Multi-parametric optimization of bifurcation points in nonlinear dynamical systems

Clément Grenat, Sébastien Baguet, Régis Dufour, Claude-Henri Lamarque

Multi-parametric optimization of bifurcation points in nonlinear dynamical systems

C. Grenat[1], **S. Baguet**[1], **R. Dufour**[1] **and C-H. Lamarque**[2]

[1]Université de Lyon, CNRS, INSA-Lyon, LaMCoS UMR5259, F-69621
Villeurbanne, France
clement.grenat@insa-lyon.fr
sebastien.baguet@insa-lyon.fr
regis.dufour@insa-lyon.fr

[2]Université de Lyon, CNRS, ENTPE, LTDS UMR5513, F-69518
Vaulx-en-Velin, France
claude.lamarque@entpe.fr

Abstract A frequency-domain method is proposed for tuning bifurcation points in nonlinear dynamical systems by means of multi-parametric bifurcation tracking and optimization criteria.

Introduction In the field of mechanical engineering, nonlinear phenomena introduced by large deflections, miniaturization, electro-mechanical couplings, nonlinear vibration absorbers, ..., are often encountered. Nonlinear systems can have multiple solutions for one unique set of parameters or, on the opposite, no stable solution for a specific parameter range. These phenomena, characterized by bifurcation points such as limit points or Neimark-Sacker bifurcations, lead to jumps in the response or changes of dynamical regime. So, being able to optimize those bifurcation points is interesting for designing nonlinear systems. The literature on bifurcation points characterization and detection comprises numerous contributions. Since then, the obtained methods have been combined with arc-length continuation in order to follow bifurcation points with respect to one parameter. This work presents a multi-parametric analysis of bifurcation points combining multi-parametric continuation and a specific optimization criterion. The original contribution of this paper lies in the multi-parametric approach. The main target application is the design of Nonlinear Energy Sinks.

Multi-parametric optimization of bifurcation points First, harmonic balance method (HBM) is carried out on the equation of motion to perform a nonlinear analysis in the frequency domain. The forced response curve is computed by applying the arc-length continuation method to the following equation, with $\boldsymbol{Q}$ the Fourier coefficient vector and ω the forcing frequency

$$\boldsymbol{R}\left(\boldsymbol{Q},\omega,\right)=\boldsymbol{Z}\left(\omega\right)\boldsymbol{Q}+\boldsymbol{F}_{nl}\left(\boldsymbol{Q}\right)-\boldsymbol{P}=\boldsymbol{0} \tag{1}$$

Then, the bifurcation points along this response curve are obtained by solving a so-called extended system made of the equation of motion (1) and an additional equation $\boldsymbol{B}$ characterizing the bifurcations of interest and based on Hill's method [1]

$$\boldsymbol{K}(\boldsymbol{Q},\boldsymbol{\Phi},\kappa,\omega)=\begin{pmatrix}\boldsymbol{R}\left(\boldsymbol{Q},\omega\right)\\ \boldsymbol{B}(\boldsymbol{Q},\boldsymbol{\Phi},\kappa,\omega)\\ l(\boldsymbol{\Phi})\end{pmatrix}=\boldsymbol{0} \tag{2}$$

where κ and $\boldsymbol{\Phi}$ are the eigenvalue and eigenmode of the bifurcation, and l is a normalization equation ensuring nontrivial solutions. Finally, the vector of system parameters $\boldsymbol{\alpha}$ is considered as a new unknown and a multi-parametric continuation of the bifurcation points with respect to $(\boldsymbol{Q},\omega,\boldsymbol{\alpha})$ combined with an optimization criterion, such as an extremal or threshold constraint, is performed. It permits to restrict the multi-dimensional map of solutions to specific points corresponding to optimal values of the system parameters.

