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About Time Measurement In A Platonic Quadridimensional Space

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Alain Jégat

Abstract

Through the observation of an elementary oscillator, concepts of synchronization, proper and improper time, this article aims to expose the reasoning that led to the definition originally made for the measurement of time by inertial frames in this space (*cf. hal-01165196, v1. A platonic (euclidean-projective) model for the special theory of relativity*).

It also offers an open reflection about different possible concepts of time in a four-dimensional Platonic space that leads, through the projective character of this space, to connect the (enlarged) Newtonian and relativistic points of view.

Résumé

À travers l'observation d'un oscillateur élémentaire, les concepts de synchronisation, de temps propre et impropre, cet article a pour objet premier d'exposer les raisonnements qui ont conduit à la définition initialement émise pour la mesure du temps par les référentiels inertiels dans cet espace (*cf. hal-01081576, v1. Un modèle platonicien (euclidien-projectif) pour la théorie de la relativité restreinte*).

Il propose également une réflexion ouverte sur différentes conceptions possibles du temps dans un espace quadridimensionnel platonicien qui conduit, via le caractère projectif de cet espace, à relier les points de vue newtoniens (élargis) et relativistes.

1. The geometrical framework

This modeling is based on the Platonic space outlined in the following articles:

« A PLATONIC (EUCLIDEAN-PROJECTIVE) MODEL FOR THE SPECIAL THEORY OF RELATIVITY » (pre-publication hal-01165196, version 1).

« UN MODÈLE PLATONICIEN (EUCLIDIEN-PROJECTIF) POUR LA THÉORIE DE LA RELATIVITÉ RESTREINTE » (pré-publication hal-01081576, version 1).

$(O, \vec{i}, \vec{j}, \vec{k}, \vec{h})$ is a frame for the four-dimensional Euclidean space whose axes are denoted (OX) , (OY) , (OZ) , (Ow) ; the direction of the projection is that of the vector \vec{h} .

We consider a punctual mobile P in rectilinear uniform motion and associate to this mobile the hyperplane H_P which contains it and is orthogonal to the direction of its motion. The points of this hyperplane, which thus have parallel trajectories to that of P , constitute what we will refer to as the "observers" of the reference frame related to P .

To simplify the following diagrams, we shall consider only mobiles whose motion is located in the plan (XOw) , as well as the "observers" connected to P located in this plan.

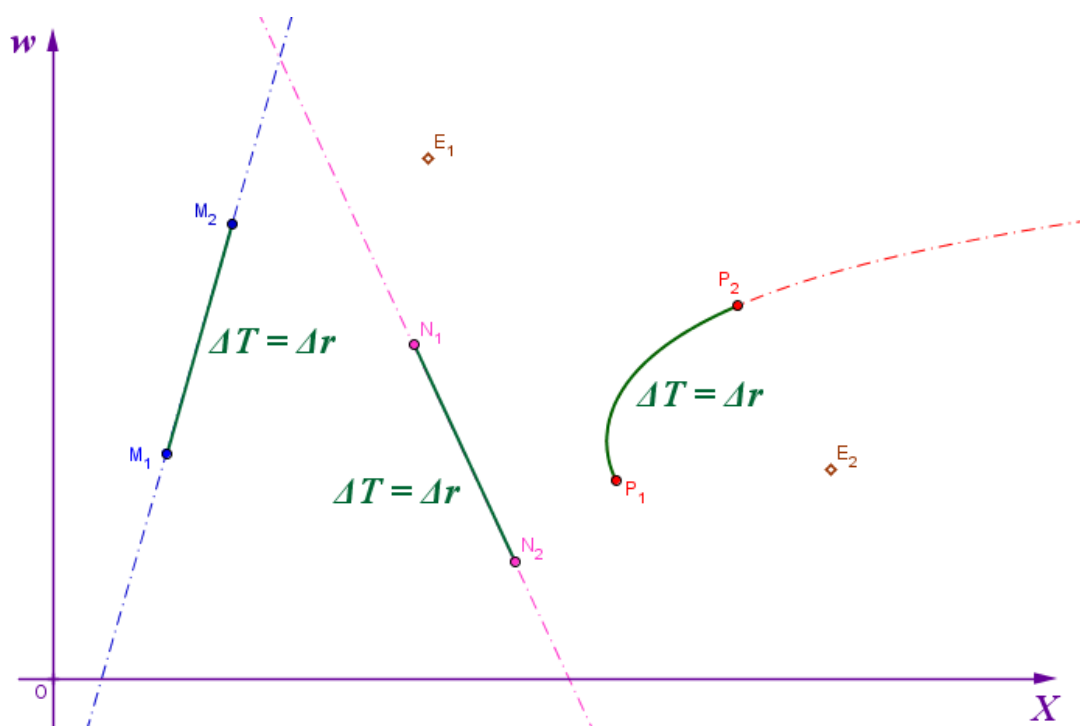
We will name d_α the straight line traveled by P , \vec{u}_α its guiding vector (oriented in the direction of the motion), R_α the reference frame related to P , α the angle $(\vec{i}, \vec{u}_\alpha)$.

2. Absolute time concept

We recall here the concept of absolute time introduced in the article hal-01340134.

Given the regular motion of the objects observed in the Platonic universe (« between two events, the distances traveled by all the observed objects are equal, regardless of their trajectory »), and with reference to the Newtonian concepts (“Absolute, true and mathematical time, of itself, and from its own nature flows equably without regard to anything external”), we can introduce a notion of absolute time in the following terms :

« **Absolute time** ΔT between two events is the distance traveled by (all) the mobiles between these two events .»



Thus, in the above diagram, if the event E_1 occurs while the mobiles M , N and P are respectively at M_1 , N_1 and P_1 and if the event E_2 occurs while the mobiles M , N and P are respectively at M_2 , N_2 and P_2 , we can define the **absolute time** ΔT between these two events by the relation $\Delta T = \Delta r$.

Note that in this Platonic space, one can thus introduce a notion of absolute time which is measured in **meters**.

