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A Low-Complexity Bit-Loading Algorithm for OFDM Systems Under Spectral Mask Constraint

Thanh Nhan Vo, Karine Amis, Thierry Chonavel, and Pierre Siohan

Abstract—This letter aims to solve the power minimization problem for interference-free OFDM systems under achievable throughput and spectral mask constraints. To this end, we use a general analytic formula to yield an efficient initial bit vector, i.e., given a total number of bits, the total power use corresponding to an efficient bit vector is minimum. We propose a low-complexity algorithm that achieves the global optimum of the problem. We theoretically prove that the number of bits required to add/remove, starting from the efficient initial bit vector, is upper bounded by the sum of half the number of active subcarriers and an arbitrary tolerance value. Simulation results show that the total power consumption is minimized with significant reduction of the computation cost as compared to other optimum algorithms.

Index Terms—Bit-loading, greedy Algorithm, Low-complexity algorithm, Margin-adaptive, OFDM, Spectral mask constraint.

I. INTRODUCTION

IN OFDM systems, with channel state information available at the transmitter, a loading algorithm can be used to allocate power and bits to the subcarriers under given constraints. It can be designed to achieve different objectives in OFDM systems, such as rate maximization, power minimization, error rate minimization, etc. Different algorithms have been proposed among which we can enumerate: adaptive-rate algorithms which maximize the capacity under the power and bit-error rate constraints (BER) [1]–[6]; margin-adaptive algorithms which minimize the consumed power under data rate and BER constraints [5], [7]–[11]. In wired communication systems such as ADSL and PLC, a spectral mask constraint must be taken into account to ensure the compatibility with other radio systems [12], [13]. Major contributions related to power use minimization under achievable rate and peak-power constraints were done by Baccarelli *et al.* in [4], by Mahmood and Jaffrot in [11] and by Papandreou *et al.* in [14].

In [15], we addressed the problem of rate-maximization under total power and spectral mask constraints. We proposed the water-filling rounding greedy-based bit-loading (WFR-GBL) algorithm and we introduced a general analytic formula to define an efficient initial bit vector depending on a given value S obtained thanks to a modified secant-based algorithm.

In this letter, we aim to solve the dual power consumption minimization problem under fixed throughput constraint. We modify the WFR-GBL by introducing a novel “water-level”

function and a tolerance value for the search of S , i.e. the equivalent water-level in the Water-filling algorithm, and by substituting the bi-section algorithm for the modified secant-based algorithm. The resulting algorithm provides the optimum solution with significantly reduced computation cost as compared to four algorithms of the literature among which two highly differ from the ones used as reference in [15].

II. POWER CONSUMPTION MINIMIZATION PROBLEM AND EXISTING SOLUTIONS

A. Power Consumption Minimization Problem

The power consumption minimization problem is given as follows:

$$\begin{aligned} & \text{minimize } \sum_n P_n \\ & \text{s.t. } P_n = \frac{(2^{b_n} - 1)\Gamma}{g_n} \leq P_n^{\max} \\ & b_n \leq A_{\max} | b_n \in \mathbb{N} \\ & \sum_n b_n = R_{\text{target}}, R_{\text{target}} \in \mathbb{N}^+ \end{aligned} \quad (1)$$

where b_n and P_n are the number of bits and the power allocated to subcarrier n , A_{\max} is the maximum number of bits defined by the maximum order constellation, g_n is the channel gain to noise ratio, R_{target} denotes the targeted total number of bits, $\Gamma \geq 1$ is the “SNR gap” that effectively estimates the gap (in terms of signal to noise ratio) between subcarrier capacity and conveyable actual rate (bits/symbol). It depends on the desired targeted error probability P_E , coding gain γ_C and required margin γ_M [16], [17]: $\Gamma = \frac{1}{3} \left[Q^{-1}\left(\frac{P_E}{4K}\right) \right]^2 \frac{\gamma_M}{\gamma_C}$, where $Q^{-1}(x)$ is the inverse tail probability of the standard normal distribution and K is an edge-effect correction factor fast approaching unity for medium and large-size QAM constellations. This SNR gap has been used in many bit loading algorithms to calculate the number of bits allocated on a subcarrier [4], [6], [7], [10], [11]. Interested readers can find more details about this “SNR gap” in [16], [18].

