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# Optimisation of spatial CSMA using a simple stochastic geometry model for 1D and 2D networks

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**Abstract**—In modern wireless networks especially in Machine-to-Machine (M2M) systems and in the Internet of Things (IoT) there is a high densities of users and spatial reuse has become an absolute necessity for telecommunication entities. This paper studies the maximum throughput of Carrier Sense Multiple Access (CSMA) in scenarios with spatial reuse. Instead of running extensive simulation with complex tools which would be somewhat time consuming, we evaluate the spatial throughput of a CSMA network using a simple model which produces closed formulas and give nearly instantaneous values. This simple model allows us to optimize the network easily and study the influence of the main network parameters. The nodes will be deployed as a Poisson Point Process (PPP) of a one or two dimensional space. To model the effect of (CSMA), we give random marks to our nodes and to elect transmitting nodes in the PPP we choose those with the smallest marks in their neighborhood. To describe the signal propagation, we use a signal with power-law decay and we add a random Rayleigh fading. To decide whether or not a transmission is successful, we adopt the Signal-over-Interference Ratio (SIR) model in which a packet is correctly received if its transmission power divided by the interference power is above a capture threshold. We assume that each node in our PPP has a random receiver at a typical distance from the transmitter i.e. the average distance between a node and its closest neighbor. We also assume that all the network nodes always have a pending packet. With all these assumptions, we analytically study the density of throughput of successful transmissions and we show that it can be optimized with regard to the carrier-sense threshold.

## I. INTRODUCTION

At the end of the twentieth century, the most common wireless networks were WiFi (IEEE 802.11) networks. The dominant architecture of such networks involved an access point where a node, called the access point, generally connected to the Internet, exchanged packets with surrounding nodes. In these networks only one packet was sent at each instant since all the nodes were within carrier sense range of the other nodes and sent their packets one after the other. More recently, great progress in wireless transmission technology has paved the way to much bigger networks with more massive transmission patterns. In these networks, which include military networks, Vehicular Ad Hoc Networks (VANETs), Wireless Sensor Networks (WSNs), applications require simultaneous transmissions and thus the model with only one access point is no longer valid. The transmissions can be multihop, for instance in military networks or WSNs, and thus the same

packet must be forwarded. In this case, and especially when there are long routes, simultaneous transmissions will increase the performance. This phenomenon is known as spatial reuse. In VANETs, extending the networks along roads leads to vast networks where spatial reuse must inevitably be present. The high density of communicating vehicles on a road using IEEE 802.11p - a CSMA-based protocol - justifies the optimization of CSMA in networks with spatial reuse.

However, the access techniques used in these recent networks remain similar to those used in the first Wireless LANs (WLANs) and are based on the well-known Carrier Sense Multiple Access techniques (CSMA). In M2M systems and in the IoT a prominent technology is IEEE 802.15.4 which is also based on CSMA.

Therefore, a deeper understanding of CSMA with spatial reuse is needed and represents the main focus of this paper. There are two main characteristics to be evaluated in spatial CSMA. The first one is to compute the probability of transmitting when the carrier sense rule is applied. The second one is to compute the probability of the packet being correctly received by a neighbor.

The remainder of this paper is organized as follows. Section II briefly reviews related work; Section III describes the model proposed to study CSMA and develops the corresponding analytical model. The results of the model evaluating the influence of the parameters are reported in Section IV. Finally Section V concludes the paper.

## II. RELATED WORK

The initial studies on CSMA date back to the mid-seventies with the seminal paper by Kleinrock [1]. This paper, together with a great number of papers using the same analytical model framework, analyzed a perfect CSMA where all the nodes are within carrier-sense range of each-other. The framework developed in [1] accurately models the carrier sense access technique but the analysis of the back-off technique remains somewhat fuzzy. In 2000, the paper by Bianchi [2] relating to the IEEE 802.11 access technique took a step further in the modeling of the CSMA backoff technique. However the model still considers a one-hop wireless network, and thus spatial reuse remained beyond the scope of the paper.

The first tentative work which tried to take into account spatial reuse in spatial networks was reported in articles devoted

to Aloha such as [3] and [4]. In 1988, Ghez, Verdu and Schwartz introduced a model for slotted Aloha [3] with multipacket reception capability. To our knowledge this paper was the first quantitative model of a wireless network with spatial reuse. This model was revisited in [4] with a more accurate evaluation of the performance of the network. In [4] Baccelli et al. show that it is possible to accurately compute the probability of successful transmission in an Aloha network with spatial reuse if the distance between the transmitter and the receiver is known. [4] also allows the density of successful transmissions to be computed if the distance between the transmitter and the receiver is known. In Aloha networks the complete and stateless randomization of the transmitters leads to a particularly simple evaluation of the pattern of the simultaneously transmitting nodes.

