New techniques for indoor positioning, combining deterministic and estimation methods

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BIOGRAPHY

Magda Chelly received her engineering diploma in Telecommunications in 2007, and a Masters degree in the Architecture of Telecommunication Systems in 2008, both from the "Ecole Supérieure des Télécommunications de Tunis", Tunisia. She is a Ph.D. student in Telecommunications at the "TELECOM & Management SudParis", Evry, France, where she is working on indoor positioning systems.

Nel Samama is member of the Navigation Group at IT-SudParis. He has been working for many years in the GPS field, on projects such as the development of new communication schemes and indoor location techniques. His current interest is to find a simple approach for a standard GPS receiver to enable indoor positioning.

INTRODUCTION

Each year, new localisation techniques elaborate new potential objectives. In recent years they have been trying to accost the hidden part of the localisation systems. In fact, each technique deployed outdoors, cannot easily be deployed indoors or when we have an environment with difficulties. This phenomenon is due to the large number of obstacles that a transmitted signal has to pass, as in the case of GPS systems.

The scientific world has carried out research to improve indoor methods, using different techniques. In the following article, we describe some of these techniques and try to understand their functionalities, study the advantages, the drawbacks, and make comparisons. The real aim of this paper is to create a quasi-exhaustive list of the existing methods to localize or position a mobile device, in extremely difficult conditions. To obtain the final position, systems use data that they compute with mathematical equations by different ways. In general, they use the distances or angles between a mobile and reference points or bases. These data are employed to reach the outcome.

I. TRIANGULATION AND TRILATERATION

In this paragraph, we describe the general approaches that could be used in localisation systems. In fact, the equipment employed varies and each one has its own characteristics. The study of the intrinsic and extrinsic environment is necessary to avoid all surprises when the process is working.

The final results are related to the effects of the neighborhood, the environment, the mobile equipment and many other things that must be taken into consideration.

One of the most used and best-known methods for localisation systems is the triangulation or trilateration method. It represents the basic mathematical method for the calculation of the position using distances or angles between transmitters and receivers.

Today, the best-known system is the GPS system, which is based on trilateration. The performance of this method is attractive and efficient in fixed conditions, achieving a precision of centimeters. Many other systems use this method to obtain the final result, meaning the coordinates of a point. We can consider the triangulation as the origins of the trilateration method. Triangulation is the process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline.

The trilateration method uses the distance between transmitter and receiver to calculate the final results.

Different methods are used to determine the

distance between the base stations or reference points and the subject. We can divide these methods into deterministic ones and probabilistic ones. First, we describe the probabilistic methods of calculating the position and second, we describe the probabilistic methods [1].

II. DETERMINISTIC METHODS

II.1 TIME OF ARRIVAL

The common process to acquire this measure is the Time of Arrival (TOA). It obtains the range information via the signal time. To use this means, accurate synchronization between equipments is necessary. To put an accurate time reference, we can add to the mobile equipment a Cesium or a Rubidium clock. Going back to the principle of this concept, the measurement of the time that the signal takes to be transmitted from a known transmitter to an unknown receiver is the key to the calculation. In fact, this time interval is multiplied by the speed of the signal. At the end, we obtain the distance transmitter -receiver.

For a two-dimensional positioning, three distances between emitter and receiver are sufficient. In the case of the GPS, we must note that a higher number of bases (emitters) are used due to fact that the two solutions given by the non linear system obtained are situated on earth and in the sky. The solution in this case is evident, in certain precise conditions. When we are based with the receiver on earth, we need four transmitters to find the coordinates of our point. The fourth satellite is in fact necessary for the synchronization processes and not to determine the point being sought.

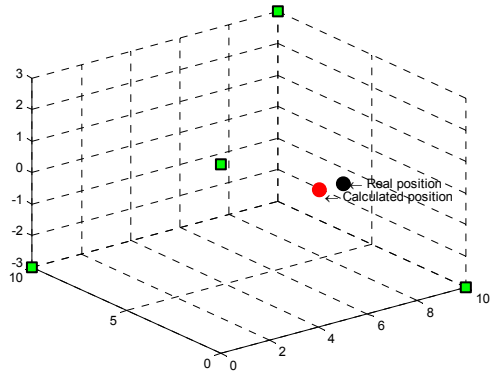

Figure 1 Resulting position with TOA method

Figure 1 shows the distribution of four mobile devices. The initial position is $x = 6.5, y = 2, z = 0.5$.

With the method used above, the calculated position in a local referential is estimated as equal to the point in red. The error is a few meters [2].

II.2 TIME OF ARRIVAL DIFFERENCE

Another technique for the trilateration process is the Time Difference of Arrival (TDOA). It's computed by transmitting a signal which is delayed. A correlation analysis is made to obtain the path difference between receivers I and j.
The receivers' signal is:

$$y_i(t) = a_i * (t - \tau_i) + e_i(t) \quad i = 1,2,3,...,n \qquad (1)$$

Where the receiver i is located at (x_i, y_i) and the transmitter is in $(x\ ,y\ )$, which are unknown.

