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Dark matter is compliant with the general relativity

Stéphane Le Corre

Abstract:

A recent publication demonstrates that statistically the gravitational potential g_{obs} deduced from the observed rotation curves of the galaxies and the gravitational potential g_{bar} deduced from the observed distribution of the baryonic mass is strongly correlated. The gravitational potential g_{bar} verifies the Poisson equation. This result implies that it is strongly likely that the observed baryonic mass must be sufficient to explain the observed rotation curves (i.e. without new exotic matter). Their study also gives accurate values for this gravitational potential and a curve that demonstrates the correlation between g_{obs} and g_{bar} . Here, we demonstrate that the equations of the general relativity allow explaining the dark matter in agreement with the results of this publication. In particular, one of their observations gives an empirical relation for weak accelerations. We are going to retrieve this relation in the frame of the general relativity. Furthermore, we will retrieve the observed value and the characteristics of this correlation's curve. These observations constrain drastically the possible gravitational potential in the frame of the general relativity to explain dark matter. In fact, the gravitational potential presented here and obtained from the relativity general is certainly the unique possible solution without modifying dynamic laws and without dark matter. This solution has already been studied with several unexpected predictions that have recently been verified.

Introduction

A recent publication [1] demonstrates:

- statistically the gravitational potential, g_{obs} , deduced from the observed rotation curves of the galaxies and the gravitational potential, g_{bar} , deduced from the observed distribution of the baryonic mass is strongly correlated,
- the fact that this potential g_{bar} is a solution of the Poisson equation,
- the value of the gravitational potential,
- the relation of the potential in the weak accelerations $g_{obs} \propto \sqrt{g_{bar}}$,
- the characteristics of the correlation's curve (deviation from the line of unity for values smaller than around $g_{bar} \sim 10^{-10}$).

This result implies that it is strongly likely that the observed baryonic mass must be sufficient to explain the observed rotation curves (i.e. without new exotic matter). Because the equations of the general relativity verify the Poisson equation, it is convenient to try to find a solution to the dark matter in the frame of the general relativity (i.e. without modifying the dynamics laws and without dark matter). We are going to see that it is possible. We are going to see that the empirical relation $g_{obs} \propto \sqrt{g_{bar}}$ can be explained in the frame of the general relativity. Furthermore, we will retrieve the observed value.

Our study will focus first on the equations of linearized general relativity, in the area of weak field (at the end of the galaxies) where the dark matter dominates.

Gravitation in linearized general relativity

From general relativity, one deduces the linearized general relativity in the approximation of a quasi-flat Minkowski space ($g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$; $|h^{\mu\nu}| \ll 1$). With the following Lorentz gauge, it gives the following field equations [3] (with $\square = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta$):

$$\partial_\mu \bar{h}^{\mu\nu} = 0 ; \quad \square \bar{h}^{\mu\nu} = -2 \frac{8\pi G}{c^4} T^{\mu\nu} \quad (I)$$

With:

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h ; \quad h \equiv h^\sigma_\sigma ; \quad h^\mu_\nu = \eta^{\mu\sigma} h_{\sigma\nu} ; \quad \bar{h} = -h \quad (II)$$

The general solution of these equations is:

$$\bar{h}^{\mu\nu}(ct, \vec{x}) = -\frac{4G}{c^4} \int \frac{T^{\mu\nu}(ct - |\vec{x} - \vec{y}|, \vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y}$$

In the approximation of a source with low speed, one has:

$$T^{00} = \rho c^2 ; \quad T^{0i} = \rho c u^i ; \quad T^{ij} = \rho u^i u^j$$

And for a stationary solution, one has:

$$\bar{h}^{\mu\nu}(\vec{x}) = -\frac{4G}{c^4} \int \frac{T^{\mu\nu}(\vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y}$$

At this step, by proximity with electromagnetism, one traditionally defines a scalar potential φ and a vector potential H^i . There are in the literature several definitions [4] for the vector potential H^i . In our study, we are going to define:

$$\bar{h}^{00} = \frac{4\varphi}{c^2} ; \quad \bar{h}^{0i} = \frac{4H^i}{c} ; \quad \bar{h}^{ij} = 0$$

With gravitational scalar potential φ and gravitational vector potential H^i :

$$\varphi(\vec{x}) \equiv -G \int \frac{\rho(\vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y}$$

$$H^i(\vec{x}) \equiv -\frac{G}{c^2} \int \frac{\rho(\vec{y})u^i(\vec{y})}{|\vec{x}-\vec{y}|} d^3\vec{y}$$

$$= -K^{-1} \int \frac{\rho(\vec{y})u^i(\vec{y})}{|\vec{x}-\vec{y}|} d^3\vec{y}$$

With K a new constant defined by:

$$GK = c^2$$

This definition gives $K^{-1} \sim 7.4 \times 10^{-28}$ very small compare to G .

