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A wheel at the origin of the center of mass motion in human walking

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Abstract

The center of mass (CoM) is a key descriptor in the understanding and the analysis of bipedal locomotion. Some approaches are based on the premise that humans minimize the CoM vertical displacement. Other approaches express walking dynamics through the inverted pendulum model. Such approaches are contradictory in that they lead to two conflicting patterns to express the CoM motion: straight line segments for the first approaches and arcs of a circle for the second ones.

In this paper we show that CoM motion is a trade-off between both patterns. Specifically CoM follows a "curtate cycloid", which is the curve described by a point rigidly attached to a wheel rolling on a flat surface. We demonstrate that all the three parameters defining a curtate cycloid only depend on the height of the subjects.

Keywords: human locomotion, analytical model, centre of mass, locomotion signature

1. Introduction

Walking aims at transporting the body from a placement on the ground to another one. Two parameters of position and one parameter of orientation

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define the body placement. Human body motions originate in the motor space
5 made of more than 600 muscles. Walking can then be viewed as a mathematical
mapping from the very high dimensional body space into the 3-dimensional
placement one. This makes walking a complex process for a simple-objective
task Whittle (2014). Facing such a complexity remains a challenge for many
disciplines in life sciences (biomechanics, neurophysiology, medicine, physical
10 therapy) and recently for computer engineering and robotics with the emergence
of humanoid robots¹. Most research approaches explore complexity reduction
principles. For example, six major determinants of gait have been identified
Inman et al. (1953); Della Croce et al. (2001) as critical features to address
walking kinematics. The introduction of gait determinants have been mainly
15 motivated by the minimization of CoM vertical displacement. On the other
hand, a popular model to express walking dynamics is the inverted pendulum
Cavagna and Margaria (1966). Inverted pendulum gives a rough approximation
of the motion of the walker center of mass (CoM) via a sequence of arcs
of a circle. The geometry of CoM motion induced by both perspectives are
20 incompatible: minimizing the CoM vertical displacement tends to a straight
line motion, which is not converging towards a motion made of a sequence of
arcs of a circle. The contradiction of both theories is deeply explored in Kuo
(2007). It is shown that both underlying premises are limited and it is proposed
to focus on mechanical work rather than the kinematics or forces of gait.

25 Nevertheless, in any case, the estimation of CoM motion plays a central role
in the study of human walking. It represents a descriptor of motion relevant
in both kinematic and dynamic point of view and may allow validating or
invalidating theories of human walking. However, reconstructing the position
of CoM is not a straightforward process, since it is not rigidly linked to any
30 limb of the body. For instance, in Whittle (1997), it is shown that CoM
moves differently from the motion of the pelvis. The importance of stance-limb

¹Beyond walking studies, the field of biomechanics of human motion is evolving rapidly
towards multidisciplinary research Alexander (2005); Latash and Zatsiorsky (2015).)

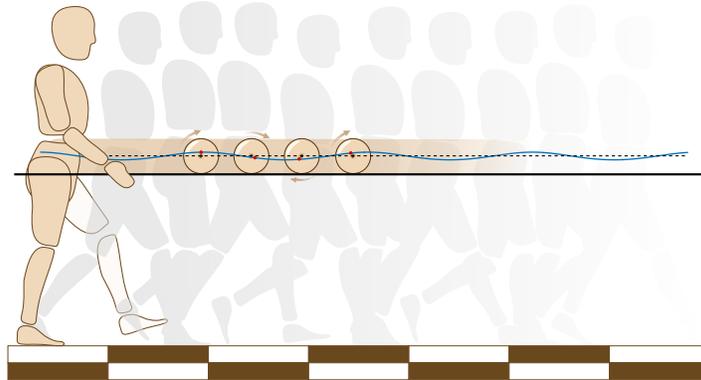


Figure 1: Illustration of the CoM trajectory in the sagittal plane during human walking. The CoM trajectory corresponds has a cycloidal pattern, described by a point on a wheel rolling at constant velocity on a flat surface.

behavior in determining the trajectory of CoM during walking and running is explored in Lee and Farley (1998). The path followed by CoM when walking on a treadmill has an upward concave figure-of-eight shape which is described
 35 in Tesio et al. (2010).