The common way to compare the signals received is to use pairs and take a correlation function.

$$\Delta d_{i,j} = v * (\tau_i - \tau_j) \quad 1 \le i \le j \le n \qquad (2)$$

Where v is the speed of sound, light or water vibrations. Here, n is the number of receivers and (i, j) is an enumeration of all K pair of receivers, where $K \le n/2$. Each $d(i, j)$ corresponds to positions $(x\ ,y\ )$ along a hyperbola.

Obtaining this system of equations, an estimation step is needed after In fact, these estimated TDOAs are transformed into range measurements [3] [4].

II.3 ANGLE OF ARRIVAL

The latest well-known method is AOA. The AOA is the angle between the propagation wave and the reference direction, with a known orientation. The orientation is represented in degrees in a clockwise direction from the north. The AOA can be relative or absolute, regarding the orientation. Two non-linear neighboring bases are required to obtain a location when the orientations are known and three neighboring bases when we want to discover the orientation.

II.4 HOP BY HOP

In many cases the techniques in wireless networks use the trajectory of the signals to obtain some metrics. In fact, in AD-Hoc networks the number of hops is frequently used.

An algorithm called hop-by-hop is based on transmitting data in a network from one piece of equipment to another to obtain the distance between the transmitter and receiver. Every device communicates with a single neighbor. This method can give us the result of the distance between two pieces of equipments. Using the method of trilateration each node can compute its own position with at least three distances. Evidently, a node can have many difficulties calculating its position. Errors, like the lack of the number of ranges used or the systems without a converging solution could be an important obstacle to obtaining the final result.

The method of Hop-By-Hop is used in relatively small networks, in which each node can only see its neighbor and not the whole architecture. This rule creates limits to this technique. To avoid this limitation, another calculation system has been evaluated.

II.5 DV-HOP

Another method used in Ad-Hoc networks is to use the number of hops correlated to the distance necessary for one hop, called Distance-Vector Hop. This distance is an average from all the distances calculated, in other terms it is the average distance between all single pairs in the network. The final distance reached is the number of hops multiplied by the average described before.

This technique comprises by three essential steps. The first step is based on the classical exchange between nodes to get the distances between each other in hops. Each node has a table with all the information needed and it must do an update very frequently. In the second step, the landmark estimates the average of one hop, which is deployed as a correction to the nodes in its neighborhood. The third step consists in calculating the position with the trilateration mode. The correction that a landmark computes is:

$$c_i = \sum \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \Big/ \sum h_i \qquad \textbf{(3)}$$

$i \neq j$

For all landmarks j

The major inconvenience of this method is that when networks are not homogenous and can present very different distances between nodes. In this case, the average used can cause a decrease in the accuracy of the final position.
In order to avoid errors, the method of DV-Hop is only used with isotropic networks, to avoid errors. In fact, in these graphs, the properties are the same in all directions. The advantages are the simplicity and the fact that it does not depend on range measurement error. The same position is taken to estimate the coordinates with this method.

In fact, the method uses the number of hops between nodes of the network to estimate the position. The result depends on the distribution of the mobile equipments and the size of the network.

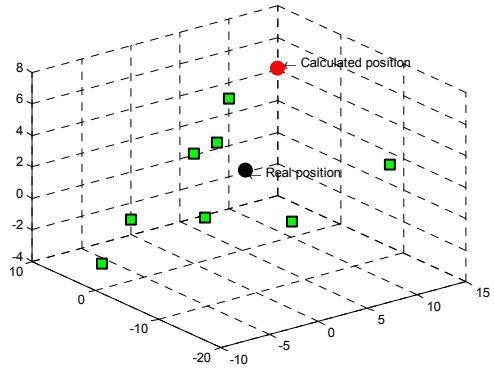

Figure 2 Resulting position with DV-Hop method

Figure 2 shows the coordinates of the position are $x = 9.0541, y = 0.9459, z = 6.7568$. The error is more important due to fact that a geographic distance could not be represented by a hop in a network with accurate results. The nodes could be distributed randomly and the number of hops could be very small and the same for different nodes far from the position.

II.6 DV-DISTANCE PROPAGATION METHOD

This method of calculating the distance uses the propagation model to calculate the distance between two nodes, using the signal strength. The next steps are identical to the DV-Hop method, described in the previous section.

II.7 EUCLIIDIAN PROPAGATION MODEL

This method uses the Euclidean distance to the landmark. A node from the network has to have

at least two neighbors, which contain their distances from the landmark.

