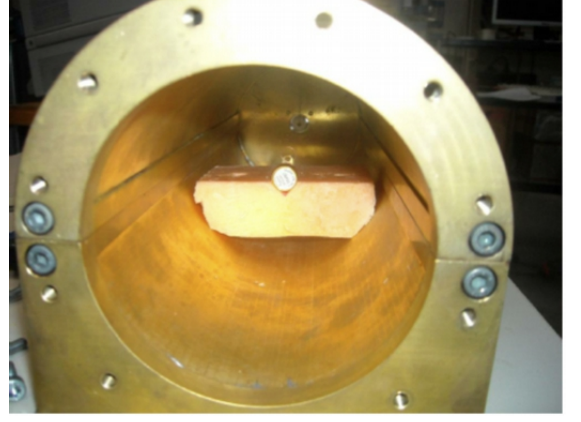


FIG. 17. Cavity photograph

α_{xy} are the coupling function between antennas and field modes, or between modes, etc. Note that we accept here that a direct coupling occurs between the antennas through α_{aa} . In [33], complete exercise was done and gives very good results in comparison with measurements, we describe now this experiment. The experiment consists in locating a small rectangular open end wire antenna on one disk of a cylindrical copper cavity. The cavity is 10 cm in diameter and 30 cm long (figure 17), and we can see on the figure the half-wave antenna centred on the bottom disk of the cavity. The half-wave antenna is 3 cm long, parallel to the metal plate of the disk, at 5 mm above it. Its base is centred and connected to an SMA connector, in order to be power-supplied using a 50Ω high frequency amplifier.

The cavity is excited at TEM_{010} first mode (cylindrical one). The coupling coefficient between the first antenna and the cavity was computed using the scalar product between the magnetomotive force of the antenna and the mode reluctance:

$$\begin{aligned} \alpha &= \frac{l^2}{\pi} \left\{ \left(\frac{1}{a+l} \right) \left[\cos\left(\frac{\pi(a+l)}{a}\right) - 1 \right] - \dots \right. \\ &\left. \left(\frac{1}{a-l} \right) \left[\cos\left(\frac{\pi(a-l)}{a}\right) - 1 \right] \right\} \end{aligned} \quad (48)$$

where a is the cavity diameter and l the antenna length. The antenna is defined using:

$$\begin{cases} Z_c = 60 \log\left(\frac{4h}{d}\right) \\ R_a = \frac{1}{l\sigma_{Cu}} \frac{l}{\pi \cdot d \cdot e} \\ Z_a = \frac{Z_c}{p \cdot \tan\left(\frac{2\pi f l}{c}\right)} \end{cases} \quad (49)$$

where h is the antenna height on the plate, σ_{Cu} the copper conductivity, d the antenna diameter and e the skin depth. c is the light speed and f the frequency.

The equivalent circuit for the mode is given by its com-

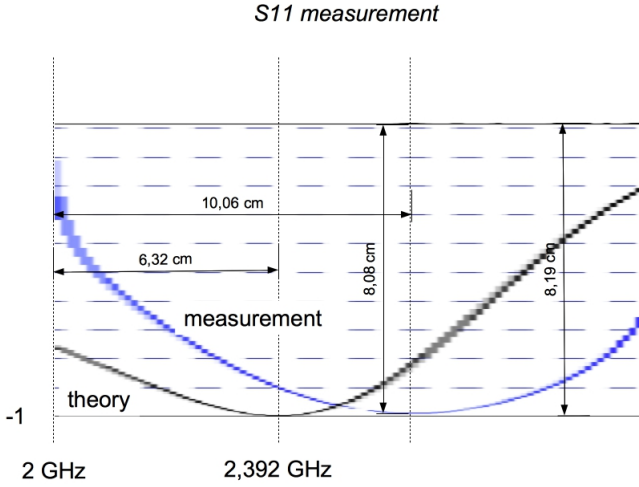


FIG. 18. Measurement versus calculation in cavity

ponents:

$$\begin{cases} L_1 = \frac{1}{2\pi f_1} \\ C_1 = L_1 \\ R_1 = L_1 2\pi \frac{f_1}{Q_r} \end{cases} \quad (50)$$

These relations lead to the coupling coefficient $\beta = \alpha\sqrt{L_a L_1}$. So the fundamental tensor for this problem directly written in the mesh space is:

$$z_m = \begin{bmatrix} R_g + R_a + \frac{1}{C_{ap}} + L_a p & -\beta \\ -\beta & R_1 + \frac{1}{C_{1p}} + L_1 p \end{bmatrix} \quad (51)$$

figure 18 shows the results. A difference, graphically measured between calculation and measurement for the S_{11} parameter obtained at the first antenna input is 10% in frequency and 2% in amplitude.

