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Energy Efficient Time-Triggered Control over Wireless Sensor/Actuator Networks

Vineeth S. Varma and Romain Postoyan

***Abstract*— We investigate the scenario where a controller is connected to a plant using a wireless network. Our objective is to design control strategies, which efficiently use the network in terms of energy, by taking into account the fluctuations of the wireless channel. In particular, we consider discrete-time systems and we focus on time-triggered control. That is, we assume that we know a controller, which ensures a desired property (such as stability), as long as two successive transmissions are not spaced by more than a fixed number of steps N. The problem formulation is generic in the sense that we do not make any assumption on the plant and the controller structures or properties, all we need to ensure is the constraint on the transmission times. We then present triggering strategies to minimize the energy expenditure based on knowledge on the channel state, while ensuring that two successive transmissions are not spaced by more than N steps. The results demonstrate that periodically communicating after exactly N time instants has passed (to minimize the number of transmissions), is not always the optimal scheme to minimize the energy cost. As a result of certain properties of the wireless channel, communicating more often when the wireless channel conditions are good, results in a smaller energy consumption overall despite a higher frequency of communication. Numerical results confirm the validity of the approach and show that a significant amount of energy can be saved.**

I. Introduction

The literature on sensor networks has extensively dealt with the extraction and communication of information obtained through the sensors. Communicating the obtained information with a high data rate or reliability, while consuming as little energy or power as possible, is the primary objective of works in this field, see e.g., [1], [2]. The approaches used to reduce the power consumption are based on the communication requirements and wireless aspects like access protocols, scheduling and transmission power control [3]. On the other hand, the literature on *networked control systems* (NCS) concentrates on applications in which actuation plays a major role, see e.g., [4], [5], [6], [7]. These are systems in which the plant and the controller communicate with each other via a communication network. In this context, the energy expenditure incurred is often assumed to be directly related to the frequency of communication through the network, meaning that the less we transmit, the less the energy expenditure due to communication.

Many researchers have recently published results which bridge the gap between the two approaches mentioned above, and develop a more unified framework for analyzing and reducing the energy consumption in NCS. In [8], [9] for instance, the implementation of event-triggered controllers over practical communication channels is investigated. The survey [10] provides a list of works that implement energy efficient sensor design in the context of NCS. Minimizing the energy consumption by a sensor, with the ability to harvest energy, communicating over a wireless channel has been done in [11]. Another work of interest among others, is [12], in which the authors look at energy efficient transmit power control in a wireless sensor/actuator network, with the power control based on efficiency of the estimation.

The primary objective of our work is to reduce, and if possible, minimize the energy consumed by a control system, in which the sensors and the actuators are connected to the controller through a wireless channel. The controller is assumed to be able to complete its task, as long as it is updated, within an interval of N time instants from the previous update. We assume that the communication is perfect (no noise through quantization) and without delay when a sufficient amount of energy is consumed. The problem formulation is generic in the sense that the system and the controller can have any dynamics and the control objective can be stability and/or optimality for instance. The only requirement for our results to hold, is on the frequency of communication as mentioned above, which is assumed to imply that a desired property for the closed-loop system holds. We propose communication strategies (determining when to communicate) that reduce the total discounted energy cost based on:

1) *the clock state,* which is the number of time instants that has passed since the previous communication. This takes a value in the set $\{1, 2, \ldots, N\}$, as communication is enforced when N instants have passed for the controller to perform its task.
2) *the channel fading,* which is a positive real number indicating the strength of the wireless connection between the wireless transmitter and the receiver (see, e.g., [11], [12]). The higher the value of the channel fading, the lower the amount of energy that has to be spent for successful communication.

Although the literature mentioned above treats the problem of energy efficient communication in NCS, the strategies obtained usually are either from a communication perspective (not considering the plant state) or from a control perspective (not considering the channel fading). Designing a power control scheme taking into account both the wireless channel fading and the plant state jointly has been done in [13], where

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the authors develop a co-designed power control policy for linear systems when the wireless channel distribution is known. In comparison, our results apply to general systems and controllers, but, which satisfy time triggering conditions. The latter enables us to separate the control and communication problems as the former is solved by restricting the set of communication strategies to the subset which satisfies the time triggering condition.

