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Kazusa Yamamoto, Pascal Moulaire, Damien Koenig, Olivier Sename. H_{∞} Filtering Design for Driver Torque Estimation in Electric Power Steering System. AVEC 2016 - 13th International Symposium on Advanced Vehicle Control, Sep 2016, Munich, Germany. hal-01367991

HAL Id: hal-01367991

<https://hal.science/hal-01367991>

Submitted on 18 Sep 2016

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H_∞ Filtering Design for Driver Torque Estimation in Electric Power Steering System

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ABSTRACT: This paper describes a method to estimate the driver torque in an Electric Power Steering (EPS) system without taking into account the torque sensor signal. First, a pinion-type EPS model is expressed by Newton's law of motion considering the effort transmitted from the road as an unknown input. Then, the design of a H_∞ filter is proposed to estimate the driver torque. The formulation aims at minimizing the estimation error in the presence of the external force induced by the road solicitations and sensor noises. The efficiency of this H_∞ filter is assessed through simulation results using input data gathered from experimental tests on a real car. Furthermore these results are compared with torque measurements obtained on the experimental car for validation purpose only.

1 INTRODUCTION

Nowadays, modern vehicles are equipped with more and more electronic systems which help the driving process. Among them, steering systems help the driver to turn the vehicle in the desired direction, improving both safety and comfort. Currently, most of car manufacturers are using Electric Power Steering (EPS) systems. Indeed, compared to Hydraulic Power Steering (HPS), they improve fuel efficiency, and are easy to integrate into vehicle, etc.

In EPS systems, an electric motor provides an assistance torque to reduce the amount of effort required from the driver to turn the wheels. An Electronic Control Unit (ECU) computes the supplied torque according to a motor torque control strategy. It mainly consists of an assistance map depending on the vehicle speed and on the measured torque sensor signal [1].

Hence, EPS assistance strategies are based on the applied steering torque depicted by torque sensor measurements. However, a failure of this sensor may lead in a loss of assistance, which must be prevented. Indeed, developing a controller based on an estimation of the driver torque would cover such a case and improve global performance since torque sensor measurements and actual driver torque differs [2], [3]. Moreover, this strategy reduces the production costs and can improve the driver feeling. Both topics represent major challenges for EPS system suppliers.

Several works have already been carried out on driver torque estimation in EPS systems. Some of them [3], [4] have developed an estimator considering as a measured input the torque sensor signal. In that case, it is straightforward to reconstruct the driver torque as it is equivalent to the measurement during static steering. Nevertheless, Marouf et al. proposed in [2] a Sliding Mode Observer (SMO) to estimate the driver torque, in which measurements are the steering wheel angle and the absolute angular position of the assist motor. Moreover, in [5], the authors have designed a Proportional Integral Observer (PIO) with the same assumptions on measurements availability.

This paper follows the same idea in order to avoid the need of torque sensor signal and the design of a H_∞ filter that estimates the driver torque is presented. Likewise, the steering wheel angle and the assist motor angle are required inputs measurements. However, no assumption is made on the driver torque dynamics is made, since it is considered as the fault to estimate. The chosen design objective is to minimize the effects of road disturbances and sensor noises on the driver torque estimation error, in an H_∞ framework.

The paper is organized as follows. In section 2, a linear EPS mechanical model is introduced. Next section presents a method to estimate driver torque through the design of a H_∞ filter. Then, section 4 is dedicated to simulation results and validation is done on experimental results obtained on a real test car.

2 EPS SYSTEM MODEL

The studied system is referred to as a pinion-type EPS (P-EPS) mainly composed by the assist motor, the pinion and the rack part. Moreover, the driver action is transmitted through the steering wheel to the torsion bar. Hence, it brings to distinguish four main parts. An illustration of a simplified mechanical model is shown in Figure 1.

