



ROM for elastodynamics including viscoelastic behaviors. From material identification to part design

Franck RENAUD, Gaël CHEVALLIER, Jean-Luc DION

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SUPMECA, EA 2336, 3 rue Fernand Hainaut, 93400 Saint-Ouen

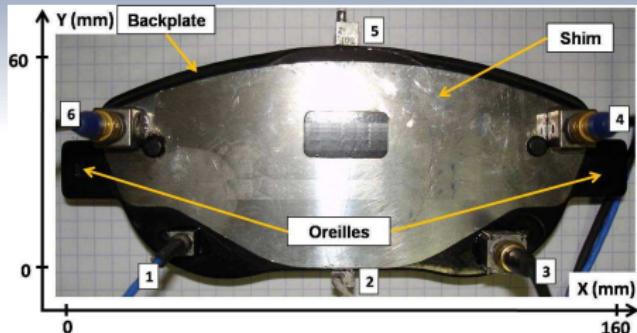
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- 1 Introduction
- 2 Viscoelastic behavior of shims
- 3 Parametric ROM : example of eigenvalue computation
- 4 Parametric ROM : example of frequency response computation
- 5 Conclusion

Context: disc brake squeal



Disc brake squeal



Pad with shim

Stability = complex eigenvalue analysis :

$$\left(p^2 \underline{\underline{M}} + \underline{\underline{K}_{el}} + \mu \underline{\underline{K}_f} \right) \underline{\underline{X}} = \underline{\underline{0}}$$

