NUMERICAL SIMULATION OF COLLISION DYNAMICS FOR FINITE SIZE PARTICLES IN TURBULENT SUSPENSION
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ICMF 2016, Firenze, March 26th 2016


## Outline

The particle laden flows

Numerical approach Collision model

Simulations
Particles dynamics
Collisions
Average flow around particles

Conclusion

## Goals and outline

The main goal of this presentation is to:

- Understand collisions for finite-size particles in a HIT.

The presentation follow those points:

- Context and motivations
- VoFLag method for finite size particles
- Lubrication phenomena and its model
- HIT characteristics
- Collisions in a HIT


## The particle laden flows

## Particle laden flows are common:

In natural flows:

- Rivers
- Volcanos
- Clouds
- Microorganism

In industrial applications:

- Fluidized bed
- Combustion Chamber


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Source:
http://geography.unt.edu

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Source: Wikipedia

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Source: Wikipedia
EFalkovich et al. - Nature - Vol. 4194 (2002).

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- Microorganism

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- Combustion Chamber


Source: Godfrey, NIWA
Durham et al. - Nature - 2148 (2013)
Denny et al. - Biological Bulletin - 203 (2002)

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- Combustion Chamber


Source: Linassier - Thèse Université de Toulouse (2012)

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In natural flows:

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- Volcanos
- Clouds
- Microorganism

In industrial applications:

- Fluidized bed
- Combustion Chamber


## The particle laden flows

Many parameters drive those flows:

- Density Ratio
- Particle size
- Shape
- Flow
- Solid fraction

Different numerical approaches exist:

- Microscopic
- Mesoscopic
- Macroscopic


## Numerical approaches

Many parameters drive those flows:

- Microscopic (Fully Resolved Particles)
- Mesoscopic (Point-wise Particles)
- Macroscopic (Continuum phase)


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## Numerical approaches

Many parameters drive those flows:

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- Mesoscopic (Point-wise Particles)
- Macroscopic (Continuum phase)


Microscopic method permits to understand surrounding fluid behaviour

## Method



## Thetis code 12 M㟔

- MAC staggered unstructured grid
- Finite volumes
- Fictitious Domain Approach

The solid behaviour is enforced by high viscosity.

Randrianarivelo et al. Int. J. Num. Meth. in Fluids (2005)

- Vincent et al. - Comp. \& Fluids (2007)

EVincent et al. - J. Comp. Phy. (2013)

## Collisions



## Collisions



Ardekani et Rangel - J. Fluid. Mech. - Vol. 596 (2008)

## Collisions



Navier-Stokes solution

## Collisions



Navier-Stokes solution

## Collisions




A model is required.

## Collisions



$$
\mathbf{F}_{\mathbf{l}}\left(\epsilon, \mathbf{u}_{\mathbf{n}}\right)=-6 \pi \mu_{f} \mathbf{R} \mathbf{u}_{\mathbf{n}}\left[\lambda(\epsilon)-\lambda\left(\epsilon_{a l}\right)\right]
$$

## Collisions



Navier-Stokes solution Lubrication model Soft-sphere model

- $\mu_{f}$ dynamic viscosity
- $R$ particle radius
$\mathbf{F}_{\mathbf{I}}\left(\epsilon, \mathbf{u}_{\mathbf{n}}\right)=-6 \pi \mu_{f} \mathbf{R} \mathbf{u}_{n}\left[\lambda(\epsilon)-\lambda\left(\epsilon_{a l}\right)\right]$
- $\mathbf{u}_{n}$ normal velocity
- $\epsilon=\frac{d}{R}$ dimensionless distance
- $\lambda$ Taylor series

```
Brenner - Chem. Eng. Sci. - Vol. 16 (1961)
```