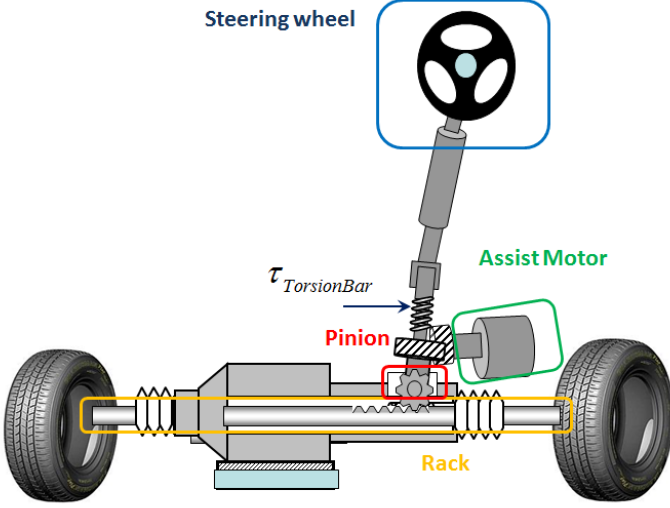


Figure 1: P-EPS simplified mechanical model

The model is obtained by applying Newton's laws of motion and neglecting dry friction. The P-EPS model is governed by the following equations (for further details see [5]):

$$J_c \ddot{\theta}_c = \tau_d - (D_{tb}(\dot{\theta}_c - \dot{\theta}_p) + K_{tb}(\theta_c - \theta_p)) - B_c \dot{\theta}_c \quad (1)$$

$$J_m \ddot{\theta}_m = \tau_m - (D_g(\dot{\theta}_m - G\dot{\theta}_p) + K_g(\theta_m - G\theta_p)) - B_m \dot{\theta}_m \quad (2)$$

$$\begin{aligned} J_p \ddot{\theta}_p = & (D_{tb}(\dot{\theta}_c - \dot{\theta}_p) + K_{tb}(\theta_c - \theta_p)) \\ & - R_p (D_{pr}(R_p \dot{\theta}_p - \dot{X}_r) + K_{pr}(R_p \theta_p - X_r)) \\ & + G (D_g(\dot{\theta}_m - G\dot{\theta}_p) + K_g(\theta_m - G\theta_p)) \end{aligned} \quad (3)$$

$$\begin{aligned} J_r \ddot{X}_r = & \tau_{road} - (D_r \dot{X}_r + K_r X_r) - B_r \dot{X}_r \\ & + D_{pr}(R_p \dot{\theta}_p - \dot{X}_r) + K_{pr}(R_p \theta_p - X_r) \end{aligned} \quad (4)$$

where the variables are θ_c the steering wheel angle (rad), θ_m the assist motor angle (rad), θ_p the pinion angle (rad), X_r the rack position (m). The mechanical parameters are described in Table 1.

Hence a state-space representation of the P-EPS system (denoted *Plant*) is the following:

$$\begin{cases} \dot{x} = Ax + Bu + Ed + Ww \\ y = Cx + Nn \end{cases} \quad (5)$$

Notation	Description	Unit
J_c	Steering column inertia	$kg.m^2$
B_c	Steering column viscous friction	$N.m/(rad/s)$
K_{tb}	Torsion bar stiffness	$N.m/rad$
D_{tb}	Torsion bar damping	$N.m/(rad/s)$
J_p	Pinion/gear inertia	$kg.m^2$
K_{pr}	Pinion/rack stiffness	N/m
D_{pr}	Pinion/rack damping	$N/(m/s)$
R_p	Pinion/rack reducer	m/rad
J_r	Rack and tie rods mass	kg
B_r	Rack viscous friction	$N/(m/s)$
K_r	Rack stiffness	N/m
D_r	Rack damping	$N/(m/s)$
G	Worm/gear reduction ratio	—
J_m	Motor inertia	$kg.m^2$
B_m	Motor viscous friction	$N.m/(rad/s)$
K_g	Worm/gear stiffness	$N.m/rad$
D_g	Worm/gear damping	$N.m/(rad/s)$