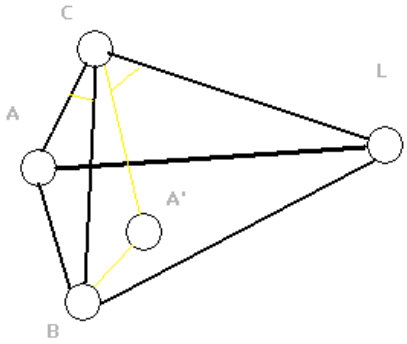

Figure 3 Principle of Euclidian method

Figure 3 explains the system. Node A has acquired distances to nodes B and C. B and C have estimates to L. The distances AB, AC and BC are known. The sides of the quadrilateral ABCL are known. A necessary condition is that A and B are also neighbors. When we establish the final calculation, we can achieve the results with two final points: A and A'. We can then with an estimate of AL determine the right point. Having the estimate for the obtained range may, in certain cases, reduce the average positioning error by up to 50 %.

$$\cos(\alpha) = AB^2 - AC^2 - BC^2 \Big/ 2 * AC * BC \quad \textbf{(4)}$$

$$\cos(\beta) = BL^2 - BC^2 - CL^2 \Big/ 2 * CL * BC \quad \textbf{(5)}$$

$$AL^2 = AC^2 + CL^2 - 2 * AC * CL * \cos(\beta \neq \alpha) \quad \textbf{(6)}$$

$$\sigma_{AL}^2 = \Sigma (\partial AL \Big/ \partial e)^2 * \sigma_e^2 \quad \textbf{(7)}$$

e = AC, CL, LB, BA, BC.

The uncertainty σ_{AL}^2 propagated together with the actual length AL to nodes which are farther from the landmark L. The advantage of this method is that it provides better accuracy under certain conditions, and there are no corrections to be deployed later, once a node has ranges from three landmarks, it may estimate the position by itself [5].

II.8 DV COORDINATE PROPAGATION METHOD

This method requires some preprocessing. In the case that the second hop deviation is available, each node can establish a local coordinate system. As we can see in the figure, node A can build its local coordinates system using its immediate neighbors and the ranges between them. A has the coordinates $(0,0)$. In the case of A, E is chosen as an indicator for x (a) axis and F for y (a). Using known ranges to eliminate ambiguities, all immediate neighbors of A are added to the local coordinate system. Every node in the network independently builds its own coordinate system centered on itself.

Each coordinate sent from one node to another, must be translated to the right coordinate system. Nodes A and B have coordinates of nodes A; B; C and D in both coordinate systems, which are used to compute associate transformation matrices used for translation from one system to another.

Nodes that have the ranges from their neighbors in their coordinate systems could not localize themselves. They have to translate all the data from one point to another. The next steps are similar to the preceding method [6].

II.9 DISTANCE CALCULATION USING RECEIVED RSSI

Some of the techniques described before, use the signal strength to calculate the distance between the transmitter and the receiver.

II.10 PROPAGATION METHOD

These techniques are based on the propagation model, which is extremely accurate outdoors but not yet perfect indoors.

The propagation model in an outdoor environment can be represented like this:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 \quad \textbf{(8)}$$

Pr is the power received by the receiver and Gr is the gain of the antenna. Pt and Gt are the same measurements but transmitted by the transmitting antenna. λ is the wavelength and d is the distance between the transmitter and the receiver. Since the measurement is typically received, it is possible to get the distance d using the following formula:

$$d = \frac{\lambda}{4\pi} \sqrt{\frac{P_t G_t G_r}{P_r}} \quad \textbf{(9)}$$

In indoor environments, this formula is slightly changed and the propagation model used is as follows:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^n \quad \textbf{(10)}$$

The theory has defined the variable n as being equal to 2 for the external environment. For the indoor, n varies from one research to another. In fact, we need experimental measurements to determine the possible value in defined conditions. With this equation we can determine the distance between the emitter and the receiver. In the case of indoor environments the radio wave are submit to different type of perturbations, this is the major negative effect on the final calculation of the distance. Our previous research defined experimentally a variable n equal to 3.5 as a best one for the indoor system base in the TELECOM & Management SudParis. This conclusion was established by doing many measurements of the signal strength and distances in the buildings.

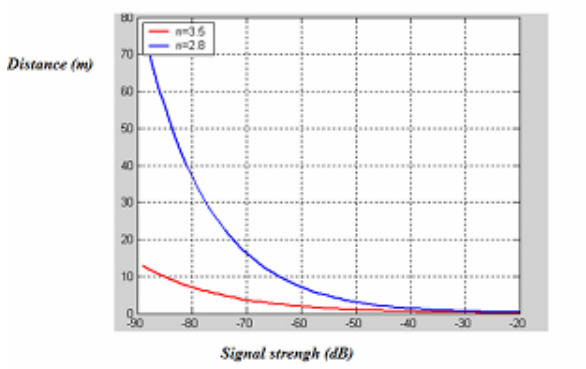

Figure 4 Distance related to the signal strength

Figure 4 below shows the evolution and the important difference of distance's measurements with various n [7].

II.11 LOCAL RANGE AND BEARING SENSING

This technique is based on a study of different devices detected in the neighborhood. After this study, we choose the two strongest signals and their transmitters. The principle of this algorithm is based on the calculation of the angle offset within the quadrant to find the bearing of the transmitter.

$$\theta = a * (v1 - v2) \quad \textbf{(11)}$$

$$\phi = Quadrant\_Angle + \theta \quad \textbf{(12)}$$

A is a calibrated constant. If $\theta \succ \pi/4$, θ is set to $\pi/4$ where the objective is to remain in the same quadrant. To calculate the range, the algorithm assumes that the signal intensity V1 is given by:

$$v1 = D * \cos(\pi/4 - \theta) \quad \textbf{(13)}$$

The equation of the range then becomes:

$$D = v1 / \cos(\pi/4 - \theta) \quad \textbf{(14)}$$

This is followed by a non-linear transformation using a look-up table to get the actual distance.