We treat three primary cases of interest, based on the wireless channel model assumed:

1) *The wireless channel conditions remain a constant for all time*: in this simple case, reducing the frequency of communication obviously minimizes the energy cost. That is, the optimal policy is transmitting only when the clock state is exactly N.
2) *The wireless channel conditions are known* for a certain number of time instants in the future: several practical scenarios have appeared over the recent years, for which forecasting the wireless channel state over a long time horizon is realistic, see [14], [15]. In this case, we develop a rollout algorithm [16] for signal power control, that consumes an amount of energy lower than or equal to that consumed by the policy given by the first case. Simulation results actually have shown that it consumes a much smaller energy cost.
3) *The probability distribution of the wireless channel conditions are also known*: this assumption is often made in wireless literature (see [3], [12], [13]) as the channel statistics can be learned. In this case, we are able to compute and minimize the minimal expected energy costs by modeling the problem as a Markov decision process.

The rest of the paper is structured in the following manner. The problem is formally stated in Section II. Section III presents the proposed communication strategies for the three cases mentioned above. Section IV provides numerical results that illustrate the energy gains observed by implementing the proposed policies. Finally, Section V concludes the paper. The proofs are omitted due to space limitations.

II. Problem statement

We investigate the scenario where a controller is implemented over a wireless network and communicates with the plant at time instants $t_i \in \mathbb{Z}_{\geq 0}$, $i \in \mathbb{Z}_{\geq 0}$ (where $\mathbb{Z}_{\geq 0} = \{0, 1, \dots\}$). In order to monitor the plant, low-energy sensors are deployed which measure the plant state and communicate this information to the controller. The primary objective of this work is to develop a communication scheduling scheme that minimizes (when possible) the energy consumption at the sensor while ensuring that the controller can consistently perform its task.

A. Control requirements

When a transmission occurs, the plant sensors send the current value of the output vector to the controller, which also sends in the meantime, the current value of the control input to the plant. We assume that the transmission delays and that quantization can be neglected and that both transmissions occur over the same channel and do not interfere. Define by $N \in \mathbb{Z}_{>0}$ (where $\mathbb{Z}_{>0} = \{1, 2, \dots\}$), the *maximum allowable transmission interval* (MATI) where

$$t_{i+1} \in \{t_i, t_i + 1, \dots, t_i + N\}, \forall i \tag{1}$$

We assume that the MATI, N as defined in (1) ensures that the control system satisfies a desired property, like stability or optimality for instance. This general assumption allows covering various situations *without imposing any model* for the plant and the controller. For instance, conditions to ensure stability properties for system, including a bound on N, are provided in [17]. The near-optimal time-triggered strategies presented in [18] can also be modeled by (1).

For the sake of convenience, we introduce the *clock state* $\tau(t)$ to count the number of steps since the last transmission.

$$\tau(t+1) = \begin{cases} 1 & \text{when} \quad \tau(t) \in \{1, \dots, N\} \\ & \qquad (\textit{transmission}) \\ \tau(t) + 1 & \text{when} \quad \tau \in \{1, \dots, N-1\} \\ & \qquad (\textit{no transmission}). \end{cases} \tag{2}$$

Where $\tau(t)$ is reset to 1 when a transmission occurs and is incremented by 1 when that is not case. We are free to trigger a transmission whenever $\tau(t) \in \{1, \dots, N\}$ in order to satisfy the control objective. As the energy consumed per transmission depends on the wireless channel, we consider a wireless channel model as described in the following section.

B. Channel fading and energy costs

The signals sent through wireless channels experience an attenuation due to various factors like shadowing, multipath propagation and path loss (distance). In the framework of wireless communications, this process is called *channel fading*. The channel fading is, in general, a time-varying quantity, and we model the gain (or loss) of the signal strength due to the channel fading at time t by the real number h_t, which is assumed to satisfy the following condition.