In [5] the authors compute the mean number of transmissions with CSMA in a linear random network of vehicles but, in contrast to the present study which takes into account the entire interference, [5] only takes into account the nearest interferer. The Matern selection process [6] was first used in [7] to evaluate the pattern of simultaneous transmissions in CSMA. The study in [8] uses a process close to the Matern process to evaluate the interference in CSMA spatial networks but it does not study the throughput of the network, which is the main purpose of this paper. Finally, the model of [7] was improved by [9], which is the model that is used and extended in this paper. Apparently, no further improvements in this field have been made in recent years.

Although a few papers [10],[11] have studied the effect of the carrier sense detection threshold in CSMA protocols, these papers do not explore the spatial effect of the carrier sense detection threshold but rather the probability of capture when all the nodes are at one hop from each other.

### III. SYSTEM MODEL

#### A. Network nodes

The nodes are randomly deployed according to a Poisson Point Process  $\Phi$ . We denote by  $\lambda$  the intensity of the process. In this paper we consider a 2D infinite plan,  $\mathcal{S} = \mathbb{R}^2$  or a 1D infinite line,  $\mathcal{S} = \mathbb{R}$ . The 2D model is for Mobile Ad-hoc NETWORKS (MANETs) or Wireless Sensor Networks (WSNs). The 1D model is more relevant to Vehicular Ad-hoc NETWORKS (VANETs).

#### B. Propagation law, fading and capture model

We suppose that the signal received in a transmission is the result of a random fading  $F$  and a power-law in the distance decay  $1/r^\beta$  where  $\beta$  is the decay factor and is generally between 3 and 6. In our study, the fading will be Rayleigh i.e exponentially distributed with parameter  $\mu$  and thus is of mean  $1/\mu$ . Thus the signal received when the transmitter and the receiver are at distance  $r$  from each other is  $F/l(r)^1$  with  $l(r) = r^\beta$ .

We use the well-accepted SIR<sup>2</sup> (Signal over Interference Ratio) with a capture threshold  $T$ .

<sup>1</sup>The power received  $P = \frac{P_0 F}{l(r)}$  and we set  $P_0 = 1$

<sup>2</sup>We omit the thermal noise but it could be easily added, as is explained below. An even more realistic model than the SIR based on a graded SIR model using Shannon's law is possible in our framework though with an increased computational cost. This will be discussed below.

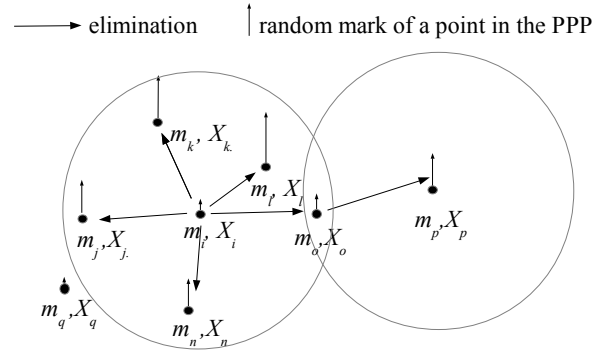


Fig. III.1. Matern CSMA selection process and an example of over-elimination.

#### C. Model for CSMA

Using the model developed in [9], we adopt a Matern selection process to mimic the CSMA selection process. The points  $X_i$  in  $\Phi$  receive a random mark  $m_i$ . We also call  $F_{i,j}$  the fading for the transmission between  $X_i$  and  $X_j$ . The idea of the Matern selection is to select the points  $X_i$  with the smallest random marks  $m_i$  in their neighborhood. To define the neighborhood of a point  $X_i$  we need to introduce the carrier sense threshold  $P_{cs}$ . We define  $\mathcal{V}(X_i) = \{X_j \in \Phi : F_{i,j}/l(|X_i - X_j|) > P_{cs}\}$  the neighborhood of  $X_i$ .  $X_i$  will be selected in the Matern selection process if and only if  $\forall X_j \in \mathcal{V}(X_i) m_i < m_j$ . In other words, this means that  $X_i$  has the smallest mark  $m_i$  in its neighborhood. The Matern selection is illustrated in Figure III.1. Node  $i$  has the smallest mark  $m_i$  within its neighborhood. Although node  $q$  does in fact have a smaller mark, it is not within node  $i$ 's neighborhood. We should point out that, for the sake of simplicity, here we have not taken into account any Rayleigh fading ( $F \equiv 1$ ) and thus the neighborhood of node  $i$  is a disc.

