

# Analysis of Heat Transfer in Dense Suspensions by Particle Resolved Numerical Simulations

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# Outline

## Context and Reviews

- Forced convection around isolated particle

- Non-isothermal flows through Static Array of particles

## Numerical method and code validation

- Tensorial penalty method

- Validation test cases

## Analysis of heat transfer in dense Suspension

- Calculation of heat transfer coefficient

- Result compared to correlations

## Random mobile Suspension

- Suspended Particle

- Heat transfer in fluidized bed

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## Context and goals

### Non-isothermal particle laden flows in dense regime

Particle-particle interaction, Particle-wall interactions

#### Heat exchange

- Between phases
- Between phases and wall
- Nusselt number, function of
  - ▶ Solid fraction
  - ▶ Structure of the suspension
  - ▶ ....

#### Industrial Applications

- Chemical Looping Combustion
- $CO_2$  Capture
- Carbon neutral-energy
  - ▶ Generation
  - ▶ Transformation

## Context and goals

### Non-isothermal particle laden flows in dense regime

Particle-particle interaction, Particle-wall interactions

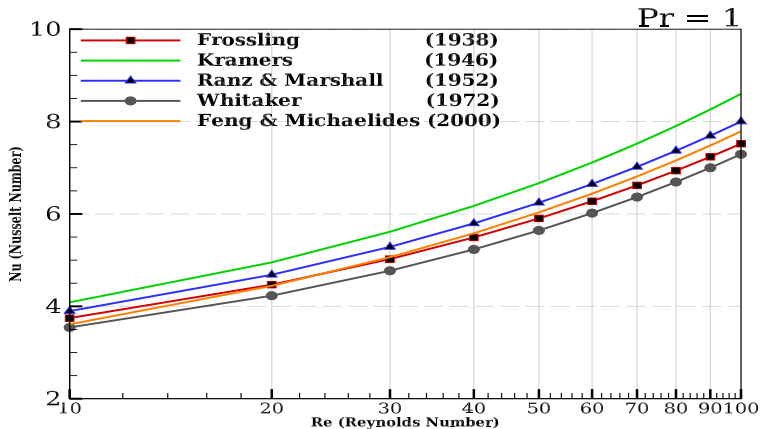
#### Heat exchange

- Between phases
- Between phases and wall
- Nusselt number, function of
  - ▶ Solid fraction
  - ▶ Structure of the suspension
  - ▶ ....
- Cases of validation for Single particle
- Fluid-particle heat transfer coefficient
  - ▶ Fixed assembly of particles, mobile suspension

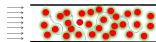
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$$Nu \sim f(Re, Pr)$$

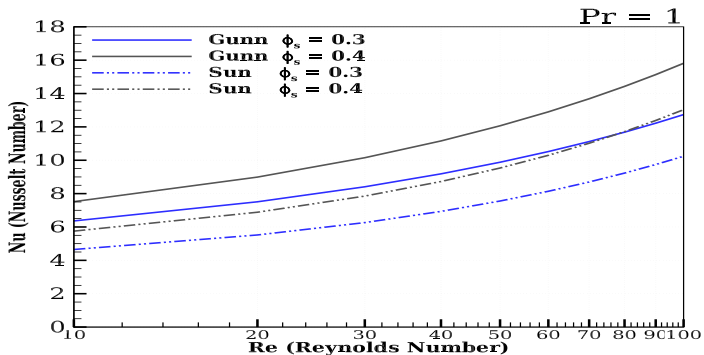


$$Nu \sim f(Re, Pr, \phi_s)$$



⊕ Gunn (Int. J. Heat and Mass Transfer 1977)

⊕ Sun & al (Int. J. Heat and Mass Transfer 2015)



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$$\nabla \cdot \mathbf{u} = 0$$

