

# SNR Improvement by Subharmonic Extraction with Hammerstein Models for Microbubble Signals

Sébastien Ménigot, Emma Kanbar, Ayache Bouakaz and Jean-Marc Girault

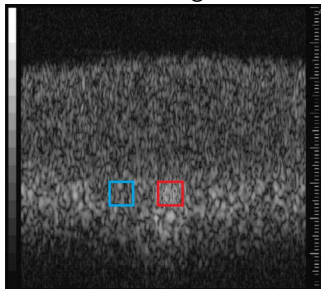
Université François-Rabelais de Tours, Inserm, Imaging and Brain UMR U930, Tours, France

September 21, 2016

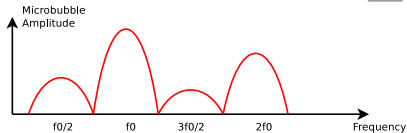
# Introduction

# Ultrasound Contrast Imaging

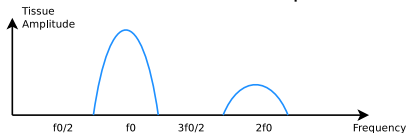
Harmonic Image at  $2f_0$



[Goertz et al., 2005]



Scheme of microbubble spectrum



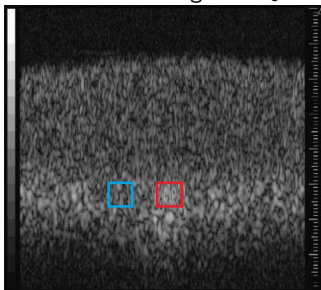
Scheme of tissue spectrum

## Ultrasound contrast imaging

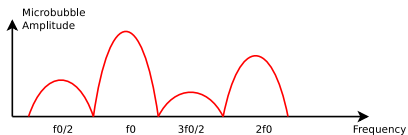
- Contrast agent: microbubble
- ↳ High nonlinear behaviour
- $CTR = \frac{E_{\text{microbubbles}}}{E_{\text{tissue}}}$

# Ultrasound Contrast Imaging

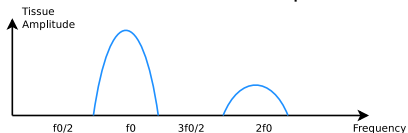
Harmonic Image at  $2f_0$



[Goertz et al., 2005]



Scheme of microbubble spectrum



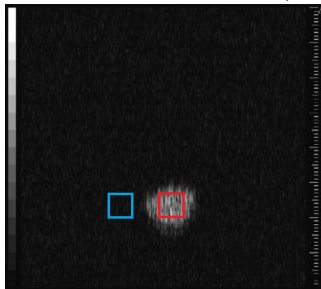
Scheme of tissue spectrum

## Subharmonic imaging

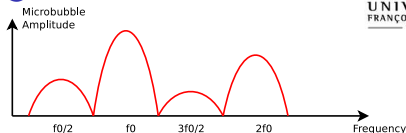
Increase the CTR by using subharmonics  
[Forsberg et al., 2000]

# Ultrasound Contrast Imaging

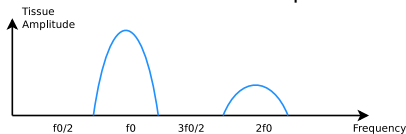
Subharmonic Image at  $f_0/2$



[Goertz et al., 2005]



Scheme of microbubble spectrum



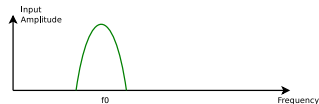
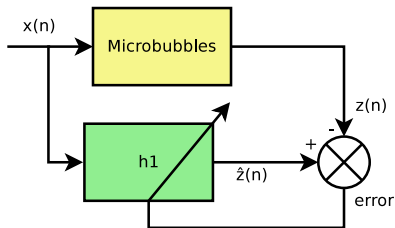
Scheme of tissue spectrum