The field equations (I) can be then written (Poisson equations):

$$\Delta\varphi = 4\pi G\rho ; \Delta H^i = \frac{4\pi G}{c^2} \rho u^i = 4\pi K^{-1} \rho u^i \quad (III)$$

With the following definitions of \vec{g} (gravity field) and \vec{k} (gravitic field), those relations can be obtained from following equations:

$$\vec{g} = -\overrightarrow{grad}\varphi ; \vec{k} = \overrightarrow{rot}\vec{H}$$

$$\overrightarrow{rot}\vec{g} = 0 ; \text{div}\vec{k} = 0 ;$$

$$\text{div}\vec{g} = -4\pi G\rho ; \overrightarrow{rot}\vec{k} = -4\pi K^{-1}\vec{p}$$

With relations (II), one has:

$$h^{00} = h^{11} = h^{22} = h^{33} = \frac{2\varphi}{c^2} ; h^{0i} = \frac{4H^i}{c} ; h^{ij} = 0$$

The equations of geodesics in the linear approximation give:

$$\frac{d^2x^i}{dt^2} \sim -\frac{1}{2}c^2\delta^{ij}\partial_j h_{00} - c\delta^{ik}(\partial_k h_{0j} - \partial_j h_{0k})v^j$$

It then leads to the movement equations:

$$\frac{d^2\vec{x}}{dt^2} \sim -\overrightarrow{grad}\varphi + 4\vec{v} \wedge (\overrightarrow{rot}\vec{H}) = \vec{g} + 4\vec{v} \wedge \vec{k}$$

This is this relation that we are going to use to explain the observed results of [1].

Remarks: Of course, one retrieves all these relations starting with the parameterized post-Newtonian formalism. From [5] one has:

$$g_{0i} = -\frac{1}{2}(4\gamma + 4 + \alpha_1)V_i ; V_i(\vec{x})$$

$$= \frac{G}{c^2} \int \frac{\rho(\vec{y})u_i(\vec{y})}{|\vec{x}-\vec{y}|} d^3\vec{y}$$

The gravitomagnetic field and its acceleration contribution are:

$$\vec{B}_g = \vec{\nabla} \wedge (g_{0i}\vec{e}^i) ; \vec{a}_g = \vec{v} \wedge \vec{B}_g$$

And in the case of general relativity (that is our case):

$$\gamma = 1 ; \alpha_1 = 0$$

It then gives:

$$g_{0i} = -4V_i ; \vec{B}_g = \vec{\nabla} \wedge (-4V_i\vec{e}^i)$$

And with our definition:

$$H_i = -\delta_{ij}H^j = \frac{G}{c^2} \int \frac{\rho(\vec{y})\delta_{ij}u^j(\vec{y})}{|\vec{x}-\vec{y}|} d^3\vec{y} = V_i(\vec{x})$$

One then has:

$$g_{0i} = -4H_i ; \vec{B}_g = \vec{\nabla} \wedge (-4H_i\vec{e}^i)$$

$$= \vec{\nabla} \wedge (4\delta_{ij}H^j\vec{e}^i) = 4\vec{\nabla} \wedge \vec{H}$$

$$\vec{B}_g = 4\overrightarrow{rot}\vec{H}$$

With the following definition of gravitic field:

$$\vec{k} = \frac{\vec{B}_g}{4}$$

One then retrieves our previous relations:

$$\vec{k} = \overrightarrow{rot}\vec{H} ; \vec{a}_g = \vec{v} \wedge \vec{B}_g = 4\vec{v} \wedge \vec{k}$$

The interest of our notation is that the field equations are strictly equivalent to Maxwell idealization. Only the movement equations are different with the factor "4". But of course, all the results of our study could be obtained in the traditional notation of gravitomagnetism with the relation $\vec{k} = \frac{\vec{B}_g}{4}$.

What the linearized general relativity predicts

The traditional computation of rotation speeds of galaxies consists in obtaining the force equilibrium from the three following components: the disk, the bulge and the halo of dark matter. More precisely, one has [6]:

$$\frac{v^2(r)}{r} = \left(\frac{\partial\varphi(r)}{\partial r}\right) \text{ with } \varphi = \varphi_{disk} + \varphi_{bulge} + \varphi_{halo}$$

Or:

$$\frac{v^2(r)}{r} = \left(\frac{\partial\varphi_{disk}(r)}{\partial r}\right) + \left(\frac{\partial\varphi_{bulge}(r)}{\partial r}\right) + \left(\frac{\partial\varphi_{halo}(r)}{\partial r}\right)$$

$$= \frac{v_{disk}^2(r)}{r} + \frac{v_{bulge}^2(r)}{r} + \frac{v_{halo}^2(r)}{r}$$

According to the linearized general relativity, the gravitational force is composed of the gravity fields (represented by $\frac{\partial\varphi_{disk}(r)}{\partial r}$ and $\frac{\partial\varphi_{bulge}(r)}{\partial r}$ in the previous equation) and by the gravitic field that we assume to be able to explain dark matter (represented by $\frac{\partial\varphi_{halo}(r)}{\partial r}$).