B. Existing Solutions

Before introducing the four selected algorithms of the state-of-art, we recall some important notations from [15]:

$$\begin{aligned} b_{\max}(n) &= \left\lfloor \log_2 \left(1 + \frac{g_n P_n^{\max}}{\Gamma} \right) \right\rfloor, \\ b_{\max}^r(n) &= \min(A_{\max}, b_{\max}(n)) \end{aligned} \quad (2)$$

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where $\lfloor \cdot \rfloor$ is the floor function, $b_{\max}(n)$ is the maximal number of bits limited by the peak-power constraint on subcarrier n and $b_{\max}^r(n)$ is the maximal effective number of bits on subcarrier n .

1) *Greedy Bit-Adding (Z-GBA) [4]*: the bit vector is initialized to the null vector. At every iteration, a subcarrier with minimum required incremental power is allocated an additional bit if the power constraints remain satisfied. The procedure is stopped when the constraint of achievable throughput is fulfilled.

2) *Greedy Bit-Removing (M-GBR) [14]*: the initial number of bits allocated on all subcarriers is set to their maximal allowable number of bits, i.e., $b_{\max}^r(n)$. Then, at every iteration, one bit is removed from a given subcarrier if its power gain is the maximum one. The iterative procedure is stopped when the constraint of achievable throughput is fulfilled.

3) *PA Algorithm*: this algorithm was proposed by Papandreou and Antonakopoulos in [7]. It consists in three phases. First, an initial loop representation bit-allocation profile is proposed. Second, they use multiple-bits insertion or removal procedure to find an intermediate bit allocation. Finally, a greedy bit-adding or bit-removing algorithm is used for the non-allocated bits, starting from the intermediate bit-allocation to obtain the required throughput.

4) *3-dB Subgroup Algorithm (3-dB Algorithm) [11]*: this algorithm was proposed by Mahmood and Jaffrot in [11]. It relies on a classification of the subcarriers in subgroups of 3 dB with respect to their bit-increment powers. Then, a subgroup-based initial bit-allocation conforming to the peak-power constraints is analytically given. Finally, a greedy bit-adding is applied till the targeted throughput is achieved.

C. Proposed Algorithm to Solve the Power Minimization Problem

1) *Theoretical Results*: let $x, S \in \mathbb{R}^+$ and let us define

$$\text{round}(x) = n \Leftrightarrow -\frac{1}{2} \leq x - n < \frac{1}{2}, n \in \mathbb{N}. \quad (3)$$

As proved in [15], (Theorem 2), the bit vector $\mathbf{b}^{WFR}(S)$ with n -th entry

$$b_n^{WFR}(S) = \text{round} \left(\left[S + \log_2(g_n) \right]_0^{b_{\max}^r(n)} \right) \quad (4)$$

where

$$[x]_0^p = \begin{cases} p & x \geq p \\ x & 0 < x < p \\ 0 & x \leq 0 \end{cases} \quad (5)$$

is an efficient bit vector. That is, its corresponding total power use is minimum (see also Theorems 3 and 4 in [15]).

If $\exists S^* \in \mathbb{R}^+$ so that

$$f(S^*) = \sum_n b_n^{WFR}(S^*) - R_{\text{target}} = 0. \quad (6)$$

then $P_n(S^*) = \frac{(2^{b_n^{WFR}(S^*)} - 1)\Gamma}{g_n}$ is the solution of (1).