3. Time measurement in a reference frame R_α

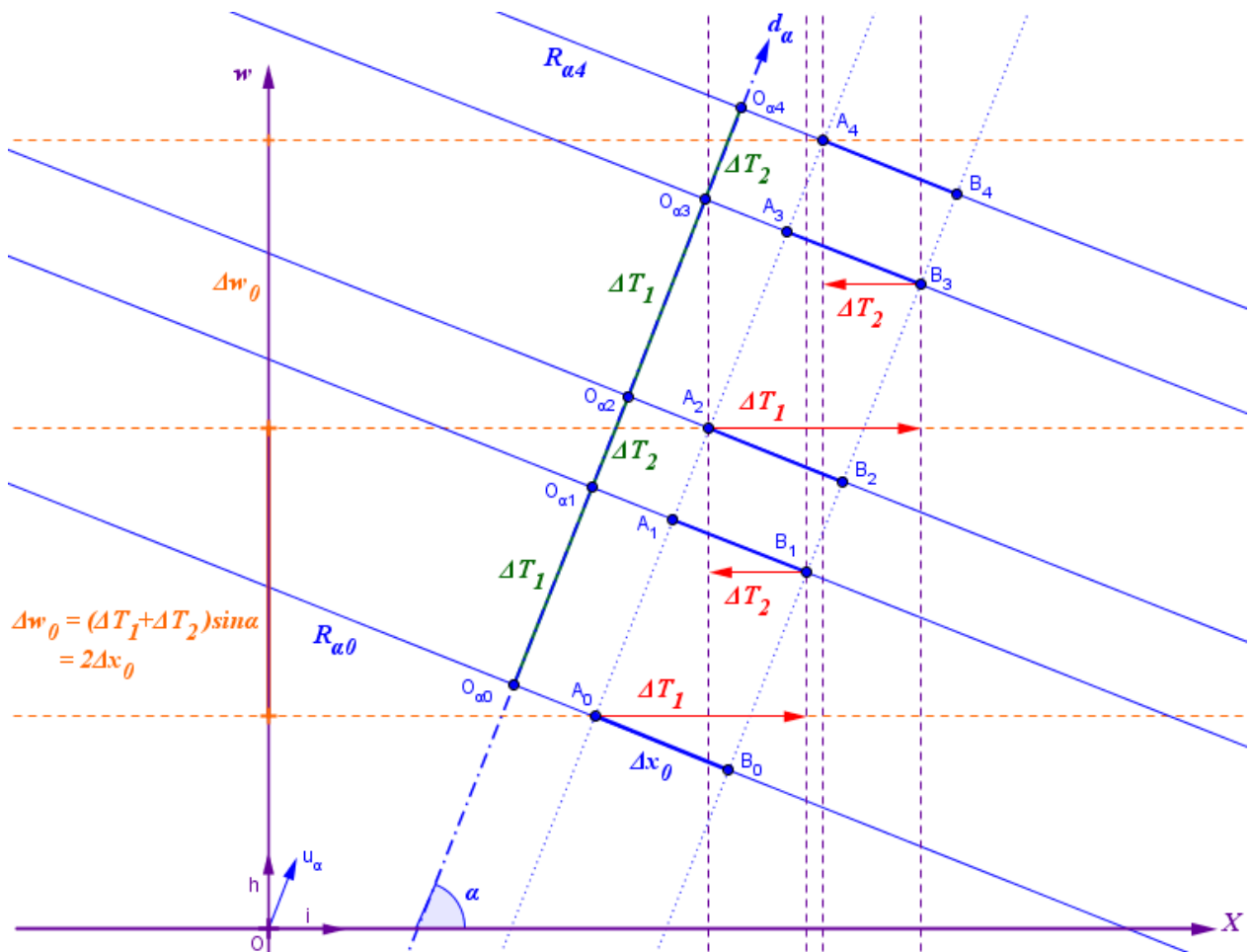
3.1 Introduction of a basic oscillator

The ideas and arguments developed in this article will be supported on the following additional *hypothesis* (which, consistent with all previous articles, was then a consequence of the definition of time measurement):

« electromagnetic waves propagate in the Platonic space according to directions orthogonal to the vector \vec{h} . »

The concept of time familiar to us is based on periodic physical phenomena.

We will present a basic oscillator whose examination will allow us, in the following paragraphs, to clarify the definition adopted for the time measurement in the Platonic model.



The diagram above shows two observers A and B of a reference frame R_α , separated in this frame by a distance Δx_0 .

An electromagnetic wave is transmitted from A to B when A is at A_0 .

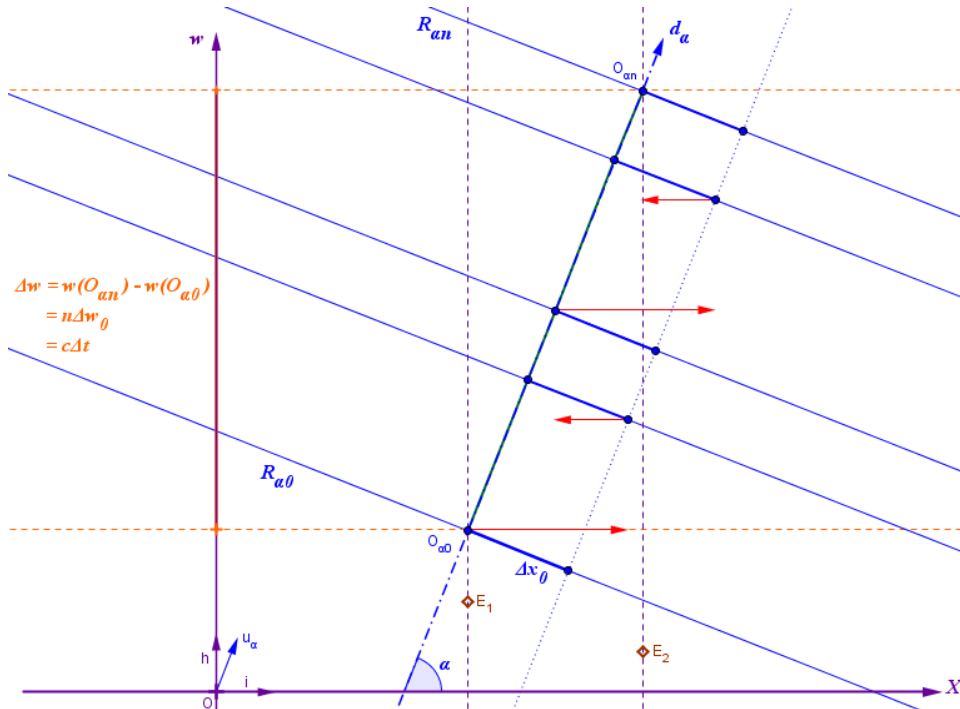
It is perceived by B when at B_1 (A is then at A_1) after an absolute duration ΔT_1 , then re-emitted to A .

So, we have : $\Delta t = n\Delta t_0 = n \frac{2\Delta x_0}{c} = n \frac{\Delta w_0}{c}$.

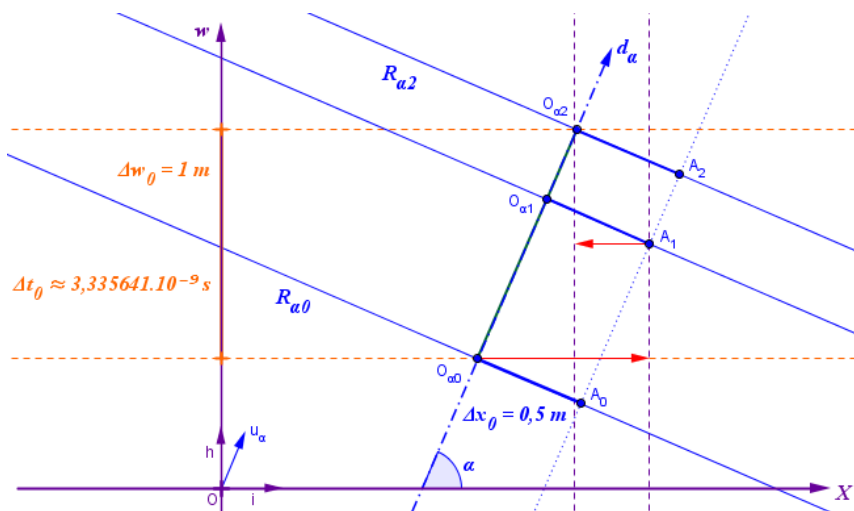
In addition, we have $\Delta w = w(O_{an}) - w(O_{a0}) = n\Delta w_0$, and we obtain :

$$c\Delta t = w(O_{an}) - w(O_{a0}).$$

Thus, the measurement of the proper relative time by an observer into a reference frame corresponds to the measurement of its movement along the axis (Ow).



Whatever the measure of the angle α (non-multiple of π), the international system of Units assigns to a distance of one meter along this axis a proper duration equal to $\frac{1}{299\,792\,458} \approx 3,335641 \cdot 10^{-9} \text{ s}$. In other words, it assigns this duration to an occurrence of the oscillator described above with $\Delta x_0 = 0,5 \text{ m}$.



3.3 Synchronizing clocks in the reference frame R_α

We assume in this section that each of the reference frame R_α observer is provided with a basic oscillator, as described above, these oscillators being identical.

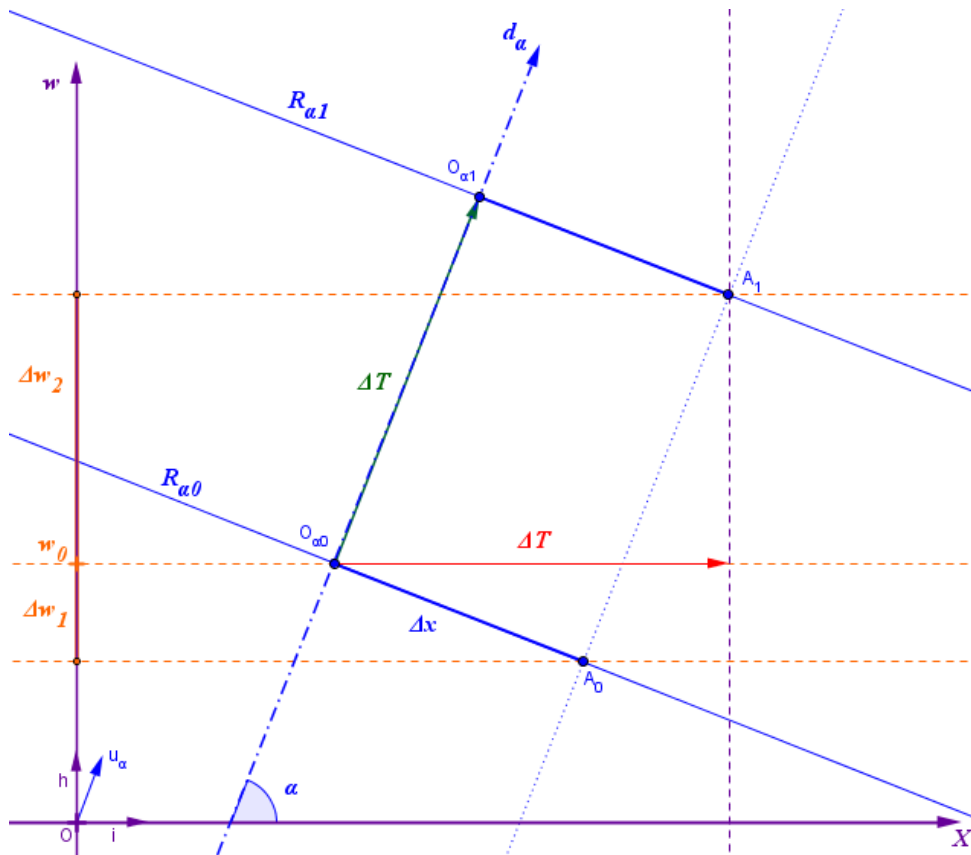
Each of these observers can measure the proper time between two events he perceives.