Assumption 1: The wireless channel fading at time $t \in \mathbb{Z}_{\geq 0}$ is given by $h_t \in \mathcal{H}$ where $\mathcal{H} := [h_{\min}, +\infty]$ and $h_{\min} > 0$. □

Assumption 1 guarantees that, at any time t, the channel h_t is larger than or equal to $h_{\min} > 0$, where $h_{\min}$ is the channel fading when the the wireless channel connection is the weakest. This property is required to ensure that communication at any time can be successful with no loss. Indeed, a real wireless device will have a bit-error rate. However, we assume that reliable communication can always be performed using error correcting codes and by using the maximum energy [19], [20]. Note that in future work, we can relax this condition by considering stochastic control. We also assume that h_t is known to the sensor, with a negligible sensing energy cost compared to cost of sending a wireless transmission. This assumption is often justified, as in practice, the receiver will typically sent pilot signals for the transmitter to estimate the channel (see [13] for instance).

Let the energy spent by the transmitter to communicate at time t be given by $E_t \in [0, E_{\max}]$, where $E_{\max} > 0$ is the maximum energy output of the transmitter. In practice, we are free to select the energy E_t at each $t \in \mathbb{Z}_{>0}$. The energy required in order to communicate with no loss depends on the wireless channel fading. We assume that the minimum amount of energy needed in order to communicate without loss is inversely proportional to the channel fading h_t as formalized below.

Assumption 2: When the transmitter consumes E_t Joules of energy at time t, then the communication is lossless if

$$E_t \geq \frac{E_0}{h_t} \tag{3}$$

where $E_0 > 0$ is a positive real number satisfying $E_0 \leq E_{\max} h_{\min}$. □

Assumption 2 is suitable when the communication performance metric is a non-decreasing function of the signal to noise ratio and the communication system implements capacity achieving codes. For satisfying a certain quality of service (like communication with no loss) the signal to noise ratio must be above a certain threshold, resulting in channel inversion for the energy (and transmit power) spent. Additional information on the validity of this assumption can be seen in in Section IV.A of [13]. Assumptions 1 and 2 guarantee that, at any time t, the communication between the sensor and the controller will be successful when an energy $\frac{E_0}{h_t}$ is consumed by the wireless transmitter.

C. Performance metric

We are interested in minimizing the total energy consumed by the transmitter over an infinite horizon. As this cost will be unbounded in general, we introduce a discount factor $\delta \in (0, 1)$ and look at the total discounted cost (often studied in literature, e.g. [18], [21]), measured in Joules as

$$e := \sum_{t=1}^{\infty} \delta^t E_t, \tag{4}$$

while ensuring that the interval between two successful communication instants is in $\{1, 2, \ldots, N\}$. From Assumption 2, we know that when $E_t \geq \frac{E_0}{h_t}$, the communication is successful. Therefore, we reduce our problem into finding a sequence $d_t \in \{0, 1\}$ for any t, where $E_t = d_t \frac{E_0}{h_t}$, which minimizes the total energy cost, i.e., the cost function,

$$e = \sum_{t=1}^{\infty} \delta^t d_t \frac{E_0}{h_t}. \tag{5}$$

Hence, when $d_t = 0$, the energy spent at time t is equal to zero, meaning that we do not transmit. When $d_t = 1$, a transmission occurs and we use $\frac{E_0}{h_t}$ Joules to successfully transmit. Minimizing e in (5) would require knowing h_t for all t, which is not reasonable in practice. Therefore, we define $\bar{h}_t := (h_t, h_{t+1}, \ldots, h_{t+M})$, the $M + 1$ dimensional vector formed by the concatenation of the current channel h_t and M future channels, where $M \in \mathbb{Z}_{\geq 0}$ at time $t \in \mathbb{Z}_{>0}$. We also introduce $\bar{h}_{|t} := (h_1, h_2, \ldots, h_t)$ that represents the value of the channel fading from 1 to $t \in \mathbb{Z}_{\geq 0}$. We assume that these M future channel fadings are known at any time t.

Assumption 3: At any time t, $\bar{h}_t$ is known by the transmitter, where $M \in \mathbb{Z}_{\geq 0}$. □

Assuming that the transmitter knows all the realizations of the system state in advance can also be seen as a way of obtaining an upper bound for the performance of scenarios that assume a reduced time horizon for forecasting. It is worth noting that several practical scenarios have appeared over the recent years as seen in[14] or [15]. Note that the case of $M = 0$, is a special case of Assumption 3 where no future channel fadings are known.