Consequently, here, $\frac{\partial\varphi_{halo}(r)}{\partial r}$ gathered the gravitic force of all the components (disk, bulge). This force due to the gravitic field \vec{k} takes the following form $\|\vec{F}_k\| = m_p 4 \|\vec{v} \wedge \vec{k}\| = m_p \frac{\partial\varphi_{halo}(r)}{\partial r}$. To simplify our computation, we idealize a situation where we have the approximation $\vec{v} \perp \vec{k}$. We can demonstrate that this perpendicularity is finally the more natural situation, meaning that this situation is very general [2].

This situation gives the following equation:

$$\frac{v^2(r)}{r} = \left(\frac{\partial\varphi_{disk}(r)}{\partial r}\right) + \left(\frac{\partial\varphi_{bulge}(r)}{\partial r}\right) + 4k(r)v(r)$$

$$= \frac{v_{disk}^2(r)}{r} + \frac{v_{bulge}^2(r)}{r} + 4k(r)v(r)$$

Far from the center of the galaxies, when the gravitational field becomes negligible, the contribution of $\left(\frac{\partial\varphi_{disk}(r)}{\partial r}\right) + \left(\frac{\partial\varphi_{bulge}(r)}{\partial r}\right)$ is negligible. It is the area where dark matter dominates. Far from the center of the galaxies, the equation becomes:

$$\frac{v^2(r)}{r} = \frac{v_{halo}^2(r)}{r} = 4k(r)v(r) \quad (IV)$$

What the observation reveals and implies in the frame of general relativity

If we get back to the results of [1], they give the following equation:

$$g_{obs} = \frac{v^2(r)}{r} = \frac{g_{bar}}{1 - e^{-\sqrt{g_{bar}/g_{\dagger}}}}$$

Let's see what happens where the dark matter dominates, i.e. at the end of the galaxies. This area is characterized by:

- very large value of $r \gg 15Mpc$,
- negligible gravity fields,
- low accelerations.

In this area, the results of [1] imply that we have:

$$g_{obs} \propto \sqrt{g_{bar}}$$

By definition, $g_{bar} = \frac{v_{bar}^2(r)}{r}$, it then gives for the low accelerations:

$$g_{obs} \propto \frac{v_{bar}(r)}{\sqrt{r}}$$

Furthermore, for very large value of $r \gg 15Mpc$, the curve \sqrt{r} is extremely flat. It evolves very slowly and can be considered as constant with a very good approximation. It means that in this area the empirical relation can be written:

$$g_{obs} \propto v_{bar}(r)$$

If we make the assumption that dark matter doesn't exist, at the ends of the galaxies the speed $v_{bar}(r)$ is in fact the speed of the ordinary mass $v(r)$. One has then:

$$g_{obs} \propto v(r) \quad (V)$$

The previous relation (IV) from the general relativity gives at the ends of the galaxies (where the effects of the bulge and disk fields are negligible):

$$g_{obs} = \frac{v^2(r)}{r} = 4k(r)v(r)$$

If we compare this relation with the empirical relation (V), the only way to conciliate them is to suppose that the gravitic field $k(r)$ at the ends of the galaxies is approximately constant. In other words, without dark matter and without modifying dynamic laws, to be in agreement with the observations, the general relativity implies the existence of an approximately uniform gravitic field $k(r) \sim k_0$.

By this way, the general relativity verifies and explains the observations of [1], as we are going to see it now.

What about this uniform gravitic field k_0

This solution has been studied in [2]. The uniform gravitic field k_0 would embed the galaxies and would come from the neighboring clusters of the galaxies. This solution gives the value $k_0 \sim 10^{-16.5}$. It leads to $g_{obs} \sim 10^{-10.5}$ (at the ends of the galaxies with $v \sim 250 km.s^{-1}$). This value is in agreement with the results of [1].

One can also note that, as demonstrated in [2], a uniform gravitic field k_0 verifies the Poisson equation. One can even note that the only way, in the frame of the general relativity, to modify the gravitational potential defined by the Poisson equation without modifying the distribution of the baryonic mass (constraints imposed by the results of [1]) is to add a uniform gravitic field. In fact, one can retrieve this situation in the accelerators of particles, with the electromagnetism (the linearized general relativity is similar to the Maxwell's idealization as seen at the beginning). The particles verify the Maxwell's equations (and then the Poisson equation) and follows circular trajectories with a uniform magnetic field. The source comes from structures of higher scale (the magnets).