## Collisions



Navier-Stokes
$\quad$ solution
Lubrication model
Soft-sphere model

## Collisions



Navier-Stokes solution

Lubrication model
Soft-sphere model

$$
\mathbf{F}_{\mathbf{s}}=\underbrace{\frac{m_{e}\left(\pi^{2}+\left[I n e_{d}\right]^{2}\right)}{\left[N_{c} \Delta t\right]^{2}}}_{\text {spring }} \mathbf{d}-\underbrace{\frac{2 m_{e}\left[\ln e_{d}\right]}{\left[N_{c} \Delta t\right]}}_{\text {dashpot }} \dot{\mathbf{d}}
$$

[^0]
## Collisions

Lubrication model
Soft-sphere model

- $m_{e}$ reduced mass

$$
\mathbf{F}_{\mathbf{s}}=\underbrace{\frac{m_{e}\left(\pi^{2}+\left[I n e_{d}\right]^{2}\right)}{\left[N_{c} \Delta t\right]^{2}}}_{\text {spring }} \mathbf{d}-\underbrace{\frac{2 m_{e}\left[\ln e_{d}\right]}{\left[N_{c} \Delta t\right]}}_{\text {dashpot }} \dot{\mathbf{d}}
$$

- $e_{d}$ coefficient of restitution in vaccum
- $N_{c}$ number of time step during collision
- $\Delta t$ time step (fluid)


## Collisions



## Collisions



Navier-Stokes solution Lubrication model Soft-sphere model

## Before Collision

After collision


$$
V=V_{a}-V_{b}
$$

$$
e=\frac{v_{\text {after }}}{v_{\text {before }}}
$$

## Collisions



# Navier-Stokes solution 

Lubrication model Soft-sphere model


$$
S t=\frac{2}{9} \frac{R V \rho_{p}}{\mu_{f}}
$$

Joseph et al. - J. Fluid. Mech. - Vol. 433 (2001)

## Collisions



Navier-Stokes solution

Lubrication model Soft-sphere model


$$
\frac{e_{I}}{e_{d}}=\exp \left(-\frac{35}{S t}\right)
$$

Legendre et al. - Phys. Fluids - Vol. 17 (2005)

## Collisions



Navier-Stokes solution

Lubrication model
Soft-sphere model


Legendre et al. - Phys. Fluids - Vol. 17 (2005)

## Collisions



Navier-Stokes solution

Lubrication model
Soft-sphere model


Legendre et al. - Phys. Fluids - Vol. 17 (2005)

## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition

$$
\begin{array}{ll} 
& R e_{\lambda}=73 \\
& R e_{\Lambda}=315 \\
L_{b}=2 \pi & \frac{\rho_{p}}{\rho_{f}}=\{1,2,4\} \\
\frac{L_{b}}{\Lambda}=2 \pi & S t_{k}=26 \frac{\rho_{p}}{\rho_{f}} \\
\frac{\Lambda}{\lambda}=4.3 & S t_{E}=1.5 \frac{\rho_{\rho}}{\rho_{f}} \\
\frac{\eta}{\lambda}=17 & \\
\frac{R_{p}}{\eta}=11 & N_{p}=512 \\
& \phi_{v}=3 \% \\
& \frac{\eta_{k}}{\Delta x}=0.56 \\
& \text { Nodes }=256^{3}
\end{array}
$$



The particles are on the inertial range.

## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition



## HIT: Initial condition



## Visualization of one simulation



512 particles
$256^{3}$ grid nodes
512 processors
$\sim 10 \mathrm{~h} / T_{E}$
(Movie)

In this section only solid collision model is activated (no lubrication).

