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Measurements and time-domain simulations of multiphonics in the trombone.

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Abstract

Multiphonic sounds of brass instruments are studied in this article. They are produced by playing a note on a brass instrument while simultaneously singing another note in the mouthpiece. This results in a peculiar sound, heard as a chord or a cluster of more than two notes in most cases. This effect is used in different artistic contexts.

Measurements of the mouth pressure, the pressure inside the mouthpiece and the radiated sound are recorded while a trombone player performs a multiphonic, firstly by playing an F_3 and singing a C_4 , then playing an F_3 and singing a note with a decreasing pitch. Results highlight the quasi-periodic nature of the multiphonic sound and the appearance of combination tones due to intermodulation between the played and the sung sound.

To assess the ability of a given brass instrument physical model to reproduce the measured phenomenon, time-domain simulations of multiphonics are carried out. A trombone model consisting in an exciter and a resonator non-linearly coupled is forced while self-oscillating to reproduce simultaneous singing and playing. Comparison between simulated and measured signals is discussed. Spectral content of the simulated pressure match very well with the measured one, at the cost of a high forcing pressures.

I INTRODUCTION

A "monodic instrument", by its etymological meaning, is an instrument designed to play "monophonic" sounds, that is sounds with a single pitch. However, most monodic instruments of the orchestra can also produce unusual sounds, called "multiphonic sounds". This term means that the listener perceives multiple notes in the sound.

30 A range of wind instruments can produce sounds called multiphonics [Castellengo, 1981]. Ac-
31 tually, this word designates two distinct phenomena. On the one hand, the term "multiphonic"
32 can refer to a multiple-pitched sound generated with an extension of the conventional playing
33 techniques: woodwind multiphonics, where quasi-periodic regimes are generated through spe-
34 cific embouchures and/or fingerings [Backus, 1978, Keefe and Laden, 1991, Dalmont et al., 1995,
35 Gibiat and Castellengo, 2000, Doc et al., 2014], belong to this category. A quasi-periodic os-
36 cillation is a deterministic oscillation whose energy is located at frequencies which are inte-
37 ger combinations of base frequencies. The ratios between the base frequencies are irrational
38 numbers. Flute-like instrument multiphonics are also based on an extended playing technique:
39 the oscillation of the air jet becomes quasi-periodic with no need to introduce another oscilla-
40 tor [Campbell and Greated, 1994, Blanc et al., 2010, Terrien et al., 2013]. Brass instrument mul-
41 tiphonics can be based on two distinct mechanisms: a spontaneous quasi-periodic self-oscillation,
42 similar to those in flutes and in reed instruments, can be involved [Castellengo et al., 1983]. But
43 on the other hand, the brass instrument player can also produce a multiphonic sound by singing
44 while he plays [Campbell and Greated, 1994, Sluchin, 1995]: two oscillators are then involved, the
45 lips and the vocal folds. This paper only focuses on this latter kind of multiphonic sound on the
46 trombone. In this situation, an intermodulation is observed, making non-harmonic frequencies
47 appear in the resulting sound.

48 According to the musical context, a multiphonic sound can be considered as a mis-
49 take made by the musician or due to a defect of the instrument: in occidental art-music,
50 a "rolling sound" is generally considered to be a mistake the musician makes and a
51 bowed instrument with "wolf notes" [Iniacio et al., 2008] is considered to be of poor qual-
52 ity. However, multiphonic sounds can also be intentional. This applies to extra-European
53 traditional instruments such as pre-Columbian *flautas de chinos* producing *sonidos raja-*
54 *dos* [Wright and Campbell, 1998, Blanc et al., 2010, Terrien et al., 2013], or the Australian
55 aboriginal *didjeridu* [Wolfe and Smith, 2008] where the musician has several options for modifying
56 the sound by singing or tuning of vocal tract resonance while he plays. But European classical
57 music also includes examples of multiphonics: an example of cadenza of a French horn concertino
58 is displayed in the score in Figure 1. The technique is mentioned in teaching methods of
59 the 19th century, as reported in [Sluchin, 1995]. Jazz and avant-garde music have popularized
60 this musical effect with artists like James Morrison, Nils Landgren, Nat McIntosh and many others.

61

62 Multiphonics produced by a musician simultaneously singing and playing a brass instrument
63 have been documented in [Campbell and Greated, 1994, Sluchin, 1995] but, to the author's knowl-
64 edge, simulations of this phenomenon have never been carried out. This paper examines the ability
65 of a simple instrument physical model to simulate trombone multiphonics. This helps a better un-
66 derstanding of the multiphonic phenomenon and defines the abilities of the considered model.
67 Comparisons between results of this model and measurements on trombone players are proposed

68 on both multiphonic sounds studied: namely, playing an F_3 while singing a C_4 (referred to here-
69 after as $F_3 - C_4$ multiphonic), and playing an F_3 while singing a note whose pitch decreases from
70 C_4 to slightly above C_3 (referred to later as "decreasing playing frequency multiphonic"). The
71 measurement and simulation tools are first presented in Section 2; then, measurements and sim-
72 ulations of a selection of multiphonic sounds are compared in Section 3 to evaluate the ability of
73 the model to reproduce multiphonics.

74 II TOOLS

75 II.A Experimental setup

76 An experimental device has been developed to measure some characteristic variables during a
77 trombone multiphonic performance. The acoustic pressure inside the instrument mouthpiece $p(t)$,
78 characteristic of the response of the instrument, is measured. Another sensor measures the blowing
79 pressure as well as the acoustic pressure inside the mouth $p_m(t)$. The radiated sound, $p_{ext}(t)$, is also
80 recorded. The measurements room has a short reverberation time, similar to that of a rehearsal
81 studio, for the musician's convenience.