Need to account for the shim viscoelasticity

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1 Introduction

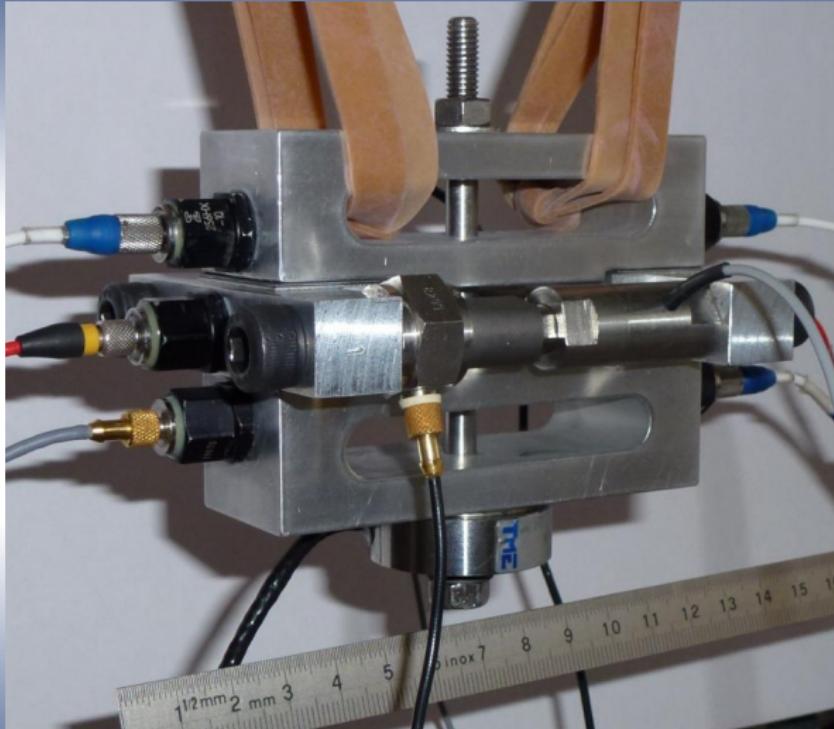
2 Viscoelastic behavior of shims

3 Parametric ROM : example of eigenvalue computation

4 Parametric ROM : example of frequency response computation

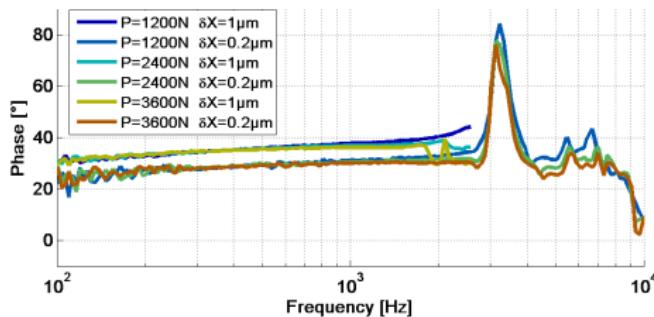
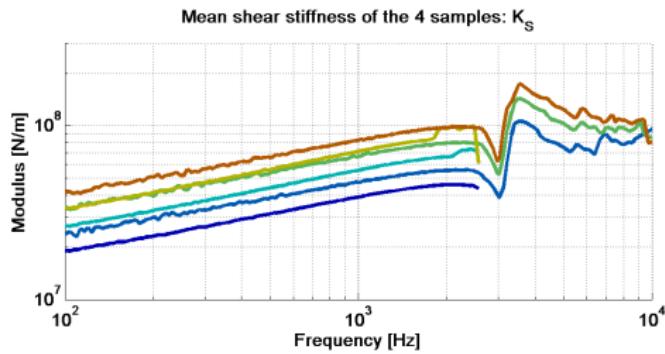
5 Conclusion

Viscoanalyzer



- 1 static force sensor.
- 2 piezoelectric actuators
- 2 dynamic force sensors.
- 6 accelerometers, one per sample face.

Dynamic stiffness of shims



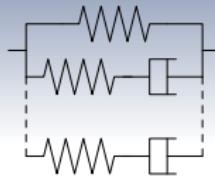
Shim behavior :

- frequency dependent shear stiffness
- high phase (about 30°) meaning high damping

Because of the first eigen mode around 3 500 Hz, results are accurate only up to 2 000 Hz.

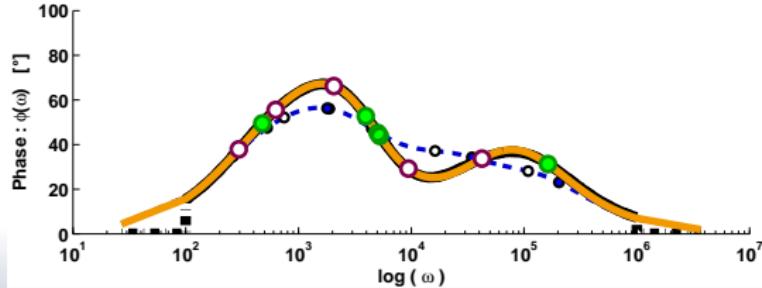
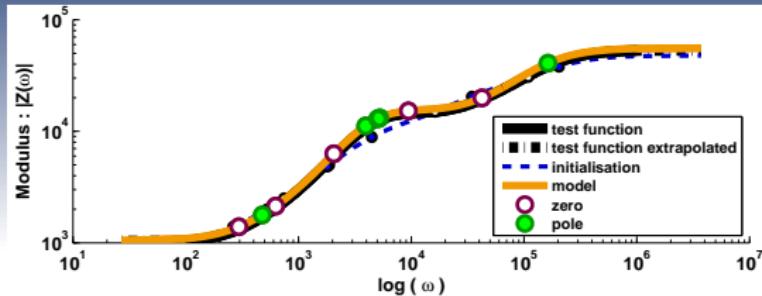
Viscoelastic constitutive law

Generalized Maxwell Model :



$$Z(\omega) = K_0 \left(1 + \sum_{i=1}^N \frac{j\omega \alpha_i \tau_i}{1 + j\omega \tau_i} \right)$$

$$Z(\omega) = K_0 \prod_{i=1}^N \frac{1 + (j\omega/\omega_{z,i})}{1 + (j\omega/\omega_{p,i})}$$



Franck Renaud, Jean-Luc Dion, Gael Chevallier, Imad Tawfiq, and Rémi Lemaire. A new identification method of viscoelastic behavior : Application to the generalized maxwell model. *Mechanical Systems and Signal Processing*, 25(3) : 991-1010, 2011.