## Collision time



## Collision time



$$
\frac{1}{\tau_{c}}=n_{p} \pi d_{p}^{2}<|\mathbf{w}|>
$$

## Collision time



$$
\begin{aligned}
& \frac{1}{\tau_{c}}=n_{p} \pi d_{p}^{2}<|\mathbf{w}|> \\
& <|\mathbf{w}|>=<\left|\mathbf{V}_{p}^{a}-\mathbf{V}_{p}^{b}\right|>
\end{aligned}
$$

## Collision time



$$
\begin{aligned}
& \frac{1}{\tau_{c}}=n_{p} \pi d_{p}^{2}<|\mathbf{w}|> \\
& \frac{1}{\tau_{c}^{\text {th }}}=n_{p} \pi d_{p}^{2} \sqrt{\frac{16}{\pi} \frac{2}{3} q_{p}^{2}}
\end{aligned}
$$

## Collision time



$$
\begin{aligned}
& \frac{1}{\tau_{c}}=n_{p} \pi d_{p}^{2}<|\mathbf{w}|> \\
& \frac{1}{\tau_{c}^{\mathrm{tr}}}=n_{p} \pi d_{p}^{2} \sqrt{\frac{162}{\pi} \frac{2}{3} q_{p}^{2}}
\end{aligned}
$$

## Collision time



$$
\begin{aligned}
& \frac{1}{\tau_{c}}=n_{p} \pi d_{p}^{2}<|\mathbf{w}|> \\
& \frac{1}{\tau_{c}^{\mathrm{th}}}=n_{p} \pi d_{p}^{2} \sqrt{\frac{16}{\pi} \frac{2}{3} q_{p}^{2}} \\
& \frac{1}{\tau_{c}^{\text {S.-T. }}}=n_{p} d_{p}^{3} \sqrt{\frac{8 \pi}{15} \frac{\epsilon}{\nu}}
\end{aligned}
$$

Saffman and Turner - J. Fluid. Mech. - Vol. 1 (1956)

## Collision time



## Collision time



$$
\begin{aligned}
& \frac{1}{\tau_{c}}=n_{p} \pi d_{p}^{2}<|\mathbf{w}|> \\
& \frac{1}{\tau_{c}^{\mathrm{th}}}=n_{p} \pi d_{p}^{2} \sqrt{\frac{16}{\pi} \frac{2}{3} q_{p}^{2}} \\
& \frac{1}{\tau_{c}^{\mathrm{S} .-\mathrm{T} .}}=n_{p} d_{p}^{3} \sqrt{\frac{8 \pi}{15} \frac{\epsilon}{\nu}}
\end{aligned}
$$



| $\rho$ | $\tau_{c}^{\text {th }} / \tau_{k}$ | $\tau_{c}^{\text {S.- }} / \tau_{k}$ | $\tau_{c}^{\mathrm{m}} / \tau_{k}$ | $\left\langle\theta_{\text {coll }}>\right.$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 21.5 | 13.9 | 24.5 | $44^{\circ}$ |
| 2 | 22.48 | - | 20.8 | $59^{\circ}$ |
| 4 | 23.1 | - | 17.1 | $69^{\circ}$ |

## Contact angle



The velocity between particles at collision are correlated.

## Collision velocity

## Effect of the lubrication on intercollision time

## Visualization of secondary collisions

$\rho=1$ with lubrication model


- = Encounter
- = Solid collision (Movie)


## Effect of lubrication in acceleration p.d.f

Collisions increase the p.d.f. for large accelerations.
Comparison for different density ratio


## Effect of lubrication in acceleration p.d.f

Collisions increase the p.d.f. for large accelerations.
Suppress accelerations during collision


## Effect of lubrication in acceleration p.d.f

Collisions increase the p.d.f. for large accelerations.
Acceleration r.m.s. is related to density.

$$
A_{0}=a_{\text {r.m.s. }}^{2} / \epsilon^{3 / 2} \nu^{1 / 2}
$$

Qureshi et al. - European Phy. Jo. B - 2008


## Effect of lubrication in acceleration p.d.f

Collisions increase the p.d.f. for large accelerations.
Suppress accelerations during collision


## Effect of lubrication in acceleration p.d.f

Collisions increase the p.d.f. for large accelerations.
Comparison for $\rho=1$


## Effect of lubrication in acceleration p.d.f

Collisions increase the p.d.f. for large accelerations.
Same but for lubricated case


The lubrication do not modify the p.d.f. for larger accelerations

## Conclusions and future work

This work highlight some important effects:

- The lubrication model is representative for collisions in viscous fluids
- The lubrication creates the secondary collisions
- This secondary collisions have an effect on collision time
- The collisions modify the acceleration p.d.f.