C. EMC analysis in a dynamic scenario

A set of available analytical equations allows us to make in-depth theoretical studies of problems. In various cases it is possible to demonstrate general results without making any numerical computation or optimisation in the design phase [36][37][38][39]. It would be difficult to write the equation of a system using nodal, direct or other methods. From this point of view, Kron's method is quite unique. In [1] for example, a simple exercise on general behaviour of a system submitted to external sources was proposed. One example can be given, inspired from Kron's work on electrical machines [40]. We imagine a first electronic device A emitting a magnetic field that can disturb a second one B seen as a receiver. In the first step we look at the coupling relation between the two electronic devices. A turns around B , so that the magnetic interaction between A and B is not only

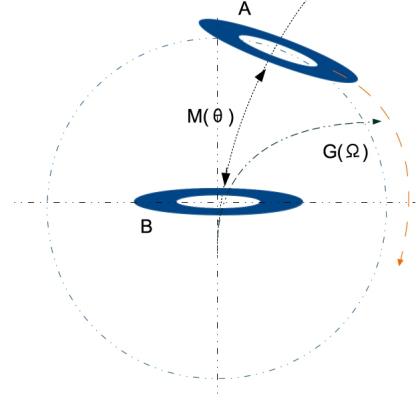


FIG. 19. Interaction between two loops in movement

reduced to the mutual inductance M_{12} between them, but also includes a cut flux coupling $G_{12}\Omega$ where G_{12} is the magnetic coefficient of the cut flux and Ω the angular speed. We suppose in this simple illustration that all parameters are constant (for example $\Omega(t) = \Omega$). figure 19 shows the system analysed.

We write the equations of the system when each electronic device is reduced to a simple mesh and in its original configuration, like a magnetic loop:

$$\begin{cases} e_A = r_{AA}i^A + L_{AA}\frac{di^A}{dt} + M_{AB}\frac{di^B}{dt} + G_{AB}\Omega i^B \\ e_B = M_{AB}\frac{di^A}{dt} + r_{BB}i^B + L_{BB}\frac{di^B}{dt} \end{cases} \quad (52)$$

This system constitutes the primitive system for Kron. Now we make a connection between currents of the same system but for a different angle at any other time:

$$\begin{cases} i^A = i'^A \cos(\theta(t)) \\ i^B = i'^B \end{cases} \quad (53)$$

This leads to the connectivity:

$$C = \begin{bmatrix} \cos(\theta(t)) & 0 \\ 0 & 1 \end{bmatrix} \quad (54)$$

System (50) can be generalized by writing:

$$e_x = r_{xx}i^x + L_{xy}\frac{di^y}{dt} + G_{xy}\Omega i^y \quad (55)$$

where L tensor includes M components. Using (53) we have $i^x = C_{\beta}^x i'^{\beta}$. With replacement in (52) this leads to:

$$e_x = r_{xx}C_{\beta}^x i'^{\beta} + L_{xy}\frac{dC_{\beta}^y i'^{\beta}}{dt} + G_{xy}\Omega C_{\beta}^y i'^{\beta} \quad (56)$$

But:

$$\frac{dC_{\beta}^x i'^{\beta}}{dt} = \frac{dC_{\beta}^x}{d\theta} \frac{d\theta}{dt} i'^{\beta} + C_{\beta}^x \frac{di'^{\beta}}{dt} \quad (57)$$

Knowing that $\dot{\theta} = \Omega$ we obtain:

$$e_x = r_{xx}C_{\beta}^x i'^{\beta} + L_{xy} \left(\frac{dC_{\beta}^y}{d\theta} \Omega i'^{\beta} + C_{\beta}^y \frac{di'^{\beta}}{dt} \right) + G_{xy}\Omega C_{\beta}^y i'^{\beta} \quad (58)$$

Now we can multiply each member on the left by C_α^x to obtain:

$$e_\alpha = r_{\alpha\beta} i'^\beta + L_{\alpha y} \frac{dC_\beta^y}{d\theta} \Omega i'^\beta + L_{\alpha\beta} \frac{di'^\beta}{dt} + G_{\alpha\beta} \Omega i'^\beta \quad (59)$$