Several changes to this method were established to offer better accuracy. The original formula would calculate an offset which was three times as large at the closest range. So the step needed to improve the results is to use the two best signal strengths received and divide them by the strongest.

$$\theta = b * (v1 - v2)/v1 \quad \textbf{(15)}$$

$$\phi = Quadrant\_Angle + \theta \quad \textbf{(16)}$$

b is still a calibrated constant. This makes the calculation of the angle much more constant for different ranges. The second aspect is that the range term is calculated using the bearing offset. This introduces the error from the bearing offset into the equation, making the result less accurate. A better approach is to use:

$$D = \sqrt{(v_1^2 + v_2^2)} \quad \textbf{(17)}$$

The third method used is based on another equation. This term takes into consideration the variation of received signal intensity. To avoid other perturbations, it is approximate like that:

$$D = \sqrt{(v_1^2 + v_2^2)} + c * v_1^2 * \cos(\theta) \quad \textbf{(18)}$$

Additional modifications can be made to these methods to improve the final result. The intensities of signals can be modelled as below:

$$v1 = D * \sqrt{\cos(\pi/4) - \theta} \quad \textbf{(19)}$$

$$v2 = D * \sqrt{\sin(\pi/4) - \theta} \quad \textbf{(20)}$$

These two equations, give the calculation of the distance:

$$D = \sqrt{(v_1^2 + v_2^2) * \cos(\pi/4 - \theta)} + c * v_1^2 * \cos(\theta) \quad \textbf{(21)}$$

The errors vary with the method used to calculate the distance. The last algorithm is the more efficient according to the results achieved when using it. All the errors were calculated using an interpolation between known points of the range term.

II.12 RECURSIVE ESTIMATION

The network in this case has several reference points, with known coordinates and the rest of the points' coordinates are unknown. This condition supposes that the distances between some emitters and some receivers are obvious and we can calculate them exactly. The system takes into consideration an arbitrary number of network nodes. The first algorithm uses four or more reference points. The steps of the algorithm consist in:

Determining the reference points of the network.

Collecting range estimates for the selected nodes and estimating the final position.

In the first phase, the node collects the list of reference points that it can receive. Then, it must choose the reference points that it will use. This selection is based on the calculation of the residual values, a measure of confidence, described as shown below:

$$\mathrm{Re}\, sidual(x, y, z) = \sum_{i \in refrence} (\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - d_i)^2 \quad \textbf{(22)}$$

The node chooses the points with the lowest values for the next phase. Once the reference points have been chosen, the node must calculate the distance estimates to each reference. The node can reject a reference point for which it can't find the distance. With the distances to the reference points, the node can estimate its position. Equations that we can achieve are given as follow:

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + \varepsilon_i \quad \textbf{(23)}$$

ε represents the error term.

The distances are calculated iteratively to find all the possible ones in the network.

II.13 MINMEAN

This technique is based on a calculation of time difference propagation between two different signals. The idea consists using a combination of the radio waves and ultrasound to enable a listener to determine the distance to the beacons. On each transmission, the transmitter sends information over the radio waves adding an ultrasonic pulse. The receiver has to analyze the radio wave first and then a little time after, it must analyze the ultrasonic pulse, which has a delay of transmission. The listener uses the time difference between the receipt of the first bit of radio wave information and the ultrasonic pulse in order to determine the distance to the beacon. It is also possible to define this distance using the velocity of the two signals.

Identifiers of the radio wave and the ultrasonic pulse are unique pairs in the system, which allows the listener to correlate the two signals properly.

The simplest algorithm simply picks the beacon with the highest frequency of occurrence in the data set. In the MinMean algorithm, the listener calculates the mean distance from each beacon for the set of data points within the data set. Then, it selects the beacon with the minimum mean as the closest one. The problem with this algorithm is that it doesn't take into consideration the multi-path effects.

III. PROBABILISTIC METHODS

In this section, we describe the methods of calculating the distances between transmitters and receivers or other parameters which could help us to solve the final system, as in the previous section. Various schemes are described below. The bases of these techniques have the probabilistic approach as a common point.

Major complications for indoor measurements are due to nonlineofsight propagation. This means that waves transmitted by the equipment are not directly transmitted to the receiver. They are subjected to many obstacles and phenomena, such as distortion. Research is being done in other

directions using estimation and probability.

III.1 A MINIMUM ENTROPY ESTIMATION

This scheme uses a different approach for the calculation of the time of arrival (TOA), taking into consideration the noise that can influence the measurements. In the absence of disturbance, the TOA measured is:

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}$$
$$t_i = {}^{d_i}\!/_{c} \quad \textbf{(24)}$$
$$i = 1...n$$

c is the speed of light.