The fact that this gravitic field is uniform also means that it cannot come from the galaxy (or it should decrease with the distance to the center). This result is in agreement with the simulations [7]. Indeed these simulations demonstrate that the own gravitic field of the galaxy is negligible and cannot explain the dark matter. In this theoretical frame, only an external field can play a role to explain the dark matter.

We are now going to see the astonishing accuracy of this solution. These results imply that we can write $k = k_1 + k_0$ [2] with k_1 the own gravitic field of the galaxy and k_0 a uniform external gravitic field. As indicated in [2], inside the galaxy, for $r < 15Mpc$, one has $k_1 \gg k_0$ and then $k \sim k_1$ and for $r > 15Mpc$, one has $k_1 \ll k_0$ and then $k \sim k_0$. It then implies that for $r < 15Mpc$ the own gravitic field of the galaxy acts, i.e. that g_{bar} depend on the baryonic mass of the galaxy and for $r > 15Mpc$ the own gravitic field is progressively replaced by an external gravitic field, i.e. that g_{bar} doesn't depend little by little on the own baryonic mass of the galaxy. It means that the solution of the general relativity implies a progressive change of nature of the correlation between g_{obs} and g_{bar} along the two areas. Concretely, for this latter area, [2] gives $10^{-16.62} < k_0 < 10^{-16.3}$ and for the interval of rotation's speeds (at the ends of the galaxies where k_0 dominates) $50km.s^{-1} < v < 300km.s^{-1}$. In term of $g_{bar} = 4k_0v$, it leads to the interval $10^{-11.32} < g_{bar} < 10^{-10.2}$. The solution [2] expects then a different behavior in the correlation of g_{obs} and g_{bar} in the interval $10^{-11.32} < g_{bar} < 10^{-10.2}$. The correlation's curve in [1] shows such a behavior. Indeed, near $g_{bar} \sim 10^{-10}$, the correlation's curve deviate from the line of unity. And one can focus on the remarkable agreement between the theoretical expectation and the experimental observations. The observed interval in [1] which deviates from the line of unity gives $10^{-11.7} <$

$g_{bar} < 10^{-10}$ (obtained from 153 galaxies). The theoretical interval is obtained from only 16 galaxies.

Conclusion

With the constraints of the observations of [1] and with the results of simulations [7] the general relativity can explain the dark matter but only by a uniform gravitic field, external to the own gravitic field. This solution has been studied in [2]. It shows that this uniform gravitic field should come from the neighboring clusters of galaxies. The agreement between the solution allowed by the general relativity (without exotic matter and without modification of the dynamic's laws) and the observation is total:

- The correlation between baryonic mass distribution and rotation's speed of the galaxies (because this solution doesn't need dark matter),
- the value of the gravitational potential,
- the fact that this potential is a solution of the Poisson equation (because it is a native solution of the general relativity),
- the relation of the potential in the weak accelerations $g_{obs} \propto \sqrt{g_{bar}}$ (demonstrated by the general relativity),
- the characteristics of the correlation's curve (deviation from the line of unity for values smaller than around $g_{bar} \sim 10^{-10}$).

Furthermore, this solution also implies several unexpected predictions that are recently verified:

- 1) The satellite dwarf galaxies are distributed according to plans
- 2) For nearby galaxies, the plans of satellite dwarf galaxies must have the same orientation
- 3) The plans must be aligned on the equatorial axis of the cluster they belong
- 4) The clusters of neighboring galaxies must have a strong tendency to align
- 5) A calculation of order of magnitude provides that these alignments can extend over distances of tens of Mpc at least
- 6) Statistically, the spin vectors of galaxies must be oriented in the same half-space (that of the cluster rotation vector)

The article [8] confirms the predictions 1, 2 and 3. The article [9] confirms the predictions 4, 5 and 6. One also could mention the observation (totally unlikely with the current assumptions) of 4 quasars perfectly aligned [10] or older papers [11] and [12] that reveal the alignment of quasars.

It also implies another predictions not yet verified, in particular, this solution implies a discrepancy in the measurement of the expected Earth's Lense-Thirring effect (for experiments such as "Gravity Probe B" or "GINGER") of a value between around 0.3 milliarcsecond/year and 0.6 milliarcsecond/year [13],

value inferior than the precision of the last experiments of measure of the Earth frame-dragging precession (39 mas/y) and of the geodetic effect (6606 mas/y). In this theoretical frame, these experiments would be the more direct way to measure the dark matter. One can also note that this solution can be adapted to explain the dark energy [14] and the result on the Hubble constant in [15] is in agreement with a prediction of this solution.

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