Table 1: EPS system mechanical parameters

with $x = (\dot{\theta}_c \ \dot{\theta}_m \ \dot{\theta}_p \ \dot{X}_r \ \theta_c \ \theta_m \ \theta_p \ X_r)^T \in \mathbb{R}^8$ are the internal states, $d \in \mathbb{R}$ is the driver torque to be estimated, $w \in \mathbb{R}$ the road reaction torque is the unknown input, and $u \in \mathbb{R}$ the assist motor torque is the control signal. The available measurements are $y = (\theta_c \ \theta_m)^T \in \mathbb{R}^2$ which are affected by white gaussian noise $n \in \mathbb{R}^2$. Moreover N is the weighting matrix associated to n in the form $N = \beta I$ with $\beta \in \mathbb{R}^+$.

3 H_∞ FILTERING APPROACH

The main objective is to estimate the unmeasured input d subject to the road disturbance w and sensor noises n affecting the system (5), using a H_∞ filter design. In the following, the known input u is not considered, but can be easily included in the filter equations.

3.1 Synthesis of H_∞ filter

In this part, the method followed to design the H_∞ filter is presented. As usual in the H_∞ approach, it is based on weighting functions that represent the specifications of the input/output signals of interest. It is worth noticing that such a scheme, as shown in Figure 2 with the extended representation with weighting functions and the H_∞ filter $F(s)$, is used for synthesis only and that the implementation only considers $F(s)$ with as inputs the measurements y .

Besides, the estimation is designed on a filtered driver torque obtained using $T(s)$. Then, a weighting function on the driver torque is introduced such that $d_f = T(s)d$. This allows to specify the frequency range on which estimation performance is expected. In practice, the driver torque average range is up to $2Hz$ and does not exceed $5Hz$ even in emergency case. Hence, in this paper the H_∞ filter problem is not designed as in the form of a state estimation like in [6], [7] to avoid assumption on driver torque dy-

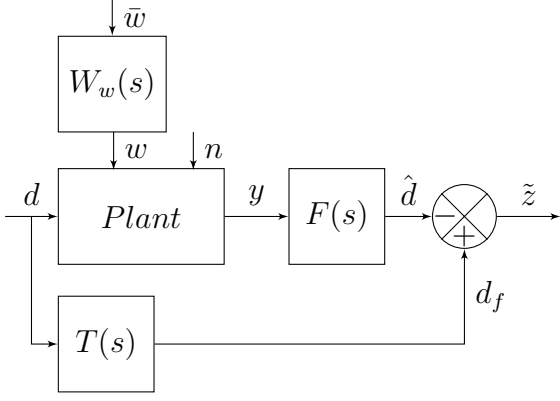


Figure 2: Block diagram of H_∞ filter synthesis

namic.

$$\begin{cases} \dot{x}_t = A_t x_t + B_t d \\ d_f = C_t x_t + D_t d \end{cases} \quad (6)$$

Moreover a weighting function is introduced on the road disturbance such that $w = W_f(s)\bar{w}$ in order to specify the frequency range and amplitude of disturbance attenuation.

$$\begin{cases} \dot{x}_w = A_w x_w + B_w \bar{w} \\ w = C_w x_w + D_w \bar{w} \end{cases} \quad (7)$$

In this application framework, both additional weighting functions are considered of first order i.e $n_t = 1$ and $n_w = 1$ which represent the number of state in (6) and (7). Hence the augmented state space representation:

$$\begin{cases} \dot{x}_a = A_a x_a + B_w w_a + 0.\hat{d} \\ \tilde{z} = C_z x_a + D_{zw} w_a - \hat{d} \\ y = C_y x_a + D_{yw} w_a + 0.\hat{d} \end{cases} \quad (8)$$

where $x_a = (x^T \ x_t^T \ x_w^T)^T \in \mathbb{R}^{10}$ is the extended state vector, $w_a = (d^T \ \bar{w}^T \ n^T)^T \in \mathbb{R}^4$ the external inputs and $\tilde{z} = d_f - \hat{d}$ is the estimation error to be minimized.

The synthesis of H_∞ filter is done, according to general control configuration shown in Figure 3.