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \rho \mathbf{g} + \underbrace{\nabla \cdot (\mu (\nabla \mathbf{u} + \nabla^t \mathbf{u}))}_{\text{viscous stress}} + \mathbf{F}_{si}$$

$$\frac{\partial \mathbf{C}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{C} = 0$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) + \ell T = \nabla \cdot (\lambda \nabla T) + S.$$

$$\nabla \cdot \mathbf{u} = 0$$

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- Tensorial Penalty Method (Vincent & Caltagirone J. Comput Phys 2011 )

$$\nabla \cdot (\mu (\nabla \mathbf{u} + \nabla^t \mathbf{u})) = \nabla \cdot [\kappa \Lambda((\mathbf{u}))] + [\zeta \Theta((\mathbf{u}))] + [\eta \Gamma((\mathbf{u}))]$$

Solid behaviour ,  $\mathbf{C} = 1 \because \eta \gg 1, \zeta = \kappa = 2\eta$

- Darcy Penalty Method (Kadra & al. Int. J. Numer Meth Fluids 1999)
- Constant temperature in the solid particle :  $\ell T, S$

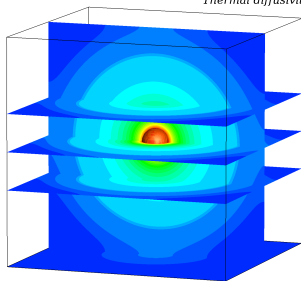
# Conduction

*Cubic Domain :  $L = 8D_p$*

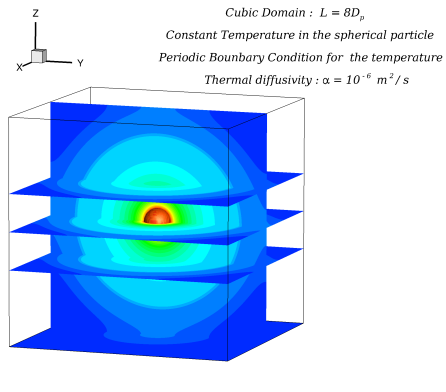
*Constant Temperature in the spherical particle*

*Periodic Boundary Condition for the temperature*

*Thermal diffusivity :  $\alpha = 10^{-6} \text{ m}^2/\text{s}$*



# Conduction



$$\frac{T(r, t) - T_\infty}{(T_s - T_\infty)} = \left( \frac{R}{r} \left( 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\xi^2} d\xi \right) \right)$$

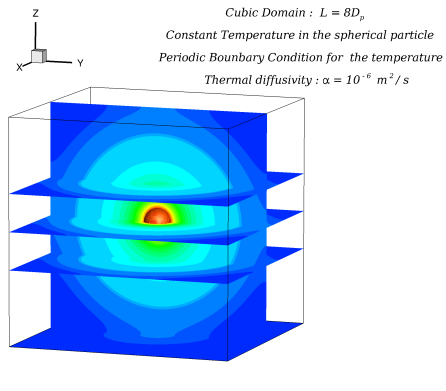
$$r \geq R, \quad \eta = \frac{r - R}{\sqrt{4\alpha t}}$$

$$Nu_a(t) = 2 + \frac{2R}{\sqrt{\pi\alpha t}}$$

$$Nu_s(t) = \frac{\Phi D_p}{4\pi R^2 (T_s - T_\infty) k_f}$$

$$E_r(\%) = 100 \frac{|Nu_a(t) - Nu_s(t)|}{|Nu_a(t)|}$$

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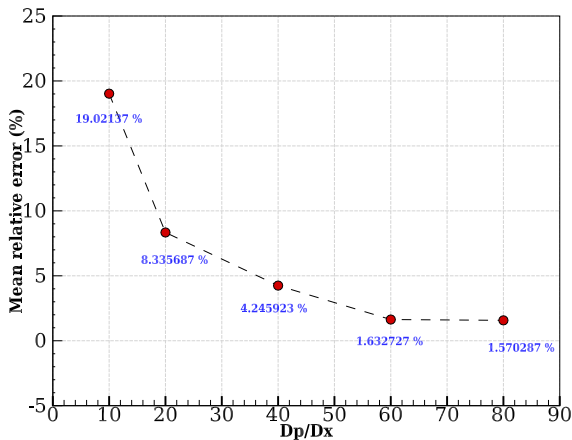
$$\Phi = \iint_{S_\nabla} -k_f \vec{\nabla} T(x_b, y_b, z_b) \cdot \vec{n}_\nabla dS_\nabla$$