The purpose of this section is to provide a method for synchronizing each of these "clocks" to a reference clock, and therefore, enable a consistent measure time between any two events perceived by two separate observers A and B into R_α .

Choose for example the clock of the observer O_α as a reference clock, and consider any observer A into R_α .

Synchronizing the clock of A with that of O_α can be performed from the emission of an electromagnetic signal directed to A , at a proper time designated at O_α by $t = t_0$.

We assume that at this moment O_α is at $O_{\alpha 0}$, with $w(O_{\alpha 0}) = w_0$, and that A is at A_0 (see diagram below).



When receiving this signal, taking into account its speed c into R_α , A sets its clock to $t_1 = t_0 + \frac{\Delta x}{c}$, with $\Delta x = O_\alpha A$. [1]

The absolute duration ΔT corresponding to the transmission of this signal is:

$$\Delta T = \Delta x \frac{\sin \alpha}{1 - \cos \alpha},$$

and we also have (see diagram above):

$$\Delta w_1 = \Delta x \cos \alpha ; \Delta w_1 + \Delta w_2 = \Delta T \sin \alpha ,$$

from where : $\Delta w_2 = \Delta x$.

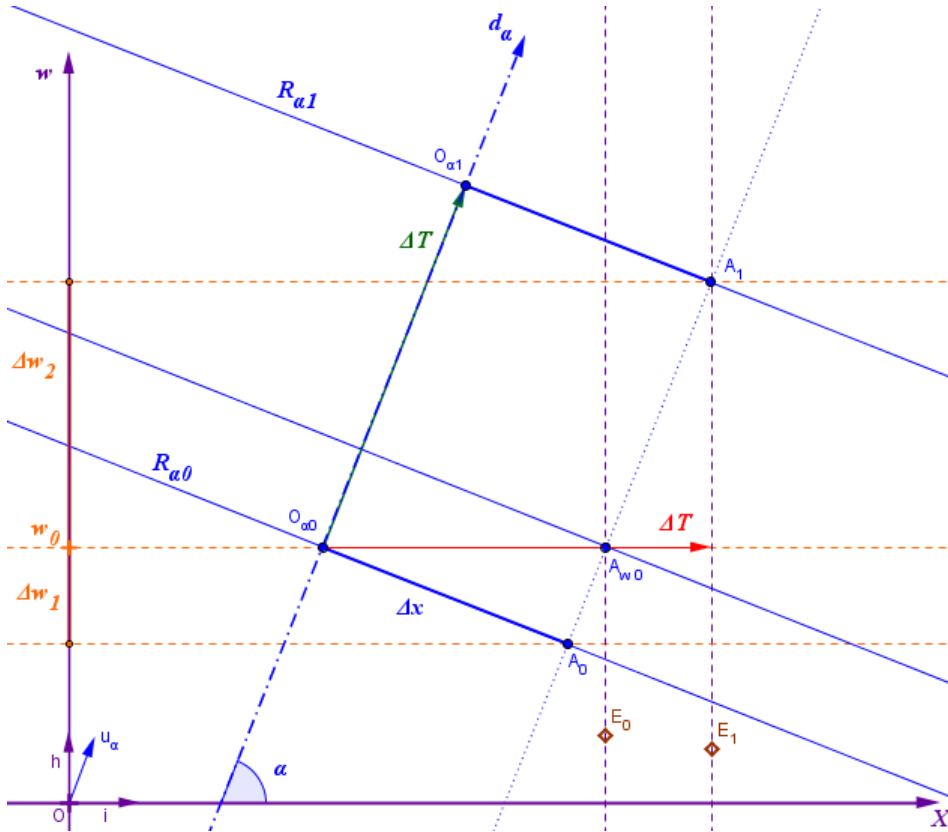
In other words, from the synchronization proposed in [1], the time t_1 corresponding to the reception of the signal A is such that:

$$t_1 = t_0 + \frac{\Delta x}{c} = t_0 + \frac{\Delta w_2}{c}, \text{ i.e. } ct_1 = ct_0 + \Delta w_2 . \quad [2]$$

This result leads to the following remark:

moving from A_0 to A_1 , observer A is passed through the hyperplane $w = w_0$.

Denote this event by E_0 , the corresponding position of A by A_{w_0} and denote the event " A is at A_1 " by E_1 (see diagram below).



We therefore obtain, applying to the proper time Δt measured by A between these events E_0 and E_1 the result obtained in 3.2. : $c\Delta t = w(A_1) - w(A_{w_0})$.

Then, with $\Delta w_2 = w(A_1) - w(A_{w_0})$ and [2], we obtain :

$$ct_1 - ct_0 = w(A_1) - w(A_{w_0}) \text{ i.e. } c\Delta t = ct_1 - ct_0.$$

As the moment t_1 corresponds to the event " A is A_1 " and the proper duration Δt corresponds to " A moved from A_{w_0} to A_1 ," it appears that, for the observer A, the event " A is at A_{w_0} " corresponds to the moment $t = t_0$.

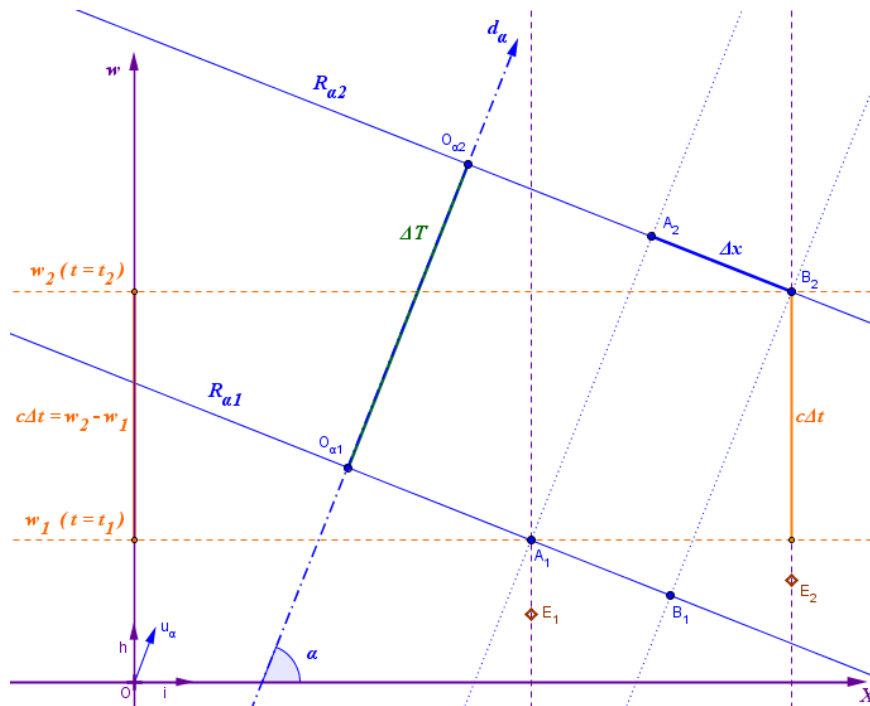
Thus, whatever the observer A into R_α , the clock of A is synchronized with the clock of O_α if and only if, for any w_0 , the events " $w(O_\alpha) = w_0$ " and " $w(A) = w_0$ " correspond to the same proper moment t_0 for O_α and A.