So, with the disturbance that can affect waves the distance could be described like this:

$$r_i = d_i + n_i \quad \textbf{(25)}$$

The parameter n is considered as the noise that attains the wave. This noise is represented as a Gaussian noise. The position of the mobile is defined as $\theta = (x, y)$ with x and y the coordinates of each point. Usually the position estimates $\hat{\theta}$ are obtained by minimizing a cost function $J(\hat{\theta})$ of the residual $e(\theta)$:

$$e_i = r_i - d_i = r_i - \|BS_i - \theta\| \quad \textbf{(26)}$$
$$i = 1...n$$

$$J(\theta) = \sum_{i=1}^{n} \varphi(e_i(\theta)) \quad \textbf{(27)}$$

The classical estimator used for this calculation is:

$$\hat{\theta}_{ML} = \arg\min_{\theta \in \Theta} [\sum_{i=1}^{n} \log \rho(e_i(\theta))] \quad \textbf{(28)}$$

To avoid problems with the use of this estimator, the minimization of the entropy of an estimate $\hat{\rho}_{n,h}(e)$ of $\rho(e)$ is needed. The $\hat{\rho}_{n,h}(e)$ is a kernel estimate based on the empirical distribution of the random sample $e_1(\theta)...e_n(\theta)$. In general, it is assumed that the convergence of $\hat{\rho}_{n,h}(e)$ to $\rho(e)$ is reached. Then, the entropy criterion is built on $\hat{\rho}_{n,h}(e)$. However, the minimization of the entropy of $\hat{\rho}_{n,h}(e)$ is hard to achieve, since the entropy of such a function is invariant by translation. Thus, the minimization of the entropy of $\hat{\rho}_{n,h}(e)$ has been shown to be more suitable, where e_E an extended vector is given by: $e_E = \{e, -e\}$. The criterion is given by the following expression:

$$J_e(\theta) = H_A(\hat{\rho}_{n,h}(e_E)) \quad \textbf{(29)}$$

Where

$$H_A(f) = -\int_{-A}^{A} f(x) * \log f(x) dx \quad \textbf{(30)}$$

Finally the minimum-entropy estimator is denoted by $\hat{\theta}_{MEE}$. Our objective function is:

$$\hat{\theta}_{MEE} = \arg\min_{\theta \in \Theta} J_e(\theta) \quad \textbf{(31)}$$

The choice of the Gaussian kernel for this calculation is very important.

III.2 ANCHORS METHOD

Another new method to calculate the distance between two points is based also on an estimation system. The network employed has to have points named anchors. The anchors represent the reference points. These points are known and have their positions. We define these points with this equation:

$$X\,Anchors = [x_i] \quad \textbf{(32)}$$
$$1 \prec i \prec m$$

The data can be employed in a matrix p x m.
The rest of the unknown points are named Xobject and they are represented as follows:

$$X\,Objects = [x_j] \quad \textbf{(33)}$$
$$m+1 \prec i \prec N$$

With a matrix p x n, representing that.
Let $\hat{x}_j$ be the estimation of x_j. The goal will be to minimize the function:

$$J(\hat{x}) = \sum_{m+1 \prec i \prec N} \sqrt{(\hat{x}_i - x_i)}^2 \quad \textbf{(34)}$$

To estimate our positions, we use the communication between points in the network.
Let

$$d_{i,j} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{p} (x_{ik} - x_{jk})^2} \quad \textbf{(35)}$$

be the Euclidian distance between two points I

and j. Let $\hat{d}_{i,j}$ be the estimation for $d_{i,j}$. The objective here is to minimize this function:

$$J2(\hat{x}) = \sum_{m+1 \prec i \prec N} \sqrt{(\hat{d}_{i,j} - \left\| \hat{x}_i - \hat{x}_j \right\|)^2} \quad (36)$$

At the beginning, the data employed are the distances between anchors. The goal is to calculate the other ones.

The estimations of distance can be based on equitable indicators:

Propagation time of a wave.

The signal at the reception (RSSI: Received Signal Strength Indicator).

The rate of errors corrected in communications (LQI: Link Quality Indicator).

If cij is the value of the indicator between nodes i and j and if the function f allows us to determine an estimate of the distance between i and j, we have:

$$\hat{d}_{i,j} = f(c_{i,j}) \quad (37)$$

The general model of estimation is based on a linear regression between the indicator cij and the distance. The data used is composed by K RSSI values for each couple (i,j) of nodes in the network. In this case, the distance $d_{i,j}$ is known.

The average value of the K moments of time is given by:

$$\bar{c}_{i,j} = \sum_{k=1}^{K} c_{i,j} \Big/ K \quad (38)$$

The estimated distance is described as follows:

$$\hat{d}_{i,j} = a * \bar{c}_{i,j} + b \quad (39)$$

Where a and b should minimize the error E_{RMSE}^{dist}.

$$E_{RMSE}^{dist} = \sqrt{\sum_{i,j \in (1,N)} (\hat{d}_{i,j} - d_{i,j})^2 \Big/ N} \quad (40)$$