CoM position and motion estimation are addressed by modern techniques of motion capture. The human body CoM depends on various parameters, which are classically reduced to articular angles and limb mass distribution. Body segments are considered as rigid bodies. In vision-based motion capture,
 40 body segments are equipped with markers. Their 3D positions are captured by vision. Articular angles are deduced from the position of body parts Wu et al. (2002, 2005). CoM is then computed from standard anthropomorphic mass distribution of body parts De Leva (1996). A second popular approach is to estimate CoM position and motion directly from force platforms Shimba
 45 (1984b); Caron et al. (1997); Barbier et al. (2003). Such platforms measure the interaction forces and moments of the body with the environment. Forces provide CoM accelerations and moments are related to CoM position through the so-called central axis of the contact wrench Latash and Zatsiorsky (2015).

It remains that both kinematics-based and dynamics-based approaches of

50 CoM estimation are subject to a lot of inaccuracy sources. In Carpentier et al.
(2016) we recently proposed a theoretical study about the observability of the
center of mass position using motion capture and force platforms. We showed
that the accuracy domain of each measurement can be easily described through a
spectral analysis. We then introduced a new approach based on complementary
55 filtering to estimate the CoM position with increased accuracy.

Based on this new CoM estimation algorithm, the present study explores the
geometric shape of the CoM path when walking. It is shown that CoM follows
a curtate cycloid in the sagittal plane, generated by a virtual wheel whose
parameters constitute original invariants of bipedal walking and illustrated on
60 Fig. 1.

Material and methods

Participants

Sixteen healthy male (age: 24.2 ± 2.3 yr, height: 1.74 ± 0.04 m, mass: $71.0 \pm$
 8.9 kg) and four female (age: 24.3 ± 3.3 yr, height: 1.71 ± 0.04 m, mass: 53.4 ± 8.9
65 kg) subjects volunteered for this investigation. The experiments were conducted
in accordance with the standards of the Declaration of Helsinki (rev. 2013), with
formal approval of the ethics evaluation committee Comité d’Evaluation Ethique
de l’INSERM (IRB00003888, Opinion number 13-124) of the Institut National
de la Santé et de la Recherche Médicale, INSERM, Paris, France (IORG0003254,
70 FWA00005831).

Data acquisition

The experiment room (dimension 6×20 m) was equipped with 12 infrared
cameras sampling at 200 Hz (Vicon, Oxford Metrics, Oxford, UK) and recording
43 reflective markers placed on the whole body of the subjects. Markers set is
75 based on Wu recommendations and approved by the International Society of
Biomechanics Wu et al. (2002, 2005). Two force plates (AMTI, Watertown,
MA, USA) embedded into the floor were used to record ground reaction forces
and ground reaction moments at 1000 Hz.

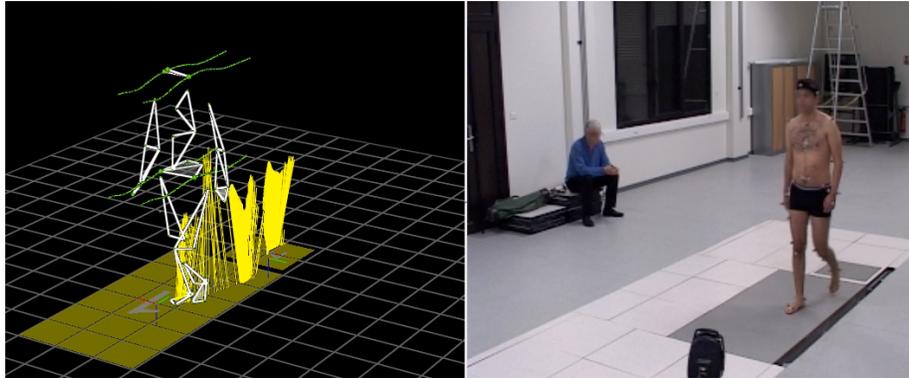


Figure 2: Capture of the experiment room during the acquisition session. A male subject was instructed to walk barefoot in straight line at his comfort walking speed on two force platforms. Two force plates are firmly embedded in the floor and allows the reconstruction of the segmentation of the walking pattern.