Example: Nonlinear Energy Sink (NES) Let us consider the strongly nonlinear vibration absorber introduced in [2] with the following set of parameters : mass ratio ϵ=0.1, damping λ=0.4, nonlinear stiffness coefficient k_{nl}=5. In [2], it is shown that quasi-periodic beating responses provide efficient vibration absorption (see Figure 1). The boundaries for the quasi-periodic beating responses can be obtained by the parametric continuation of Neimark-Sacker (NS) bifurcations. This NS tracking is plotted in Figure 2 in the case when the amplitude of the periodic excitation A is used as the varying parameter.
The two blue dots, corresponding to local minima of this curve, are very interesting for the tuning of the NES. Indeed, no quasi-periodic motion, i.e. no effective energy transfert is possible below the lower point. Therefore, this point defines a threshold of excitation level triggering the quasi-periodic motion. The upper point corresponds to an optimal excitation level where the motion is quasi-periodic over the widest range of frequencies. Therefore, this point defines an optimal excitation level.
The tuning of the NES is performed via a multi-parametric analysis of these two points in order to find the set of parameters (λ, k_{nl}) generating the lowest threshold of excitation level, or the maximum optimal excitation level for instance. The distance between these two levels can also be maximized in order for the NES to be efficient on the widest possible range of excitation level. Other optimization criteria such as a specific energy level of the system can also be considered.

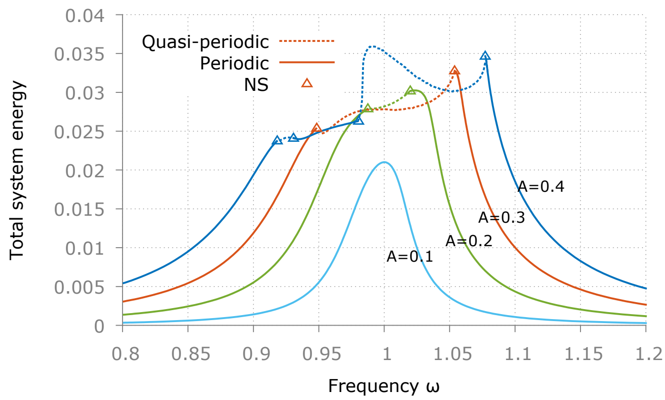

Figure 1: Average total system energy for ϵ=0.1, λ=0.4, k_{nl}=5 and A=0.1, 0.2, 0.3, 0.4.

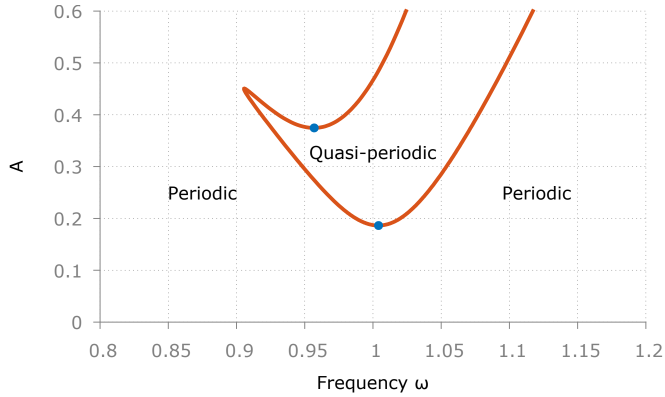

Figure 2: NES : Neimark-Sacker tracking curve for ϵ=0.1, λ=0.4, k_{nl}=5 and varying A.

References

[1] L. Xie, S. Baguet, B. Prabel and R. Dufour, Numerical Tracking of Limit Points for Direct Parametric Analysis in Nonlinear Rotordynamics, Journal of Vibration and Acoustics, In press. doi:10.1115/1.4032182

[2] Y. Starosvetsky and O. Gendelman, Attractors of harmonically forced linear oscillator with attached nonlinear energy sink. ii: Optimization of a nonlinear vibration absorber, Nonlinear Dynamics 51, 4757, 2008. doi: 10.1007/s11071-006-9168-z