With the last square min and these data $\bar{c} = \sum_{i,j} \bar{c}_{i,j}$ and $\bar{d} = \sum_{i,j} \bar{d}_{i,j}$, we have this solution:

$$a = \sum_{i,j} (\bar{c}_{i,j} - \bar{c}) * d_{i,j} \Big/ \sum_{i,j} (\bar{c}_{i,j} - \bar{c}) * c_{i,j} \quad (41)$$

$$b = \bar{d} - a * \bar{c} \quad (42)$$

Another model can be used. Each node gets an estimate of distance with all its neighbors using Gaussian noise. The model used is:

$$\hat{d}_{i,j} = \begin{cases} \phi \_\_\_\_\_ for\ d_{i,j} \succ d\max \\ N(d_{i,j}, \sigma) \_\_\_\_ for\ d_{i,j} \leq d\max \end{cases} \quad (43)$$

III.3 DISCRETE METHOD

In many complicated cases, when the localisation system could not obtain the coordinates with the classical method, probabilistic algorithms are used. In this paragraph, we describe discrete methods. The localisation problem involved only non-deterministic uncertainty. In addition, it was assumed that nature does not interfere with the state transition. Otherwise, the I-states are not predictable.

For the active localisation problem, this means that a localisation plan must use information feedback. In other words, the actions must be conditioned on I-states so that the appropriate decisions are taken after new observations are made. An update is necessary. The passive localisation problem involves computing probabilistic I-states from the sensing and action histories. Only the non-deterministic I-space I_{ndet} will be considered, for the explanations.

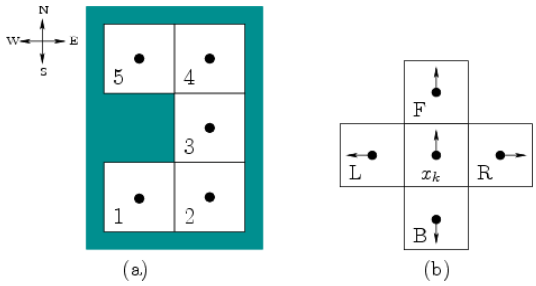

Figure 5 Possible actions

Figure 5.a This map is given to the robot for localisation purposes Figure 5.b The four possible actions each take one step, if possible, and re-orient the robot as shown.

Figures 5.a and 5.b show a map that can be created by acquainting the coordinates of each point detected for a period of time. The equipment detecting its successive positions does not have its initial coordinates and it creates a grid using these acquainted points' coordinates.

To formulate this problem, we must include in the state both the position and the orientation of the mobile equipment. We suppose that the mobile equipment is represented by a robot. The robot may be oriented in one of four directions, which are labeled N, E, W, and S, for ``north,'' ``east,''

``west," and ``south," respectively. The state of the robot is determined by $x \in X, x = (p,d)$, where p is a position and d is a direction.

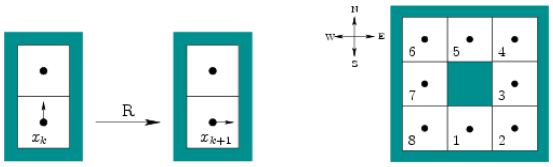

Figure 6 Possible directions

Figure 6.a If a direction is blocked because of an obstacle, then the orientation changes, but the position remains fixed. In this example, the R action is applied. Figure 6.b Another map is given to the robot for localisation purposes.

In this case, the robot cannot localise itself exactly.

The robot is given four actions,

$$U = \{F, B, R, L\} \quad \textbf{(44)}$$

which represent ``forward,'' ``backward,'' ``right motion,'' and ``left motion,'' respectively. These motions occur with respect to the current orientation of the robot, which may be unknown. For the F action, the robot moves forward one grid element and maintains its orientation. For the B action, the robot changes its orientation by 180 degrees and then moves forward one grid element. For the R action, the robot turns right by 90 degrees and then moves forward one grid element. The L action behaves similarly. If it is not possible to move because of an obstacle, it is assumed that the robot changes its orientation (in the case of B, R, or L) but does not change its position. The robot has one simple sensor that can only detect if it was able to move in the direction that was attempted. The sensor space is $Y = \{0,1\}$, the sensor mapping is $h : XxX - - > Y$. This yields $y = h(x_{k-1}, x_k)$ if x_{k-1} and x_k place the robot at different positions, and $h(x_{k-1}, x_k) = 0$ otherwise. Thus, the sensor indicates whether the robot has moved after the application of an action, or not. Non deterministic uncertainty will be used, and the initial I-state η_0 is always assumed to be X (this can easily be extended to allow us to start with any non-empty subset of X. A history I-state at stake k in its general form appears as

$$\eta_0 = (X, \tilde{u}_{k-1}, y_2, ..., y_k) \quad \textbf{(45)}$$

There is no observation y_1 because the sensor mapping requires a previous state to report a value. Thus, the observation history starts with y_2. An example history I-state for stage $k = 5$ is

$$\eta_R = (X, R, R, F, L, 1, 0, 1, 1) \quad \textbf{(46)}$$

in which $\eta_0 = (X), \tilde{u}_4 = (R, R, F, L)$, and $(y_2, y_3, y_4, y_5) = (1,0,1,1)$. The passive localisation problem starts with a given map, such as the one shown before, and a history I-state η_k, and computes the non-deterministic I-state $X_k(\eta_k) \subseteq X$. The active localisation problem is to compute some k and sequence of actions $(u_1, ..., u_{k-1})$, such that the non-deterministic I-state is as small as possible. In the best case, $X_k(\eta_k)$ might become a singleton set, which means that the robot knows its position and orientation on the map. However, due to symmetries, which will be presented shortly in an example, it might not be possible.