Our goal is to design "policies" which determine d_t, given $\bar{h}_t$ (the vector of known channels) and $\tau(t)$ (the clock state), such that the energy cost e in (5) is minimized when possible, while ensuring $\tau(t) \leq N$ for any t, hence ensuring the desired control task. We define a policy as $\pi_M : \mathbb{Z}_{>0} \times \mathcal{H}^{M+1} \to \{0, 1\}$ ($\mathcal{H}$ defined in Assumption 1), which is a function that determines whether to communicate or not given $\tau(t)$ and $\bar{h}_t$.

Given a policy π_M and channel fadings $\bar{h}_{|t+M}$, let the communication decision made at time t be given by $d_t(\pi_M, \bar{h}_{|t+M})$, and the clock state as a result of these decisions be given by $\tau(t) = S_t(\pi_M, \bar{h}_{|t+M})$, where S_t is some function to be designed. Denote by Π the set of all policies satisfying the time triggering rule (1), i.e.,

$$\Pi := \{\pi_M | \pi_M(N', \bar{g}) = 1 \, \forall \bar{g} \in \mathcal{H}^{M+1}, \, \forall N' \geq N\}.$$

Our objective is to find policies $\pi_M \in \Pi$ that (when possible) minimize the cost function

$$e(\pi_M, \bar{h}_{|\infty}) := \sum_{t=1}^{\infty} \delta^t d_t(\pi_M, \bar{h}_{|t+M}) \frac{E_0}{h_t}. \tag{6}$$

Since knowing all the future channel values, i.e., $\bar{h}_{|\infty}$ is not generally possible in practice, we are interested in optimizing the decisions in order to minimize the expected cost. Let $h_t \sim \mathcal{P}$ where $\mathcal{P}$ is the probability distribution of h_t, and the probability distribution function (PDF) of h_t is $P(\cdot)$. The expected energy cost $\bar{e}(\pi_M)$ (over channel realizations) while using a policy π_M is given by

$$\bar{e}(\pi_M) := \mathbb{E}_{\bar{h}_{|\infty}} e(\pi_M, \bar{h}_{|\infty}), \tag{7}$$

where $\mathbb{E}_{\bar{h}_{|\infty}} := \int_{\mathcal{H}^\infty} \Pi_{t=1}^{\infty} P(h_t) d\bar{h}_{|\infty}$ is the expectation over all the channels.

The objective of our work is to minimize $\bar{e}(\pi_M)$, when P_t is known and otherwise, find policies that reduce the consumed energy $e(\pi_M, \bar{h}_{|\infty})$ compared to a base policy defined in the following section.

III. Main results

A. Constant channel fading

In this subsection, we look at the trivial case where the channel fading is a constant for all time. This kind of model is suitable when both the transmitter and receiver are stationary and is at line-of-sight with no obstructions. This

results in a Rician channel fading with a small variance, i.e., approximately a constant.

Assumption 4: There exists $h_0 \geq h_{\min}$ such that $h_t = h_0$ for any $t \in \mathbb{Z}_{>0}$. □

Note that h_0 is necessarily known in view of Assumption 3. We will now introduce a policy π_M^B which we will refer to as the "base heuristic".

Definition 1: The base heuristic π_M^B is given by, for any $\tau \in \{1, \ldots, N\}$ and $\bar{g} \in \mathcal{H}^{M+1}$,

$$\pi_M^B(\tau, \bar{g}) := \begin{cases} 1 & \text{if } \tau = N \\ 0 & \text{if } \tau < N \ . \end{cases} \tag{8}$$

□

This policy minimizes the communication frequency by transmitting the measurement only when the clock state is N. Additionally, this policy does not require any information on the channel fading (present or future). Therefore, the implementation is very straightforward and requires almost no computational resources or information. Moreover, such a policy is optimal under Assumptions 1-4, as we show in the next proposition. Note that such a policy would also be naturally used when ignoring the channel fadings fluctuations and considering only the control requirements.