It is possible to identify several material constrained to have the same poles

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Parametric ROM $(p^2 \underline{\underline{M}} + \underline{\underline{K}}_{\text{el}} + \mu \underline{\underline{K}}_{\text{f}} + \sum_{m=1}^M \underline{\underline{K}_{\text{visco}}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right)) \underline{X} = \underline{0}$

Stability analysis of a brake system with viscoelastic materials :

Step 1 : Problem with friction and viscoelasticity

$$\left(p^2 \underline{\underline{M}} + \underline{\underline{K}}_{\text{el}} + \mu \underline{\underline{K}}_{\text{f}} + \sum_{m=1}^M \underline{\underline{K}_{\text{visco}}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right) \right) \underline{X} = \underline{0}$$

Parametric ROM $(p^2 \underline{\underline{M}} + \underline{\underline{K}}(\mu, \alpha, \tau))\underline{\underline{X}} = 0$

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Step 2 : Subspace initialization : eigenvectors of a simplified problem

$$(p_i^2 \underline{\underline{M}} + \underline{\underline{K}}_{\text{el}}) \underline{\varphi}_i = \underline{\underline{0}} \implies {}^t \underline{\varphi}_i \underline{\underline{M}} \underline{\varphi}_j = \delta_{ij}, \quad {}^t \underline{\varphi}_i \underline{\underline{K}}_{\text{el}} \underline{\varphi}_j = \lambda_i \delta_{ij}$$

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Step 3 : Projection on the subspace $\underline{\underline{\Phi}} = [\dots \underline{\varphi}_i \dots] : \underline{\underline{X}} = \underline{\underline{\Phi}} \underline{\underline{q}}$

$$\left(p^2 \underline{\underline{I}} + \underline{\underline{\Lambda}} + \mu {}^t \underline{\underline{\Phi}} \underline{\underline{K}_f} \underline{\underline{\Phi}} + \sum_{m=1}^M {}^t \underline{\underline{\Phi}} \underline{\underline{K}_{visco}}^m \underline{\underline{\Phi}} \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right) \right) \underline{\underline{q}} = \underline{\underline{0}}$$

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Step 4 : Complex eigenvalue computation s_i and $\underline{\psi}_i$

Parametric ROM $(p^2 \underline{\underline{M}} + K(\mu, \alpha, \tau))\underline{\underline{X}} = 0$

Stability analysis of a brake system with viscoelastic materials :

Step 1 : Problem with friction and viscoelasticity

$$\left(p^2 \underline{\underline{M}} + \underline{\underline{K}_{el}} + \mu \underline{\underline{K}_f} + \sum_{m=1}^M \underline{\underline{K}_{visco}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right) \right) \underline{\underline{X}} = \underline{\underline{0}}$$

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Step 4 : Complex eigenvalue computation s_i and $\underline{\psi}_i$

Step 5 : Evaluation of the **solution accuracy**, if required : **subspace improvement** and reanalysis (Step 1) until convergence

Subspace improvement

Step 1 : Force residu computation for each couple s_i and $\underline{\psi}_i$

$$\left(s_i^2 \underline{\underline{M}} + \underline{\underline{K}_{\text{el}}} + \mu \underline{\underline{K}_{\text{f}}} + \sum_{m=1}^M \underline{\underline{K}_{\text{visco}}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m s_i}{1 + \tau_u^m s_i} \right) \right) \underline{\Phi} \underline{\psi}_i = \underline{R}_{F,i}$$

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Step 2 : Displacement residu computation $\underline{R}_{X,i} = \underline{\tilde{K}}^{-1} \underline{R}_{F,i}$ thanks to an arbitrarily chosen operator, for example $\underline{\tilde{K}} = \underline{\underline{K}_{\text{el}}} + \mu \underline{\underline{K}_{\text{f}}}$