- 82 • The mouth pressure $p_m(t)$ is measured with an Endevco 8510B-5 miniature pressure sensor,
83 through a capillary tube (1.5 mm inner diameter) inserted in the mouth of the musician.
84 The capillary tube is glued to a short piece of silicone tubing (4 mm diameter) connected
85 to the pressure sensor. The assembly of tubes forms a Helmholtz resonator which bandpass-
86 filters the signal. An *ad hoc* convolution filter is applied to the measured signal in order to
87 compensate for the transfer function of the tube.
- 88 • The pressure inside the mouthpiece $p(t)$ is measured through the same sensor model. The
89 sensor is screwed into the mouthpiece wall. The microphone is small enough for the shape
90 of the mouthpiece cup not to be significantly altered.
- 91 • The radiated sound $p_{ext}(t)$ is recorded with a *B&K* 1/4" microphone, placed about 40 cm
92 downstream from the center of the trombone's bell. The vicinity with the bell limits the
93 influence of the reflections in the room.

94 These sensors are connected to their respective conditioners/amplifiers. The signals are simul-
95 taneously recorded by a computer through NI-9234 and 9215 acquisition modules. The sampling
96 frequency is 51200 Hz.

97 II.B Time-domain simulation of a trombone's physical model

98 All along this article, measurements are compared with time-domain simulations from a trombone
99 physical model. The retained self-sustained model relies on a linear exciter which is non-linearly
100 coupled with a linear resonator. Each of these three elements is described below. This kind of

101 model has been widely used for wind instruments [Fletcher, 1993] including brass instruments
 102 [Elliott and Bowsher, 1982, Yoshikawa, 1995, Cullen et al., 2000], since the seminal work of von
 103 Helmholtz [von Helmholtz, 1877].

104

105 For brass instruments, the exciter consists of the lips of the musician, represented by a linear,
 106 oscillator-like valve, linking the height between the lips $h(t)$ and the pressure difference across the
 107 lips $\delta p(t) = p_m(t) - p(t)$. A one degree of freedom valve (referred to hereafter as "1-DOF valve")
 108 [Fletcher, 1993] is enough to model the lips for common playing situations [Yoshikawa, 1995] with
 109 a tractable number of parameters. Two kinds of 1-DOF valves can be considered : "striking
 110 outward", which tends to open when δp grows, and "striking inward" which presents the opposite
 111 behavior. For the reasons detailed in [Velut et al., 2016] we chose a "striking outward" valve to
 112 model the lips of the trombonist. This model relies on the following equation:

$$\frac{d^2h}{dt^2} + \frac{\omega_l}{Q_l} \frac{dh}{dt} + \omega_l^2(h - h_0) = \frac{1}{\mu}(p_m - p(t)), \quad (1)$$

113 where $\omega_l = 2\pi f_l$ (rad/s) is the lip resonance angular frequency; Q_l the (dimensionless) quality
 114 factor of the lips; h_0 the value of $h(t)$ at rest (m); μ an equivalent surface mass of the lips ($\text{kg}\cdot\text{m}^{-2}$).

115

116 Lip parameters very similar to those chosen in [Velut et al., 2016] after a thorough bibliograph-
 117 ical review are used. These parameters are given in Table I. The only difference is the quality
 118 factor Q_l lessened from 7 to 5. Results previously obtained in [Velut et al., 2016] were satisfactory,
 119 with reasonable threshold blowing pressures in the [1kPa : 15kPa] range for commonly played
 120 notes. The refining of the Q_l value results in periodic playing on a larger range of f_l on the Jupiter
 121 JSL 232l trombone used in the experiment.

122

$h_0(\text{m})$	$L(\text{m})$	$1/\mu(\text{m}^2\text{kg}^{-1})$	Q_l
$5\cdot 10^{-4}$	$12\cdot 10^{-3}$	0.11	5

123

124 Table I: Lip parameters retained for this article.

125 In most studies about brasswinds, p_m is assumed to be constant in usual playing, corresponding
 126 to the blowing pressure. However, in order to simulate a multiphonic, a forcing component is added
 127 to the static value of p_m , corresponding to the acoustic pressure produced by the vibrating vocal
 128 folds. For multiphonic simulations with a constant singing frequency, the p_m signal takes the
 129 following form:

$$p_m(t) = p_m^0 + p_m^1 \cdot \sin(2\pi t f_s), \quad (2)$$

130 where p_m^0 and p_m^1 are respectively the constant blowing pressure and the amplitude of the singing
 131 signal, and f_s the singing frequency.

132 For simulations with a decreasing frequency, the signal is divided into 3 parts: first, the mouth
 133 pressure is a constant blowing pressure for 6 seconds: this gives time to reach a steady-state

134 oscillating regime. Then, the p_m signal is similar to Eq. (2) for 6 seconds with $f_s = f_1$. Then, the
 135 frequency decreases from f_1 to f_2 taking this form:

$$p_m(t) = p_m^0 + p_m^1 \cdot \sin \left[2\pi t \left(f_s + \frac{(f_2 - f_s) \cdot t}{2d} \right) \right], \quad (3)$$

136 where d is the duration of the frequency decrease (s).

137 In the measurements, the p_m^1/p_m^0 ratio is about 0.02, but higher values are used in simulations,
 138 to get computed spectra as close as possible to experimental ones. A value of $p_m^1 = 0.3p_m^0$ is
 139 used all along this paper. This difference in p_m^1 values between simulation and measurement is a
 140 limitation of our model.

141 This exciter is non-linearly coupled to a resonator: the bore of the trombone. Propagation in
 142 brass instruments, particularly the trombone, is known to be nonlinear for loud tones. This causes
 143 nonlinear distortion of the waveform, possibly leading to shockwaves [Hirschberg et al., 1996,
 144 Myers et al., 2012]. In musical terms, the sound becomes "brassy" at louder playing dynam-
 145 ics. However, since this study focuses on low and moderate playing dynamics, a linear model of
 146 propagation is sufficient. Thus, the resonator is modeled by its input impedance Z . By definition,
 147 Z is the ratio, in the frequency domain, of the pressure at the input of the resonator $P(\omega)$ to the
 148 acoustic flow $U(\omega)$ taken at this same point :

$$Z(\omega) = \frac{P(\omega)}{U(\omega)}. \quad (4)$$

149 The input impedance of the Jupiter JSL 232l tenor trombone used for experi-
 150 ments (with the slide fully pulled in) is measured with the impedance sensor described
 151 in [Macaluso and Dalmont, 2011], then fitted by a sum of 13 Lorentzian functions -representing
 152 the 13 first modes of the trombone- using a least squares method similar to that in [Silva, 2009].