Experimental protocol

80 Participants were instructed to walk barefoot in straight line at their comfort walking speed. The walking distance was about 8 ± 1 m. At 4 m from the starting point, the subjects had to walk on the two consecutive force platforms. For each subject, 10 valid trials were recorded. A trial was considered valid as soon as the stance foot was completely located on the force plates, allowing the full
85 measurement of the external forces and wrenches.

Center of mass reconstruction

To fully reconstruct the center of mass position, we used a new method recently introduced by the authors and described in Carpentier et al. (2016). Improving CoM reconstruction methods currently used in Biomechanics Shimba
90 (1984a); Gard et al. (2004), this approach efficiently merges three different inputs: the external forces, external wrenches and the center of mass position computed from the marker positions and anthropomorphic tables Dumas et al. (2007). All those signals carry noises and errors, but with different frequency
95 bandwidths for each signal. Those measurements are then merged together according to their respective bandwidth accuracy thanks to a complementary

filtering approach. As output, we obtained an estimation of the center of mass position which is more accurate and consistent than previous measurement methods. This method offers the ability to both operate in online and off-line mode, thanks to optimization and averaging of forward and backward passes, resulting in a zero-delay output signal. We used this second mode to estimate the CoM trajectory.

1.1. The curtate cycloid

A cycloid is a curve corresponding to the path followed by a point c attached to the radius of a wheel rolling without slipping on a plane surface. This curve is defined in the sagittal plane by the parametric cartesian equation:

$$x = R\theta - r \sin(\theta) \quad (1)$$

$$z = z_0 - r \cos(\theta) \quad (2)$$

where θ is the angle between the radius of the wheel and the vertical direction, R is the radius of the wheel, r is the distance of the point c to the wheel center and z_0 is the altitude of the wheel center. x and z are the coordinates of c regarding the forward and vertical spatial axis. All in all, the cycloid is described by a set of 3 parameters denoted $\mathbf{p} = [R, r, z_0]$ and one variable θ which evolves according to time. The first time derivative of the trajectory θ corresponds to the angular velocity ω of the cycloid. We may distinguish three cases which are illustrated by the Fig. 3. In the following, we are interested by the curtate cycloid. This curve has the property of being cyclic and asymmetric, similarly to CoM trajectories for which the convex lower part of the cycle is longer than the concave higher part.

Segmentation of the gait

The use of two synchronized force plates enable us to efficiently and precisely segment the walking motion into single and double support phases.