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Step 3 : Error estimation

$$\delta E_i = \frac{|{}^t R_{X,i} | R_{F,i}|}{|{}^t \underline{\psi}_i | \underline{\Phi} \underline{\underline{K}} \underline{\Phi} \underline{\psi}_i |} \stackrel{?}{<} \epsilon$$

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Step 4 : Orthonormalisation of $\underline{R}_{X,i}$ versus mass matrix

$$\underline{R}_{X,i \perp} = \underline{R}_{X,i} - \underline{\Phi} \left({}^t \underline{\Phi} \underline{\underline{M}} \underline{R}_{X,i} \right) \quad \text{then} \quad \underline{R}_{X,i \perp} := \underline{R}_{X,i \perp} / \sqrt{{}^t \underline{R}_{X,i \perp} \underline{\underline{M}} \underline{R}_{X,i \perp}}$$

Subspace improvement

Step 1 : Force residu computation for each couple s_i and $\underline{\psi}_i$

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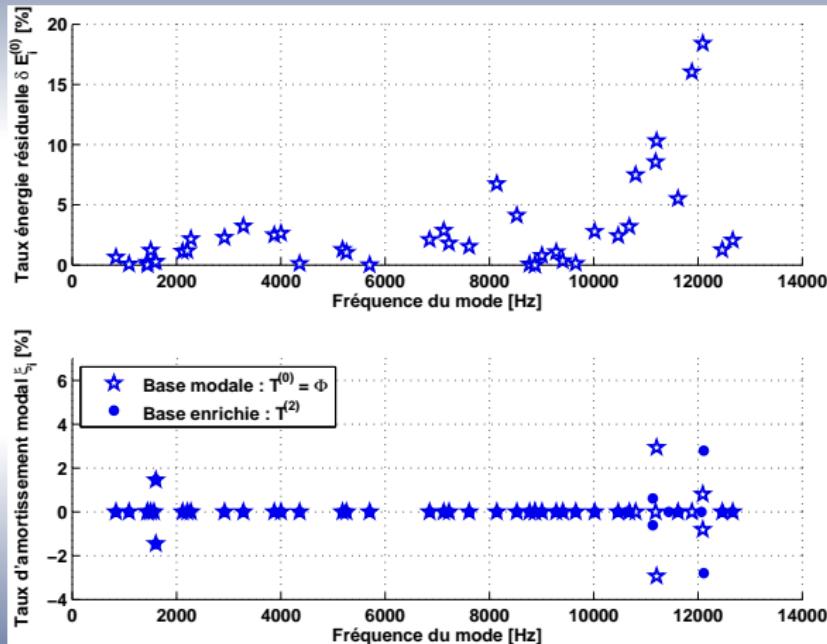
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Step 5 : Subspace improvement $\underline{\Phi} := [\underline{\Phi}, \underline{R}_{X,i \perp}]$

Effect of subspace improvement on :

$$\left(p^2 I + \underline{\Delta} + \mu^t \underline{\Phi} \underline{K_f} \underline{\Phi} \right) q = 0$$



Internal state formulation for viscoelasticity

$$\left(p^2 \underline{\underline{I}} + \underline{\underline{\Lambda}} + \color{red}{\mu}^t \underline{\underline{\Phi}} \underline{\underline{K_f}} \underline{\underline{\Phi}} + \sum_{m=1}^M \underline{\underline{t}} \underline{\underline{\Phi}} \underline{\underline{K_{visco}}}^m \underline{\underline{\Phi}} \left(\sum_{u=1}^N \frac{\color{green}{\alpha_u^m} \color{black}{\tau_u^m} p}{1 + \color{black}{\tau_u^m} p} \right) \right) \underline{\underline{q}} = \underline{\underline{0}}$$

Internal state formulation for viscoelasticity

$$\left(p^2 I + \underline{\Lambda} + \underline{\mu}^t \underline{\Phi} \underline{K_f} \underline{\Phi} + \sum_{m=1}^M \underline{\underline{\Phi}} K_{\text{visco}}^m \underline{\underline{\Phi}} \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right) \right) \underline{q} = \underline{0}$$

