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Gabriel Lagasque

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# A note on simple randomly switched linear systems

Gabriel LAGASQUIE

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## Abstract

We construct a planar process that switches randomly between the flows of two linear systems built from two Hurwitz matrices (all eigenvalues have negative real parts). The goal here is to study the long time behaviour according to the switching rates. We will see that, even if the two systems are stable, it is possible to obtain a blow up if we choose the switching rates wisely. Finally we will see a connection, between the tail of the invariant measure (when the switching times follow an exponential law) and the existence of a deterministic control that makes the process explode.

**AMS Classification:** 60J99, 34A60

## 1 Introduction

Piecewise Deterministic Markov Processes (PDMPs) is a huge class of stochastic processes introduced by Davis [8]. In this note we are studying a particular subclass of these processes. we consider  $(X_t, I_t)_{t \geq 0}$  evolving on the space  $\mathbb{R}^d \times \{0, 1\}$  solving the equation:

$$\dot{X}_t = F(X_t, I_t)$$

where  $F$  is a smooth function and  $(I_t)$  is a jump process on  $\{0, 1\}$ . We call  $\lambda_i$  the jump rate of the process  $(I_t)$  from state  $i$  to state  $1 - i$ . Such processes play a role in modeling problems in various fields such as molecular biology [13], population dynamics [7], Internet traffic [10]. Even if these processes are easy to define, their stability is never clear.

More precisely, we are here interested in the long time qualitative behaviour of the planar randomly switched process  $(X_t, I_t)_{t \geq 0} \in \mathbb{R}^d \times \{0, 1\}$  solving the equation

$$\begin{cases} \dot{X}_t = A_{I_t} X_t \\ X_0 = x_0, \end{cases} \quad (1)$$

where  $A_0, A_1$  are  $d \times d$  real matrices, and  $(I_t)$  is a Markov process on  $E = \{0, 1\}$  with constant jump rates:

$$\begin{aligned} \lambda_0 &= \beta u, \\ \lambda_1 &= \beta(1 - u) \end{aligned}$$

where  $\beta > 0$  and  $u \in (0, 1)$ . The process  $((X_t, I_t))_{t \geq 0}$  is a piecewise deterministic Markov process as defined in [8] and several examples have been studied in [12] or [5]. It appears that the following quantity dictates the large time behaviour of this process according to the parameters  $A_0, A_1, \beta$  and  $u$ :

$$\chi := \chi(A_0, A_1, \beta, u) = \lim_{t \rightarrow +\infty} \frac{1}{t} \log (\|X_t\|) \quad \text{a.s.}$$