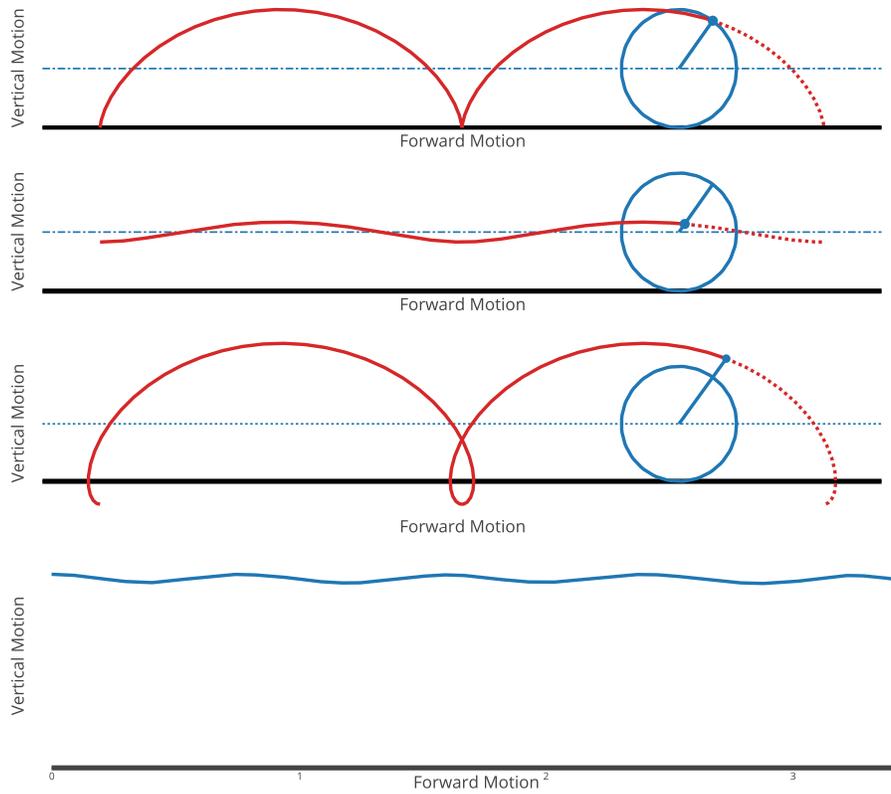


Figure 3: Illustration of the three types of cycloid. From top to bottom: normal cycloid, curtate cycloid and prolate cycloid. The last plot corresponds to the CoM trajectory in the sagittal plane. Its shape is very similar to the curtate cycloid.

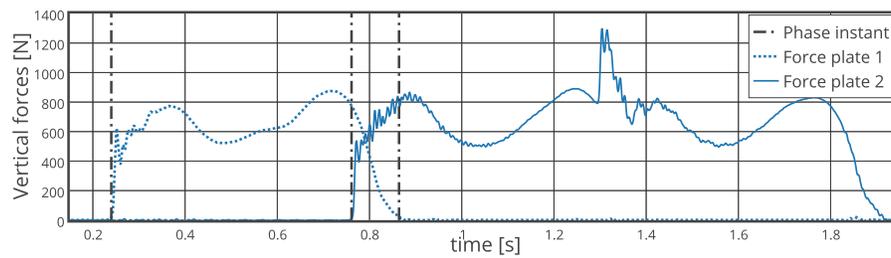


Figure 4: Illustration segmentation of the gait into single and double support phases. The two force plates allows an efficient detection of initial and final contact instant of the gait.

The fitting process is based on numerical optimization. It is set up as a nonlinear least-square problem, where we try to minimize the distance between the CoM trajectory (the measurement) and the cycloidal model composed of three parameters $\mathbf{p} = [R, r, z_0]$ and one variable θ which evolves according to time. The fitting problem is written as:

$$\min_{\mathbf{p}, \theta} \sum_{k=1}^N \|\mathbf{y}_{\text{mes}}^k - f_{\text{model}}(\theta^k, \mathbf{p})\|_2^2 \quad (3)$$

where f_{model} corresponds to the parametric models exposed in Eq. (2) and $\mathbf{y}_{\text{mes}}^k$ is the k^{th} sample measurement of the CoM trajectory in the sagittal plane.

This problem is efficiently solved with standard nonlinear least-square solvers. In this study we use the function *lsqnonlin* provided with MATLAB, The MathWorks, Inc., Natick, Massachusetts, United States.

Even though the natural walking is considered as a cyclic process, both the amplitude of the CoM trajectory and the cycle duration vary slightly even between two consecutive steps. To overcome those natural fluctuations, the data concerning each subject is made of a collection of ten single steps. The standard intra-subject deviation is represented in Fig. 5. Therefore, to overcome those natural fluctuations, we chose to operate the fitting procedure only for one step composed of a single support and double support phases. This choice allows to reach precisions finer than the intra-subject variability, and even to study this variability in terms of few parameters as we present later in this paper. In the following, we call this interval on which the optimization operates the fitting interval.

From the optimization result, we finally fit a polynomial of degree 1 to the trajectory θ according to time. The monomial of degree 1 finally corresponds to the angular velocity ω of the wheel. With this model, the equation θ is then given by:

$$\theta(t) = \theta_0 + \omega t \quad (4)$$

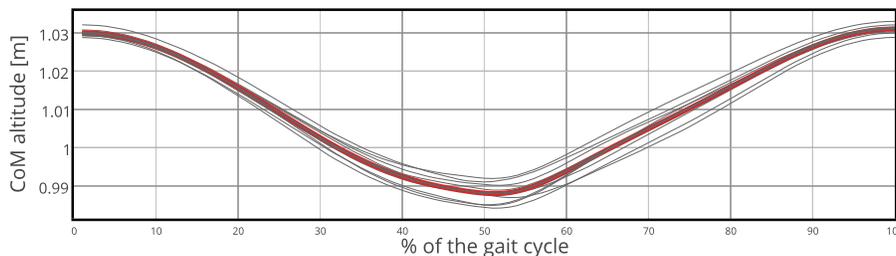


Figure 5: Illustration of the variability of the CoM on the vertical direction during one single step. The black curves represent the estimated trajectory of the CoM according to the percentage of the gait cycle and the red curve is the mean of all those trajectories.