153 The coupling between this resonator and the aforementioned exciter is non linear. It is provided
 154 by the airflow through the lip channel. The air jet is assumed to be laminar in the lip channel,
 155 then turbulent in the mouthpiece, all its kinetic energy being dissipated without pressure recovery.
 156 Applying the Bernoulli law and the mass conservation law between the mouth and the lip channel
 157 gives the following expression of the flow between the lips, depending on the pressure difference
 158 and the height of the lip channel [Elliott and Bowsher, 1982, Hirschberg et al., 1995]:

$$u(t) = \sqrt{\frac{2}{\rho}} L h(t) \cdot \text{sign}(p_m - p(t)) \cdot \sqrt{|p_m - p(t)|}, \quad (5)$$

159 where $u(t)$ is the airflow rate ($\text{m}^3 \cdot \text{s}^{-1}$), $\rho = 1.19 \text{ kg} \cdot \text{m}^{-3}$ the air density at $20 \text{ }^\circ\text{C}$ and L the
 160 width of the lip channel (m).

161

162 Simulations based on this model are carried out with MoReeSC [Silva, 2013]. This open-access
 163 Python library solves the equations of the model numerically, based on the modal decomposition of
 164 the pressure signal in the instrument. This provides values of p , the lip opening h and the airflow

165 between the lips u at each time sample. It features the possibility of modifying input parameters
166 during the simulation, which is particularly useful in this study for defining time-varying mouth
167 pressure signals. In order to get a simulated pressure, a measurement of the transfer function
168 of the trombone, between p and p_{ext} is made. Filtering the simulated p with the given transfer
169 function results in a simulated p_{ext} .

170 II.C Preliminary measurement on vocal folds

171 A preliminary experiment is carried out to assess the choice of a forcing term to model the singing
172 like in Eq. (2) and (3). The production of a multiphonic sound requires two exciters: the lips of
173 the musician and his vocal folds. In order to evaluate the independence of the vocal fold oscillation
174 with respect to the lip oscillation, an estimation of the vocal fold oscillation is carried out by
175 measuring the electrical conductivity of the glottis, similarly to what was done in the experimental
176 campaign conducted on the didjeridu [Wolfe and Smith, 2008]. The glottis conductivity is assumed
177 to be approximatively proportional to the contact area of the lips [Hezard et al., 2014]. Since we
178 are interested in comparing orders of magnitude, this approximative proportionality is sufficient.

179 An electroglottograph from VoceVista is used to measure the conductivity of the vocal folds
180 while the musician plays an $F_3 - C_4$ multiphonic. The spectrum of the resulting signal is shown
181 in Fig. 3.

182 Spectral components can be observed at the singing frequency $f_s = 262.5$ Hz but also at
183 the playing frequency $f_{trb} = 174.2$ Hz, showing a coupling between the lips and the vocal folds.
184 However, the amplitude of the f_{trb} component is 20 dB lower than the amplitude of the f_s one: this
185 indicates that the oscillation of the vocal folds is not much altered by the acoustic feedback. Thus,
186 since we are interested in identifying the simplest model simulating multiphonic sounds, modeling
187 the contribution of vocal folds through a forcing term seems to be a decent approximation, much
188 simpler than a model that would take into account the vocal folds, the vocal tract and the lips .
189 However, a time-domain simulation tool which would simulate the coupling with the impedance
190 of the vocal tract would probably be of some interest. The implementation of such a model could
191 be realized with the tools presented here.

192 III RESULTS

193 III.A F_3 - C_4 multiphonic

194 III.A.1 Experiment

195 The study firstly focuses on the F_3 - C_4 multiphonic, which is one of the most commonly played by
196 trombonists and proposed as a first exercise in [Sluchin, 1995]. Producing an $F_3 - C_4$ multiphonic
197 consists in playing an F_3 while singing a C_4 , i.e. a fifth above. In physical terms, this means
198 playing on the third register of the trombone, while simultaneously singing the note whose

199 frequency is 1.5 times higher ($f_s/f_{trb} = 1.5$). The musician is asked to successively sing a C_4 , then
 200 play an F_3 , then perform an $F_3 - C_4$ multiphonic.

201

202 Figure 4 shows the spectrograms corresponding to this experiment and calculated for the
 203 time-domain signals of p_m in Fig. 4 (a), p in Fig.4 (b) and p_{ext} in Fig.4 (c). The successive tasks
 204 - singing, playing, multiphonic - successively appear on the spectrograms. During the singing,
 205 a component appears at $f_s = 259.8Hz$ with its harmonics. Similarly, $f_{trb} = 173.6Hz$ and its
 206 harmonics appear while the musician is playing an F_3 . A component at f_{trb} can be observed
 207 in Fig. 4 (b) and (c) but also in the p_m spectrogram, because of the coupling with the vocal
 208 tract of the musician [Chen et al., 2012, Fréour and Scavone, 2013]. When the multiphonic is
 209 played, $p(t)$ and $p_{ext}(t)$ contain the fundamental and harmonics of both f_s and f_{trb} . In addition,
 210 other frequency components also appear, which do not belong to either the harmonic series
 211 of f_s or that of f_{trb} . These components are shown by arrows in Fig. 4 (b). Note that one of
 212 these components has its frequency under f_{trb} . Figure 5 superimposes the spectra of $p(t)$ during
 213 the three phases of the performance: singing, playing, multiphonic. This highlights that some
 214 peaks of the multiphonics spectrum clearly do not belong to the played signal or to the sung signal.