These effects have to be considered when the collision frequency is treated. New models have to be put in place.

## Thanks

Thanks for your attention. Some questions ?


## Acknowledgements:

P. Fedem, W.-P. Breugem, P. Costa, J. Derksen, ...

Supercomputer support: CALMIP, CINES

## Appendix

Appendix:

$$
\begin{aligned}
& \text { Method } \\
& \text { VoFLag method }
\end{aligned}
$$

## Simulation

Collision
Particle Stats
Average flow calculation
Validations
Poiseuille
Rotation
Sedimentaition
Job time

## Method



| . | . | . |  |  | : | - |  |  | . |  |  | - | . |  | . |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| : | : | : |  |  | - | - | : | - | - |  |  | - | : |  | : |  | . |
| - |  |  |  |  | : |  |  |  |  |  |  | - | - |  |  |  |  |
|  | - |  |  |  | : | $:$ |  |  |  |  |  | : |  |  | $\because$ |  |  |
| $\therefore$ | - |  |  |  | - | $\cdots$ | - |  | - |  |  | - | . |  | - |  |  |
| $\because$ | - | - |  |  | - | - | . | - | - |  |  | - | . |  | . |  |  |
| $\square$ | - | - |  |  | - | - | . | . | - |  |  | - | . | - | . |  | - |
| : | - | - |  |  | - | - | - | - | - |  |  | - | - |  | . |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  | . |  |  | $\square$ |  |  |
|  |  |  |  |  | : |  |  |  |  |  |  | : | : |  | : |  |  |
| - | - | - |  |  | - | - | - |  | - |  |  | - | - |  | - |  |  |
| : |  | : |  |  | : | - | : |  | - |  |  | : | - |  | : |  |  |
| $\because$ | - | - |  |  | : | . | . | - | - |  |  | - | - |  | . |  |  |
| : |  | : |  |  | : | : | : | - | - |  |  | : | : |  | : |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Thetis code $\mathbf{1}$ Ma

- MAC stagered grid
- Finite volumes
- Fictitious Domain Approach


## Method




## Method



The solid behaviour is enforced by high viscosity.

Randrianarivelo et al. Int. J. Num. Meth. in Fluids - Vol. 47 (2005)

- Vincent et al. - Comp. \& Fluids - Vol. 36 (2007)


## Method



$$
C=\frac{\sum_{1}^{25^{N}} \delta_{P \in R}}{25^{N}}
$$

## Method



$$
\begin{gathered}
\rho=C \rho_{s}+(1-C) \rho_{l} \\
\mu=\frac{\mu_{f} \mu_{s}}{C_{\mu_{f}+(1-C) \mu_{s}}}
\end{gathered}
$$

## Method




## Method




$$
\begin{aligned}
& \tilde{\mathbf{X}}_{i}^{n+\frac{1}{2}}=\mathbf{X}_{i}^{n}+\frac{\Delta t}{2} \mathbf{V}_{i}^{n} t \\
& \tilde{\mathbf{v}}_{i}^{n+\frac{1}{2}}=\frac{3}{2} \mathbf{V}_{i}^{n}-\mathbf{V}_{i}^{n-1}
\end{aligned}
$$

## Method



## CPU block

|  | 0 |  |  |  |  | 0 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 |  |  |  |  | 0 |  |  |
|  |  |  |  | 0 |  |  |  |  |
|  | 0 | 0 | 0 |  |  |  | 0 |  |
|  | 0 | 0 | 0 | 0 | 0 |  |  |  |
|  |  |  | 0 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  | 0 | 0 | 0 |  |  |  |
| 0 |  |  |  |  |  |  | 0 |  |
|  |  |  |  |  |  |  |  |  |

To find collisions Cell-list approach is implemented.