The technique based on marks used by the Matern selection process results in an over-elimination of nodes. When a node is eliminated by a node with a smaller mark, the node which has the smallest back-off in its neighborhood can start transmitting. The nodes which have been eliminated should not further eliminate other nodes. But this over-elimination can occur, as shown in Figure III.1. Node  $o$  is eliminated by node  $i$ , but node  $o$  eliminates node  $p$  in the Matern selection process, whereas in a CSMA system, node  $o$  is correctly eliminated by node  $i$ , but, being eliminated, node  $o$  can not eliminate another node. We do not take this case into account in our model.

We note the medium access indicator of node  $X_i$   $e_i = \mathbb{1}(\forall X_j \in \mathcal{V}(X_i) m_i < m_j)$

**Proposition III.1.** *The mean number of neighbors of a node is:*

$$\mathcal{N} = \lambda \int_{\mathcal{S}} P\{F \geq P_{cs} l(|x|)\} dx.$$

In a 2D network we have :

$$\mathcal{N} = \frac{2\pi\lambda\Gamma(2/\beta)}{\beta(P_{cs}\mu)^{2/\beta}}.$$

In a 1D network we have :

$$\mathcal{N} = \frac{\lambda\Gamma(1/\beta)}{\beta(P_{cs}\mu)^{1/\beta}}.$$

This result is very simple. Let  $F_j^0$  be the fading between the node at the origin  $X_i$  and node  $X_j$

This is just the application of Slivnyak's theorem and Campbell's formula, see [12], [9]

$$\begin{aligned} \mathcal{N} &= E^0 \left[ \sum_{X_j \in \Phi} \mathbb{1}(F_j^0 l(|X_j - X_i|) \geq P_{cs}) \right] \\ &= \lambda \int_{\mathcal{S}} P\{F \geq P_{cs} l(|x|)\} dx \end{aligned}$$

A straightforward computation provides the explicit value of  $\mathcal{N}$  in the 1D and 2D cases.

**Proposition III.2.** *The probability  $p$  that a given node  $X_0$  transmits i.e.  $e_0 = 1$  is:*

$$p = \mathbf{E}^0[e_0] = \frac{1 - e^{-\mathcal{N}}}{\mathcal{N}}.$$

*Proof:* The proof is obtained by computing the probability that a given node  $X_0$  at the origin with a mark  $m = t$  is allowed to transmit. The result is then obtained by deconditioning on  $t$ . The details of the proof can be found in [9]. ■

Thus  $p$  measures the probability of transmission in a CSMA network. If  $p$  is close to 1 this means that the carrier sense does not restrain transmissions. In contrast, if  $p$  is small, this means that the carrier sense imposes a severe restriction on transmissions.

**Proposition III.3.** *The probability that  $X_0$  transmits given that there is another node  $X_j \in \Phi$  at distance  $r$  is  $p_r$  with*

$$p_r = p - e^{-P_{cs}\mu l(r)} \left( \frac{1 - e^{-\mathcal{N}}}{\mathcal{N}^2} - \frac{e^{-\mathcal{N}}}{\mathcal{N}} \right)$$

*Proof:* The proof is the same as that of Proposition III.2. ■

**Proposition III.4.** *Let us suppose that  $X_1$  and  $X_2$  are two points in  $\Phi$  such that  $|X_1 - X_2| = r$ . We suppose that node  $X_2$  is retained by the selection process. The probability that  $X_1$  is also retained is:*

$$h(r) = \frac{\frac{2}{b(r)-\mathcal{N}} \left( \frac{1-e^{-\mathcal{N}}}{\mathcal{N}} - \frac{1-e^{-b(r)}}{b(r)} \right) (1 - e^{-P_{cs}\mu l(r)})}{\frac{1-e^{-\mathcal{N}}}{\mathcal{N}} - e^{-P_{cs}\mu l(r)} \left( \frac{1-e^{-\mathcal{N}}}{\mathcal{N}^2} - \frac{e^{-\mathcal{N}}}{\mathcal{N}} \right)}$$

with

$$b(r) = 2\mathcal{N} - \lambda \int_{\mathcal{S}} e^{-P_{cs}\mu(l(|x|)+l(|r-x|))} dx.$$