215

216 III.A.2 Simulation

217 This experiment (playing an F_3 on a trombone while singing a C_4) is simulated, using the
 218 method described in Section II.B with the parameters given in Table I: the physical model of
 219 trombone is set to play an F_3 , on its 3^d register, with a lip resonance frequency $f_l = 140$ Hz
 220 and a steady blowing pressure set to $p_m^0 = 4500$ Pa. This value is slightly above the threshold
 221 pressure calculated by linear stability analysis, as in [Velut et al., 2016]. Then, the "sung" note
 222 is included to simulate the multiphonic: a forcing sinusoidal component is added to the static
 223 blowing pressure, at a frequency f_s corresponding to the upper fifth, as written in Eq. (2). The
 224 amplitude of the forcing sinusoidal component is set to 30% of p_m^0 so that $p_m^1 = 1350$ Pa. This
 225 forcing component starts 3 seconds after the blowing pressure, once the self-oscillation of the
 226 instrument model has reached its steady state. This avoids interferences between the forcing
 227 component and the transitory phase of the self-sustained oscillation.

228

229 Spectrograms of the simulation results for p_m , p and p_{ext} are displayed in Fig. 6 (a), (b) and (c),
 230 respectively. First, the model auto-oscillates on its own until $t = 3s$; then the forcing component
 231 is added. Fig. 6 (a) does not display any spectral component at f_{trb} , because the retained model
 232 does not take the coupling with the vocal tract into account. Before $t = 3s$, Fig. 6 (b) and (c) only
 233 display spectral components at $f_{trb} = 189$ Hz and its harmonics. The oscillation frequency of the
 234 simulation is higher than the experimentally recorded playing frequency in Fig. 4. This is consistent
 235 with a well-known limitation of this brass model, known to oscillate at sharper frequencies than the
 236 tempered scale notes [Campbell, 2004, Silva et al., 2007, Chaigne and Kergomard, 2016, p.547].

237 Then, at $t = 3$ s, the forcing component at $f_s = 282.7$ Hz appears. As in the experiment, $p(t)$ and
 238 $p_{ext}(t)$ show frequency components which are neither f_s , nor f_{trb} , nor their harmonics. This is also
 239 to be seen in the p spectra displayed in Fig. 7: some peaks of the multiphonic signal do not match
 240 f_s , f_{trb} (solid lines) or their harmonics (dashed lines).

241 III.A.3 Discussion

242 Frequencies of the peaks appearing in the spectra of multiphonics, either simulated (Fig. 6 and 7)
 243 or measured (Fig. 4 and 5), match very well integer combinations of f_s and f_{trb} : the relative error
 244 is less than 3% for the measured frequencies and less than 0.5% for the simulated frequencies.
 245 Table II reports the frequencies appearing in the simulation and in the measurement, and
 246 proposes one or two integer combinations giving the same frequency. A given integer combination
 247 matches the peak of the same rank in the experiment and in the simulation. These frequency
 248 components at integer combinations of f_{trb} and f_s are hereafter called combination tones (CT),
 249 as in [Campbell and Greated, 1994, p.64]. In terms of amplitude, the peaks in the spectra of the
 250 simulated and the measured multiphonics are not the same, even if the shapes of the spectral
 251 envelopes remain comparable.

Peak No.	1	2	3	4	5	6	7	8	9	10
$f_{exp}(Hz)$	86.1	176	259.8	347.3	433.4	521	605.1	694.8	778.7	868.7
$f_{sim}(Hz)$	94.15	189	282.7	377.9	471.4	567.6	661.1	756.6	850	946.2
lin. comb.	$f_s - f_{trb}$ $2f_{trb} - f_s$	f_{trb}	f_s $3f_{trb} - f_s$	$2f_{trb}$ $4f_{trb} - f_s$	$f_s + f_{trb}$	$3f_{trb}$ $2f_s$	$3f_s - f_{trb}$ $5f_{trb} - f_s$	$2f_s + f_{trb}$ $4f_{trb}$	$3f_s$	$5f_{trb}$ $4f_s - f_{trb}$

252 Table II: (color online) Frequencies of first peaks of spectrum of $p(t)$ measured (f_{exp}) and simulated
 (253 f_{sim}) during an $F_3 - C_4$ multiphonic. Integer combinations of f_s and f_{trb} corresponding to each
 254 combination tone are indicated.

255
 256 Signals recorded during this multiphonic playing are periodic signals. However, the fundamental
 257 frequency of these signals is an octave below the played note, at $f_{trb}/2$.

258
 259 The radiated pressure signals p_{ext} of the recorded [sound1,] and the simulated [sound2,]
 260 multiphonic are both heard as chords rather than as a single note. The playing and singing
 261 frequencies appear, along with other notes, notably the F_2 one octave below f_{trb} . Though,
 262 informal listening tests highlights some perceptive differences. First, the sung note is heard louder
 263 in the experimental recording than in the simulation. Then, while listening to the recording of
 264 the musical performance, a third note, namely an A_4 , can be heard, whose fundamental frequency
 265 is $f_s + f_{trb}$. This note cannot be clearly heard in the simulated p_{ext} signal, although its frequency
 266 components are present. One reason might be the differences in spectral balance between the
 267 experiment and the simulation, which could be related to the simplicity of the forcing signal: this
 268 latter hypothesis will be investigated later in the article.

As a conclusion, the $F_3 - C_4$ multiphonic studied here appears to be quite a peculiar periodic regime of oscillation. The spectral components are f_s , f_{trb} , their harmonics, and the combination tones of frequencies $f_{CT} = qf_{trb} \pm f_s$, $q \in \mathbb{N} \cup \{-1\}$ (except negative frequencies). The simulation model, based on a self-oscillating system, sinusoidally forced to model the contribution of the singing voice, reproduces the emergence of a regime which is very similar in terms of frequency content, but with some differences in the peak amplitudes.

Though it is periodic, this regime is not a usual self-sustained oscillation of a brass instrument: the fundamental frequency is not the trombone's oscillation frequency, but a combination tone. Several harmonics are also combination tones, each one matching with several integer combinations of f_s and f_{trb} . This fact is related to this specific situation where f_{trb}/f_s is a rational value. The system undergoes a 3:2 synchronization (also called an internal resonance in the dynamic system terminology) which makes combination tones integer multiples of $f_s - f_{trb}$. It is a particular behavior of a forced self-sustained oscillator. Academic case studies presenting this situation are developed in [Nayfeh and Balachandran, 1995, p.156] with forced Van Der Pol oscillators for instance.