Spherical particle surface is approximated by triangular elements and the flux is computed as :

- ▶  $\Phi = \sum -k_f \vec{\nabla} T(x_b, y_b, z_b) \cdot \vec{n}_{\nabla} dS_{\nabla}$
- ▶  $(x_b, y_b, z_b)$  barycenter of each triangle
- ▶  $\vec{n}_{\nabla}$  normal vector of each surface element
- ▶  $dS_{\nabla}$  surface of each triangle

$$\left. \frac{dT}{dx} \right|_i = \frac{-T_{i+2} + 8(T_{i+1} - T_{i-1}) + T_{i+2}}{12Dx} + \mathcal{O}(h^4)$$

Mean relative error  $Nu$ ,  $f\left(\frac{D_p}{D_x}\right)$

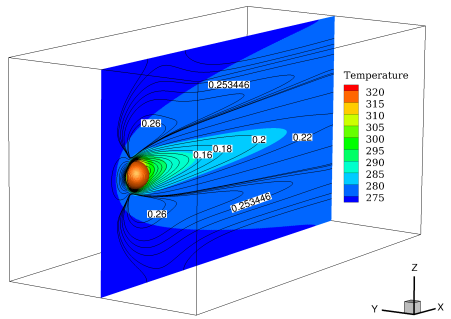


## Forced Convection

*Forced Convection around a stationary spherical particle*

Domain :  $L_x = 15D_p$  ;  $L_y = 8D_p$  ;  $L_z = 8D_p$

Reynolds number  $Re = 50$





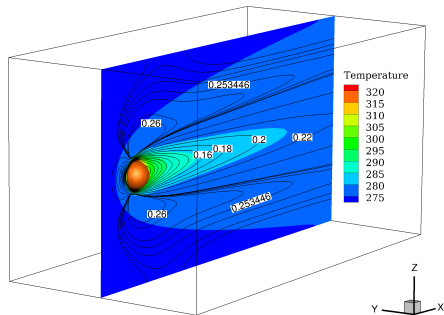
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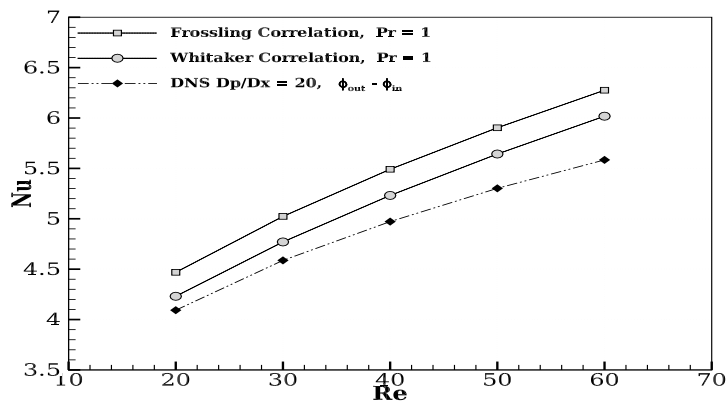
- Darcy Penalty Method (DPM)
- $Re = 10 \dots 60$ ,  $Pr = 1$

$$Nu = \frac{h_f D_p}{k_f}$$

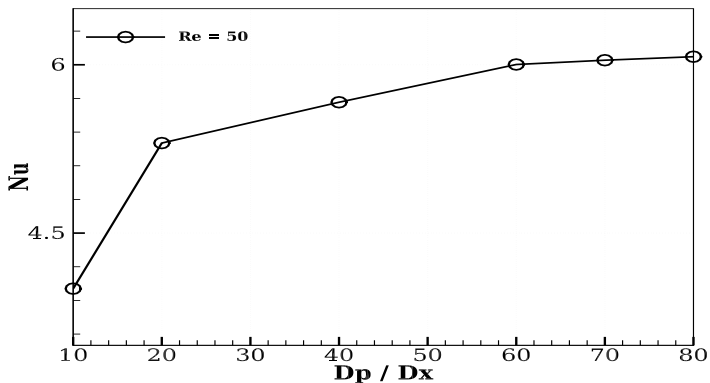
$$h_f = \frac{\Phi}{\Delta T (4\pi R^2)}$$