## Method




A collision model gives the force applied on each particle.
The model developed is explained in later.

Brändle et al. - Phys. Fluids - (2013)

- Breugem - ASME Summer Meeting - Montreal (2010)


## Method




The force is distributed on the solid volume and becomes a source term for N.-S. solution.

## Method




## An augmented Lagrangian method is implemented. With BiCG Stab solver.

## Method




The new velocity field is calculated. The solid behaviour is respected by this field.

EVincent et al. - Comp. \& Fluids - Vol. 36. (2007)

## Method



## Method



## The particle velocity is calculated by interpolating the velocity

## Method



## Method




$$
\mathbf{X}_{i}^{n+1}=\mathbf{X}_{i}^{n}+\Delta t\left(\frac{\mathbf{V}_{i}^{n+1}+\mathbf{V}_{i}^{n}}{2}\right)
$$

## Validations



Dennis et al. - JFM - Vol 101 (1980)
Mordant et al. - Europ. Physical
Journal B - Vol. 18 (2000)

$\downarrow g$

## Validations



## Validations



## Constraints

## Physics:

## Numerical:

- $\frac{\eta}{\Delta x} \sim 1$
- Scale separation
- $\frac{D}{\Delta x} \sim 20$
- Moderate concentrated
- $N_{x} \sim 256$
- Not too inertial
- As many particles as possible


## Constraints

Physics:

- Scale separation
- Moderate concentrated
- Not too inertial


## Numerical:

- $\frac{\eta}{\Delta x} \sim 1$
- $\frac{D}{\Delta x} \sim 20$
- $N x \sim 256$
- As many particles as possible


## Constraints

Physics:

- Scale separation
- Moderate concentrated
- Not too inertial


## Numerical:

- $\frac{\eta}{\Delta x} \sim 1$
- $\frac{D}{\Delta x} \sim 20$
- $N x \sim 256$
- As many particles as possible

Consequences:

- $60<R e_{\lambda}<90$

Collisions in turbulence
J.C.B. de Motta

## Constraints

Physics:

- Scale separation
- Moderate concentrated
- Not too inertial


## Numerical:

- $\frac{\eta}{\Delta x} \sim 1$
- $\frac{D}{\Delta x} \sim 20$
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## Constraints

## Physics:

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- Moderate concentrated
- Not too inertial
- $\frac{\eta}{\Delta x} \sim 1$


## Numerical:

- $\frac{D}{\Delta x} \sim 20$
- $N_{x} \sim 256$
- As many particles as possible


## HIT: Forcing

The turbulence forcing is done in physical space by maintaining the total energy

$$
u_{n}=\sqrt{\frac{E_{0}}{E_{n}}}
$$

(the density of particles is not taken into account)
Rosales et. al - Phys. Fluids - Vol. 17 (2005)

## HIT: Turbulence modulation



Turbulence increases for high wave numbers

## HIT: Turbulence modulation




Lucci et al. - J. Fluid. Mech. -
Vol. 650 (2010)

Turbulence increases for high wave numbers (Numerical effect?)