Then, $P(s)$ including the extended state is described below:

$$P = \begin{pmatrix} A_a & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{pmatrix} \quad (9)$$

where:

$$A_a = \begin{pmatrix} A & 0 & WC_w \\ 0 & A_t & 0 \\ 0 & 0 & A_w \end{pmatrix}, \quad B_w = \begin{pmatrix} E & WD_w & 0 \\ B_t & 0 & 0 \\ 0 & B_w & 0 \end{pmatrix},$$

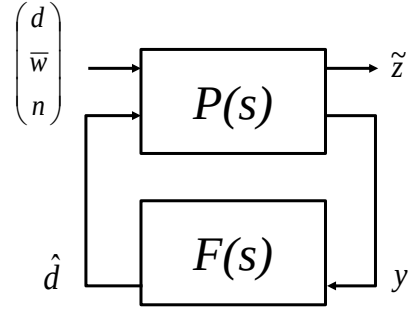


Figure 3: H_∞ general control configuration

$$B_u = (0 \ 0 \ 0)^T, \quad C_z = (0 \ C_t \ 0), \quad D_{zw} = (D_t \ 0 \ 0), \quad D_{zu} = -1, \quad C_y = (C \ 0 \ 0), \quad D_{yw} = (0 \ 0 \ N), \quad D_{yu} = 0.$$

Denoting the H_∞ filter $F(s)$ as:

$$\begin{cases} \dot{x}_f = A_f x_f + B_f y \\ \hat{d} = C_f x_f + D_f y \end{cases} \quad (10)$$

Now the problem is to design the standard H_∞ filter $F(s)$ that minimizes the H_∞ norm of the overall system from w_a to $\tilde{z}$. The system is given by:

$$\begin{cases} \dot{x}_{ag} = A_{ag} x_{ag} + (E_{ag} \ W_{ag} \ N_{ag}) w_a \\ \tilde{z} = C_{ag} x_{ag} + D_{ag} w_a \end{cases} \quad (11)$$

where:

$$A_{ag} = \begin{pmatrix} A & 0 & WC_w & 0 \\ 0 & A_t & 0 & 0 \\ 0 & 0 & A_w & 0 \\ B_f C & 0 & 0 & A_f \end{pmatrix}, \quad E_{ag} = \begin{pmatrix} E \\ B_t \\ 0 \\ 0 \end{pmatrix},$$

$$W_{ag} = \begin{pmatrix} WD_w \\ 0 \\ B_w \\ 0 \end{pmatrix}, \quad N_{ag} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ B_f N \end{pmatrix}, \quad C_{ag} =$$

$$(-D_f C \ C_t \ 0 \ -C_f), \quad D_{ag} = (D_t \ 0 \ -D_f N),$$

$$x_{ag} = (x^T \ x_t^T \ x_w^T \ x_f^T)^T \in \mathbb{R}^{18}, \quad \tilde{z} = d_f - \hat{d}.$$

The H_∞ filter synthesis problem is as follows: given the system (11), find matrices A_f , B_f , C_f and D_f such as $\|T_{\tilde{z}w_a}\|_\infty \leq \gamma_\infty$. This is solved by using a Linear Matrix Inequality (LMI) approach following [8]. If there exist symmetric matrices R , S satisfying the following LMI system:

$$\begin{pmatrix} N_R & 0 \\ 0 & I_{n_{w_a}} \end{pmatrix}^T \begin{pmatrix} AR + RA^T & RC_z^T & B_w \\ * & -\gamma I_{n_z} & D_{zw} \\ * & * & -\gamma I_{n_{w_a}} \end{pmatrix} \begin{pmatrix} N_R & 0 \\ 0 & I_{n_{w_a}} \end{pmatrix} < 0 \quad (12)$$

$$\begin{pmatrix} N_S & 0 \\ 0 & I_{n_z} \end{pmatrix}^T \begin{pmatrix} A^T S + SA & SB_w & C^T \\ * & -\gamma I_{n_{w_a}} & D_{zw}^T \\ * & * & -\gamma I_{n_z} \end{pmatrix} \begin{pmatrix} N_S & 0 \\ 0 & I_{n_z} \end{pmatrix} < 0 \quad (13)$$