Proposition 1: When Assumptions 1-4 hold, the expected energy cost $\bar{e}(\pi_M)$ is minimized by implementing the base heuristic, i.e., $\pi_M^* = \pi_M^B$ and results in a total energy consumption of $\bar{e}(\pi_M^B) = \frac{\delta^N E_0}{h_0(1-\delta^N)}$. □

B. Channel statistics are unknown

In most practical systems Assumption 4 does not hold true, as h_t typically varies in time. The energy cost from when the base heuristic is implemented for any general channel model (h_t is not necessarily a constant) is given by

$$e(\pi_M^B, \bar{h}_{|\infty}) = \sum_{i=1}^{\infty} \delta^{iN} \frac{E_0}{h_{iN}}. \tag{9}$$

In this section, we only suppose that Assumptions 1-3 hold and we do not make any assumption on the channel statistics, i.e., the PDF is unknown. We propose a rollout algorithm (see [16]), which outperforms the base heuristic. Recently, some works like [13], [21] have utilized the rollout algorithm in the field of event-triggered control. Although the latter does not result in an optimal solution, it may produce considerable (and often dramatic) performance improvements over the base heuristic, as confirmed in the simulation results presented in Section IV.

The rollout algorithm assumes that the algorithm only affects a certain horizon of time, and that the base heuristic is implemented after this horizon. The related costs of the system beyond the horizon are calculated assuming that the base heuristic given in Definition 1 is implemented. The total energy cost within the time period $\{t, t+1, ..., t+M\}$ is minimized, while ensuring that the base heuristic will be implemented after $t+M$.

Definition 2: The rollout policy π_M^R is defined as, for clock state $\tau \in \mathbb{Z}_{>0}$ and channel vector $\bar{g} \in \mathcal{H}^{M+1}$

$$\pi_M^R(\tau, \bar{g}) := \arg_{b_0} \min_{\bar{b}} \sum_{j=0}^{M} \delta^j b_j \frac{E_0}{g_j} \tag{10}$$

where $b_j \in \{0, 1\}$ for all $j \in \{0, 1, .., M\}$ and $\bar{b} = (b_0, b_1, \ldots, b_M)$ has to satisfy the following constraints. Let $S'_j(\bar{b})$ be the clock state after j time steps, where $S'_0(\bar{b}) = \tau$ by definition, and $S'_j(\bar{b})$ is defined for $j \in \{1, 2, \ldots, M\}$ as

$$S'_j(\bar{b}) := \begin{cases} j + \tau \quad \text{if } b_k = 0 \quad \forall k \in \{0, 1, .., j-1\} \\ j - \max\{k \in \{1, \ldots, j-1\} | b_k = 1\} \\ \qquad\qquad\qquad\qquad \text{otherwise.} \end{cases} \tag{11}$$

The vector $\bar{b}$ is such that

$$S'_j(\bar{b}) \leq N \qquad \forall j \in \{0, 1, .., M-1\} \tag{12}$$

and

$$S'_M(\bar{b}) \leq N - (N\lceil \frac{\tau + M}{N} \rceil - \tau - M). \tag{13}$$

where the notation $\lceil y \rceil$ denotes the smallest integer greater than or equal to y (ceiling function). □

In Definition 2, $\bar{b}$ represents the vector of decisions that will be taken from t to $t+M$ if the rollout policy is implemented when the clock state at t is τ. Assuming the decisions $\bar{b}$ from t to $t+M$ are taken, then $S'_j(\bar{b})$ represents the clock state after j time steps and can be estimated by (11). As the time triggering rule (1) has to be satisfied, we impose the constraint (12). We also enforce an additional constraint (13) to avoid transmitting after the rollout horizon and before the schedule of the base heuristic. Implementing the rollout policy π_M^R results in an energy cost, which is at most, the energy cost of the base heuristic. This result is detailed in the following proposition.

Proposition 2: When Assumptions 1-3 hold, the rollout policy π_M^R leads to an energy cost that is less than or equal to the energy cost implementing π_M^B. □

Implementing π_M^R when M is large, requires an exhaustive search over all the possible decisions with a computational complexity of 2^M steps and dynamic programming can be used in order to reduce the computational complexity and solve the minimization problem in (10). Our numerical results, for the considered channel fading model, indicate that this may not essential, as the gain from increasing M is not significant beyond $M = 5$ when $N = 10$.

C. Channel distribution is known

While the rollout strategy may reduce the energy expenditure, the obtained cost is not optimal a priori. In this subsection, we aim at providing optimal policies. We assume for this purpose that the channel distribution information is known for all time and are independent and identically distributed random variables.