In a 2D network, we have:

$$b(r) = 2\mathcal{N} - \lambda \int_0^\infty \int_0^{2\pi} e^{-P_{cs}\mu(l(\tau)+l(\sqrt{\tau^2+r^2-2r\tau\cos(\theta)}))} d\tau d\theta.$$

In a 1D network, we have:

$$b(r) = 2\mathcal{N} - \lambda \int_{-\infty}^\infty e^{-P_{cs}\mu(l(\tau)+l(|r-\tau|))} d\tau$$

*Proof:* The proof can be found in [9] ■

**Proposition III.5.** *Given the transmission of a packet, we denote by  $p_c(r, P_{cs})$  the probability of successfully receiving*

*this packet at distance  $r$  in a CSMA system (modeled by a Matern selection process with a carrier sense threshold  $P_{cs}$  and with a capture threshold  $T$ ). We have:*

$$p_c(r, P_{cs}) \simeq \exp \left( -\lambda \int_{\mathcal{S}} \frac{h(|x|)}{1 + \frac{l(|x-r|)}{Tl(r)}} dx \right)$$

In a 2D network, we have:

$$p_c(r, P_{cs}) \simeq \exp \left( -\lambda \int_0^\infty \int_0^{2\pi} \frac{\tau h(\tau)}{1 + \frac{l(\sqrt{\tau^2+r^2-2r\tau\cos(\theta)})}{Tl(r)}} d\tau d\theta \right)$$

In a 1D network, we have:

$$p_c(r, P_{cs}) \simeq \exp \left( -\lambda \int_{-\infty}^\infty \frac{h(\tau)}{1 + \frac{l(|r-\tau|)}{Tl(r)}} d\tau \right)$$

*Proof:* The idea of the proof is to consider a transmitter at the origin and to compute the probability of successful reception by a receiver at distance  $r$ . To do so, we condition by the presence of another transmitting node at distance  $\tau$ . According to proposition III.4, the density of such nodes is  $\lambda h(\tau)$ . We approximate the interference by the interference of a Poisson Process of density  $\lambda h(\tau)$ . The result is obtained by integrating on  $\tau$ . The details of the proof can be found in [9]. ■

It is easy to add a thermal noise  $W$  to the model. The expression of  $p_c(r, P_{cs})$  must then be multiplied by  $\mathcal{L}_W(\mu T l(r))$  where  $\mathcal{L}_W(\cdot)$  is the Laplace Transform of the noise.

In a more advanced model using Shannon's law, we have the average transmission rate for  $X_0$

$$\begin{aligned} \mathbf{E}^0(\log(1+SIR)|e_0 = 1) &= \int_0^\infty P^0(\log(1+SIR) > t | e_0 = 1) dt \\ &= \int_0^\infty p_c(r, P_{cs}, e^t - 1) dt \end{aligned}$$

with  $p_c(r, P_{cs}, x) = p_c(r, P_{cs})$  where  $T$  is substituted by  $x$ , see [9]. Although more complicated, such an approach seems computationally achievable, and will form the subject of a more extensive study of spatial CSMA.

We resume with the capture model in the SIR model.

**Proposition III.6.** *The spatial density of successful transmissions is:*

$$\lambda p p_c(r, P_{cs})$$

*This spatial density has a 1D and a 2D version and the values of  $p$  and  $p_c(r, P_{cs})$  are chosen accordingly.*

*Proof:* Proposition III.6 is just the exploitation of propositions III.2 and III.5. ■

## IV. RESULTS OF THE MODEL

In this section, the model is used to analyze the network performance and the influence of the model's parameters. We study the transmissions for pairs of source-destination nodes at distance  $r$ .  $r$  is set at  $1/\sqrt{\lambda}$  or  $1/\lambda$  for 2D and 1D networks respectively.  $r$  can be seen as a typical distance in these

networks since it is the average distance between a node and its closest neighbor. Thus the transmitters are in the Poisson Point Process and for each transmitter, we create a random receiver at distance  $r$ .