According to the oscillator theory, the system studied should generate a quasi-periodic oscillation when there is no internal resonance. The following part will investigate multiphonic situations with a variable singing frequency, leading to multiple situations where f_s/f_{trb} is irrational.

III.B Variable singing frequency

III.B.1 Experiment

The trombone player is now asked to play a multiphonic with f_{trb} as stable as possible, while decreasing his singing frequency. He starts playing the same $F_3 - C_4$ multiphonic as before. Then, he lowers its singing frequency f_s as linearly as possible, until he reaches a frequency just above f_{trb} , while he keeps playing an F_3 . He then holds these playing and singing frequencies for a few seconds.

Spectrograms of the resulting p_m , p and p_{ext} are shown in Fig. 8 (a), (b) and (c), respectively. Fig. 8 (a) exhibits the evolution of the f_s and f_{trb} components in the mouth. The f_{trb} component is fairly stable all along the measurement. The singing component remains stable between 0 and 5 seconds, then decreases between 5 and 10 seconds, and stays 152 musical cents above f_{trb} until the end ($f_s/f_{trb} = 1.09$). After $t = 6$ s, a new spectral component emerges in the mouth, at a frequency growing towards f_{trb} . The peak at f_s has the highest amplitude all along the measurement.

The phenomenon that appears during the first 6 seconds of the spectrogram in Fig. 8 (b) and (c) is very similar to the one appearing after 12s in Fig. 4. Then, when f_s decreases, some frequency peaks seem to "split" in two components, one with a decreasing frequency, the other one with an increasing frequency. While the frequencies get closer to one another, other components with increasing or decreasing frequencies become stronger in amplitude, collecting towards f_{trb} and its harmonics. This leads to a quite rich spectrum with several secondary peaks around f_{trb} and its

306 harmonics, after $t = 8$ s when the singing frequency stabilizes.
 307 To follow the evolution of the components more easily, spectra are computed at each second on
 308 the mouthpiece signal and plotted in Fig. 9. The splitting of certain frequency components is
 309 noticeable: for illustration, the first peak at $t = 3.12$ s at 85 Hz (Fig. 9 (a)) splits progressively into
 310 two distinct peaks of frequency 75 Hz and 100 Hz, respectively, at $t = 5.06$ s (Fig. 9 (c)). The last
 311 spectrum, at $t = 10.12$ s (Fig. 9 (h)), shows numerous peaks on either side of f_{trb} and its harmonics.
 312

313 III.B.2 Simulation

314 The same simulation model as in Section III.A is used to reproduce this second experiment. Start-
 315 ing with the same parameter values as in the previous section ($f_l = 140$ Hz, $p_m^0 = 4500$ Pa,
 316 $p_m^1 = 1350$ Pa, $f_s = 282.7$ Hz i.e. a fifth above f_{trb}), the singing frequency is decreased linearly
 317 (sweep signal) between $t = 6$ s and $t = 12$ s, to 152 cents above the playing frequency: $p_b(t)$ follows
 318 Eq. (3) with $f_1 = 282.7$ Hz, $f_2 = 206$ Hz and $d = 6$ s. The final frequency is chosen to stick
 319 with the experiment, where the final singing frequency of the musician is also 152 cents above the
 320 playing frequency. During the last two seconds, $f_s = 206$ Hz. Just as above, spectrograms of p_m ,
 321 p and p_{ext} are shown in Fig. 10 (a), (b) and (c), respectively. Fig. 11 represents the spectra of p
 322 computed each second. Fig. 10 and 11 for simulation are equivalent to Fig. 8 and 9 for experiment.
 323 According to Eq. (3), the mouth pressure only contains the forcing term, providing a convenient
 324 view on the evolution of f_s . The first seconds of the p and p_{ext} spectrograms of Fig. 10 (b) and (c)
 325 are very similar to the spectrograms of Fig. 6 with components of f_s and f_{trb} plus the combination
 326 tones. Then, when f_s starts decreasing, each component at f_s or its harmonics splits into two
 327 components moving away from one another, one having a decreasing frequency and the other
 328 having an increasing frequency. The amplitude of increasing-frequency components is weaker than
 329 that of decreasing-frequency components. Between $t = 7$ s and $t = 8$ s, a regime briefly emerges
 330 at $38,5 Hz = f_{trb}/5$ and its harmonics, while $f_s = 265 Hz$: it can be noticed under the form of
 331 evenly spaced lines (at 38.5 Hz, 77 Hz, 115.5 Hz, 154 Hz, 192.5 Hz, 231 Hz etc.) in Fig. 10 (b)
 332 (between the dash-dotted lines) and (c). This is a periodic regime due to a 5 : 1 internal resonance.
 333 Indeed, at this point the f_s/f_{trb} ratio is rational and the resulting regime is periodic. The resulting
 334 frequency is very close to the first acoustic resonance frequency of the trombone (38.9 Hz) so the
 335 first mode may sustain this oscillation. This will be addressed in Section III.C. After $t = 10$ s, the
 336 existing components strengthen and new ones appear, to end up with several secondary peaks on
 337 each side of the components of the auto-oscillation. These new peaks are particularly visible in
 338 the last spectra in Fig. 11.

339 III.B.3 Discussion

340 As in the previous experiment, several frequency components appear to be harmonics of neither
 341 f_{trb} nor f_s . At the beginning, the frequencies are the same as in Section III.A. Certain integer
 342 combinations result in the same combination tones, like for example $3f_{trb} = 2f_s$. Then, when f_s

343 decreases, these integer combinations are no longer equal since f_s and f_{trb} are no longer in a 3 : 2
 344 ratio: the combination tone frequencies either increase or decrease, depending on the sign in front
 345 of f_s in the integer combination. The frequencies contained in the signal are no longer integer
 346 multiples of the lowest frequency. The oscillation becomes quasi-periodic as soon as f_s/f_{trb} is no
 347 longer rational.