$$\Phi = \Phi_{out} - \Phi_{in}.$$

$$\Phi = \rho_f C_p \iint_{S_{out}} U_x(L_x, y, z) T(L_x, y, z) dydz - \rho_f C_p \iint_{S_{in}} U_x(0, y, z) T(0, y, z) dydz$$



$$\Phi = \rho_f C_p \iint_{S_{out}} U_x(L_x, y, z) T(L_x, y, z) dydz - \rho_f C_p \iint_{S_{in}} U_x(0, y, z) T(0, y, z) dydz$$



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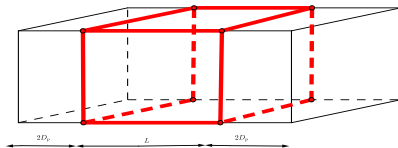
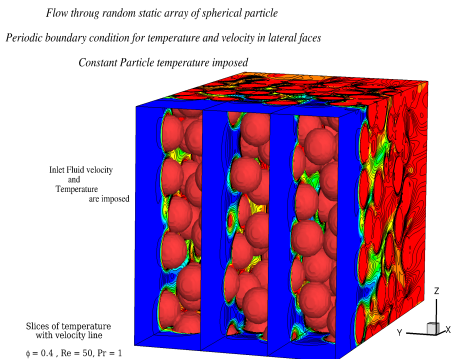
Result compared to correlations

## Random mobile Suspension

Suspended Particle

Heat transfer in fluidized bed

## Conclusion



- Darcy Penalty Method (DPM)
- Imposed Inlet (T,V)
- Randomly distributed particle
- $\phi_s = 0.1, 0.2, 0.3, 0.4$
- $Re = 10, 50, 100 ; Pr = 1$

- $\chi(x, y, z) = 1 - \mathbf{C}(x, y, z)$  Phase Function
- $T_b$  Cup-mixing bulk temperature

$$T_b(x) = \frac{\iint_{A_c} \chi(x, y, z) U_x(x, y, z) T(x, y, z) dydz}{\iint_{A_c} \chi(x, y, z) U_x(x, y, z) dydz}$$

- Total convective heat flux

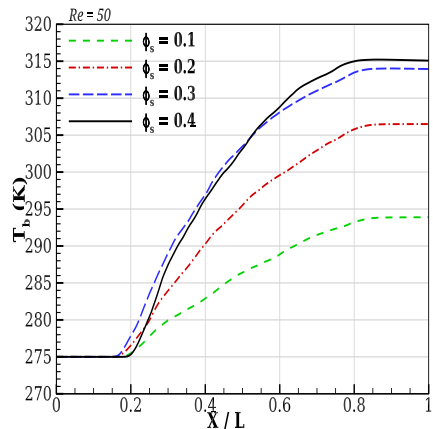
(Deen & al Chem. Eng. Sci 2012; Tavassoli & al Int. J. Multiph. Flow 2013)

$$Q = U_\infty \rho_f C_p \frac{dT_b(x)}{dx} = h_f \frac{6\phi_s}{D_p} (T_s - T_b(x)).$$

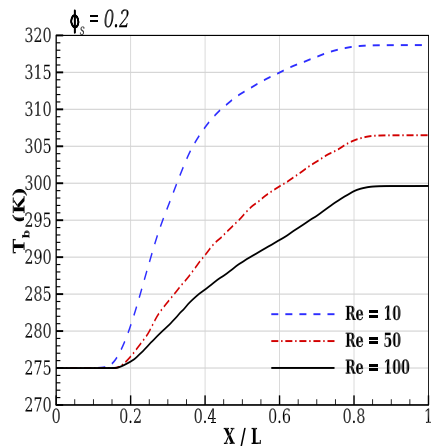
- The heat transfer coefficient assumed to be constant

$$h_f = -\ln\left(\frac{T_s - T_{b_{out}}}{T_s - T_{b_{in}}}\right) \left(\frac{U_\infty \rho_f C_p}{\frac{6\phi_s}{D_p} L}\right)$$