Results

This section is organized as follows: we first present the results of the fitting process, we then expose the correlations between the fitting parameters and the height of the subjects. We conclude this section by showing the extraction of the temporal segmentation from the data of this model.

1.2. Fitting of the model

Hereinafter, we start by presenting the example of the fitting of one subject. Subsequently, we show statistical data about the quality of the reconstruction of all the subjects.

Fig. 6 shows both the result of the fitting for one fitting interval which corresponds to a full step composed of one single support and one double support. We see that the fitting is able to closely follow the trajectory of the CoM and to reproduce its asymmetry.

The error between the reconstructed CoM and the fitted trajectory is displayed in Fig. 7 on the forward and vertical motion axes. We can observe that the fitting error is lower than 1.5mm on the entire duration of the cycle.

This level of fitting quality does not vary a lot among subjects. Fig. 8 shows the mean and the standard deviation of the reconstruction error for all subjects. In general, the mean reconstruction is less 3.5mm with maximal

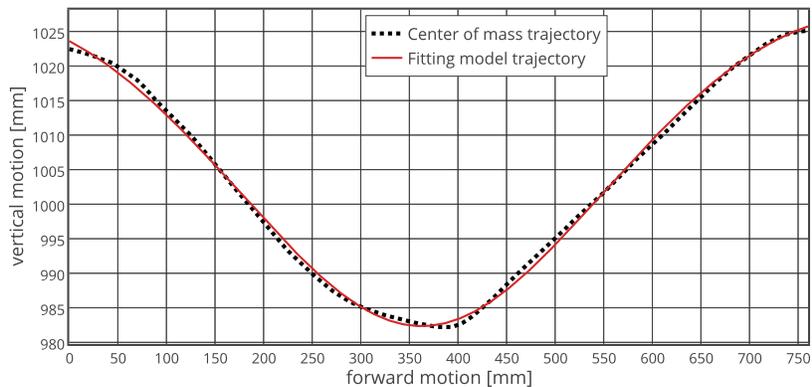


Figure 6: Center of mass trajectory fitted with the proposed model. The fitting closely follows the CoM trajectory and reproduces its asymmetry. Notice that, for a better reading, the axis coordinates are rescaled.

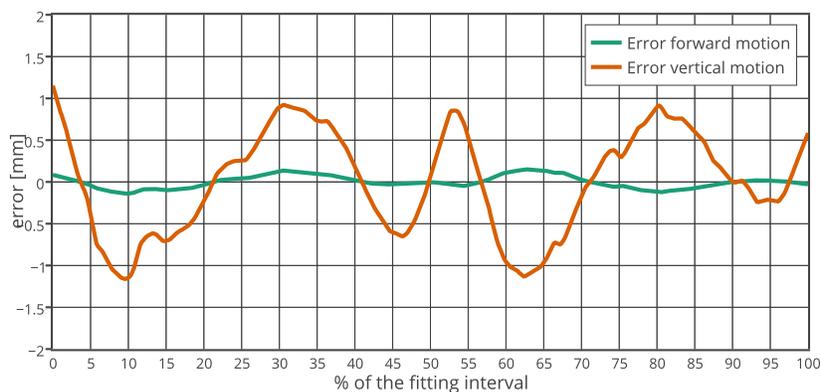


Figure 7: Reconstruction error between the CoM trajectory and its fitting with the proposed model. Over the while cycle, the reconstruction error remains below 1.5mm and it is mainly contained in the vertical direction.

standard deviations of 1mm. At this stage, it is worth to notice that for numerous subjects the mean error is less than 1mm with very low standard deviation, less than 0.5mm.