## HIT: Turbulence modulation


(a) Particle effect


Lucci et al. - J. Fluid. Mech. -
Vol. 650 (2010)

Turbulence increases for high wave numbers (Numerical effect?)
$\Rightarrow$ We need to analyze the Lagrangian spectrum

## HIT: Lagrangian spectrum

Fluid particles


## HIT: Lagrangian spectrum



Fluid particles


## HIT: Lagrangian spectrum



Fluid particles


The turbulence is not modified by the particles

## Intercollision time



## Statistics

| $\rho$ | $q_{p}^{2}$ <br> $\left(m^{2} s^{-2}\right)$ | $q_{p}^{2}$ <br> $q_{f}^{2}$ | $T_{L}$ <br> $(s)$ | $T_{L}$ <br> $T_{L}^{t}$ <br> - | $D$ <br> $\left(m^{2} s^{-1}\right)$ | $\frac{D}{D_{f}}$ <br> - <br> 1 1.27 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.85 | 1.22 | 1.31 | 3.10 | 1.13 |  |
| 2 | 1.17 | 0.79 | 1.35 | 1.45 | 3.16 | 1.15 |
| 4 | 1.10 | 0.74 | 1.53 | 1.65 | 3.37 | 1.23 |

## Statistics

| $\rho$ | $\begin{gathered} q_{p}^{2} \\ \left(m^{2} s^{-2}\right) \end{gathered}$ | $\frac{q_{p}^{2}}{q_{f}^{2}}$ | $\begin{aligned} & T_{L} \\ & (s) \end{aligned}$ | $\overline{\frac{T_{L}}{T_{L}^{f}}}$ | $\begin{gathered} D \\ \left(m^{2} s^{-1}\right) \end{gathered}$ | $\frac{D}{D_{f}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.27 | 0.85 | 1.22 | 1.31 | 3.10 | 1.13 |
| 2 | 1.17 | 0.79 | 1.35 | 1.45 | 3.16 | 1.15 |
| 4 | 1.10 | 0.74 | 1.53 | 1.65 | 3.37 | 1.23 |

The $q_{p}^{2}$ agitation decreases with $\rho$.

## Statistics

| $\rho$ | $q_{p}^{2}$ | $\frac{q_{\rho}^{2}}{q_{f}^{2}}$ | $T_{L}$ <br> $\left(m^{2} s^{-2}\right)$ | - | $\frac{T_{L}}{T_{L}^{T}}$ | $D$ <br> - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left.-m^{2} s^{-1}\right)$ | $\frac{D}{D_{f}}$ |  |  |  |  |  |
| - |  |  |  |  |  |  |
| 1 | 1.27 | 0.85 | 1.22 | 1.31 | 3.10 | 1.13 |
| 2 | 1.17 | 0.79 | 1.35 | 1.45 | 3.16 | 1.15 |
| 4 | 1.10 | 0.74 | 1.53 | 1.65 | 3.37 | 1.23 |

The velocity autocorrelation time $T_{L}$ increases with $\rho$.

## Statistics

| $\rho$ | $q_{p}^{2}$ <br> $\left(m^{2} s^{-2}\right)$ | $q_{p}^{2}$ <br> $q_{f}^{2}$ | $T_{L}$ <br> $(s)$ | $\frac{T_{L}}{T_{L}^{t}}$ <br> - | $D$ <br> $\left(m^{2} s^{-1}\right)$ | $\frac{D}{D_{f}}$ <br> - <br> - <br> 1 1.27 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1.17 | 0.85 | 1.22 | 1.31 | 3.10 | 1.13 |
| 4 | 1.10 | 0.74 | 1.35 | 1.45 | 3.16 | 1.15 |



## Statistics

| $\rho$ | $q_{p}^{2}$ | $\frac{q_{p}^{2}}{q_{f}^{2}}$ | $T_{L}$ | $\frac{T_{L}}{T_{L}^{f}}$ | $D$ | $\frac{D}{D_{f}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | $\left(m^{2} s^{-2}\right)$ | - | $(s)$ | - | $\left(m^{2} s^{-1}\right)$ | - |
| 1 | 1.27 | 0.85 | 1.22 | 1.31 | 3.10 | 1.13 |
| 2 | 1.17 | 0.79 | 1.35 | 1.45 | 3.16 | 1.15 |
| 4 | 1.10 | 0.74 | 1.53 | 1.65 | 3.37 | 1.23 |

$q_{p}^{2}$ and $T_{L}$ balance and give similar diffusion coefficient.