# Bulk temperature

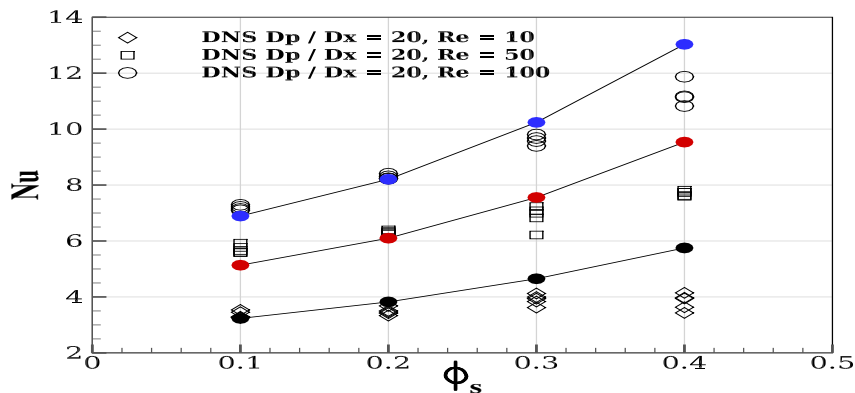


## Bulk temperature





$$Nu \sim f(Re, \phi, Pr = 1)$$



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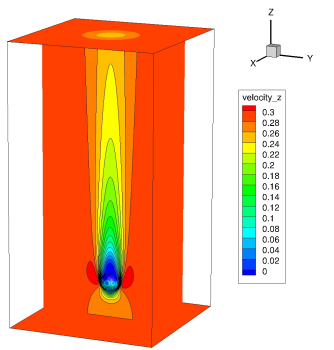
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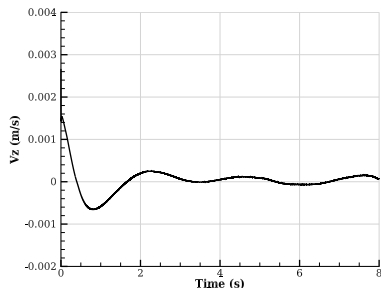
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- Tensorial Penalty Method (TPM)

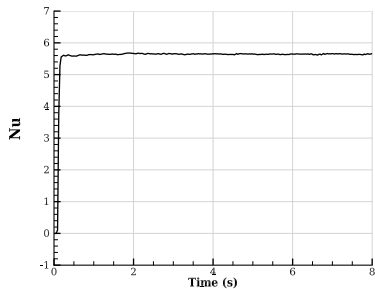
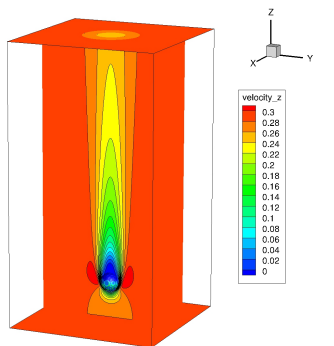


- Inlet velocity and gravity
- Balance of forces
- $Re = 58$



## Suspended Particle

- Tensorial Penalty Method (TPM)



$$Re = 58, Nu = 5.4 \text{ (DPM)}$$

## Heat transfer in fluidized bed

- Periodic and Circulating
- Porous Wall
- Soft Sphere collision
  - ▶ Particle-particle
  - ▶ Particle-wall

J. C. B de Motta & al (Physics of Fluids 2013)

- Several solid fraction
  - ▶ Settling velocity
  - ▶ Fluidization velocity

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  - Conduction
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- ▶ Convergence study and code validation
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  - Heat transfer Analysis
  - Statistical dependence
- ▶ Random mobile suspension
  - Local heat transfer coefficient
  - Instantaneous distribution of particles
  - More random configurations of particles

*Tanks to Université Paul Sabatier de Toulouse for the funding.  
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Thank you for your attention

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