348 Frequencies of the components of p during the phase where f_s decreases are recorded, and plotted
 349 with marks in Fig. 12 (a) (experimental measurement) and (b) (simulation). Some integer com-
 350 binations of f_s and f_{trb} , of positive frequency $qf_{trb} \pm f_s, q \in \mathbb{N} \cup \{-1\}$ are also plotted with plain
 351 lines on the same figures. The match between each peak frequency and one integer combination
 352 is remarkable, with a maximum relative error of 2.5%. Therefore, the emerging frequency compo-
 353 nents are integer combinations of f_s and f_{trb} , just as the $F_3 - C_4$ multiphonic. It can be concluded
 354 that this more complex multiphonic regime is due to the same phenomenon as in Section III.A,
 355 emerging from the coupling between a self-sustained oscillator and a forcing term. However, here
 356 the oscillation regime is quasi-periodic.

357 The simulated p_{ext} and the musical performance were heard as similar during the experiment, yet
 358 differences exist. In the recorded multiphonic [sound3,], an A_3 can be heard at the beginning.
 359 When f_s begins to change, this A_3 rapidly decreases in loudness. During the decrease in f_s , certain
 360 notes become audible while others disappear. These notes can also be heard in the simulated
 361 p_{ext} [sound4,] but their overall loudness is weaker. At the end of this multiphonic, f_s and f_{trb} are
 362 close to each other. There, a "rolling" or "beating" sound can be heard, both in the experimental
 363 and in the simulated p_{ext} signals. This could be explained by the very low $f_s - f_{trb} = 16Hz$
 364 component (16.3 Hz in measurements, 17.3 Hz in simulation), perceived as a modulation of the
 365 sound.

366 These observations can be related to the spectral envelopes of the measured and the simulated
 367 p : comparatively with the peaks at f_{trb} , the combination tones and the harmonics of f_s have a
 368 smaller amplitude in simulation than in experimental measurements. The f_s component of p is an
 369 exception as it is weaker in measurement than in simulation. At the end of the multiphonic, the
 370 peaks at harmonics of f_{trb} are flanked by smaller peaks on both sides. These secondary peaks are
 371 significantly weaker in measurement than in simulations.

372 It can be assumed that these amplitude differences between experiment and simulation are related
 373 to the sinusoidal nature of the forcing whereas the singing is a more complex periodic signal. To
 374 assess this hypothesis, another simulation is carried out, this time with a forcing signal reproducing
 375 the three first harmonics of the recorded singing signal. The oscillating component of p_m now
 376 consists of three sinusoids, with the same relative amplitudes and phases as the first components
 377 of the measured sung signal. Figure 13 compares spectra of p simulated with a forcing signal
 378 consisting of one or three harmonics (blue and red plots, respectively), at the beginning (Fig. 13 (a))
 379 and the end (Fig. 13 (b)) of a simulation with decreasing f_s . All parameters of the three-harmonic
 380 forcing simulation are equal to those of the simulation presented in Fig. 10 and Fig. 11 except the
 381 p_m signal.

382 Adding harmonics to the forcing signal only results in minor changes in the spectral envelope of the
383 signal. Apart from the components at $2f_s$ and $3f_s$ being logically stronger, the major difference
384 between these simulations is the relative height of the peaks as compared to the numerical
385 noise. Between amplitude peaks, the minima are visibly weaker with a three-component forcing
386 signal, particularly at the end of the simulation (Fig.13 (b)). The influence on the amplitude
387 of combination tones is not significant. The lack of major difference could be explained by the
388 weakness of the harmonics of the forcing, the second and third harmonics of f_s being respectively
389 17.5 dB and 27.5 dB weaker than the fundamental.

390

391 From the results on multiphonics with a sliding f_s described in this section the conclusions from
392 the previous section can be generalized: the self-oscillating model with an additional forcing term
393 is sufficient to simulate multiphonics, resulting in the same frequency components as the measured
394 ones. The frequencies of the components of both the simulated and the measured p match very
395 well with integer combinations of the instrument self-oscillation frequency and the singing or
396 forcing frequency: these components are either harmonics of f_s and f_{trb} or combination tones.
397 This consolidates the idea that multiphonic regimes of brass instruments are either quasi-periodic
398 regimes or, when the f_{trb}/f_s ratio is rational, periodic regimes. Both behaviors come from the
399 same phenomenon. The $F_3 - C_4$ multiphonic considered in III.A is a particular case, where all
400 frequencies in the signal are harmonic because of the 3:2 internal resonance.

401 The simulation model tested here reproduces this behavior accurately, in spite of the simplicity
402 of the model. The main difference is the spectral envelope. A forcing signal closer to the singing
403 signal does not significantly change this limitation.

404 III.C Simulations with one acoustic resonance

405 New simulations with an even simpler model are carried out. Not only the forcing is sinusoidal
406 as seen in Fig. 14, but the resonator is reduced to one resonance, the third acoustic mode of the
407 trombone - the one used for playing an F_3 .

408 A simulation with a decreasing sinusoidal forcing is carried out. The playing frequency with this
409 resonator is $f_{trb} = 180.5$ Hz, so the playing frequency decreases from $f_s = 270$ Hz to $f_2 = 196.4$
410 Hz for 6 seconds to keep the same frequency ratios as in Section III.B.