$$\begin{pmatrix} R & I_n \\ I_n & S \end{pmatrix} \geq 0 \quad (14)$$

with $N_R = \text{Ker} \begin{pmatrix} B_u^T \\ D_{zw}^T \end{pmatrix}$, $N_S = \text{Ker} \begin{pmatrix} C_y \\ D_{yw} \end{pmatrix}$. Then, a positive definite matrix X is obtained as:

$$\begin{pmatrix} S & I_n \\ N^T & 0 \end{pmatrix} = X \begin{pmatrix} I_n & R \\ 0 & M^T \end{pmatrix}, \quad MN^T = I_n - RS \quad (15)$$

Hence, (16) is a LMI in $\begin{pmatrix} A_f & B_f \\ C_f & D_f \end{pmatrix}$, the solution gives the parameter of the H_∞ filter.

$$\begin{pmatrix} A_{ag}^T X + X A_{ag} & X(E_{ag} & W_{ag} & N_{ag}) & C_{ag}^T \\ * & -\gamma_\infty I & & D_{ag}^T \\ * & & * & -\gamma_\infty I \end{pmatrix} < 0 \quad (16)$$

3.2 Analysis of H_∞ filter synthesis

A compromise between estimation performance and disturbance rejection shall be found through the design parameter of the H_∞ filter. Indeed, the ideal case is to get $T(s) = 1$ which means to estimate the driver torque regardless of his frequency content. Hence, Table 2 shows the influence of the weighting function $T(s)$ on H_∞ filter synthesis. Variations of the cut-off frequency and order of $T(s)$ have been performed. Then, the estimation performance has been assessed through the steady state gain $T_{dd}(\omega = 0)$ (that should be close to 1) and the maximal disturbance rejection $\|T_{zw}\|_\infty \leq \gamma_\infty$ (i.e value of γ_∞ is expected to be as small as possible).

$T(s)$ 1 st order			
w_n	5Hz	10Hz	25Hz
γ_∞	0.15	0.27	0.51
steady state gain	0.91	0.75	0.49
$T(s)$ 2 nd order			
$\zeta = 0.7$ and w_n	5Hz	10Hz	25Hz
γ_∞	0.12	0.12	0.34
steady state gain	0.98	0.98	0.69

Table 2: Influence of weighting function $T(s)$ on the H_∞ filter synthesis

In view of Table 2, increasing the bandwidth of $T(s)$ leads to a degradation of estimation, (as γ_∞ gets close to 1). The previous overview shows that a second order filter $T(s)$ is convenient. From (11), the transfer matrix $T_{zw} = [T_{zd}; T_{zw}; T_{zn}]$ of the sensitivity functions is as follows:

$$T_{zd}(s) = C_{ag}(sI - A_{ag})^{-1}E_{ag} + D_t \quad (17)$$

$$T_{zw}(s) = C_{ag}(sI - A_{ag})^{-1}W_{ag} \quad (18)$$

$$T_{zn}(s) = C_{ag}(sI - A_{ag})^{-1}N_{ag} + D_k N \quad (19)$$

Figures 4, 5 and 6 illustrates the sensitivity function deduced from (17) – (19) with different tuning for $T(s)$. It is again noticed that a cut-off frequency at 25Hz gives less efficient results than with 5Hz or 10Hz. Indeed, the optimization problem is harder to be solved when considering a greater frequency range. As both remaining synthesis show similar performance, the one designed with 10Hz is preferable.