A state is once again expressed as $x = (p,d)$. The initial condition is a probability distribution, $P(x_1)$ over X. One reasonable choice is to make $P(x_1)$ a uniform probability distribution, which makes each direction and position equally likely. The robot is once again given four actions, but now assumes that nature interferes with state transitions. For example, if $u_k = F$, then perhaps with high probability the robot moves forward, but with low probability it may move right, left, or possibly not move at all, even if it is not blocked. The sensor mapping indicated whether the robot moved. In the current setting, nature can interfere with this measurement. With low probability, it may incorrectly indicate that the robot moved, when in fact it remained stationary. Conversely, it may also indicate that the robot remained still, when in fact it moved. Since the sensor depends on the previous two states, the mapping is expressed as

$$y_k = (x_k, x_{k-1}, \psi_k) \quad \textbf{(47)}$$

With a given probability model, $P(\psi_k)$, this can be expressed as $P(y_k | x_k, x_{k-1})$.

To solve the passive localisation problem, the expressions for computing the derived I-states are applied. If the sensor mapping used only the current state, then successive expressions would apply without modification. However, since h

depends on both x_k and x_{k-1}, some modifications are needed. Recall that the observations start with y_2 for this sensor. Therefore $P(x_1|\eta_1) = P(x_1|y_1) = P(x_1)$.

After each stage, $P(x_{k+1}|\eta_{k+1})$ is computed from $P(x_k|\eta_k)$. It reduces using marginalization as

$$P(y_k|\eta_{k-1},u_{k-1},x_k)= \sum_{x_{k-1}\in X} P(y_k|\eta_{k-1},u_{k-1},x_{k-1},x_k)*P(x_{k-1}|\eta_{k-1},u_{k-1},x_k)$$

(48)

The first factor in the sum can be reduced to the sensor model,

$$P(y_k|\eta_{k-1},u_{k-1},x_{k-1},x_k) = P(y_k|x_{k-1},x_k)$$

(49)

because the observations depend only on x_{k-1},x_k, and the nature sensing action, ψ_k. The second term below can be computed using Bayes's rule as

$$P(y_k|\eta_{k-1},u_{k-1},x_k) = P(x_k|\eta_{k-1},u_{k-1},x_{k-1})*P(x_{k-1}|\eta_{k-1},u_{k-1}) : \sum_{x_{k-1}\in X} P(x_k|\eta_{k-1},u_{k-1},x_{k-1})*P(x_{k-1}|\eta_{k-1},u_{k-1})$$

(50)

in which $P(x_k|\eta_{k-1},u_{k-1},x_{k-1})$ simplifies to $P(x_k|u_{k-1},x_{k-1})$. This is directly obtained from the state transition probability, which is expressed as $P(x_{k+1}|u_k,x_k)$ by shifting the stage index forward. The term $P(x_{k-1}|\eta_{k-1},u_{k-1})$ is given before. Then the computation of the probabilistic I-states is necessary which solves the passive localisation problem.

Solving the active localisation problem is substantially harder because a search occurs on I_{prob}. The same choices exist as for the discrete localisation problem. Computing an information-feedback plan over the whole I-space I_{prob} is theoretically possible but impractical for most environments.

III.4 CONTINOUS METHOD

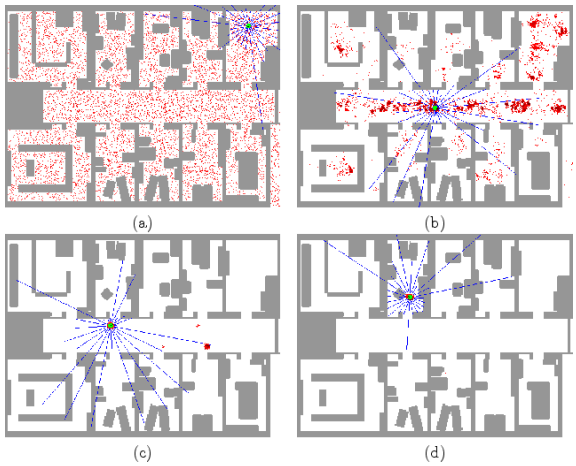

Figure 7 Continous space

Localisation in a continuous space using probabilistic models has received substantial attention in recent years. It is often difficult to localize mobile equipment. Probabilistic modeling and the computation of probabilistic I-states have been quite successful in many experimental systems, both for indoor and outdoor mobile robots. Figure 7 shows localisation successfully being solved using sonars only. The vast majority of work in this context involves passive localisation because the robot is often completing some other task, such as reaching a particular part of the environment. Therefore, the focus is mainly on computing the probabilistic I-states, rather than performing a difficult search on I_{prob}.