## Particles diffusion



## Average flow calculation

## Average flow calculation

## Average flow calculation



## Average flow calculation



## Average flow calculation



## Validations

Ici je vais mettre les slides que j'avais fait sur les validations afin de répondre aux questions

## Validations: Poiseuille



Segré and Silberberg - Nature - Vol. 189 (1961)

## Validations: Poiseuille



## Equilibrium

Segré and Silberberg - Nature - Vol. 189 (1961)

## Validations: Poiseuille

A : Inamuro et al. - IJMF - Vol 26-2000
B : Pan et al. - JCP - Vol 181-2002

|  | Author | Case1 | Case2 | Case3 | Case4 | Case5 | Case6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R e=\frac{L \overline{u_{f}}}{\mu}$ |  | 12.78 | 27.72 | 43.40 | 55.55 | 69.13 | 96.74 |
| $\bar{Y}$ | A | 0.2745 | 0.2733 | 0.2728 | 0.2723 | 0.2716 | 0.2706 |
|  | B | 0.2732 | 0.2725 | 0.2723 | 0.2720 | 0.2719 | 0.2722 |
|  | VofLag | 0.2643 | 0.2636 | 0.2637 | 0.2643 | 0.2650 | 0.2752 |
|  | Error | 3.8\% | 3.6\% | 3.4\% | 3.0\% | 2.5\% | 2.0 \% |
| $\bar{U}$ | A | 0.04137 | 0.04131 | 0.04091 | 0.04118 | 0.04110 | 0.04101 |
|  | B | 0.04155 | 0.04159 | 0.04122 | 0.04166 | 0.04148 | 0.04144 |
|  | VofLag | 0.04599 | 0.04605 | 0.04570 | 0.04628 | 0.04623 | 0.04682 |
|  | Error | 10.6\% | 10.9\% | 11.1\% | 11.7\% | 11.7\% | 13.2\% |
| $\bar{\omega}$ | A | -0.054675 | -0.054694 | -0.053772 | -0.053765 | -0.053101 | -0.051607 |
|  | B | -0.05345 | -0.05343 | -0.05264 | -0.05264 | -0.05206 | -0.05052 |
|  | VofLag | -0.05500 | -0.05331 | -0.05082 | -0.04966 | -0.04761 | -0.05276 |
|  | Error | 0.6\% | 2.6\% | 5.6\% | 7.9\% | 10.9\% | 2.2\% |

With 6 nodes per diameter we obtain a low relative error

## Validations : Dennis 1980



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## Validations : Dennis 1980

Dimensionless torque $M=\frac{2 T}{\rho_{f} R^{5} \omega^{2}}$ vs Reynolds number $R e_{\omega}=\frac{R^{2} \omega \rho}{\mu_{f}}$


Symbols by
$\frac{\Delta x}{D}=\{10,20,40\}$
Colors by
T

## Validations: Ten Cate 2002



## Validations: Ten Cate 2002



Dashed lines $\frac{D}{\Delta T}=8$, straight line $\frac{D}{\Delta T}=16$.
ten Cate et al. - Phys. Fluids - Vol 14 (2002)

## HIT : Jobs time

## With 512 procs

|  | $T_{\text {sim }} / T_{E}$ | Percentage |
| :--- | ---: | :---: |
| Navier-Stokes Resolution | 26828 s | $75 \%$ |
| Postraitement (Out + Spectra) | 3977 s | $11 \%$ |
| Fluid Particles | 2121 s | $6 \%$ |
| Advection | 2105 s | $6 \%$ |
| Collisions | 39 s | $<1 \%$ |
| Others | 714 s | $2 \%$ |
| Total | $35784 \mathrm{~s} \sim 10 \mathrm{~h}$ |  |


[^0]:    Schäfer et al. - Jo. de Phys. - Vol 6 (1996)