It is thus found that the clocks are *synchronized* into R_α if the moment $t = t_0$ corresponds, for each observer into R_α , to his crossing of the hyperplane $w = w_0$.

3.4 Measurement of an improper duration into a reference frame R_α

Clocks being so synchronized into R_α , consider two events E_1 and E_2 respectively perceived into R_α by the observer A (then located at A_1) at the moment t_1 and by the observer B (then located at B_2) at the moment t_2 .

The duration Δt between these two events into R_α being defined by $\Delta t = t_2 - t_1$, it immediately follows from the previous observation that $c\Delta t = w(B_2) - w(A_1)$.



3.5 Link between absolute time and relative time

Consider two events E_1 and E_2 respectively perceived into R_α by the observer A (then located at A_1) at the moment t_1 and by the observer B (then located at B_2) at the moment t_2 .

Note Δt and Δx the duration and the distance separating these two events into R_α (see previous diagrams).

From the results $\Delta x = \frac{\Delta X - \Delta T \cos \alpha}{\sin \alpha}$ and $c\Delta t = \frac{\Delta T - \Delta X \cos \alpha}{\sin \alpha}$ (1)

we obtain : $\Delta T = \frac{\Delta x \cos \alpha + c\Delta t}{\sin \alpha}$ (and $\Delta X = \frac{\Delta x + c\Delta t \cos \alpha}{\sin \alpha}$).

In particular, for any observer A , the proper time between two events corresponds to $\Delta x = 0$ and therefore $\Delta T = \frac{c\Delta t}{\sin \alpha}$, or $\Delta t = \frac{\Delta T \sin \alpha}{c}$.

This means, in fact, that the measurement of proper time which is familiar to us (the time measured by our clocks) corresponds into the Platonic space to the measurement of the component along the (Ow) -axis of the absolute time (divided by c).

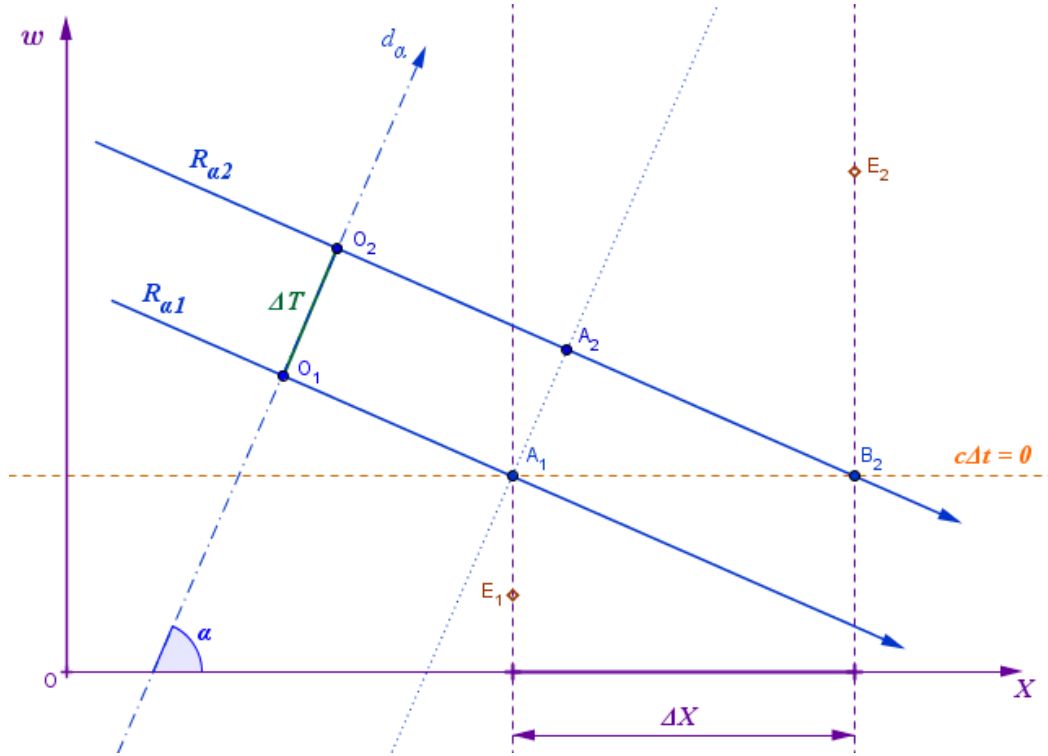
Finally, in the very special case of an observer who would be linked to a photon, any proper duration is equal to zero ($\sin \alpha = 0$), which corresponds to a photon movement whose direction is contained in a hyperplane orthogonal to \vec{h} (since the component along the (Ow) -axis of absolute time is then equal to zero).

Remark : formulas (1) provide a link between the absolute Newtonian concepts (adapted to the Platonic space) and familiar relativistic concepts. This link is developed into paragraph 3.7 and will allow us to demonstrate the Lorentz transformation.

3.6 Simultaneity – Causality

3.6.1. Simultaneity

By definition, two events E_1 and E_2 perceived respectively by A_1 and B_2 are called *simultaneous* in the reference frame R_α if and only if the time interval Δt between them is zero; ie. $w(B_2) = w(A_1)$.



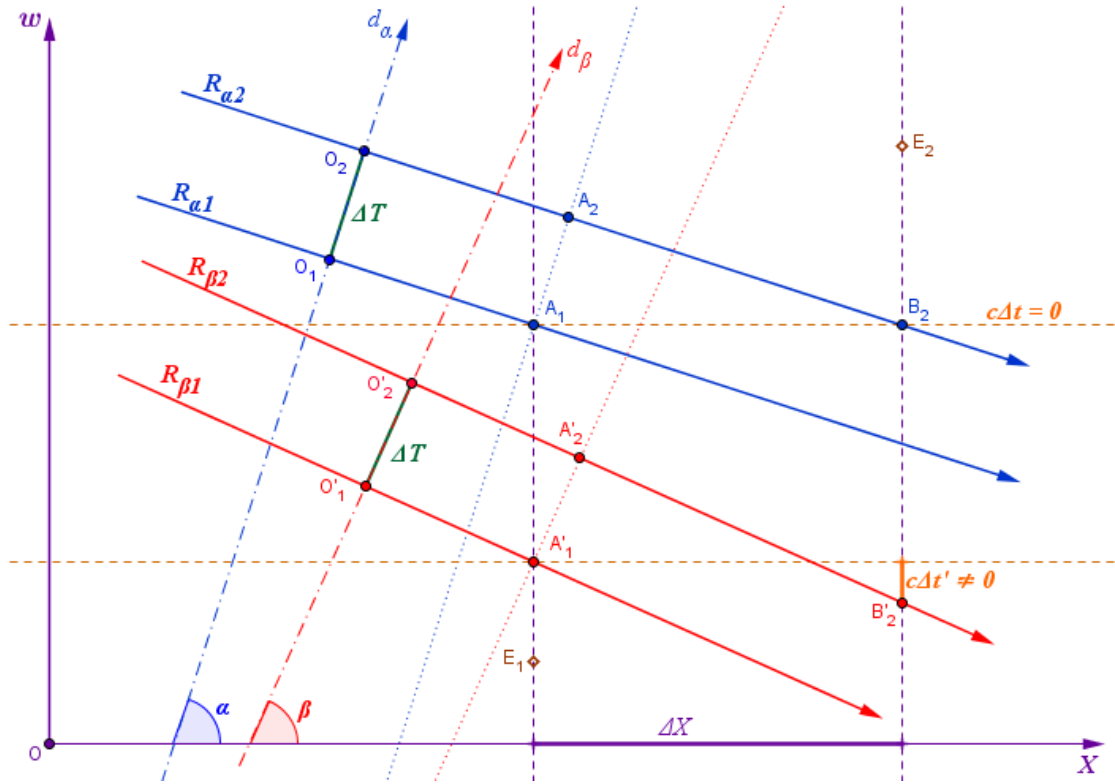
This situation corresponds for these two events E_1 and E_2 to the equality:

$$\cos \alpha = \frac{\Delta T}{\Delta X}, \text{ where } \Delta T \text{ is the absolute time between them.}$$

Thus, two events E_1 and E_2 *can not* be perceived simultaneously into any reference frame R_α if $|\Delta T| > |\Delta X|$ (such events can be bound by a *causal link* – see next paragraph –).