Probabilistic localisation in continuous spaces most often involves the definition of the probability densities $P(x_{k+1}|u_k,x_k)$ and $P(y_k x_k)$ (in the case of a state sensor mapping). If the stages represent equally spaced times, then these densities usually remain fixed for every stage. The state space is usually $X = SE(2)$ to account for translation and rotation, but it may be $X = \Re^2$ for translation only. The density $P(x_{k+1}|x_k,u_k)$ accounts for the unpredictability that arises when controlling a mobile robot over a fixed time interval. A method for estimating this distribution for non-holonomic robots by solving stochastic differential equations can be used to calculate the final result.

The density $P(y_k|x_k)$ indicates the relative likelihood of various measurements when given

the state. Most often this models distance measurements that are obtained from a laser range scanner, an array of sonars, or even infrared sensors. Suppose that a robot moves around in a 2D environment and takes depth measurements at various orientations. In the robot body frame, there are n angles at which a depth measurement is taken. Ideally, the measurements should look like those in the figure above; however, in practice, the data contain substantial noise. The observation $y \in Y$ is an n-dimensional vector of noisy depth measurements.

One common way to define $P(y \mid x)$ is to assume that the error in each distance measurement follows a Gaussian density. The mean value of the measurement can easily be calculated as the true distance value once x is given, and the variance should be determined from the experimental evaluation of the sensor. If it is assumed that the vector of measurements is modeled as a set of independent, identically distributed random variables, a simple product of Gaussian densities is obtained for $P(y \mid x)$.

Once the models have been formulated, the computation of probabilistic I-states directly follows. The initial condition is a probability density function, $P(x_1)$ over X. The marginalization and Bayesian update rules are then applied to construct a sequence of density functions of the form $P(x_k \mid \eta_k)$ for every stage k.

In some limited applications, the models used to express $P(x_{k+1} \mid x_k, \mid u_k)$ and $P(y_k \mid x_k)$ may be linear and Gaussian. In this case, the Kalman filter can easily be applied. In most cases, however, the densities will not have this form. Moment-based approximations can be used to approximate the densities. If second-order moments are used, then the so-called extended Kalman filter is obtained, in which the Kalman filter update rules can be applied to a linear-Gaussian approximation to the original problem. In recent years, one of the most widely accepted approaches in experimental mobile robotics has been to use sampling-based techniques to directly compute and estimate the probabilistic I-states. The particle-filtering approach appears to provide a good experimental performance when applied to localisation. The application of particle filtering in this context is often referred to as Monte Carlo localisation [8] [9].

IV. COMPARATIVE BALANCE OF ALL THE TECHNIQUES

Studying the techniques and approaches employed for the localisation systems; we can conclude that each one has its advantages and drawbacks.

Establishing a real comparison is not possible in practice because approaches are not based on the same fundamentals. Each technique use data that are not the same and tries to analyze them as possible.

A general table, figure 8, with the achievements of all methods will be implemented next to give an idea about achieved results. As a general analysis we can assume that the probabilistic methods are based on previous states of the mobile, in other words the historical steps. Often the equipment to be localized needs a global localisation and does not have the previous positions. This is the most important drawback of these methods.

	Historical data	Size of the environment	Accuracy	Reliability	Outdoors	Indoors
Time of arrival	NOT USED	NM	GOOD	GOOD	USED	NOT USED
Time Difference of arrival	NOT USED	NM	GOOD	GOOD	USED	NOT USED
Angle of Arrival	NOT USED	NM	GOOD	GOOD	USED	NOT USED
Hop-By-Hop	NOT USED	SMALL	POOR	POOR	USED	USED
DV-Hop	NOT USED	SMALL	POOR	POOR	USED	USED
DV-Distance propagation method	NOT USED	SMALL	POOR	POOR	USED	NOT USED
"Euclidean" propagation model	NOT USED	NM	MEDIUM	GOOD	USED	USED
DV-coordinate propagation method	NOT USED	SMALL	POOR	POOR	USED	NOT USED
Propagation model	NOT USED	NM	MEDIUM	POOR	USED	USED
Local range and bearing sensing	NOT USED	NM	MEDIUM	POOR	USED	USED
Recursive estimation	NOT USED	SMALL	GOOD	POOR	USED	NOT USED
MinMean	NOT USED	NM	POOR	POOR	USED	NOT USED
A minimum entropy estimation	USED	LARGE	POOR	POOR	USED	USED
Anchors method	USED	LARGE	POOR	GOOD	USED	USED
Discrete method	USED	LARGE	POOR	POOR	USED	USED
Continuous method	USED	LARGE	POOR	POOR	USED	USED

Figure 8 Comparison table between the methods

CONCLUSION

In this paper, we have set out the different approaches that have been established recently for positioning.

Each method is characterized by advantages, drawbacks and is restricted by the environmental conditions. However, probabilistic methods are generally based on historical data and the first established position may not be accurate.

This article allows us to have a broad idea of the implementation approaches and give us the possibility to draw up a new approach of localisation, based essentially on the spatial relations between mobile equipments. Our recent work is based on modeling an environment composed by mobile networks using graphs. It would be interesting to make divers researches towards this direction.

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