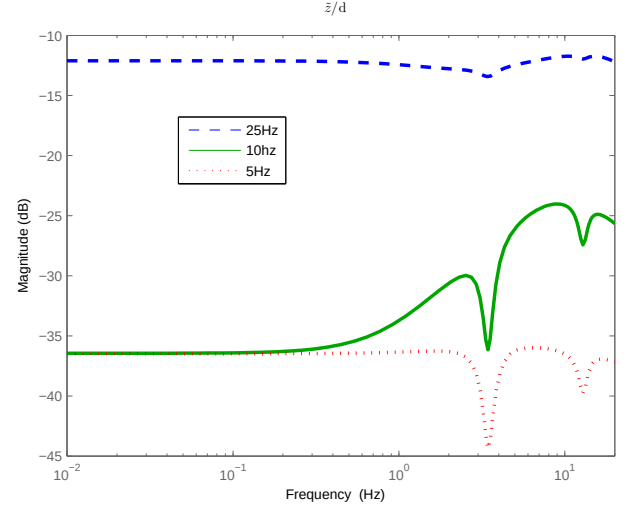


Figure 4: Influence of various $T(s)$ design on $T_{zd}(s)$

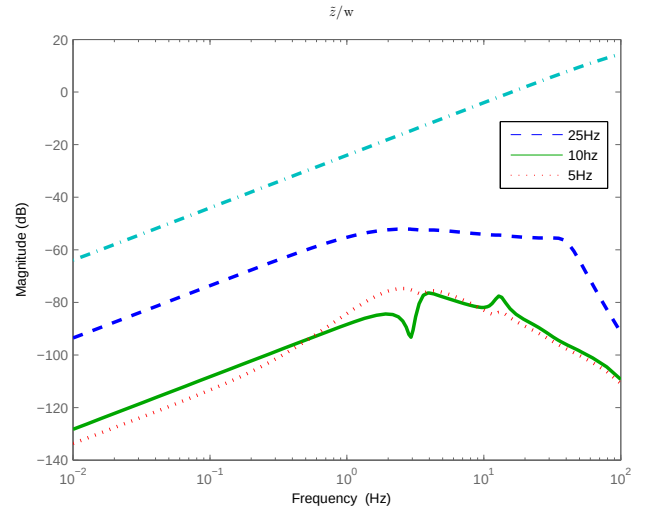


Figure 5: Influence of various $T(s)$ design on $T_{zw}(s)$

For simulation part, the driver torque is estimated following a H_∞ filter $F(s)$ design method where $T(s)$ is a 2nd order filter with cut off frequency at 10Hz.

4 SIMULATION RESULTS

In this section, simulation results are presented using the H_∞ filter designed in the previous section. The Figure 7 illustrates the experimental implementation scheme. It is worth noting that, to evaluate the estimation error, a sensor is available on the experimental test car for driver torque measurement, referred to as $d_{measured}$.

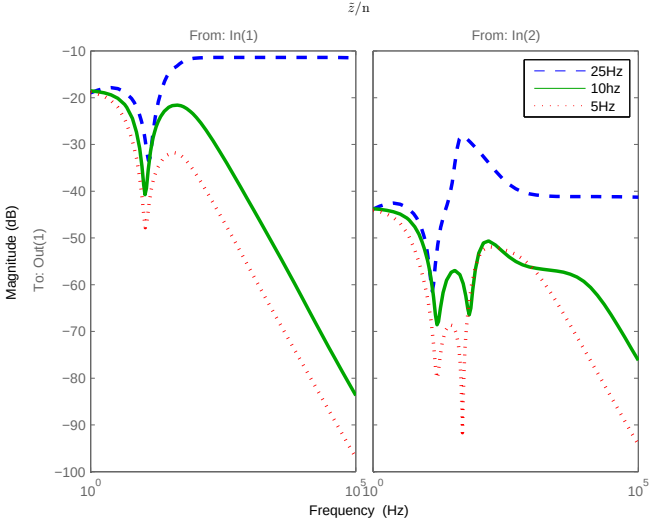


Figure 6: Influence of various $T(s)$ design on $T_{zn}(s)$

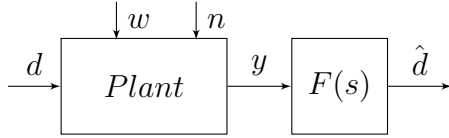


Figure 7: Block diagram of H_∞ filter implementation

4.1 Simulation case 1

According to the representation in Figure 7, the following simulation scenario has been considered. A sinusoidal driver torque of $d = 5Nm$ at $0.5Hz$ is applied as input. The disturbance affecting the plant is represented by the road effort. At the beginning, it is a constant of $w = 1kN$ and changed to $w = 5kN$ at $2.5s$. Also sensor noises of amplitude $N = 1e^{-2}$ is added to the measured output.