We note $H_{\alpha\beta,\theta} = L_{\alpha y} \frac{dC_\beta^y}{d\theta}$ to obtain:

$$e_\alpha = r_{\alpha\beta} i'^\beta + H_{\alpha\beta,\theta} \Omega i'^\beta + L_{\alpha\beta} \frac{di'^\beta}{dt} + G_{\alpha\beta} \Omega i'^\beta \quad (60)$$

Now we can wonder what the added term $H_{\alpha\beta,\theta} \Omega i'^\beta$ represents when compared to a classic EMC evaluation of coupling between two loops in static configuration. For a static system, the equation is: $e_a = r_{ab} i'^b + L_{ab} \frac{di'^b}{dt}$. The ratio r between the electromotive force received in the moving loop compared to the electromotive force received in a static one in the harmonic domain becomes:

$$r = \frac{r_{\alpha\beta} + L_{\alpha\beta} p + L_{\alpha y} p C_\beta^y \Omega^2 + G_{\alpha\beta} \Omega}{r_{\alpha\beta} + L_{\alpha\beta} p} \quad (61)$$

If $r_{\alpha\beta}$ is low, $G \approx L$, this leads for $\theta = 0$ to:

$$r = \Omega^2 + \frac{\Omega}{p} + 1 \quad (62)$$

If $\Omega \ll \omega$, $r \approx 1 + \Omega^2$, it shows that the influence increases when the angular speed is squared. The phenomenon may be significant in magnitude. In this case, its contribution should be taken into account for risk prediction. This kind of theoretical analysis can be done thanks to Kron's formalism and linked with tensorial algebra, giving access to complex situations, even relativistic ones[41][42]. Far beyond EMC, Kron's work has influenced many communities through his approach for systems[43]. His formalism makes it possible to synthesize general interactions in an easier way than any other, even in multiphysics, since his fundamental tensor leads directly to Lagrange's equations of dissipative systems.

V. CONCLUSION

It is little known that the nodal method usually employed for circuits was replaced by the "modified nodal analysis" in order to take into account driven sources. But in [50], it was demonstrated that this problem can be taken into account in classical nodal analysis, through the mesh space! In fact, Kron's mesh space is able to solve any circuit today. Furthermore the method makes it possible to carry out in-depth analyses of a problem

using the geometrical approach in the real mathematical meaning of topology. We have shown through three examples that Kron's method applied to EMC gives the technique to study complex systems using all the theoretical knowledge developed in EMC. It can be the first task done by EMC engineers before using numerical tools to validate their designs. No doubt many other techniques will be found based on Kron's works usable partly for EMC, such as the recent "xTAN" method to take into account the human factor in the studying of complex systems from the EMC point of view [41][51][52][53].

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Appendix A: Kron's diakoptics

Diakoptics is perhaps the best known technique developed by Kron[54][55]. There are various ways to apply diakoptics. We consider here a general principle among the simplest ones.

From a first set of solutions, we obtain mesh current values depending on sources: $i^k = (G_{mk})^{-1} E_m$. Then we add loads on the boundaries of the previous network. This can be written:

$$\begin{bmatrix} E_m \\ 0 \end{bmatrix} = [(G_{mk} \oplus Z_{kf}) + S_{mf}] \begin{bmatrix} i'^k \\ i'^f \end{bmatrix} \quad (A1)$$

because loads do not have self sources and i'^f are the boundary currents. Which leads to the system

$$\begin{cases} E_m = G_{mk} i'^k - S_{mf} i'^f \\ 0 = -S_{mk} i'^k + Z_{mf} i'^f \end{cases} \quad (A2)$$

but $E_m = G_{mk} i^k$. Writing $i'^f = -y^{fm} \mu_{mk} i^k$ we find:

$$G_{mk} i^k = G_{mk} i'^k + S_{mf} y^{fm} \mu_{mk} i^k \quad (A3)$$

The new currents with added loads can be obtained from the previous circuit computing:

$$i^k = [g_{mk} + \mu_{mf} y^{fm} \mu_{mk}]^{-1} g_{mk} i^k \quad (A4)$$

Yuri Sohor has developed very efficient techniques to use diakoptics for large problems[51].

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