#### A. Optimizing the density of successful transmissions with the carrier sense threshold $P_{cs}$

We consider that the parameters of the model  $\lambda$ ,  $T$  and  $\mu$  are constant and we vary  $P_{cs}$  to maximize the density of successful transmissions. It is easy to show that  $p$  is an increasing function of  $P_{cs}$ . When the carrier threshold increases, the probability of transmission in CSMA increases. VANETS As the carrier threshold increases, transmission becomes easier and thus  $p$  increases. This can be verified using the equation of proposition III.2. In contrast, when  $P_{cs}$  increases then  $p_c(r, P_{cs})$  decreases. This can be shown using the equation of proposition III.4. When  $P_{cs}$  increases,  $h(\tau)$  increases and thus  $p_c(r, P_{cs})$  decreases. Since the density of successful transmissions is upper-bounded by  $\lambda$  we know that there is an optimal value of  $P_{cs}$  which optimizes the density of successful transmissions. Studying a few examples, we have seen that the density of successful transmissions always has the same behavior, as shown in Figure IV.1. For small value of  $P_{cs}$  and when we increase  $P_{cs}$ ,  $p$  increases faster than  $p_c(r, P_{cs})$  decreases, and thus the density of successful transmissions is an increasing function of  $P_{cs}$ . This density reaches a maximum for a given value of  $P_{cs}$  and then becomes a decreasing function of  $P_{cs}$ . We assume that this is always the case although it seems difficult to show it mathematically. We use Maple to numerically compute this optimum of the density of successful transmissions.

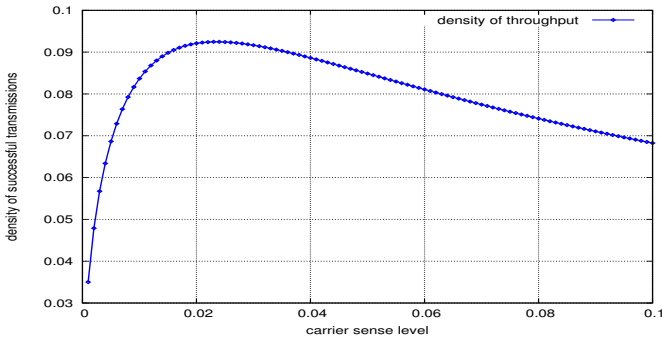


Fig. IV.1. Density of successful transmissions versus carrier threshold ( $T=1$ ,  $\mu = 10$ ,  $\beta = 4$ ).

In Figures IV.2 and IV.3, we present the carrier sense threshold versus the density of nodes when the density of successful transmissions is optimized.

#### B. Effect of the fading rate $\mu$

We note that in the probability of transmission  $p$  found in Proposition III.2, we can isolate  $\mu P_{cs}$ . It is the same for  $p_c(r, P_{cs})$ . This means that if we multiply  $\mu$  by 10, exactly the same performance can be obtained with  $P_{cs}$  divided by 10. Thus there is no influence of  $\mu$  on the global performance of the system; the optimum density of successful transmissions, the probability of capture  $p_c(r, P_{cs})$  and the probability of transmission  $p$  at the optimum value of  $P_{cs}$ . This remark regarding  $\mu$  is valid for both 1D and 2D networks. In the following we use  $\mu = 10$ .

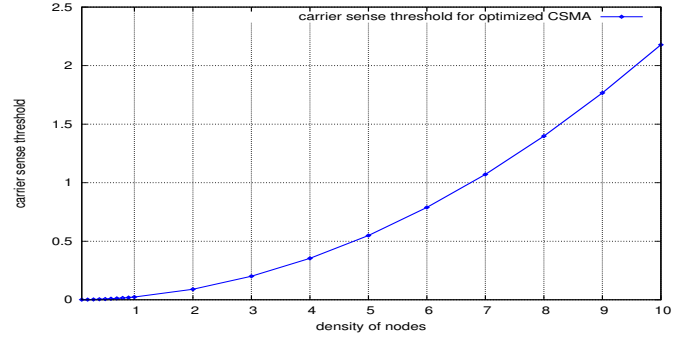


Fig. IV.2. Optimized carrier threshold versus density, 2D network ( $T=1$ ,  $\mu = 10$ ,  $\beta = 4$ ).

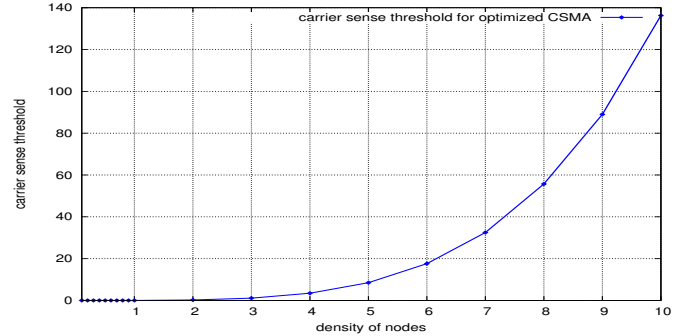


Fig. IV.3. Optimized carrier threshold versus density, 2D network ( $T=1$ ,  $\mu = 10$ ,  $\beta = 4$ ).