## How to extract subharmonics ?

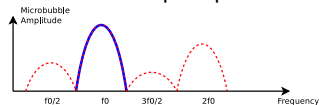
- ↪ How to avoid the filter setting (cutoff frequencies...) ?
- ⇒ **Optimal filtering**
  - Goal: Experimental validation [Sbeity, 2013]

# Hammerstein Decomposition: Optimal filtering

# Optimal Wiener Filtering



Scheme of input spectrum



Scheme of output spectrum

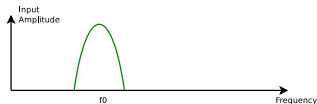
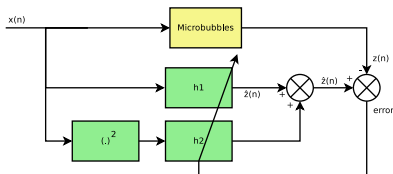
## Linear Model

- $\hat{z}(n) = \sum_{m=1}^M h_p(m)x(n-m)$
- Linear optimal filtering

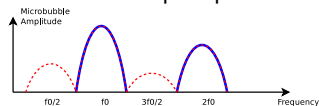
## Optimal Extraction of Components

- ☺ Fundamental modelling
- ☹ Harmonic modelling
- ☹ Subharmonic modelling

# Optimal Harmonic Modelling



Scheme of input spectrum



Scheme of output spectrum

## Hammerstein Model

$$\hat{z}(n) = \sum_{p=1}^P \sum_{m=1}^M h_p(m) x^p(n-m)$$

Polynomial model

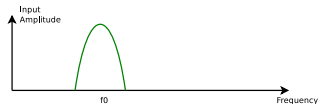
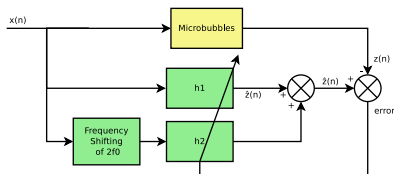
Frequency Shifting  $\rightarrow$  Modulation

## Optimal Extraction of Components

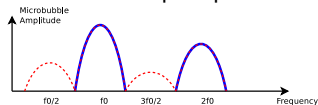
- 😊 Fundamental modelling
- 😊 Harmonic modelling
- 😞 Subharmonic modelling



# Optimal Harmonic Modelling



Scheme of input spectrum



Scheme of output spectrum

## Hammerstein Model

$$\hat{z}(n) = \sum_{p=1}^P \sum_{m=1}^M h_p(m) x(n-m) \cdot$$

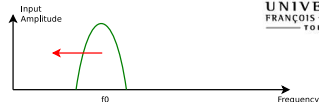
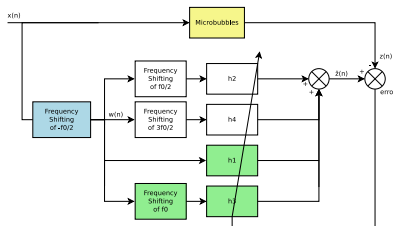
$$\cos\left(2\pi \frac{(p-1)f_0 n}{F_s}\right)$$

Frequency Shifting  $\rightarrow$  Modulation

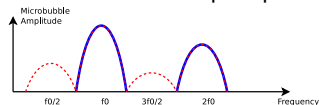
## Optimal Extraction of Components

- 😊 Fundamental modelling
- 😊 Harmonic modelling
- 😞 Subharmonic modelling

# Solution 1: Input Modification



Scheme of modified input spectrum



Scheme of output spectrum

## Subharmonic Hammerstein Model from [Boaghe, 2013] and [Sbeity, 2013]

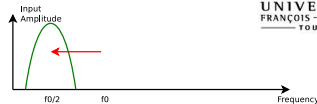
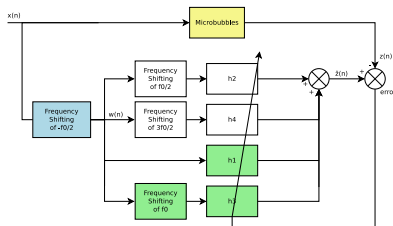
- Method based on