411

412 The frequency components of the internal pressure p are harmonics of f_{trb} and f_s or combination
413 tones, as in the previous simulations and measurements, for all values of f_s tested. Thus, the
414 multiphonic behavior appears with this model. However, the amplitude of most frequency compo-
415 nents is weaker. This is illustrated by the amplitude differences reported in Table III: while the
416 amplitude of the component at f_{trb} remains fairly constant, all other components are significantly
417 weaker for the 1-mode simulation. Harmonics of f_{trb} are no longer supported by the modes 6, 9
418 and 12, which affects the amplitude of combination tones. The f_s component in $p(t)$ is weaker,
419 even though the amplitude of the forcing is equal: the modulus of the resonator impedance is 13

420 times weaker at f_s with only one mode than with 13 modes, which reduces the amplitude of the
421 f_s component in the mouthpiece. The lowest component $f_s - f_{trb} = 15.9$ Hz at the end of the
422 simulation is also weaker, but it is perceived as a modulation of the sound, making a "rolling"
423 sound just like in previous simulations and in measurement. The brief regime at fundamental
424 frequency $f_{trb}/5$ which can be seen between $t = 7s$ and $t = 8s$ in Fig 10 does not occur here,
425 because of the absence of the first mode. Another simulation with all acoustic modes except the
426 first one does not make this regime appear, which strengthens this hypothesis. This simulation
427 result is not shown here for the sake of brevity.

frequency	$f_s - f_{trb}$	f_{trb}	f_s	$2f_{trb}$	$f_s + f_{trb}$	$3f_{trb}$	$3f_s - f_{trb}$
difference (dB) at $t = 5$ s	0.6	0.3	10.7	27	11.7	23.2	16
difference (dB) at $t = 9$ s	7.6	0.5	4.8	27.4	14.2	27	8.7
difference (dB) at $t = 13$ s	26	1.2	4.5	17.9	12.7	25.1	17.4

428 Table III: Differences (in dB) in the amplitudes of the first peaks for one-mode and 13-mode
simulations, measured on the data displayed in Fig. 14, at three time points: $t = 5s$ when $f_s = 270$
429 Hz, $t = 9s$ when $f_s = 233$ Hz and decreases, and $t = 13s$ when $f_s = 196.4$ Hz. While amplitudes
of the peaks at f_{trb} are equivalent, other components are much weaker when the resonator has a
single mode.

430 The quasi-periodic regime related to multiphonics therefore occurs even with a very simple in-
431 strument model: a one-DOF mechanical exciter non-linearly coupled with a one-mode acoustic
432 resonator, self-oscillating and forced with a sinusoidal signal.

433 IV CONCLUSIONS

434 Both measurement and simulation results presented in this paper confirm the type of oscillating
435 regime of the multiphonic sounds studied. When the musician sings and plays different notes
436 simultaneously, the resulting pressure signal inside the instrument contains harmonics of the sung
437 and played frequencies, as well as combination tones, whose frequencies are integer combinations
438 of f_s and f_{trb} . This is verified not only in the particular case of an $F_3 - C_4$ multiphonic as in
439 Section III.A but also in the case of a time-variable frequency. This generally corresponds to
440 quasi-periodic oscillation, except when an internal resonance occurs and the ratio between playing
441 and singing frequencies is rational. In this latter case, the oscillation is periodic, though it is
442 different from the usual periodic self-oscillation of a brass instrument.

443 To some extent, the trombone physical model used in this paper is able to simulate this phe-
444 nomenon. Even if the simulated and measured f_{trb} are not exactly the same (as usual with an
445 outward-striking lip model), the frequency content of the simulated internal pressure of the in-
446 strument p is very similar to the probe microphone measurements inside the mouthpiece, with
447 harmonics of both the sung and played signals along with combination tones. This similarity

448 also applies when f_s decreases with time, and when the instrument model is simplified at most.
449 The major difference between simulation and measurement consists in differences in the spectral
450 envelopes: the amplitude of the peaks corresponding to combination tones is generally weaker in
451 measurement than in simulation when the forcing is sinusoidal. This is particularly true when
452 f_s and f_{trb} are close to each other, the secondary peaks being much weaker in measurement. A
453 richer forcing signal with three harmonics, closer to the measured sung signal, does not dramati-
454 cally change the results: changing the forcing signal does not seem the best way to improve the
455 simulation.

456 While the simulated blowing pressure p_m^0 is of the same order of magnitude as blowing pressures
457 usually measured on trombone players [Bouhuys, 1968, Fréour and Scavone, 2013], the amplitude
458 of the forcing signal p_m^1 used in our simulations is about 15 times higher than the amplitude
459 measured in the recorded mouth pressure signal p_m . This is the main limitation of this model
460 for multiphonic simulation. Yet, these very different input parameters give comparable results in
461 terms of internal pressure p , the difference in amplitude between the peaks at f_{trb} and f_s being of
462 the same order of magnitude in the measured and simulated p spectra. This is the main limitation
463 of our model. A more complex model, taking into consideration the couplings between the lips,
464 the vocal folds, the resonances in the instrument and those in the vocal tract would possibly give
465 a better simulation of multiphonics in terms of amplitude of the frequency components, yet at the
466 expense of a far greater complexity.

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477 References

478 [Backus, 1978] Backus, J. (1978). Multiphonic tones in the woodwind instruments. *J. Acoust.*
479 *Soc. Am*, 32(2):591–599.

480 [Blanc et al., 2010] Blanc, F., de la Cuadra, P., Fabre, B., Castillo, G., and Vergez, C. (2010).
481 Acoustics of the *Flauta de Chinos*. In *Proceedings of the 20th International Symposium on*
482 *Music Acoustics*, Sydney and Katoomba, Australia.