#### C. Effect of the density of nodes $\lambda$

We compute the optimum density of successful transmissions when  $P_{cs}$  is optimized versus  $\lambda$  the density of nodes in the network. We use the following parameters  $T = 1$ ,  $\mu = 10$  and  $\beta = 4$ . The results of these computations are shown in Figure IV.4 for 2D networks and in Figure IV.5 for 1D networks. Our numerical study shows that the density of successful transmissions is linear in  $\lambda$ . This means that the maximum of the product of  $pp_c(r, P_{cs})$  does not depend on  $\lambda$ . This is an interesting result and one which is not easily apparent in the analytical formulas of  $pp_c(r, P_{cs})$ .

Figures IV.4 and IV.5 also show the density of successful transmissions when the carrier sense threshold is constant and taken as the optimal value for  $\lambda = 1$ . The loss is significant for small values of  $\lambda$ : 26% for  $\lambda = 0.1$  and much more significant for large values of  $\lambda$ : 80% for  $\lambda = 10$  in 2D networks and 85% for  $\lambda = 10$  in 1D networks. For instance, this means that, in a VANET, the channel cannot be used efficiently if the carrier sense threshold is not properly optimized according to the density of vehicles. When  $\lambda = 0.1$  and if we use the optimization for  $\lambda = 1$  we do not have any restriction on the transmission rights and we actually have an excess of transmission rights. The problem comes from the probability of success for a given transmission. When  $\lambda = 10$  and if we use the optimization for  $\lambda = 1$  we have a stringent restriction<sup>3</sup> on the transmission rights, whereas a given transmission is very well protected by the CSMA scheme and thus every transmission is nearly always successful. The model shows

<sup>3</sup>the access right of CSMA (excess or stringent restriction) is determined by the equation given in Proposition III.2.

that the problem concerning the access right is much more detrimental to the global throughput than collisions would have been if the density of nodes had been overestimated.

We have studied the probability of capture when the throughput is optimized. We observed that the optimum throughput is not obtained when most of the transmissions are successful but rather when the success rate is around 55% in 2D networks and around 70% in 1D networks. The numerical results we obtained show that, at the optimum,  $p_c(r, P_{cs})$  does not depend on  $\lambda$  and we also deduce that  $p$  does not depend on  $\lambda$ . This is an interesting result which is not brought to light using the analytical formulas.

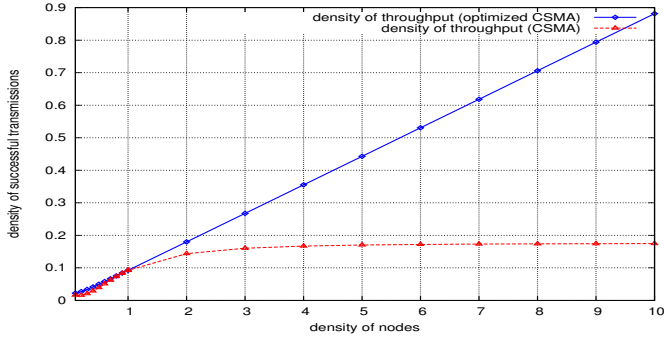


Fig. IV.4. Density of successful transmissions versus density of nodes ( $T=1$ ,  $\mu = 10$ ,  $\beta = 4$ ). Spatial network (2 D)

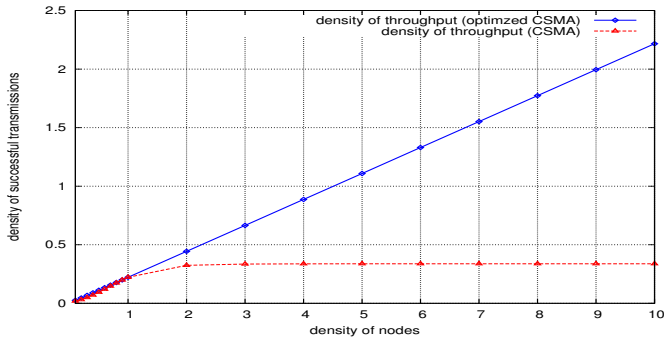


Fig. IV.5. Density of successful transmissions versus density of nodes ( $T=1$ ,  $\mu = 10$ ,  $\beta = 4$ ). Linear network (1 D)