Without assumption : $N \times M + 2$ states

$$\underline{q_v} = p \underline{q}, \quad \underline{q_u^m} = \frac{\tau_u^m}{1 + \tau_u^m p} \underline{q_v}$$

$$p \underline{q_v} + \left(\underline{\Lambda} + \underline{\mu}^t \underline{\Phi} \underline{K_f} \underline{\Phi} \right) \underline{q} + \sum_{u=1}^N \sum_{m=1}^M \left(\alpha_u^m \underline{\underline{\Phi}} K_{\text{visco}}^m \underline{\underline{\Phi}} \underline{q_u^m} \right) = \underline{0}$$

Internal state formulation for viscoelasticity

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Without assumption : $N \times M + 2$ states

$$\underline{q}_v = p \underline{q}, \quad \underline{q}_u^m = \frac{\tau_u^m}{1 + \tau_u^m p} \underline{q}_v$$

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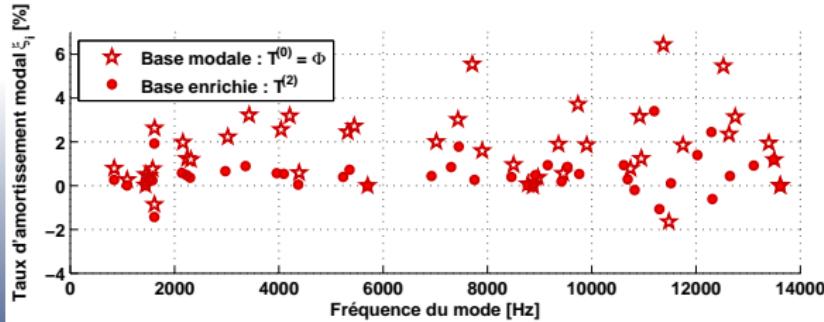
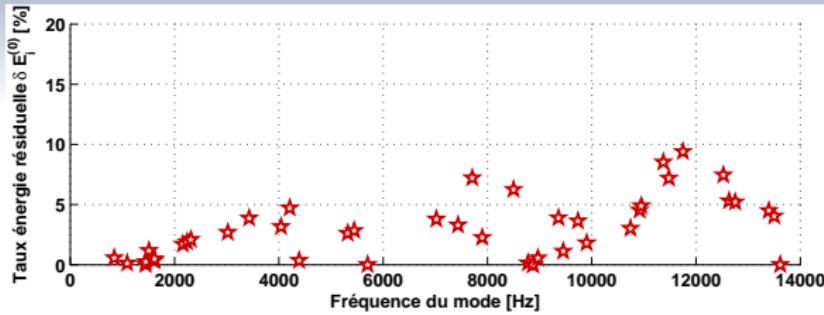
With $\tau_u^m = \tau_u$ assumption : $N + 2$ states

$$\underline{q}_v = p \underline{q}, \quad \underline{q}_u = \frac{\tau_u}{1 + \tau_u p} \underline{q}_v$$

$$p \underline{q}_v + \left(\underline{\Lambda} + \underline{\mu}^t \underline{\Phi} K_f \underline{\Phi} \right) \underline{q} + \sum_{u=1}^N \left(\sum_{m=1}^M \alpha_u^m \underline{\underline{\Phi}} K_{\text{visco}}^m \underline{\underline{\Phi}} \right) \underline{q}_u = \underline{0}$$

Effect of subspace improvement on :

$$p \underline{q}_v + \left(\underline{\Lambda} + \mu^t \underline{\Phi} \underline{K}_f \underline{\Phi} \right) \underline{q} + \sum_{u=1}^N \left(\sum_{m=1}^M \alpha_u^m \underline{\Phi}^t \underline{K}_{\text{visco}}^m \underline{\Phi} \right) \underline{q}_u = 0$$