Conversely, if $|\Delta T| < |\Delta X|$, there is always a reference frame R_α in which these events can be perceived *simultaneously* (and they *can not* therefore be bound by *causality*).

Note that if $\alpha \neq \beta$, then simultaneously perceived phenomena into R_α are not simultaneous into R_β (in the diagram below, the events E_1 and E_2 are perceived *simultaneously* by observers A_1 and B_2 into R_α , but are separated into R_β by the interval of time $\Delta t' \neq 0$, with $c\Delta t' = \Delta X \frac{\cos \alpha - \cos \beta}{\sin \beta} = \frac{-v/c \Delta x}{\sqrt{1 - \frac{v^2}{c^2}}}$):



3.6.2. Causality

In accordance with the approach adopted in the theory of relativity, we will consider that if two events can not be reached at a speed lower or equal to the speed of light, they have *no direct causal link*.

This results in any reference frame R_α by inequality $\Delta x > c\Delta t$,

which leads to $\Delta x(1 - \cos \alpha) > c\Delta t(1 - \cos \alpha)$

and then to $\Delta x + c\Delta t \cos \alpha > \Delta x \cos \alpha + c\Delta t$,

and therefore, with the formulas set out in paragraph 3.5: $|\Delta X| > |\Delta T|$.

So there is a *possible causality* between the events E_1 and E_2 only if $|\Delta T| \geq |\Delta X|$.

Note that the case $|\Delta T| = |\Delta X|$ corresponds to the case where the events E_1 and E_2 may be linked by an electromagnetic signal.

As expected, it is thus observed that if two events may be linked by a causal link in a Galilean reference frame R_α , then they can also be in any inertial frame. Indeed :

$$|\Delta T| \geq |\Delta X| \Rightarrow |c\Delta t| \geq |\Delta x|,$$

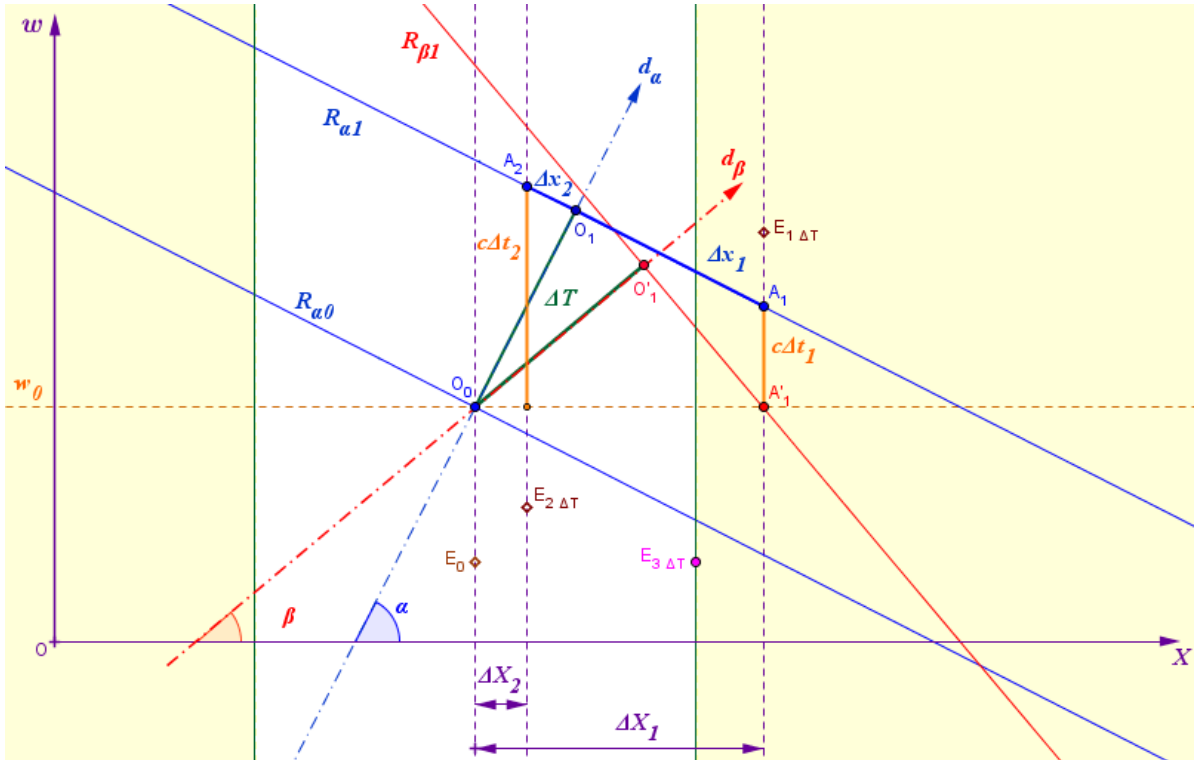
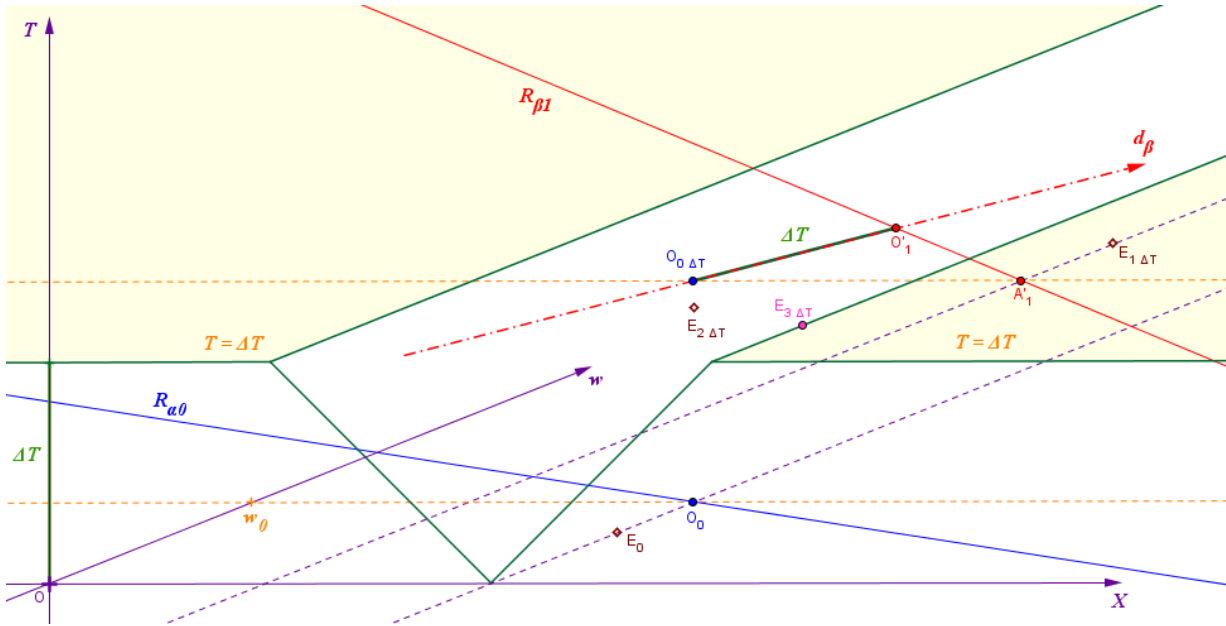
whatever is α , not a multiple of π (cf. above calculation).

Finally, to restate a fact mentioned in the previous paragraph, we recall that *two events can not be bound by a causal relationship if there is a reference frame R_α where they are perceived simultaneously (and vice versa)*.

3.6.3. Light prism

By analogy with the "light cone" mentioned in the theory of relativity, we can represent by a "light prism" (in a five-dimensional space, the fifth dimension identifying the flow of absolute time T) the zone without any causal link with the observer, i.e. the outside of the prism.

This "prism" is here simplified in a three-dimensional representation, whose hyperplane $T = \Delta T$, shown in the second diagram in two dimensions, is discussed in the next page.



An event E_0 is perceived by the observer O into R_α at the absolute moment $T = 0$. Consider the three events $E_{1\Delta T}$, $E_{2\Delta T}$, $E_{3\Delta T}$, occurring after an absolute duration ΔT .

✓ Regarding $E_{1\Delta T}$, we have $|\Delta X_1| > |\Delta T|$: thus *there is* a reference frame R_β where E_0 and $E_{1\Delta T}$ are perceived *simultaneously* (with here $\cos \beta = \frac{\Delta T}{\Delta X_1}$, the perception of $E_{1\Delta T}$ being done in R_β by the observer A' , then located at A'_1); E_0 and $E_{1\Delta T}$ therefore *can not* be bound by a *causal link*.