Pseudocode of the modified WFR-GBL algorithm for power minimization problem

```

1: Calculate  $b_{\max}(n), b_{\max}^r(n), \forall n$ .
2: if  $R_{\text{target}} \geq \sum_n b_{\max}^r(n)$  then
3:    $b_n = b_{\max}^r(n)$ 
4: else
5:   Solve  $|h(S)| \leq \Delta$  to find  $S$  by using bi-section algorithm
   for a given  $\Delta > 0$ .
6:   Calculate  $b_n^{WFR}(S)$ .
7:   if  $\sum_n b_n^{WFR}(S) < R_{\text{target}}$  then
8:     Use greedy-based bit-adding to add the number of bits
     on the subcarriers that have not reached yet their
     effective maximal number of bits  $b_{\max}^r(n)$ .
9:   else if  $\sum_n b_n^{WFR}(S) > R_{\text{target}}$  then
10:    Use greedy-based bit-removing to remove the number
    of bits on the subcarriers that have not reached 0 yet.
11:  else
12:     $b_n = b_n^{WFR}(S)$ 
13:  end if
14: end if

```

Pseudocode of the bi-section algorithm to find S

```

1: Set  $\Delta \in \mathbb{N}^+$  to a desired tolerance value.
2: Set  $S_0 = -\log_2(\max_n g_n)$ ,  $S_1 = \log_2 \left( \max_n \left\{ \frac{P_{\max}^r(n)}{\Gamma} + \frac{1}{g_n} \right\} \right)$ .
3: Set  $h_0 = h(S_0) = -R_{\text{target}} < 0$ ,  $h_1 = h(S_1) = \sum_n b_{\max}^r(n) - R_{\text{target}} > 0$ .
4: while  $|h| > \Delta$  do
5:    $S = \frac{h_1 S_1 - h_0 S_0}{h_1 - h_0}$ 
6:    $h = \sum_n c_n^{WFR}(S) - R_{\text{target}}$ .
7:   if  $h > 0$ 
8:      $S_1 = S, h_1 = h$ 
9:   else
10:     $S_0 = S, h_0 = h$ 
11:   end if
12: end while

```

For the rate-maximization in [15], to define the efficient initial bit vector, we solve the equation $\sum_n \left[S - \frac{\Gamma}{g_n} \right]_0^{P_{\max}^r(n)} - P_{\text{tot}} = 0$ that has at least one root. For the power consumption minimization problem, we cannot ensure that (6) has at least one root since $f(S)$ is not continuous (e.g. no solution when g_n is constant and modulo $(R_{\text{target}}, L) \neq 0$), where L denotes the number of active subcarriers. In the following, we substitute $f(S)$ by a novel “water-level” function $h(S)$ and we modify the WFR-GBL algorithm of [15] to solve (1) in all cases.

2) *Implementation*: Let us define a novel “water-level” function h by

$$h(S) = \sum_n c_n^{WFR}(S) - R_{\text{target}} \quad (7)$$

where $c_n^{WFR}(S) = \left[S + \log_2(g_n) \right]_0^{b_{\max}^r(n)}$. We now introduce a desired tolerance value, denoted by Δ , to search for S such that $|h(S)| \leq \Delta$ thanks to the bi-section algorithm whose pseudocode is given above. As in [15], we use a combination of

TABLE I
COMPARISON IN TERMS OF NUMBER OF OPERATIONS

| | |
|------------------|---|
| Z-GBA | $5L + (L + 3)R_{target}$ |
| M-GBR | $5L + (L + 3)(\sum_n b_{max}^r(n) - R_{target})$ |
| 3-dB | $10L + (L + 3)(R_{target} - R_{init}^{DB})$ |
| PA | $7L + 5\ell^{PA}L + (L + 3)(R_{target} - R_{init}^{PA})$ |
| Modified WFR-GBL | $10L + 4\ell^{WR}L + (L + 3)(R_{target} - R_{init}^{WR})$ |

greedy-based bit adding and removing to solve (1) and refer the resulting algorithm to as modified WFR-GBL.

In the pseudocode of the bi-section algorithm, since h is a continuous, incremental function and $h(S_0) < 0$ and $h(S_1) > 0$, it always yields one value of S whenever $0 \leq \Delta \leq h(S_1)$. The number of iterations to find S in the bi-section algorithm are all the smaller as Δ is large. However, according to the upper-bound computed at Eq. (10), a small value of Δ limits the number of iterations of the greedy-based bit-adding or bit-removing algorithm. The value of Δ should result from a trade-off between the run-time of the bi-section and of the greedy-based algorithms. In this letter, we choose $\Delta = 20$ that is neither too big nor too small.