Example of a parametric study

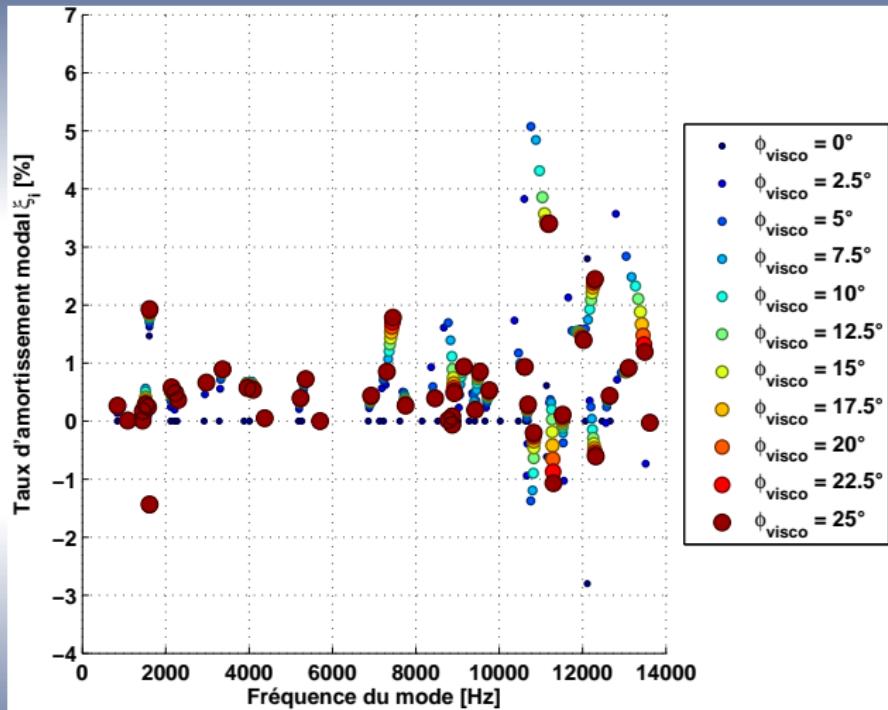


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A good subspace initialization

In the case of a forced response, we have :

$$\left(p^2 \underline{\underline{M}} + \underline{\underline{K}_{\text{el}}} + \mu \underline{\underline{K}_{\text{f}}} + \sum_{m=1}^M \underline{\underline{K}_{\text{visco}}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right) \right) \underline{X} = \underline{F}$$

Let us define $\underline{\underline{K}}(p) = \underline{\underline{K}_{\text{el}}} + \sum_{m=1}^M \underline{\underline{K}_{\text{visco}}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right)$

A good subspace initialization

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Subspace T_1 : eigenvectors of a simplified problem at low frequency

$$(p_i^2 \underline{\underline{M}} + \underline{\underline{K}}(0)) \underline{\varphi}_i^0 = \underline{0} \quad \Rightarrow \quad T_1 = [\Phi^0]$$

A good subspace initialization

In the case of a forced response, we have :

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Subspace T_2 : eigenvectors of a simplified problem at low and high frequency

$$(p_i^2 \underline{\underline{M}} + \underline{\underline{K}}(\infty)) \underline{\varphi_i^\infty} = \underline{0} \implies T_2 = [\Phi^0, \Phi^\infty]$$

A good subspace initialization

In the case of a forced response, we have :

$$\left(p^2 \underline{\underline{M}} + \underline{\underline{K}_{\text{el}}} + \textcolor{red}{\mu} \underline{\underline{K}_{\text{f}}} + \sum_{m=1}^M \underline{\underline{K}_{\text{visco}}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right) \right) \underline{X} = \underline{F}$$

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Subspace T_1 : eigenvectors of a simplified problem at low frequency

$$(p_i^2 \underline{\underline{M}} + \underline{\underline{K}}(0)) \underline{\varphi}_i^0 = \underline{0} \implies T_1 = [\Phi^0]$$