Assumption 5: The channel fading distribution $P(h) : \mathbb{R}_+ \to \mathbb{R}_+$ is known such that $h_t \sim \mathcal{P}$ for all t, and the PDF of h_t is given by $P(h)$. □

Assumption 5 is often made in the framework of wireless communications [3], [12], [13]. For any M, when the prob-

ability distribution $P(h)$ is known, we derive the *optimal* policy (optimal given the available information of M future channel states). Let $v_{\tau,\bar{g}}(\pi_M)$ represent the expected energy cost while using policy π_M for all future times while the clock state is τ and $\bar{g}$ is the vector of known future channels. Define $\bar{v}_\tau(\pi_M) := \mathbb{E}_{\bar{g}}[v_{\tau,\bar{g}}(\pi_M)]$.

Proposition 3: Under Assumptions 1-3 and 5, the problem resolves into a Markov decision process (MDP) and the optimal policy π_M^* is given by iteratively solving the following equations

$$\pi_M^*(\tau,\bar{g}) = \arg_{b_0}\min_{\bar{b}}\left\{b_0\frac{E_0}{g_0} + \sum_{m=1}^{M}\delta^m b_m\frac{E_0}{g_m} + \delta^{M+1}\bar{v}_{S'_{M+1}(\bar{b})}(\pi_M^*)\right\} \tag{14}$$

where $b_j \in \{0,1\}$ for all $j \in \{0,1,..,M\}$ and $\bar{b} = (b_0, b_1, \ldots, b_M)$ satisfies the constraint (12) and $S'_j(\bar{b})$ is from Definition 2, and

$$\bar{v}_\tau(\pi_M^*) = \mathbb{E}_{\bar{g}}\left[\pi_M^*(\tau,\bar{g})\frac{E_0}{g_0} + \delta\pi_M^*(\tau,\bar{g})\bar{v}_1(\pi_M^*) + \delta(1-\pi_M^*(\tau,\bar{g}))\bar{v}_{\tau+1}(\pi_M^*)\right] \tag{15}$$

□

Although iteratively solving equations (14) and (15) can be computationally heavy, this can be done offline and $\bar{v}_\tau(\pi_M^*)$ can be computed. The expected energy cost for the policy π_M^* is $\bar{e}(\pi_M^*) = \bar{v}_1(\pi_M^*)$ (as we initialize with communication at $t = 0$). In general, the minimization problem in (14) is similar to the problem in the previous subsection, but also yields non-trivial results for $M \in \{0,1\}$. Of special interest is the case of $M = 0$, where the future channel states are unknown, but their statistics are known, in which case the policy is given as follows.

Optimal policy for $M = 0$: The optimal policy obtained from (14) has a simple form in this case. Let $\tilde{g}_\tau := \frac{E_0}{\delta\bar{v}_{\tau+1}(\pi_0^*) - \delta\bar{v}_1(\pi_0^*)}$ and (14) can be simplified to be a threshold policy on the channel for each state τ:

$$\pi_0^*(\tau, g_0) = \begin{cases} 1 & \text{if } g_0 \geq \tilde{g}_\tau \text{ or } \tau = N; \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Hence, we compute $\tilde{g}_\tau$ for all $\tau \in \{1,2,\ldots,M\}$ offline by iteratively solving (14) and (15), and we simply check on-line whether h_t is bigger than $\tilde{g}_{\tau(t)}$, in which case a transmission is triggered.

When $M > 0$, it becomes more complex (in terms of computation) to find the optimal policy as we also have information on $h_{t+1}, h_{t+2}, \ldots, h_{t+M}$. The total number of decision variables are $M+1$, and as in the previous section, this results in a computational complexity of 2^{M+1}. Dynamic programming can be used to find the optimal decision policy π_M^* given the expected minimal energy cost function $\bar{v}_\tau(\pi_M^*(\cdot,\cdot))$ with less computational complexity.

So far, we have provided several policies to reduce or minimize the energy costs in a time triggered control system. In the next section, we provide some numerical results that illustrate the advantages and gains resulting from our proposed policies.

IV. Numerical results

We use a Rayleigh model of channel fading for our numerical study, and generate channel realizations with the PDF

$$P(h_t) = \exp(-h_t)$$

by considering unit variance (note that h for us, is in the signal power domain, and not amplitude, resulting in the exponential distribution). Additionally, we consider $h_{\min} = 0.01$ and consider all randomized $h_t = 0.01$ if the randomly selected $h_t < 0.01$. We take the parameters $E_0 = 1$ and $\delta = 0.99$.