This fitting quality allows to study human walking trajectories in a reduced number of parameters. One example is presented hereafter where we study the correlations that lie between the trajectory parameters and the height of the

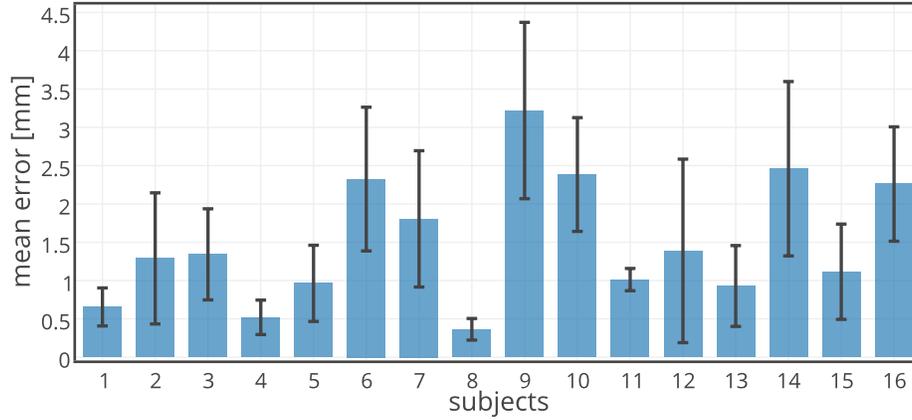


Figure 8: Mean and standard deviation of the reconstruction error for each subject. The mean reconstruction for all the subjects remains below 3.5mm with a maximal standard deviation of 1.5mm.

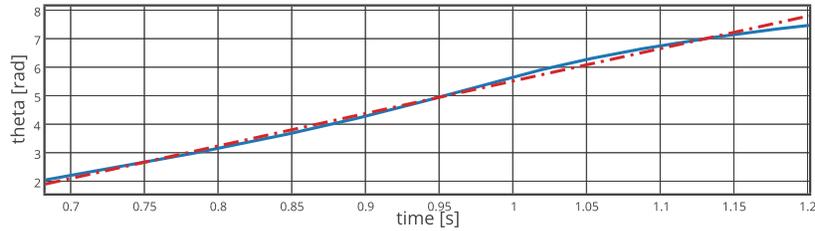


Figure 9: Evolution of θ according to time and the corresponding linear fitting. It follows that θ can be well approximated by an affine function of time represented by Eq. (4).

subjects.

1.3. Link between model parameters and the subject's height

From one step to another, the found fitting parameters are different due to the variability of the gait cycle. Nevertheless, the variation of these parameters is very small and may even be characteristic of each subject. But to properly support this claim, a higher number of subjects is necessary.

The following results study how these parameters correlate with the height of the subjects. Fig. 10 highlights the linear correlation between the radius parameters R and r of the cycloid and the height of the subjects. For both

correlations, the computed p-value is lower than 0.01 and the coefficient of correlations is 0.67 and 0.61 respectively.

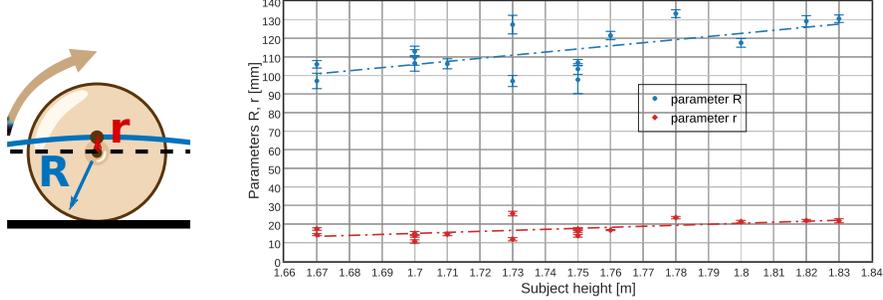


Figure 10: On the left, scheme of the wheel with the notations of the model: R is the radius of the wheel while r is the distance of the point to the wheel center. On the right, evolution of means of the the radius parameters R and r according the subjects' size. The standard deviation of the parameters is weak (below 5mm) for all the subjects. It appears that those two parameters are correlated to size of the subjects.