Figure 8 shows the measured driver torque $d_{measured}$, the estimated driver torque $\hat{d}$, the filtered driver torque d_f (synthesized signal) and the resulting estimation error $\tilde{z} = d_f - \hat{d}$ and $z = d_{measured} - \hat{d}$.

The root mean squared error (RMSE) is calculated as:

$$RMSE(z) = \sqrt{\frac{\sum_{i=1}^m z(i)^2}{m}} \quad (20)$$

where $z = d_{measured} - \hat{d}$ i.e difference between the real data and the estimated value, m is the number of data points. The normalized RMSE (NRMSE) on the whole range is computed by:

$$NRMSE(z) = \frac{RMSE}{d_{measured}^{max} - d_{measured}^{min}} \quad (21)$$

The obtained errors are: $RMSE(z)=0.27Nm$ (resp. $NRMSE(z)=2.67\%$) and $RMSE(\tilde{z})=0.076Nm$ (resp. $NRMSE(\tilde{z})=0.74\%$). The H_∞ filter estimates accurately the applied driver torque $d_{measured}$, since the

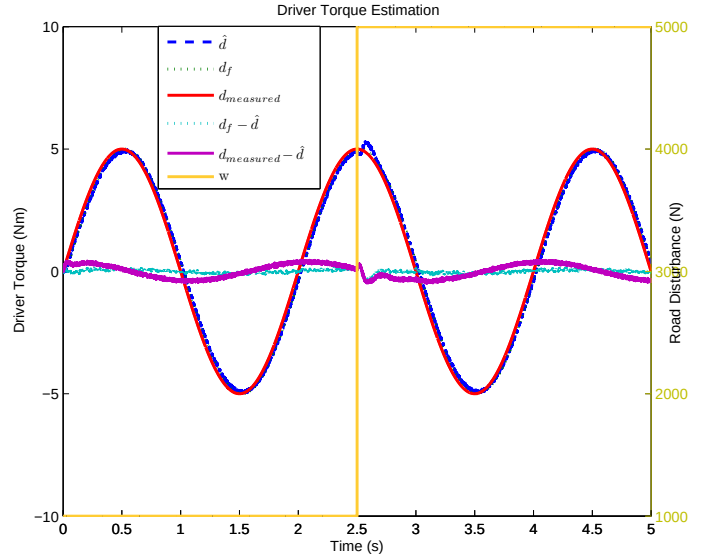


Figure 8: Driver torque estimation -Sinusoidal driving case 1

computed error is small enough. Thus, the H_∞ filter have an acceptable performance level. It is worth noticing that the H_∞ filter has been designed introducing the weighting function $T(s)$ and that the optimization problem has been solved for $\tilde{z}$, which explains this slightly better estimation result.

4.2 Simulation from experimental input data

In this case, the simulation environment takes only the H_∞ filter $F(s)$ part from Figure 7, to validate the performance of the estimation using as inputs real measurements i.e $y = (\theta_c \ \theta_m)^T$. Data acquisition has been done on a development car equipped with a P-EPS system and several specific sensors (motor resolver, torque sensor, steering sensor and driver torque sensor).

The first test case comes from a particular driving condition where the driver follows an '8' shaped pattern on the track at a constant speed of 20kph. This involves to get a sinusoidal driver torque. Figure 9 shows the comparison between the measured driver torque and the estimated torque.

Another test measurements come from parking maneuver. The driver turns slowly the steering wheel with a small steering angle amplitude. As it is in parking, the effort required from the driver is high. The comparison between the measured driver torque and the estimated torque shown in Figure 10 emphasizes the efficiency of the estimation.