The global optimality of the modified WFR-GBL lies on the efficiency of the bit vector at any step of the algorithm. Indeed, the power use corresponding to the initial bit vector is minimum and each step of the greedy-based bit-adding (respectively bit-removing) algorithm chooses the subcarrier that induces the least power increase (respectively highest power reduction) [15]. Thanks to the mutual independence between the power variation on the subcarriers, this leads to the global optimum solution of (1).

Theorem 1: To obtain the global optimum bit allocation, starting from bit vector \mathbf{b}^{WFR} , the number of iterations used in greedy-based bit-adding or bit-removing are upper bounded by $\Delta + L/2$.

Proof: See Appendix. ■

D. Complexity Analysis

In this section, we compare the algorithms in terms of computation cost. The number of iterations for the Z-GBA and M-GBR algorithms are given in [15]. In Table I, we denote by R_{init}^{PA} , R_{init}^{DB} and R_{init}^{WR} the total number of bits obtained in the initialization step of the PA, 3-dB algorithms and modified WFR-GBL algorithm; by ℓ^{PA} the number of iterations to find the intermediate bit allocation (i.e., to find α in Algorithm 4, page 11 of [14]) and by ℓ^{WR} the number of iterations used to find S in the bi-section step of the proposed algorithm. The number of operations (i.e., multiplications, additions and comparisons) for each algorithm is given in Table I.

III. SIMULATION RESULTS

The simulation parameters are the following:

- Multi-path PLC channels for different classes [19] and background Gaussian noise [20].
- Number of active subcarriers $L = 917$.
- The maximum number of bits allowable on the subcarriers, $A_{max} = 12$.

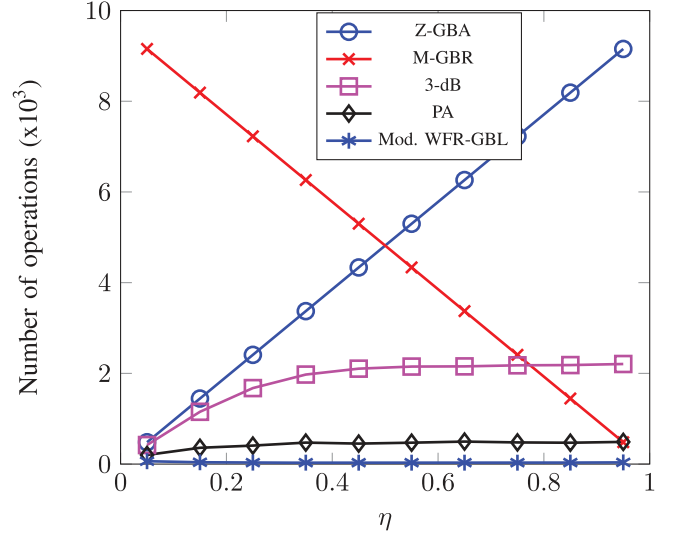


Fig. 1. Number of operations per subcarrier for different algorithms.

TABLE II
NUMBER OF OPERATIONS PER SUBCARRIER, RUN-TIME AND
 $R_{target} - R_{init}$ AVERAGED FOR DIFFERENT η

| | Number of operations | Run-time (ms) | $R_{target} - R_{init}$ (bits) |
|------------------|----------------------|---------------|--------------------------------|
| Z-GBA | 4817 | 48.4 | 4817 |
| M-GBR | 4818 | 48.6 | 4818 |
| 3-dB | 1820 | 23.2 | 1810 |
| PA | 429 | 5.3 | 413 |
| Modified WFR-GBL | 40 | 0.72 | 12 |

- $P_n^{max} = 1, \forall n \in \{1, 2, \dots, L\}$ (normalized to $P_0 \Delta f$ where $P_0 = -55$ dBm (1 Hz) is the spectral mask value defined by the IEEE P1901 standard and Δf is the subcarrier spacing between two consecutive subcarriers).
- $R_{target} = \lfloor \eta \sum_n b_{max}^r(n) \rfloor$, η varies from 0.05 to 0.95.
- $\Gamma = 7$, for a target symbol error rate of 10^{-5} [21].
- $\Delta = 20$.
- Number of channel realizations: 1000.

