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Harmonic Magnification by Time Reversal based on a Hammerstein Decomposition

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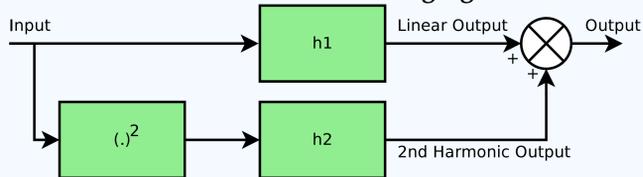


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Introduction

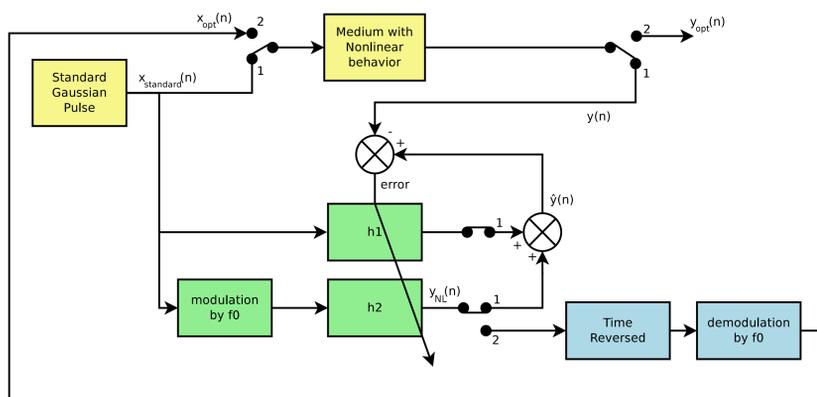
The medical ultrasound imaging systems have been improved by taking into account the nonlinear wave propagation. These improvements leads to increase the signal-to-noise ratio and the focusing in tissue harmonic imaging (THI). Since the backscattered nonlinearities are a function of the transmitted signal, enhancing these nonlinearities means to design the best wave [1]. Time reversal process is well-known to optimize the SNR and free itself to phase aberration, by combining a waveform design in space and in time thanks a physical matched filter [2]. However, it is not well-adjusted for THI, because the propagation of the time reversed signal destroys the harmonics.

How can we optimize the SNR in tissue harmonic imaging ?



As we want to guarantee a SNR optimization with a good focusing, the method has to include time reversal. The solution firstly consists in extracting the harmonic component at $2f_0$ by a Hammerstein filter. Then this time reversed harmonic component at $2f_0$ is frequency shifted to the fundamental component at f_0 and retropropagated in the medium.

Methods



1. Sending a first standard excitation $x_{standard}(n)$
2. Harmonic extraction based on a Hammerstein model where the nonlinear function is a frequency shifting by modulation (since it is a bijective function on \mathbb{R}):

$$\hat{y}(n) = \sum_{p=1}^2 \sum_{m=1}^M h_p(m) x_{standard}(n-m) \cdot C_p(n), \quad (1)$$

n the discrete time, M the memory of the Hammerstein model and $C_p(n) = \cos\left(2\pi\frac{(p-1)f_0 n}{F_s}\right)$ with F_s the sample frequency.

The model can be solved by a pseudo-inversion:

$$\mathbf{h} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}. \quad (2)$$

$\mathbf{h} = [h_1(1), \dots, h_1(M), h_2(1), \dots, h_2(M)]$, $\hat{\mathbf{y}} = [y(M+1), \dots, y(N)]^T$ with N the sample quantity, the matrix of input signals $\mathbf{X} = [\mathbf{x}_1 \mathbf{x}_2]$ with

$$\mathbf{x}_p = \begin{pmatrix} v_p(M) & v_p(M+1) & \dots & v_p(N) \\ v_p(M+1) & v_p(M+2) & & 0 \\ \vdots & & \ddots & \\ v_p(N) & 0 & & 0 \end{pmatrix}, \quad (3)$$

and $v_p(n) = x(n)C_p(n)$. Finally, the second harmonic signal is: $\mathbf{y}_{NL} = \mathbf{x}_2^T [h_2(1), \dots, h_2(M)]^T$.