- $$\hat{z}(n) = \sum_{p=1}^{2P} \sum_{m=1}^M h_p(m) x(n-m) \cos\left(2\pi \frac{(p-1)f_0}{2} n\right)$$

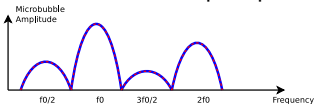
- Frequency Shifting of the Input of  $-\frac{f_0}{2}$

😊 Fundamental    😊 Harmonics    😞 Subharmonics

# Solution 1: Input Modification



## Scheme of modified input spectrum



## Scheme of output spectrum

## Subharmonic Hammerstein Model from [Boaghe, 2013] and [Sbeity, 2013]

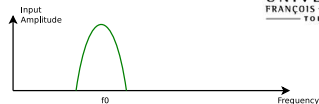
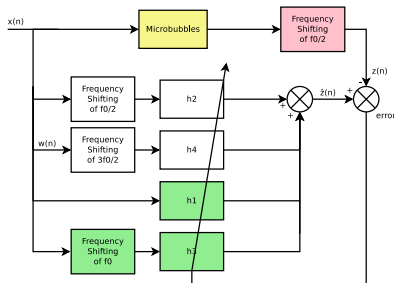
- Method based on

- $$\hat{z}(n) = \sum_{p=1}^{2P} \sum_{m=1}^M h_p(m) x(n-m) \cos\left(2\pi \frac{(p-1)\frac{f_0}{2}n}{F_s}\right)$$

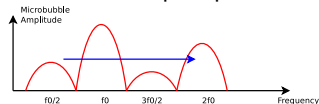
- Frequency Shifting of the Input of  $-\frac{f_0}{2}$

😊 Fundamental    😊 Harmonics    😞 Subharmonics

# Solution 2: Output Modification



Scheme of input spectrum

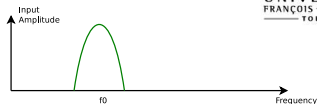
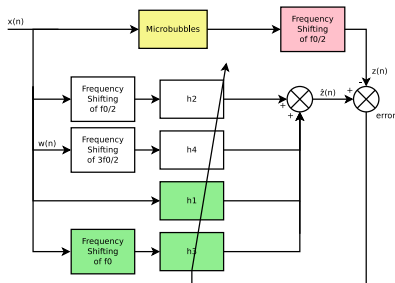


Scheme of modified output spectrum

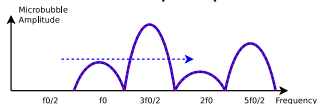
## Subharmonic Hammerstein Model from [Sbeity, 2013]

- $$\hat{z}(n) = \sum_{p=1}^{2P} \sum_{m=1}^M h_p(m) x(n-m) \cos\left(2\pi \frac{(p-1)f_0 n}{F_s}\right)$$
- Frequency Shifting of the Output of  $+\frac{f_0}{2}$
- Required to frequency shift the model to recover  $z(n)$
- 😊 Fundamental    😊 Harmonics    😞 Subharmonics

# Solution 2: Output Modification



Scheme of input spectrum



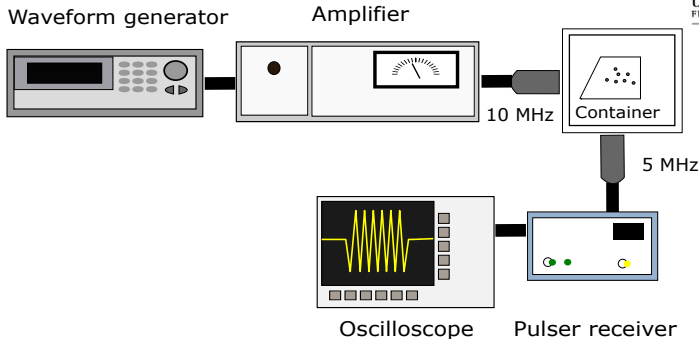
Scheme of modified output spectrum

## Subharmonic Hammerstein Model from [Sbeity, 2013]

- $$\hat{z}(n) = \sum_{p=1}^{2P} \sum_{m=1}^M h_p(m) x(n-m) \cos\left(2\pi \frac{(p-1)f_0 n}{2F_s}\right)$$
- Frequency Shifting of the Output of  $+\frac{f_0}{2}$
- Required to frequency shift the model to recover  $z(n)$
- 😊 Fundamental   😊 Harmonics   😞 Subharmonics