#### D. Evaluation of exclusion area when the system is optimized

We computed the optimum value of  $P_{cs}$ :  $P_{cs}(opt)$  in the previous section. CSMA is optimum when a transmission at a given point forbids a transmission where the signal power exceeds  $P_{cs}(opt)$ . This means that on average, around a transmitter, any transmission at distance  $R_{cs}$  is forbidden such that  $\frac{1}{\mu(R_{cs})} = P_{cs}(opt)$ . We study the ratio of  $R_{cs}$  by the distance between the transmitter and its receiver. We recall that this distance is  $1/\sqrt{\lambda}$  in 2D networks and  $1/\lambda$  in 1D networks. In Figure IV.6, we show the ratio of  $R_{cs}$  by the distance between the transmitter and its receiver for 2D networks versus the density of nodes  $\lambda$ . We see that the average exclusion area around a given transmitter ranges, on average, from 0.92 to 1.47 times the distance between the source and destination nodes for 2D networks  $T=1$ ,  $\mu = 10$  and  $\beta = 4$ . For 1D networks as there is only one degree of freedom, the nodes are more grouped and the average exclusion area around a given

transmitter is larger; it ranges from 1.47 to 1.63 times the distance between the source and destination nodes depending on the density of the nodes.

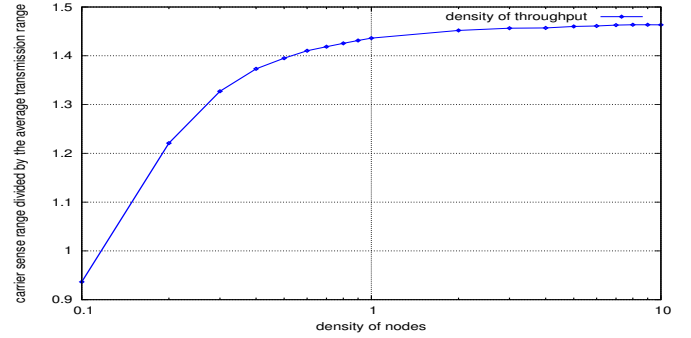


Fig. IV.6. Ratio of carrier sense range and transmission range versus  $\lambda$  density of nodes for  $T=1$ ,  $\mu = 10$ ,  $\beta = 4$  and a 2D network.

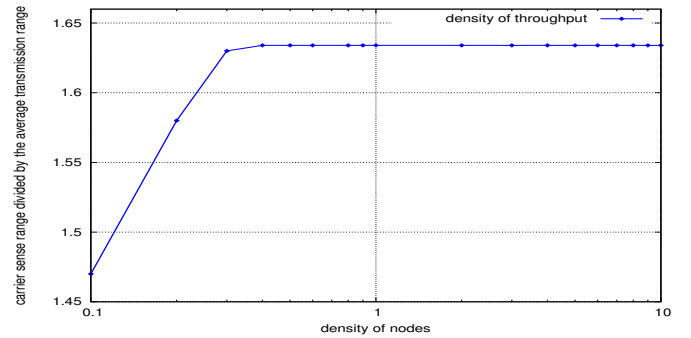


Fig. IV.7. Ratio of carrier sense range and transmission range versus  $\lambda$  density of nodes for  $T=1$ ,  $\mu = 10$ ,  $\beta = 4$  and a 1D network.

#### E. Effect of the capture threshold $T$

We study the effect of the capture threshold on the maximum density of successful transmissions. In Figure IV.8 and Figure IV.9 we plot the maximum density of successful transmissions for  $T$  varying from 0.01 to 10 respectively for 2D and 1D networks. We observe that dividing the capture threshold by 100 leads to multiplying the density of successful transmissions by 5.6 and 1.9 for 2D and 1D networks. This means that a small capture threshold is much more beneficial in 2D networks. The study of the analytical model does not show any obvious scaling of the density of successful transmissions with the capture threshold  $T$ .

#### F. Effect of the transmission decay $\beta$

In Figures IV.10 and IV.11, we plot the maximum density of successful transmissions for  $\beta$  varying from 2 to 6 respectively for 2D and 1D networks. In 2D networks, we observe that the maximum density of successful transmissions is multiplied by 1.91 when  $\beta$  varies from 2.5 to 6. For linear networks (1D) the maximum density of successful transmissions is multiplied by 1.32 when  $\beta$  varies from 2.5 to 6. As for the capture threshold, the effect of a large transmission decay is less beneficial for 1D networks than for 2D networks. The study does not show any apparent scaling of the density of successful transmissions with the capture threshold  $\beta$ .