Fig 11 shows the evolution of the mean value of the parameter z_0 (the height of the wheel) according to the height of the subjects. Unsurprisingly, a strong correlation is observed with $p \leq 0.001$ and a correlation coefficient with value 0.87.

Finally, Fig 12 shows the mean and the standard deviation of the angular velocity of the cycloid according to the subject heights. It highlights the weak correlation ($p \geq 0.83$) between the angular velocity of the cycloidal and the size of the subjects.

1.4. The segmentation is embedded in the model

The gait cycle has a natural segmentation due to the transitions between single and double support phases, and the extraction of this segmentation is relatively easy for the case of our model. The curtate cycloid has a specific shape with one minimum and two crossing points with the horizontal axis during one period. If we look at the two time instants where the cycloid crosses the horizontal axis at level z_0 , they approximatively match the time of start and end of the double support phase respectively. this observation is assessed by Fig. 13

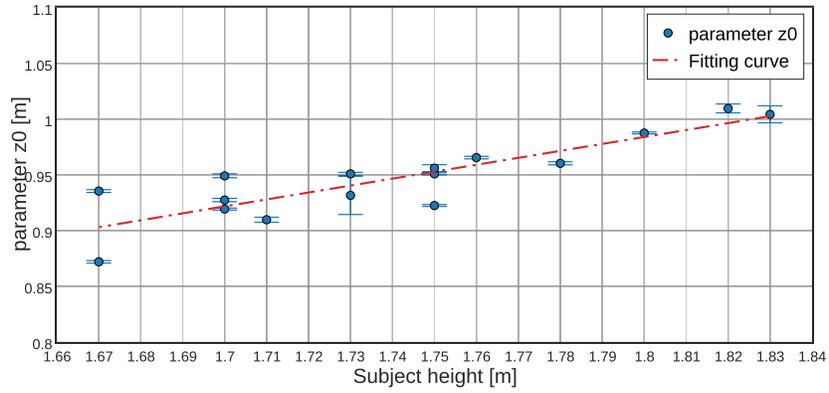


Figure 11: Evolution of the mean altitude z_0 according to the subject's size. The standard deviation of this parameters for each subject is very weak (below 2mm). Furthermore, the altitude is strongly correlated to the size of the subjects $p \leq 0.001$ with a correlation coefficient of 0.87.

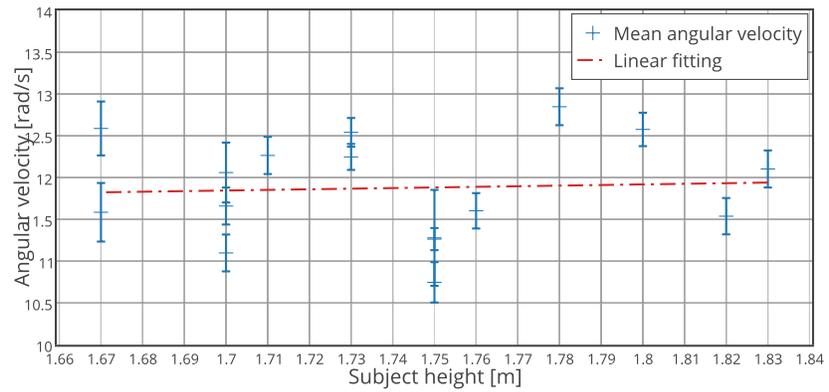


Figure 12: Variation of the mean angular velocity ω of the cycloid according to the subjects' size.

190 which shows the mean prediction error of the start and end time instants of the double support phases for each subjects. We can observe that in average the two instants are well captured by the model.