Subspace T_2 : eigenvectors of a simplified problem at low and high frequency

$$(p_i^2 \underline{\underline{M}} + \underline{\underline{K}}(\infty)) \underline{\varphi}_i^\infty = \underline{0} \implies T_2 = [\Phi^0, \Phi^\infty]$$

Subspace T_3 : static solution

$$X^0 = \underline{\underline{K}}(0)^{-1} \underline{F} \implies T_3 = [\Phi^0, \Phi^\infty, X^0]$$

A good subspace initialization

In the case of a forced response, we have :

$$\left(p^2 \underline{\underline{M}} + \underline{\underline{K}_{\text{el}}} + \textcolor{red}{\mu} \underline{\underline{K}_{\text{f}}} + \sum_{m=1}^M \underline{\underline{K}_{\text{visco}}}^m \left(\sum_{u=1}^N \frac{\alpha_u^m \tau_u^m p}{1 + \tau_u^m p} \right) \right) \underline{X} = \underline{F}$$

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$$(p_i^2 \underline{\underline{M}} + \underline{\underline{K}}(0)) \underline{\varphi}_i^0 = \underline{0} \implies T_1 = [\Phi^0]$$

Subspace T_2 : eigenvectors of a simplified problem at low and high frequency

$$(p_i^2 \underline{\underline{M}} + \underline{\underline{K}}(\infty)) \underline{\varphi}_i^\infty = \underline{0} \implies T_2 = [\Phi^0, \Phi^\infty]$$

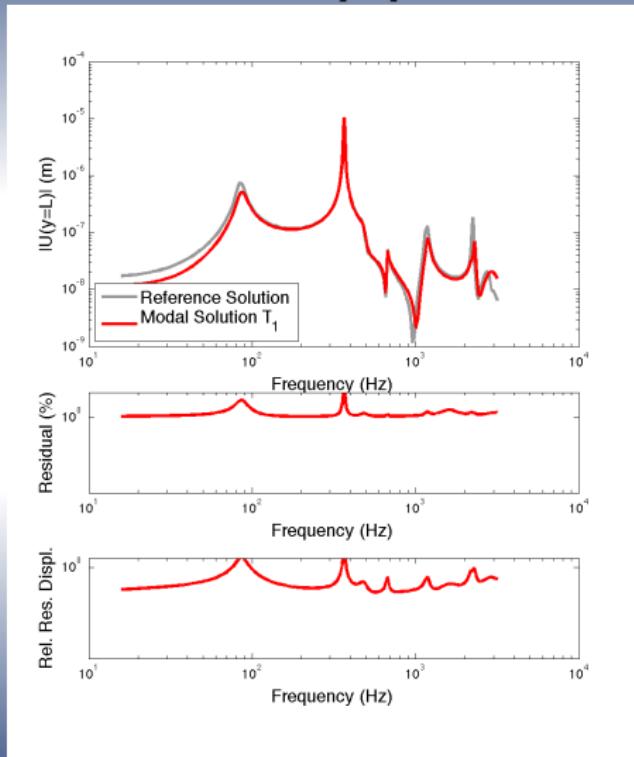
Subspace T_3 : static solution

$$X^0 = \underline{\underline{K}}(0)^{-1} \underline{F} \implies T_3 = [\Phi^0, \Phi^\infty, X^0]$$

