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SYMMETRY-BREAKING IN A THREE-NANOMECHANICAL-RESONATOR ARRAY FOR MASS DETECTION

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Summary Due to their accuracy and their size, M/NEMS resonant sensors are present in a lot of domain such as automotive, aerospace and biotechnology. They are used as accelerometers, gyroscopes for inertial navigation and also mass detection which is the target application of this investigation. The literature essentially focus on improving the sensitivity of a single resonator. Mass sensors are now able to detect a mass around a few zeptograms. This work presents a three-nanomechanical-resonator array analysis. The original contribution lies in the use of the array symmetry-breaking for mass detection.

ARRAY OF THREE NANOMECHANICAL RESONATORS

An array of three clamped-clamped beams is considered, as sketched in Fig. 1. All beams are assumed to have identical dimensions (length l, width b, height h, moment of inertia I, gap g between two adjacent beams) and identical material properties (Young's modulus E and material density ρ). Each beam is subjected to the electrostatic forces due to its two adjacent beams. The two beams at both ends of the array (beam 0 and $n+1$) are clamped and non-deformable.

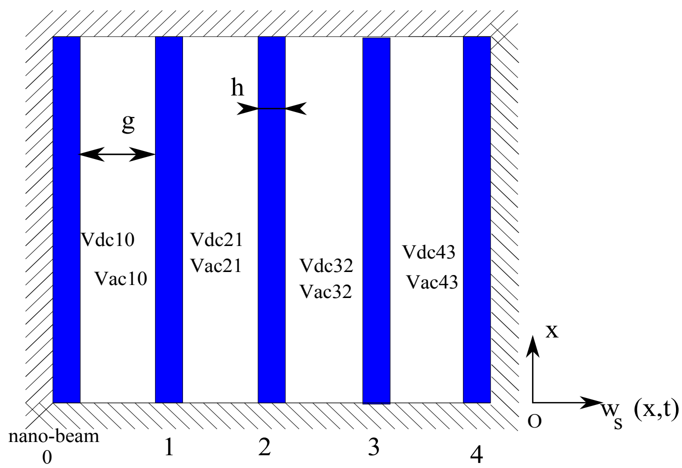

Figure 1: Array of three clamped-clamped M/NEMS beams.

Let $\tilde{w}_s$ be the lateral displacement in the plane $x-z$ along the z-axis of the s-th beam and $V_{s,s+1} = Vdc_{s,s+1} + Vac_{s,s+1}\cos(\Omega t)$ the voltage between s-th and s+1-th beams. The s-th resonator motion equation can be written as [1, 2]:

$$EI\frac{\partial^4 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}^4} + \rho bh\frac{\partial^2 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{t}^2} - \Big[\tilde{N} + \frac{Ebh}{2l}\int_0^l \big(\frac{\partial \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}}\big)^2 \,\mathrm{d}\tilde{x}\Big]\frac{\partial^2 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}^2}$$
$$= \frac{\epsilon_0 b C_n}{2}\left[\frac{V_{s,s+1}^2}{\big(g+\tilde{w}_{s+1}-\tilde{w}_s\big)^2} - \frac{V_{s-1,s}^2}{\big(g+\tilde{w}_s-\tilde{w}_{s-1}\big)^2}\right] \quad (1)$$

where $s = 1,..,3$ and $\tilde{N}_s$ is the axial force acting on the s-th beam and resulting from an externally applied load or from residual manufacturing stress. ϵ_0, C_N are the dielectric constant and fringing field coefficient respectively. First and last resonators are clamped, therefore the following conditions apply:

$$\tilde{w}_0(\tilde{x},\tilde{t}) = \tilde{w}_{N+1}(\tilde{x},\tilde{t}) = 0. \quad (2)$$

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By assumption the three identical beams have the same eigenmodes. First, a Galerkin expansion is used to eliminate the spatial dependence of the lateral displacement. Then the electrostatic forces in Eq. (1) are treated by either multiplying by the denominator or either using Taylor series expansions. Then the Harmonic Balance Method associated with the Asymptotic Numerical Method (HBM+ANM) [3] is used to solve Eq. (1). The ANM is preferred to a more conventional Newton- Raphson method because of the robustness of the ANM-based continuation and its ability to follow very complicated solution branches.

MASS DETECTION USING SYMMETRY-BREAKING OF NANOMECHANICAL RESONATOR ARRAYS

In order to use the symmetry-breaking of the resonator array for mass detection, a symmetric configuration is considered, with symmetric voltages as defined in Table 1. The electrostatic forces acting on both sides of the central beam are therefore compensating each others. Consequently, the central resonator will not vibrate. The symmetry-breaking appears when a mass fall on either the first or the third resonator. The symmetry is then broken and the central resonator starts to vibrate. The presence of a mass is thus detected.

Depending on the value of the added mass, different responses are obtained. The corresponding curves are plotted in Fig. 2. The central figure represents the response of the central beam. Without added mass ($m = 0$) the central beam does not vibrate. As soon as a mass is added on the first or the third beam, the symmetry is broken and the vibration amplitude of the central beam is not null anymore and increases with the value of the added mass.

Vdc_{10}	Vac_{10}	Vdc_{21}	Vac_{21}	Vdc_{32}	Vac_{32}	Vdc_{43}	Vac_{43}
0	0	5.3	1	5.3	1	0	0

Table 1: Voltage configuration of the three array beams.

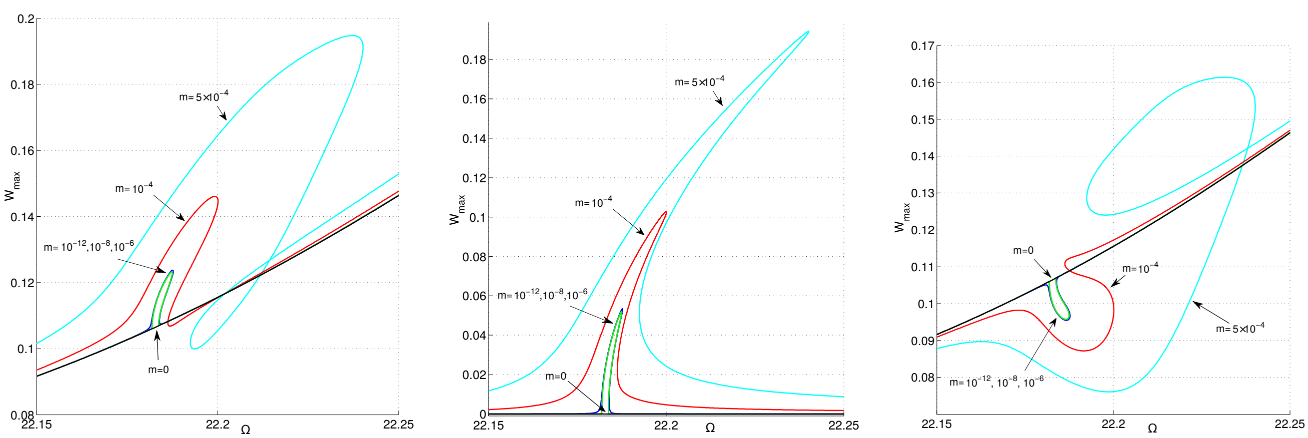

Figure 2: Array of three clamped-clamped M/NEMS beams.

CONCLUSIONS

A new mass detection method using symmetry-breaking of a M/NEMS array has been proposed. This method gives an original way to detect particles and to determine their mass. This work is a first step towards MEMS-based mass spectrometry via the implementation of thousands resonators in parallel.

References

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