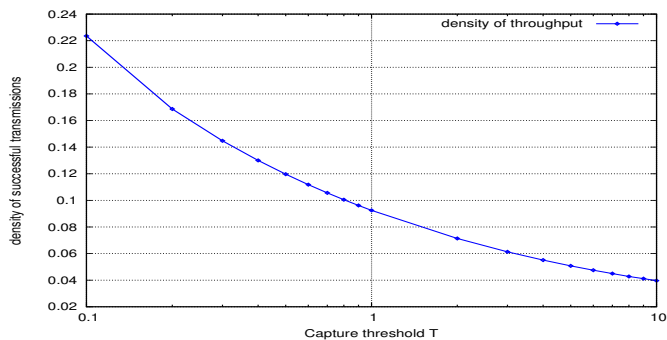


Fig. IV.8. Density of successful transmissions versus capture threshold  $T$  for 2D networks ( $\lambda=1, \mu = 10, \beta = 4$ ).

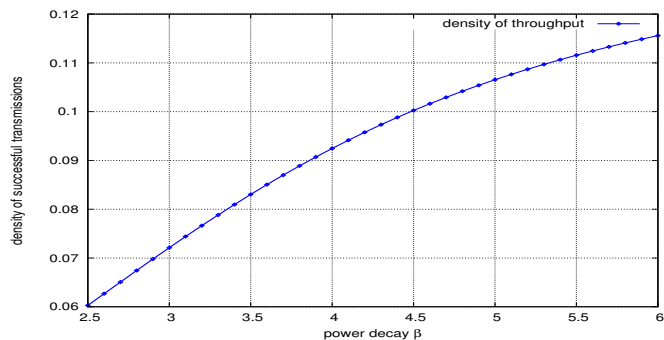


Fig. IV.10. Density of successful transmissions versus the decay factor  $\beta$  for 2D networks ( $\lambda=1, \mu = 10, T = 1$ ).

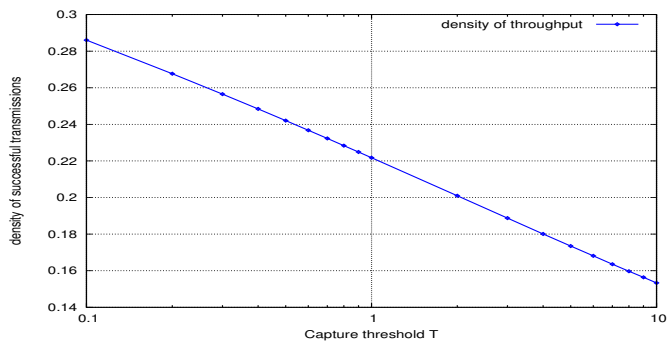


Fig. IV.9. Density of successful transmission versus the capture threshold  $T$  ( $\lambda=1, \mu = 10, T = 1$ ). Linear network (1 D)

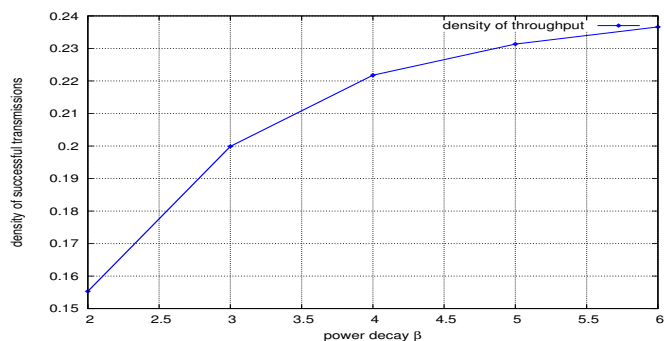


Fig. IV.11. Density of successful transmissions versus the decay factor  $\beta$  ( $\lambda=1, \mu = 10, T = 1$ ). Linear network (1 D)

## V. CONCLUSION

In this paper, we present a simple model of CSMA and we show the importance of optimizing it according to the density of nodes. We have shown that the optimized density of successful transmissions scales linearly with the density of nodes. We have observed that using a constant carrier threshold leads to a very significant loss in the network's global throughput. This effect is much more penalizing when the density of nodes in the network is underestimated than when it is overestimated. The numerical computations we have carried out show that the best performance of the network is not reached when transmissions are nearly always successful but when there is a success rate of around 0.6. We have also studied the influence of the model's parameters :  $\mu$ ,  $T$  and  $\beta$ . The rate of fading does not influence the performance of the network if it is optimized. We show that  $T$  and  $\beta$  have a greater impact on 2D networks than on 1D networks. The results of this study have yet to be compared with simulation results, preliminary tests show a good matching between the results of both approaches. This will form the subject of our future work. In addition the approximation of CSMA induced by the Matern selection process should be further investigated.

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