In Figure 1, we compare the energy cost by implementing the rollout policy with the optimal policy (w.r.t the available information) developed in Section III C. Observe that for $N = 10$, even when $M = 0$, the expected energy cost is reduced by about 74% when compared to the base policy (which is equivalent to rollout with $M = 0$). Figure 1 also shows that the reduction in energy cost by using $M > 5$ is not significant for neither the rollout and nor the optimal policy. Beyond this point, both the policies converge to costs that are not far from the optimal policy obtained by knowing all values of h_t. This indicates that an exhaustive search is feasible to solve (10) or (15), and it is not essential to use advanced techniques like dynamic programming.

Figure 2 plots the thresholds $\tilde{g}_\tau$ against the time since the last transmission (τ), for the optimal policy with $M = 0$, i.e., for $\pi_0^*(\tau, g_0)$. This figure clearly shows how the optimal policy can be implemented when $M = 0$. If the channel realization $h_t \geq \tilde{g}_\tau$ (with $\tau = \tau(t)$), the optimal decision would be to transmit, and not transmit otherwise. For a larger N, the channel realization much be much larger to transmit efficiently for the same τ, as you have the possibility to wait longer for a better channel.

V. Conclusion

We have studied the time triggered control of systems in which controller and the plant are connected through a wireless channel. The closed-loop system is such that, the controller requires an update on system state within N time instants from the previous update, in order to ensure desired properties, such as stability. This update is provided across the wireless network, which incurs an a minimum

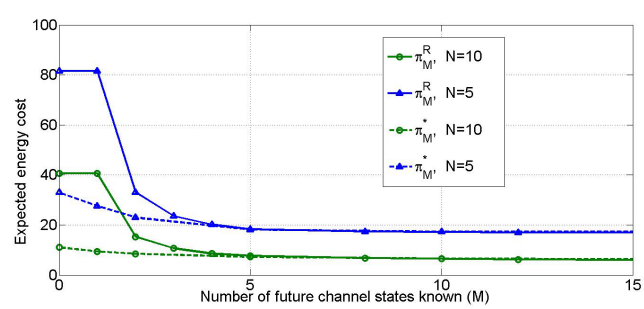

Fig. 1. Expected energy cost with the rollout scheme $\bar{e}(\pi_M^R)$ (defined in Section III. B) or the optimal policy $\bar{e}(\pi_M^*)$ (defined in Section III. C) vs the number of known future channel values M.

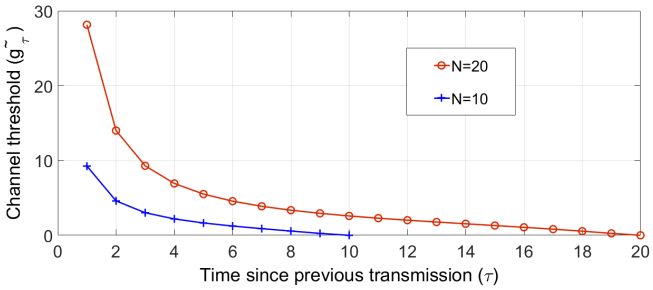

Fig. 2. Channel thresholds ($\tilde{g}_\tau$) against time since last transmission (τ) for the optimal policy with $M = 0$, i.e., for $\pi_0^*(\tau, g_0)$.

energy cost $\frac{E_0}{h_t}$ for successful communication, where h_t models the channel fading (the strength of the wireless channel). We study three models for the wireless channel and provide policies that determine when communication should be done, in order to minimize (when possible) the total discounted energy cost. The first case of interest is when the channel fading is a constant and we prove that to communicate after exactly N time instants from the previous communication instant is optimal. In the second case, the channels fading values are unknown in general but known for M future instants and we propose a sub-optimal rollout policy which is shown to provide large gains in energy cost through simulations. Finally, when the channel statistics remain a constant through time and are known, we find the optimal policy by modeling the problem as a Markov decision process. An important property of our analysis is that, the energy gains are independent of every property of the control system except for N, the MATI.

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