3. Time reversing \mathbf{y}_{NL}
4. Annihilation of the second harmonic effects of the second harmonics:

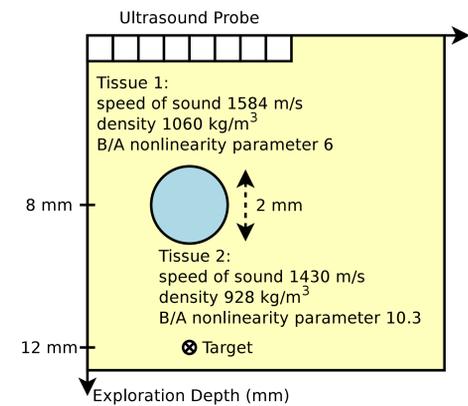
$$x_{opt}(n) = A \cdot y_{NL}(n) \cdot \cos\left(2\pi f_0 \frac{n}{F_s} + \phi\right), \quad (4)$$

A preserve the transmit power to $x_{standard}(n)$ and $\phi = \arctan\left(\frac{\phi(\mathbf{y}_{NL})}{\phi(\cos)}\right)$, with $\phi(\mathbf{y}_{NL})$ the phase of the the second harmonic signal \mathbf{y}_{NL} and the $\phi(\cos)$ the phase of a cosinus at the frequency f_0 .

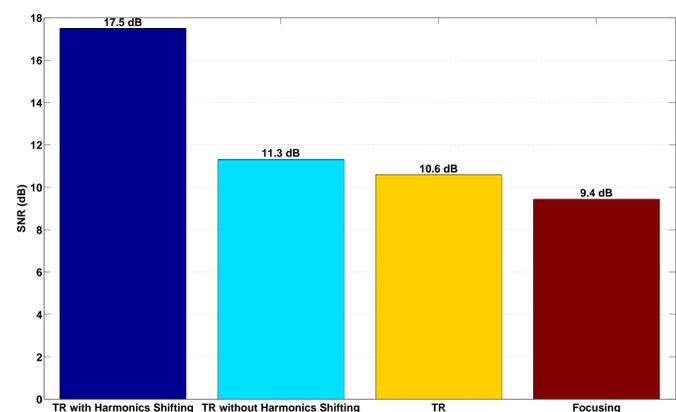
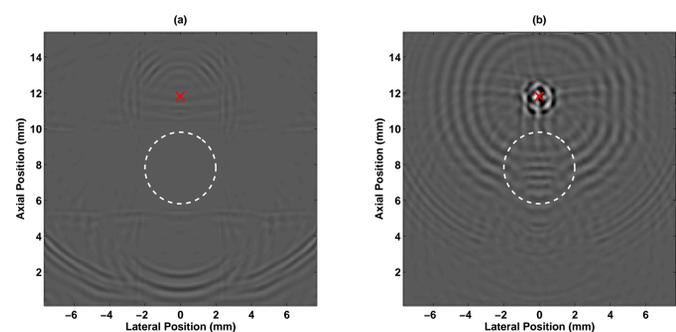
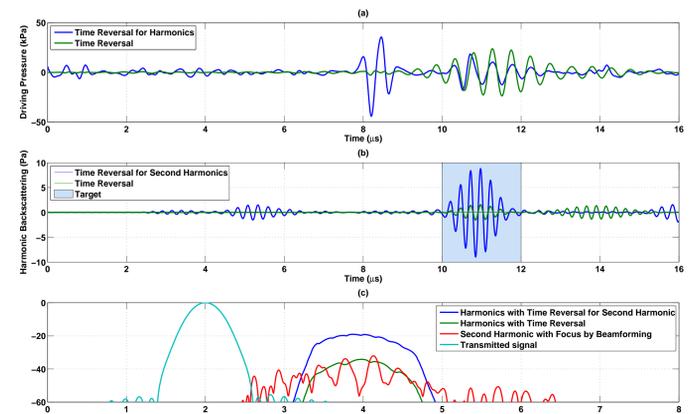


Simulation

- 2D nonlinear wave propagation in a cavity [3]
- 8-element probe centred at $f_c = 4$ MHz
- Initial Gaussian pulse $x_{standard}(n)$ centred at $f_0 = 22$ MHz and with a bandwidth of 50%



Results



Discussions and Conclusion

- Magnify the second harmonics and improve the SNR
- Hammerstein model using frequency shifting with the good phase
→ Annihilation of the harmonic effects before sending the time-reversed signal
- Matched filter for harmonic components
→ Extension of the time reversal principle to second harmonics

[1] Ménigot S. & Girault, J.-M. Optimization of Contrast Resolution by Genetic Algorithm in Ultrasound Tissue Harmonic Imaging. Ultrasonics. 2016.
[2] Fink, M. Time-reversal of Ultrasonic Fields. 1. Basic Principles. IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control. 1992 (39).
[3] Anderson, M. E. A 2D Nonlinear Wave Propagation Solver Written in Open-Source MATLAB Code. Proceeding IEEE Ultrasonic Symposium. 2000.