✓ Regarding $E_{2\Delta T}$, we have $|\Delta X_2| < |\Delta T|$: thus *there is no* reference frame R_β where E_0 and $E_{2\Delta T}$ can be perceived *simultaneously*; E_0 and $E_{2\Delta T}$ therefore *can* be bound by a *causal link*.

✓ Regarding $E_{3\Delta T}$, we have $|\Delta X_3| = |\Delta T|$: E_0 and $E_{3\Delta T}$ therefore *can* be bound by a *causal link* (e.g. a light signal).

To expand the view expressed in paragraphs 3.6.1 and 3.6.2, however, one could also imagine in this case limit the existence of such a photon linked reference frame R_β where E_0 and $E_{3\Delta T}$ are perceived simultaneously, as in this case all proper duration is zero (see section 3.5).

Remark : in the situation diagrammed above, the simultaneity of events E_0 and $E_{1\Delta T}$ into the frame R_β , which corresponds to the equality $\cos \beta = \frac{\Delta T}{\Delta X_1}$, is also

reflected in the equality $\frac{v}{c} = \frac{c\Delta t_1}{\Delta x_1}$, where v is the speed of R_β measured by

R_α (with $\frac{v}{c} = \frac{\cos \beta - \cos \alpha}{1 - \cos \alpha \cos \beta}$), Δx_1 refers to the distance between the events

E_0 and $E_{1\Delta T}$ in the frame R_α , Δt_1 the duration between events E_0 and $E_{1\Delta T}$ in the frame R_α .

If $\Delta t'$ is the time between the events E_0 and $E_{1\Delta T}$ measured in the reference

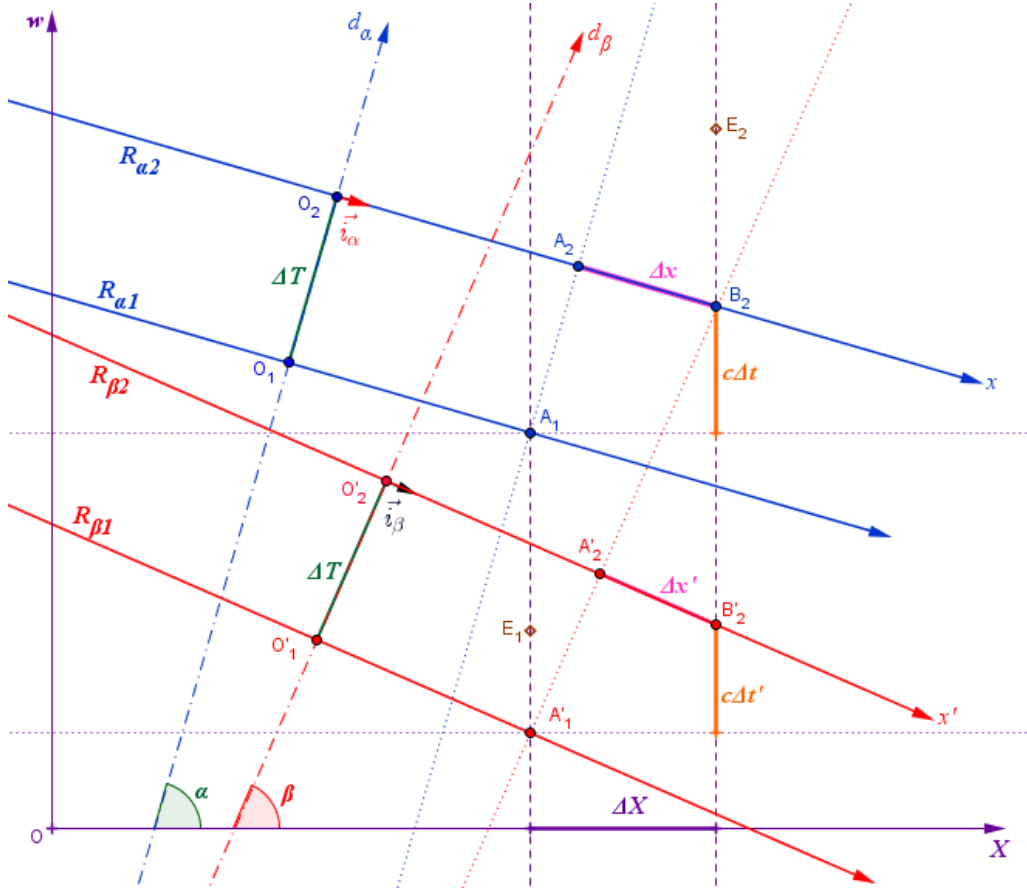
frame R_β , the equality $\frac{v}{c} = \frac{c\Delta t_1}{\Delta x_1}$ corresponds to $\Delta t' = 0$, according to the

Lorentz transformation recalled into the next paragraph.

3.7 Lorentz transformation

3.7.1. Summary of the article hal- 01165196

Two reference frames R_α and R_β in uniform translation (i.e. with *rectilinear trajectories*) are here considered, and two events E_1 and E_2 are observed.



$$\text{We have: } \Delta x = \frac{\Delta X - \Delta T \cos \alpha}{\sin \alpha} \quad \text{and} \quad c\Delta t = \frac{\Delta T - \Delta X \cos \alpha}{\sin \alpha} .$$

$$\text{Similarly, we find: } \Delta x' = \frac{\Delta X - \Delta T \cos \beta}{\sin \beta} \quad \text{and} \quad c\Delta t' = \frac{\Delta T - \Delta X \cos \beta}{\sin \beta} .$$

$$\text{With } \frac{v}{c} = \frac{\cos \beta - \cos \alpha}{1 - \cos \alpha \cos \beta}, \text{ we obtain: } \Delta x - \frac{v}{c} c\Delta t = \frac{(\Delta X - \Delta T \cos \beta) \sin \alpha}{1 - \cos \alpha \cos \beta} ;$$

$$\text{Supposing that } \sin \alpha \sin \beta > 0, \text{ we have } \sqrt{1 - \frac{v^2}{c^2}} = \frac{\sin \alpha \sin \beta}{1 - \cos \alpha \cos \beta}, \text{ then :}$$

$$\frac{\Delta x - \frac{v}{c} c\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta X - \Delta T \cos \beta}{\sin \beta}, \text{ i.e. } \Delta x' = \frac{\Delta x - v\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} .$$

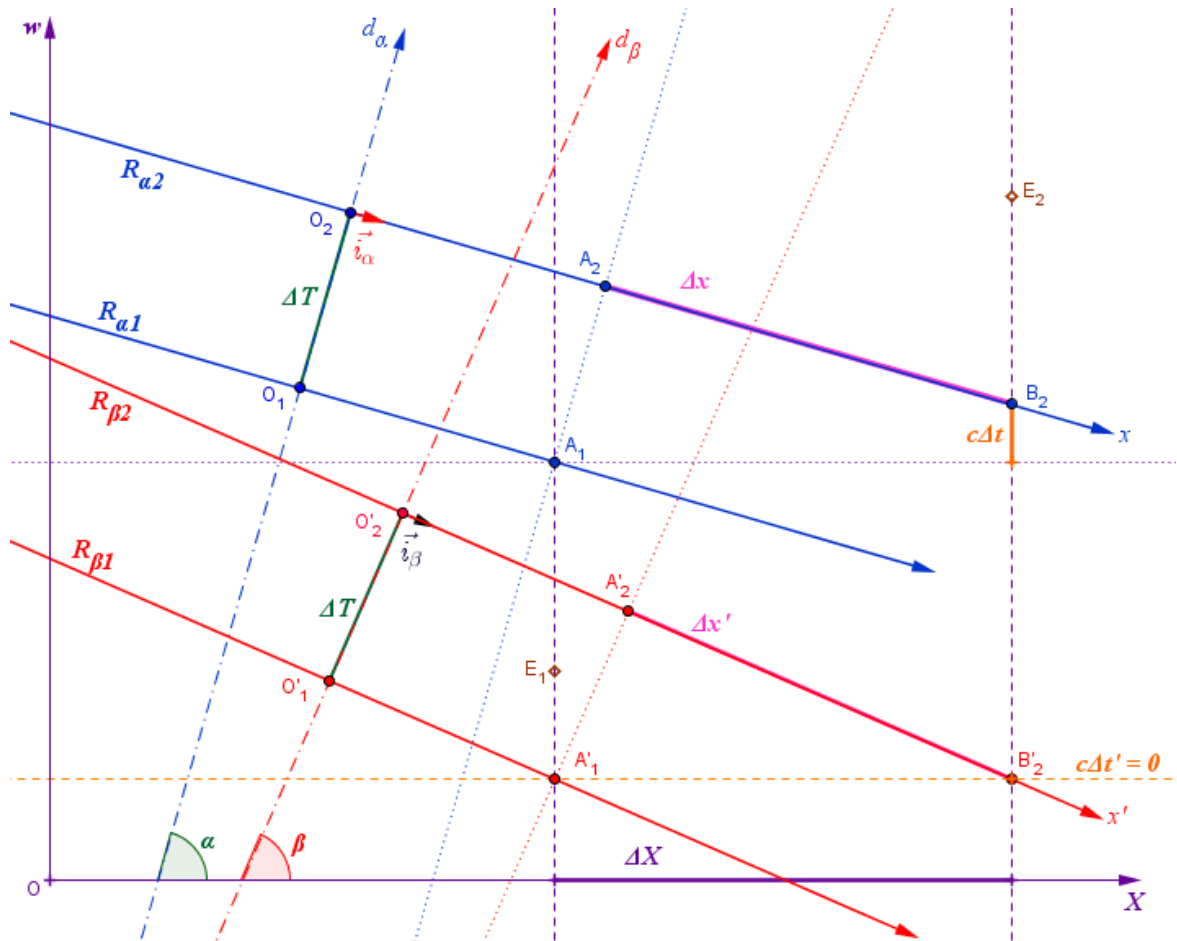
Similarly, we obtain : $c\Delta t - \frac{v}{c}\Delta x = \frac{(\Delta T - \Delta X \cos \beta) \sin \alpha}{1 - \cos \alpha \cos \beta}$,