The resulting errors are: for experimental data 1 is $RMSE(z)=3.18 Nm$, $NRMSE(z)=6.63\%$ and for experimental data 2 is $RMSE(z)=1.1Nm$, $NRMSE(z)=4.94\%$. Since experimental data are gathered from real system where non-linearity (mainly friction) are influent, whereas in the H_∞ filter design this element has been neglected, the estimation results differ from the two types of simulation. Moreover, road disturbance condition differs

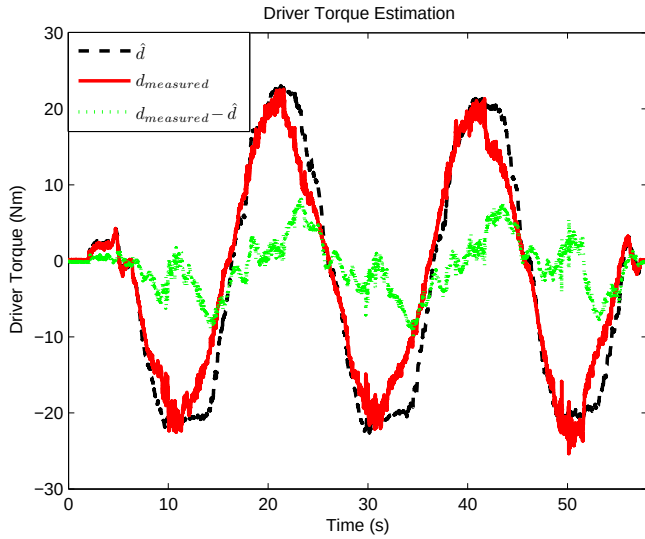


Figure 9: Driver torque estimation using experimental data- Sinusoidal driving

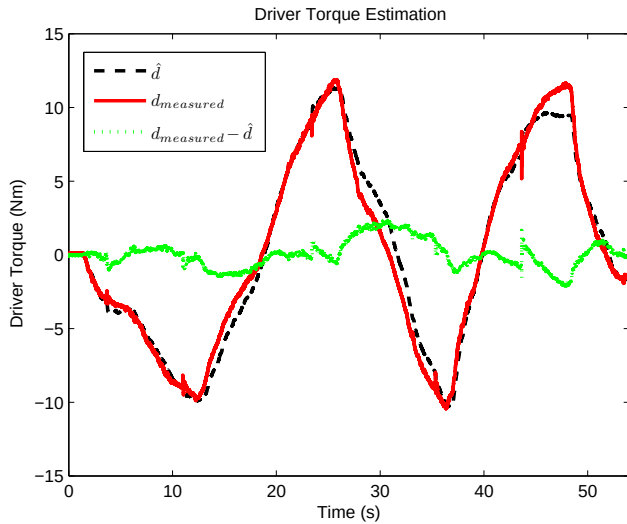


Figure 10: Driver torque estimation using experimental data- Parking maneuver

from parking to rolling, this explains the variation in performance level. However, according to the above simulation results, good performance of the driver torque estimation by H_∞ filter strategy is obtained using experimental data as inputs.

5 CONCLUSION

In this paper, an H_∞ filter has been designed subject to minimization objective on road disturbance and sensor noises. Then, performance has been evaluated by simulation using experimental measurements.

The estimation of driver torque presented in this paper is application-oriented and aims to be a back-up calculation to cover failure of torque sensor signal. As this is a software strategy, it could be easily implemented in the ECU. Moreover, it only requires measurements from sensors existing on board of most of mass produced vehicle equipped with an EPS system.

Even though driver torque estimation might be affected by friction on EPS system in practice (due to its

non-linearity), the design of a robust controller could result in reducing the unwanted effect to the driver. Hence, future works will concentrate on the whole design of an observer-controller system to provide assistance torque without using torque sensor.

ACKNOWLEDGMENT

This study has been supported by JTEKT corporation in Japan: Miyama Shinpukujicho Okazaki-shi, Aichi-ken, 444-2106 Japan.

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