# Experiments

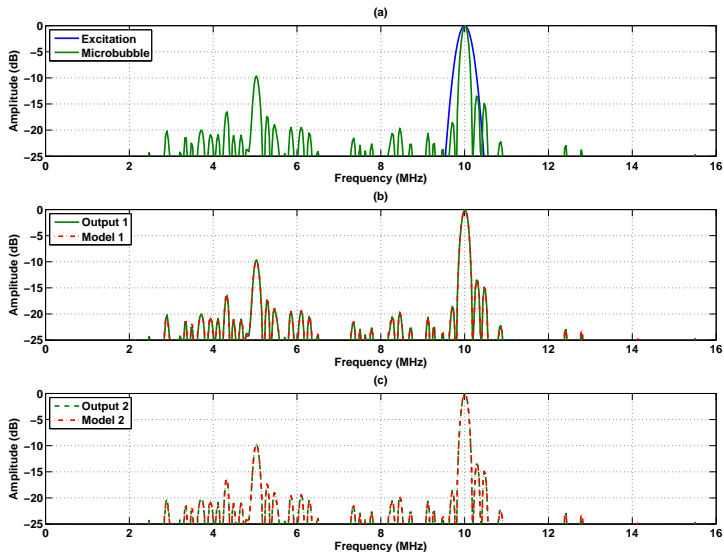
# Experimental Setup



## Transmission / Reception

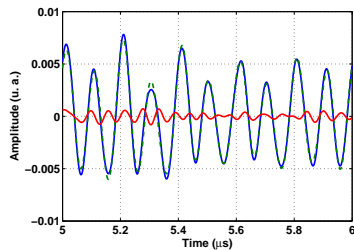
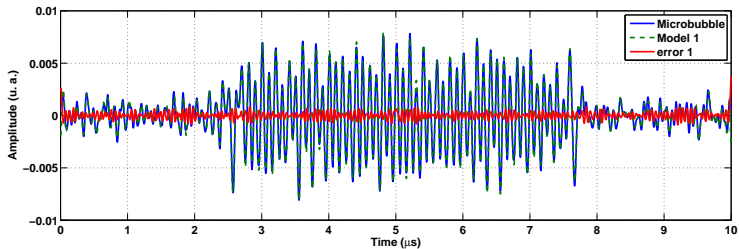
- Transmission of a wave at 10 MHz by a 10 MHz-centred transducer
- Reception centred on subharmonic components at 5 MHz by a 5 MHz-centred transducer
- Diluted solution of Sonovue<sup>TM</sup> microbubbles

## Results

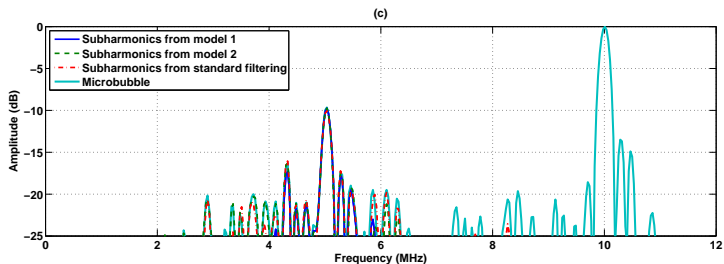
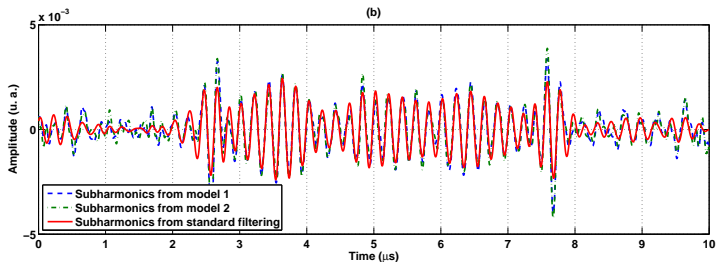