then finally: $\frac{c\Delta t - \frac{v}{c}\Delta x}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta T - \Delta X \cos \beta}{\sin \beta}$, i.e. $c\Delta t' = \frac{c\Delta t - \frac{v}{c}\Delta x}{\sqrt{1 - \frac{v^2}{c^2}}}$.

3.7.2. Length contraction

With $c\Delta t' = 0$, we obtain : $c\Delta t = \frac{\cos \beta - \cos \alpha}{1 - \cos \alpha \cos \beta} \Delta x$, from which we deduce:

$$\Delta x' = \frac{\sin \alpha \sin \beta}{1 - \cos \alpha \cos \beta} \Delta x, \quad \text{i.e.} \quad \Delta x' = \Delta x \sqrt{1 - \frac{v^2}{c^2}}.$$



This length contraction is shown above from $c\Delta t' = 0 \Leftrightarrow \Delta X = \frac{\Delta T}{\cos \beta}$.

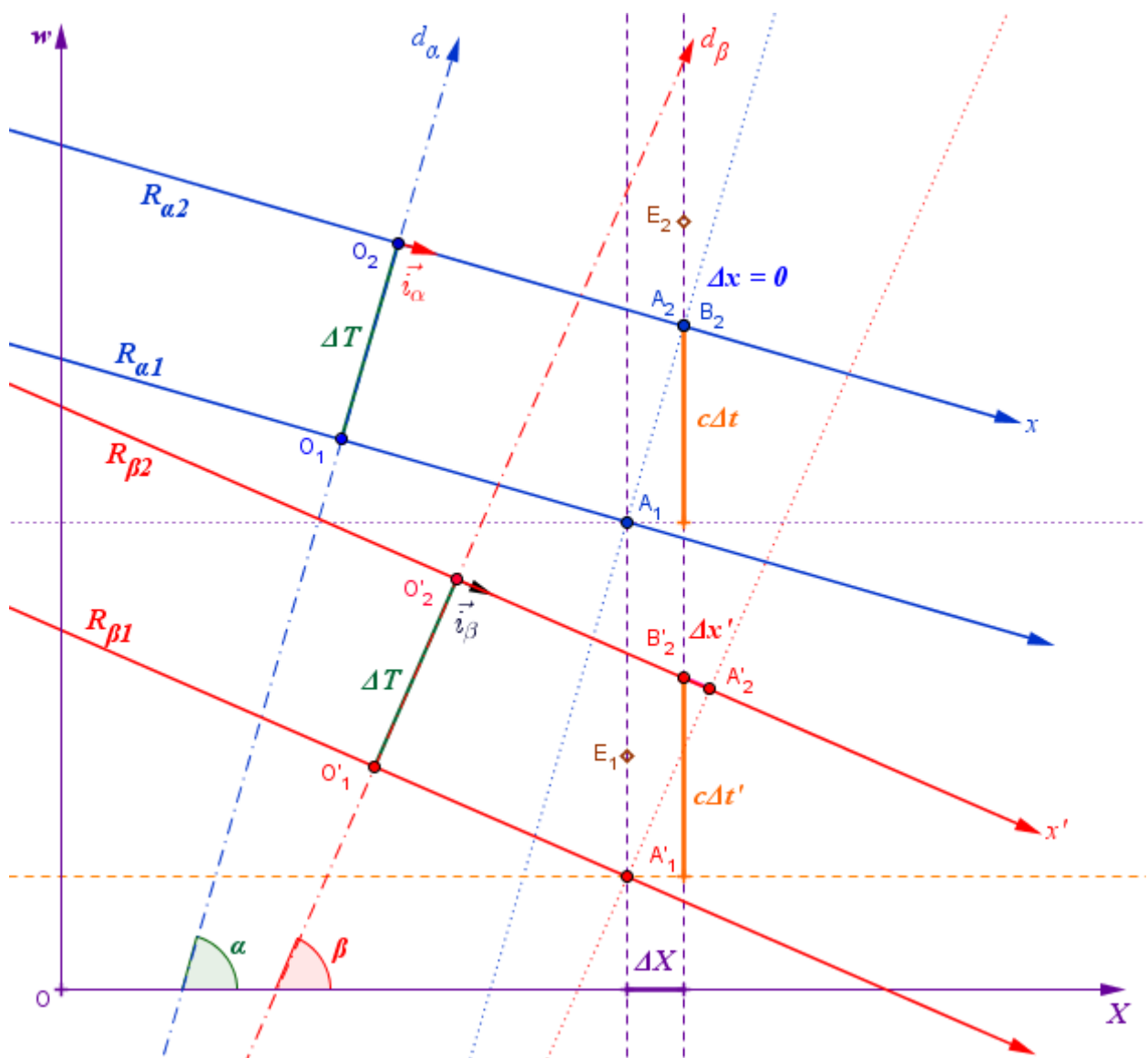
3.7.3. Time dilation

Similarly, with $\Delta x = 0$, we obtain :

$$c\Delta t' = \frac{1 - \cos \alpha \cos \beta}{\sin \alpha \sin \beta} c\Delta t, \quad \text{i.e.} \quad c\Delta t' = \frac{c\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

This time dilation is illustrated below from:

$$\Delta x = 0 \Leftrightarrow \Delta X = \Delta T \cos \alpha.$$



3.7.4. Illustration : lifetime of the muon

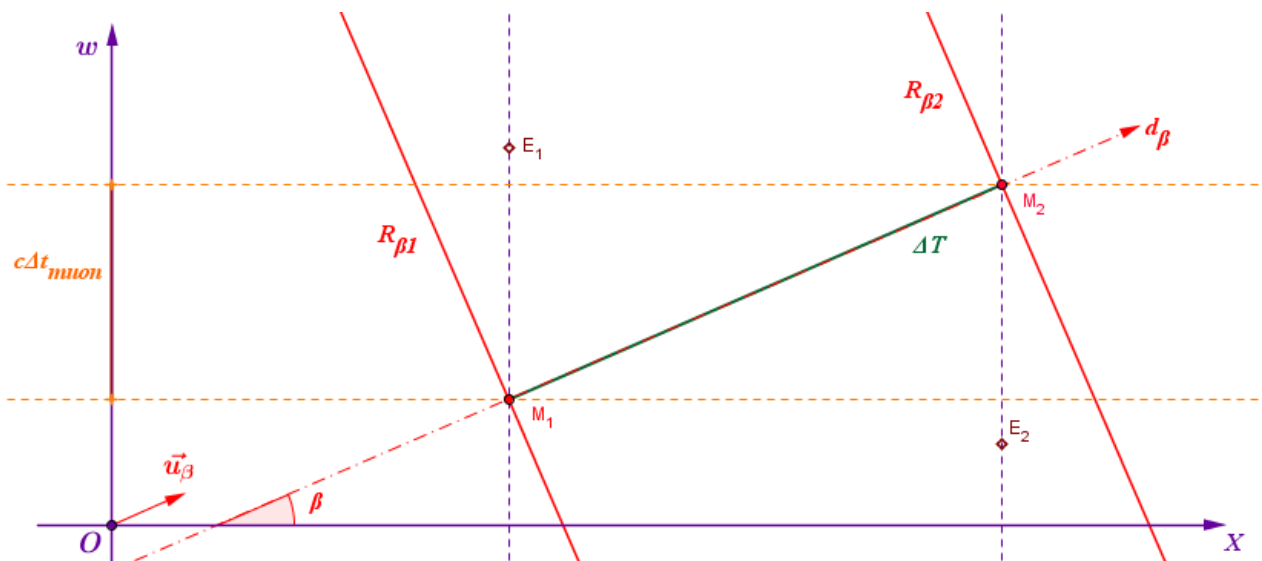
An illustration of the above relations concerns the lifetime of the muon, unstable particle whose rest lifetime is about $\Delta t = 2,2 \mu s$.

The lifetime of a muon moving at a speed such as $v \approx 0,992c$ in a particle accelerator becomes:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} = 17,4 \mu s.$$

To simplify the following diagram, we shall consider a muon whose motion is located in the plan (XOw) .

- Consider to start a muon M moving along a trajectory led by a vector \vec{u}_β , with $\beta = (\vec{i}, \vec{u}_\beta)$. We mean by "proper lifetime" of this muon the duration, noted Δt_{muon} , between the events E_1 : "muon creation" (M is at M_1) and E_2 : "muon decay" (M is then at M_2) measured in a rest frame R_β .



We have : $c\Delta t_{muon} = w(M_2) - w(M_1)$ (cf. paragraph 3.2).

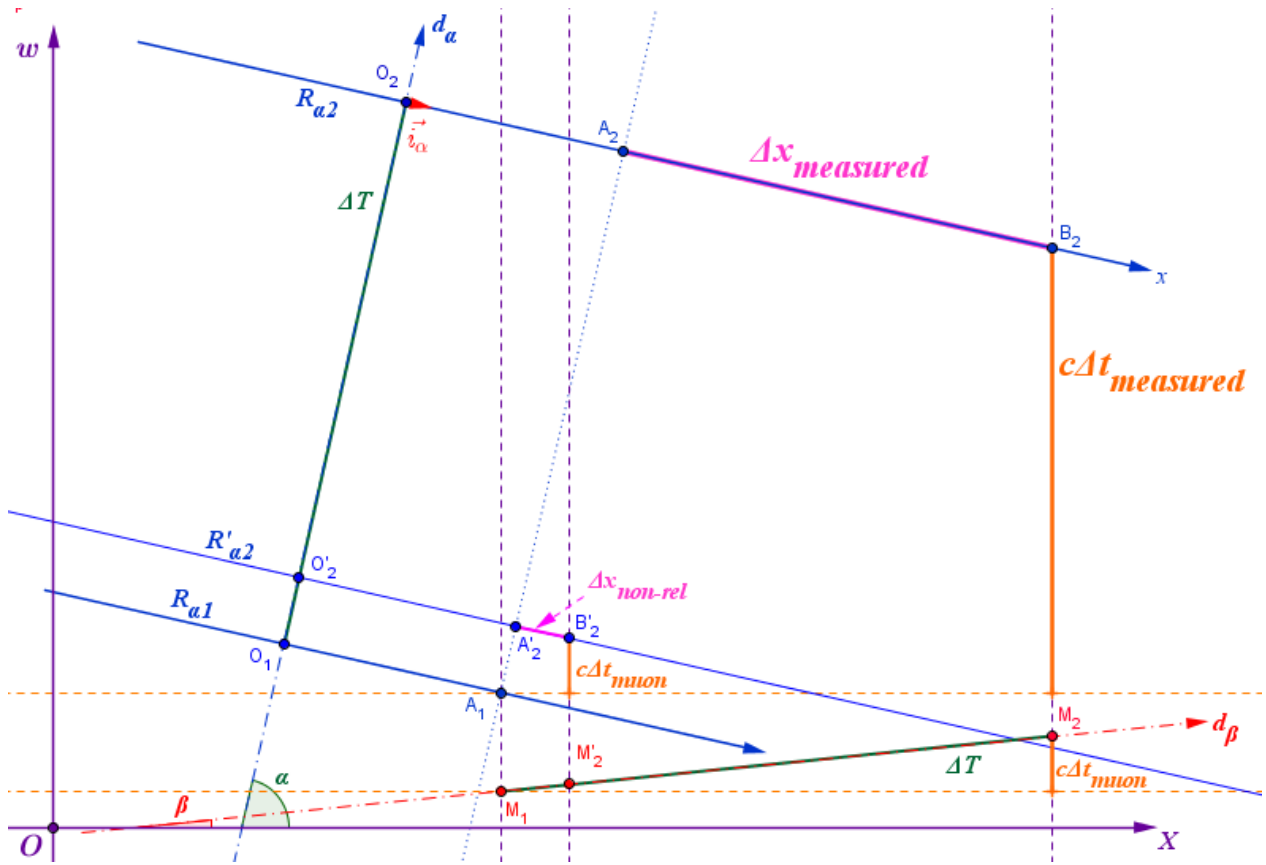
The absolute duration ΔT corresponding to the proper lifetime of the muon is given by:

$$\Delta T = \frac{c\Delta t_{muon}}{\sin \beta} \quad (\text{cf. paragraph 3.5}).$$

The hypothesis that will underpin this paragraph is that *the proper lifetime of the muon is independent of its trajectory in the Platonic space.*

Thus, regardless of the reference frame R_α , if the muon is at rest in that frame, its lifetime measured would be equal to Δt_{muon} .

- Now consider the observation of the muon by any reference frame R_α :



Comments

The muon is moving in the direction d_β . Its proper lifetime is denoted by « Δt_{muon} ». The muon is initially located at M_1 . Into the reference frame R_α , it is then perceived by the observer A_1 . In this diagram, $v \approx 0,992c$.

In a non-relativistic way, after a time lapse measured into R_α equal to the proper lifetime Δt_{muon} , the muon is supposed to be perceived at M'_2 by the observer B'_2 , after covering in R_α the distance labeled « $\Delta x_{non-rel}$ », with $\Delta x_{non-rel} = v\Delta t_{muon}$.

But its proper lifetime Δt_{muon} leads the muon at M_2 , after an absolute duration equal to $\Delta T = \frac{c\Delta t_{muon}}{\sin \beta}$.

It is then perceived into R_α by the observer B_2 .

The distance traveled by the muon, measured into R_α , is labeled $\Delta x_{measured}$ and its lifetime measured into R_α is labeled $\Delta t_{measured}$.

We obtain in this illustration the usual relativistic relations:

$$c\Delta t_{measured} = \frac{c\Delta t_{muon}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad \Delta x_{measured} = \frac{\Delta x_{non-rel}}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

4. Conclusion

As we have seen, this article, through the presentation of some generating ideas of the Platonic model, allows to propose a geometric approach of the concept of time which we are familiar.

Since the Michelson-Morley experiment (whose modeling was proposed in Article hal-01247385) and the birth of the theory of relativity, our time measurement is based on the counting of occurrences of an electromagnetic periodic phenomenon.

For a given observer measuring the proper time Δt between two events, this article shows that it actually leads to measure the axis (Ow) component of the distance travelled (i.e. the absolute time ΔT measured) by this observer in the Platonic space. Our system of units assigns to a distance of one meter covered on that axis a proper duration approximately equal to $3,3356 \cdot 10^{-9}$ second, regardless of its trajectory.

Between two given events, the apparent relativity of durations (and distances) measured by different reference frames appears as a direct consequence of their different trajectories ... and shows that the absolute Newtonian concepts and relativistic concepts are not so irreconcilable that could the thinking, the latter being the result of a partial (projective) measure of the first (adapted to a Platonic space).

Is it possible to measure the intrinsic flow of absolute time between two specified events (not just its (Ow) axis component)?

If so, our contemporary vision of physics will be again profoundly changed.

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