Subspace T_4 : $T_4 = T_3$ plus iteration on residu

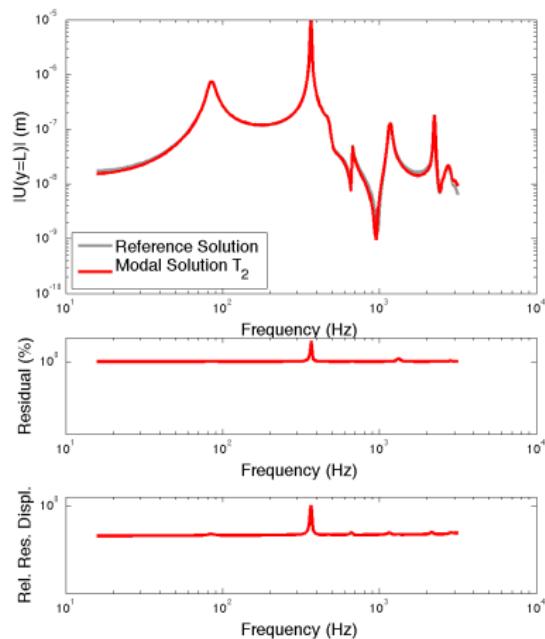
A good subspace initialization

$$T_1 = [\Phi^0]$$

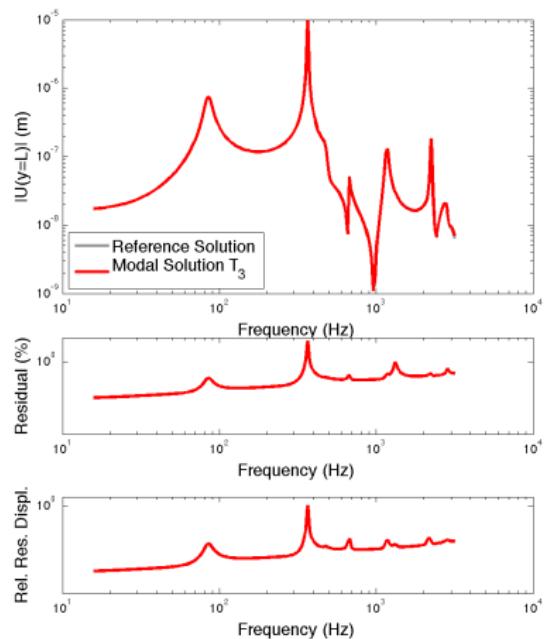


A good subspace initialization

$$T_2 = [\Phi^0, \Phi^\infty]$$



$$T_3 = [\Phi^0, \Phi^\infty, X^0]$$



A good subspace initialization

$$T_4 = [\Phi^0, \Phi^\infty, X^0] + \text{iteration on residu}$$

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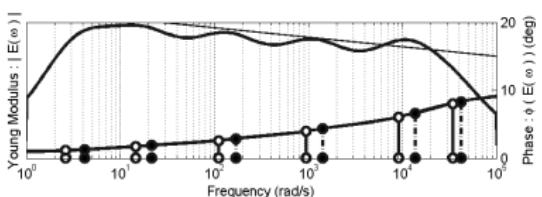
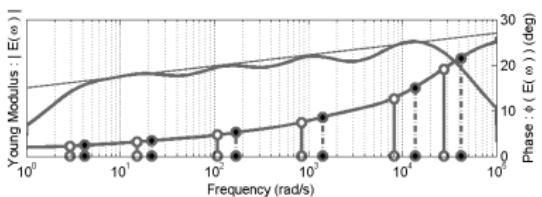
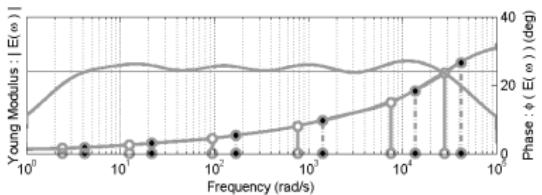
- 1 Introduction
- 2 Viscoelastic behavior of shims
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Conclusion

- A viscoanalyser allows shear stiffness characterization of shims.
- Viscoelastic material are frequency dependent. The stiffness increase versus frequency.
- The Generalized Maxwell model is suitable for viscoelasticity.
- The problem with friction and viscoelasticity is too complex for direct complex eigen value computation. It has to be reduced.
- The choice of the subspace for reduction is of great importance. The subspace of the eigen modes computed without friction or viscoelasticity is not good enough and has to be improved.
- Residu computation is a good way for subspace improvement.
- Finally, the problem with friction and viscoelasticity is reduced on an improved subspace and the effect of viscoelasticity is studied. Most of eigen modes are damped but few become unstable due to mode coupling.

Generalized Maxwell identification

Common poles assumption



No assumption