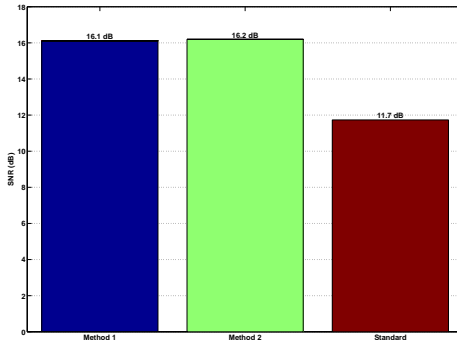
## Results



## Results



# Results

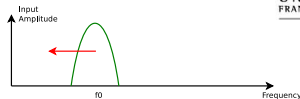
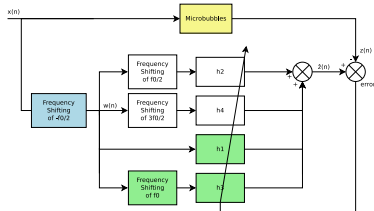


## Signal-to-Noise Ratio(SNR)

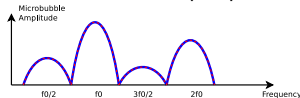
- SNR Improvement by 4.5 dB
- Comparison with a Butterworth filter
  - ▶ Centre frequency 5 MHz
  - ▶ Bandwidth of 40% at  $-3$  dB

## Discussions and Conclusion

# Discussions and Conclusion



Scheme of modified input spectrum



Scheme of output spectrum

- Optimal and automatic extraction of all the components (harmonics, subharmonics and ultraharmonics)
- Increase the SNR by 4.5 dB compared to a non-optimal frequency filtering
- *A priori* knowledge: only the transmitted signal and the order of the subharmonics
- Reasonable computation time ( $\approx$  1 minute per 1000 samples)
- Prospect: Subharmonic extraction on a whole image

# Discussions and Conclusion

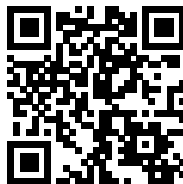
# Thank you for your attention

sebastien.menigot@univ-tours.fr

emma.kanbar@etu.univ-tours.fr

ayache.bouakaz@univ-tours.fr

jean-marc.girault@univ-tours.fr



*Program example available on [Runmycode.org](http://Runmycode.org)*



# Discussions and Conclusion

## Algebraic form

$$\hat{\mathbf{z}} = \mathbf{W}^T \mathbf{h}$$

- Model  $\hat{\mathbf{z}} = [z(M+1), \dots, z(N)]^T$
- Coefficients  $\mathbf{h} = [h_1(1), \dots, h_1(M), h_2(1), \dots, h_4(M)]^T$
- Input Matrix  $\mathbf{W} = [\mathbf{w}_1 \mathbf{w}_2 \mathbf{w}_3 \mathbf{w}_4]$

$$\mathbf{w}_p = \begin{pmatrix} v_p(M) & v_p(M+1) & \dots & v_p(N) \\ v_p(M+1) & v_p(M+2) & & 0 \\ \vdots & & \ddots & \\ v_p(N) & 0 & & 0 \end{pmatrix} \text{ and } v_p(n) = w(n) \cos\left(2\pi \frac{(\rho-1)\frac{f_0}{F_s} n}{2}\right)$$

$$\mathbf{h} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \mathbf{z}$$

## Subharmonic extraction

$$\mathbf{z}_{sub} = \mathbf{w}_1^T \mathbf{h}_{sub}$$

with the odd coefficients  $\mathbf{h}_{sub} = [h_1(1), \dots, h_1(M), h_3(1), \dots, h_3(M)]^T$