In all cases, we checked that all algorithms achieve the same bit and power allocations. This supports the optimality of the proposed algorithm to solve (1). Fig. (1) illustrates the complexity. The modified WFR-GBL significantly reduces the complexity as compared to the others. The number of operations as well as the run-time averaged for different values of η are given in Table II. The proposed algorithm outperforms the other two computational efficient PA and 3-dB algorithms. The average value of $R_{target} - R_{init}$ is also given in Table II. We observe a smaller value of $R_{target} - R_{init}$ for the modified WFR-GBL as compared to PA and 3-dB. In other words, the initial bit allocation in the modified WFR-GBL is much closer to the optimal bit allocation. This explains why the number of iterations and thus the complexity are lower.

IV. CONCLUSION

In this letter, we proposed a novel low-complexity and optimal algorithm to solve the power minimization problem under throughput and spectral mask constraints in OFDM systems.

We modified the WFR-GBL algorithm proposed in [15] to solve the dual throughput maximization problem under power constraints. We introduced a novel “water-level” function and a tolerance value Δ to obtain the value of S , i.e. the equivalent water-level in the Water-Filling algorithm, that defines an efficient initial bit vector of the greedy-based procedure step. We also proved that the number of bits required to add/remove to obtain the optimal bit allocation is upper-bounded by the sum of Δ and half the number of subcarriers. The advantage in terms of computation cost was theoretically analyzed. Simulations carried out in the context of PLC systems showed the efficiency of the proposed algorithm in terms of complexity and run-time with different configurations, such as different PLC channel classes and different achievable rate constraints. In all cases, the proposed algorithm significantly outperforms the reference algorithms. As a conclusion, combining [15] and this letter, it appears that the analytic formula used to define an efficient initial bit vector enables to obtain two versions of the WFR-GBL algorithm that yield the global optimum of the rate-maximization and the power-minimization problems in OFDM systems.

APPENDIX

As soon as $|R_{\text{target}} - \sum_{n=1}^L c_n^{WF}(S)| \leq \Delta$, the bi-section algorithm ends.

As $b_n^{WFR}(S) = \text{round}(c_n^{WF}(S))$, by definition, we get $|c_n^{WF}(S) - b_n^{WFR}(S)| \leq \frac{1}{2}$.

We thus deduce that

$$|R_{\text{target}} - \sum_{n=1}^L b_n^{WFR}(S)| \leq |R_{\text{target}} - \sum_{n=1}^L c_n^{WF}(S)| + |\sum_{n=1}^L c_n^{WF}(S) - \sum_{n=1}^L b_n^{WFR}(S)| \quad (8)$$

$$\leq \Delta + \sum_{n=1}^L |c_n^{WF}(S) - b_n^{WFR}(S)| \quad (9)$$

$$\leq \Delta + \frac{L}{2} \quad (10)$$

We have proved that $|R_{\text{target}} - \sum_{n=1}^L b_n^{WFR}(S)|$ is upper bounded by $\Delta + \frac{L}{2}$.

Remark: In practice, the value of $|\sum_{n=1}^L c_n^{WF}(S) - \sum_{n=1}^L b_n^{WFR}(S)|$ in Eq. (8), is very small as compared to $L/2$. On average, half the subcarriers satisfies $c_n^{WF}(S) - \text{round}(c_n^{WF}(S)) \leq 0$ and the other half satisfies $c_n^{WF}(S) - \text{round}(c_n^{WF}(S)) \geq 0$. Then

$$\sum_{n=1}^L (c_n^{WF}(S) - b_n^{WFR}(S)) \approx 0 \quad (11)$$

In simulations, we observed that $|\sum_{n=1}^L c_n^{WF}(S) - \sum_{n=1}^L b_n^{WFR}(S)| \ll L/2$.

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