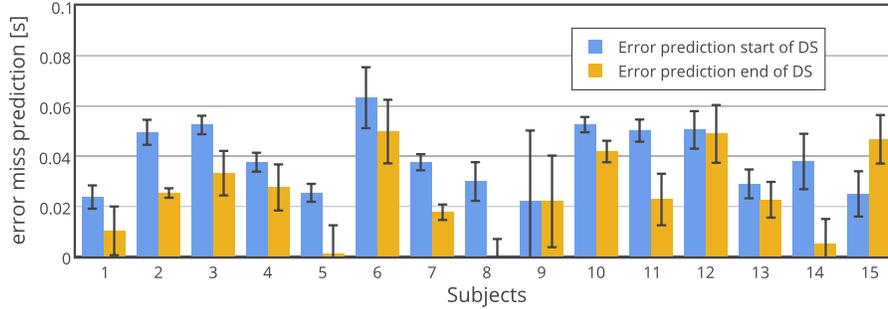


Figure 13: Bar graph of the prediction error of the time instants of start and end of the double support phases. In average, the two instants defining the double support are well captured by the model with only few milliseconds of errors.

Discussions

Our study shows that the center of mass of a walker follows the trajectory of a point attached to a virtual wheel moving on an horizontal plane at a constant
 195 velocity.

Accuracy of the model

The most important feature of our model is that the level of error in the vertical motion shown in Fig. 8 is one order of magnitude less than the existing models in the literature (Hayot et al., 2016; Zijlstra and Hof, 1997; Sakka et al.,
 200 2010), which overestimate the vertical position by up to 2cm.

In fact, our model fits the reconstructed trajectories with a higher precision than the accuracy of the measurement systems which is around 2mm with classic and accurate motion capture systems Winter (2009). In other words, it is not possible to go beyond this quality of fitting for this state-of-the-art
 205 measurement system. Moreover, even using force and moment sensors, the observability conditions of the center of mass are very weak along the direction of the contact forces Carpentier et al. (2016), which leads mostly to CoM height misestimation.

An intuitive model with few parameters

210 Only three parameters are necessary to describe all the CoM trajectory with our model. This is to be compared to the other models in the literature Hayot et al. (2016), where the models are composed of pendulum and inverted pendulum, but they necessarily need more parameters, for instance the location of the pendulum pivot points.

215 It is worth to emphasize that the few number of the parameters of our model keeps a simple intuitive geometrical interpretation, in contrast with possible purely numeric parametrization such as Fourier transforms or other approaches based on moments or frequency-domain representations Minetti et al. (2011).

A stable descriptor and reliable predictor

220 Our model allows to study both a single step and an average steady gait. Indeed, the parameters can be identified from one single step, and our experimental results show that these parameters have relatively low variability regarding the variability of the gait: every subject samples their parameters following a tight probability distribution which spans a space comparable to the variability of the gait itself. Therefore, this also turns our model into a reliable predictor for the 225 motion of the center of mass, given this distribution.

A segmentation-free model

To the best of our knowledge, our model is the first time-domain model which is free of segmentation. In other words, the curtate cycloid is a single curve. It 230 is not composed of various patterns defined on multiple intervals. The model does not require any distinction between single and double support phases. ²

More than that, the model contains the segmentation of the locomotion. Indeed, the minima of the cycloidal trajectory and its crossing with the wheel axis define together the beginning and the end of the double support.

²The rolling of an egg on a table has already been considered as model to explain the shape of the center of mass trajectory during walking Nigg et al. (2000). As the best of author's knowledge, no results have confirmed yet this hypothesis on a population of individuals.

235 This gives an interesting echo to the observation made by Sternad and
Schaal that "Segmentation of endpoint trajectories does not imply segmented
control" Sternad and Schaal (1999). This suggests that a possible continuous
control of the CoM position can be achieved using this kind of models.

A useful model to study walking gait

240 The 3-dimensional space defining the curtate cycloids allows to study walking
gait in a simple way. The correlations between these parameters and the height
of the subjects are easy to capture. It is then possible to make use of this
model can be used to study other features such as the effect of sensory-motor
impairments on walking motion generation.

245 Finally, one striking feature of our model is that it creates also naturally
another moving reference frame which is the center of the virtual wheel. In this
reference frame the CoM produces orbiting trajectories at a constant distance
from the origin, which hovers in the space at constant height and low-varying
velocity. The center of the wheel produces then smooth and regular locomotor
250 trajectories, easy to exploit to study broader properties of walking motion
generation.

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260 Conflict of interest

The authors have no financial or personal relationships with other people or
organizations that could have inappropriately influenced this research.

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