- 483 [Bouhuys, 1968] Bouhuys, A. (1968). Pressure-flow events during wind instrument playing. *Annals*
484 *of New York Academy of Sciences*, 155(1):264–275.
- 485 [Campbell, 2004] Campbell, M. (2004). Brass instruments as we know them today. *Acta Acustica*
486 *united with Acustica*, 90:600–610.
- 487 [Campbell and Greated, 1994] Campbell, M. and Greated, C. (1994). *The musician’s guide to*
488 *acoustics*. Oxford University Press.
- 489 [Castellengo, 1981] Castellengo, M. (1981). Sons multiphoniques aux instruments à vent (multi-
490 phonic sounds on woodwind instruments). *Rapports de l’IRCAM*.
- 491 [Castellengo et al., 1983] Castellengo, M., Caussé, R., and Sluchin, B. (1983). étude acoustique
492 de l’émission multiphonique aux cuivres (acoustical study of multiphonic emission on brass
493 instruments. In *Proceedings of International Conference on Acoustics 1983*, pages 355–385,
494 Paris-Lyon-Toulouse.
- 495 [Chaigne and Kergomard, 2016] Chaigne, A. and Kergomard, J. (2016). *Acoustics of musical in-*
496 *struments*. Springer-Verlag, New York, 869 pages.
- 497 [Chen et al., 2012] Chen, J.-M., Smith, J., and Wolfe, J. (2012). Do trumpet players tune reso-
498 nances of the vocal tract ? *J. Acoust. Soc. Am.*, 131(1):722–727.
- 499 [Cullen et al., 2000] Cullen, J., Gilbert, J., and Campbell, M. (2000). Brass instruments: Linear
500 stability analysis and experiments with an artificial mouth. *Acta Acustica united with Acustica*,
501 86:704–724.
- 502 [Dalmont et al., 1995] Dalmont, J., Gazengel, B., Gilbert, J., and Kergomard, J. (1995). Some
503 aspects of tuning and clean intonation in reed instruments. *Applied Acoustics*, 46(1):19–60.
- 504 [Doc et al., 2014] Doc, J., Vergez, C., and Missoum, S. (2014). A minimal model of a single-reed
505 instrument producing quasi-periodic sounds. *Acta Acustica united with Acustica*, (3):543–554.
- 506 [Elliott and Bowsher, 1982] Elliott, S. and Bowsher, J. (1982). Regeneration in brass instruments.
507 *Journal of Sound and Vibration*, 83:181–217.
- 508 [Fletcher, 1993] Fletcher, N. (1993). Autonomous vibration of simple pressure-controlled valves in
509 gas flows. *J. Acoust. Soc. Am.*, 93(4):2172–2180.
- 510 [Fréour and Scavone, 2013] Fréour, V. and Scavone, G. (2013). Acoustical interaction between
511 vibrating lips, downstream air column, and upstream airways in trombone performance. *J.*
512 *Acoust. Soc. Am.*, 134(5):3887–3898.
- 513 [Gibiat and Castellengo, 2000] Gibiat, V. and Castellengo, M. (2000). Period doubling occurrences
514 in wind instruments musical performance. *Acta Acustica united with Acustica*, 86:746–756.

- 515 [Hezard et al., 2014] Hezard, T., Freour, V., Caussé, R., Helie, T., and Scavone, G. (2014). Syn-
516 chronous multimodal measurements on the lips and glottis: comparison between two human-
517 valve oscillating systems. *Acta Acustica united with Acustica*, 100:1172–1185.
- 518 [Hirschberg et al., 1996] Hirschberg, A., Gilbert, J., Msallam, R., and Wijnands, P. (1996). Shock
519 waves in trombones. *J. Acoust. Soc. Am.*, 99:1754.
- 520 [Hirschberg et al., 1995] Hirschberg, A., Kergomard, J., and Weinreich, G. (1995). *Mechanics of*
521 *musical instruments*. Springer-Verlag, Wien, Austria.
- 522 [Iniacio et al., 2008] Iniacio, O., Antunes, J., and Wright, M. (2008). Computational modelling of
523 string-body interaction for the violin family and simulation of wolf notes. *Journal of Sound and*
524 *Vibration*, (310):260–286.
- 525 [Keefe and Laden, 1991] Keefe, D. and Laden, B. (1991). Correlation dimension of woodwind
526 multiphonic tones. *J. Acoust. Soc. Am.*, 90(4):1754–1765.
- 527 [Macaluso and Dalmont, 2011] Macaluso, C. and Dalmont, J.-P. (2011). Trumpet with near-
528 perfect harmonicity: Design and acoustic results. *J. Acoust. Soc. Am.*, 129(1):404–414.
- 529 [Myers et al., 2012] Myers, A., Pyle, R., Gilbert, J., Campbell, M., Chick, J., and Logie, S. (2012).
530 Effects of nonlinear sound propagation on the characteristic timbres of brass instruments. *J.*
531 *Acoust. Soc. Am.*, 131(1):678–688.
- 532 [Nayfeh and Balachandran, 1995] Nayfeh, A. and Balachandran, B. (1995). *Applied nonlinear*
533 *dynamics: Analytical, computational and Experimental methods*. Wiley-VCH Verlag KGaA,
534 Weinheim.
- 535 [Silva, 2009] Silva, F. (2009). *Émergence des auto-oscillations dans un instrument de musique à*
536 *anche simple (Emergence of self-sustained oscillations in a single reed music instrument)*. PhD
537 thesis, Université de Provence Aix-Marseille 1.
- 538 [Silva, 2013] Silva, F. (2013). Source website of the moreesc software. [http://moreesc.lma.
539 cnrs-mrs.fr/](http://moreesc.lma.cnrs-mrs.fr/). Accessed: 2015-06-22.
- 540 [Silva et al., 2007] Silva, F., Kergomard, J., and Vergez, C. (2007). Oscillation thresholds for
541 "striking outwards" reeds coupled to a resonator. In *Proceedings of ISMA 2007*, Barcelona.
- 542 [Sluchin, 1995] Sluchin, B. (1995). *Jeu et chant simultanés sur les cuivres (Playing and singing*
543 *simultaneously on brass instruments)*. Editions Musicales Européennes, 127 rue Amelot, 75011
544 Paris.
- 545 [sound1,] sound1. Link to the simulated radiated pressure with constant singing frequency, stored
546 online.

547 [sound2,] sound2. Link to the measured radiated pressure with constant singing frequency, stored
548 online.

549 [sound3,] sound3. Link to the measured radiated pressure with variable singing frequency, stored
550 online.

551 [sound4,] sound4. Link to the simulated radiated pressure with variable singing frequency, stored
552 online.

553 [Terrien et al., 2013] Terrien, S., Vergez, C., Cuadra, P. D. L., and Fabre, B. (2013). Is the jet-
554 drive flute model able to produce modulated sounds like flautas de chinos ? In *Proceedings of*
555 *the Stockholm Music Acoustics Conference*, pages 465–472, Stockholm.

556 [Velut et al., 2016] Velut, L., Vergez, C., Gilbert, J., and Djahanbani, M. (2016). How well
557 can linear stability analysis predict the behavior of an outward valve brass instrument
558 model? hal-01245846; Submitted to *Acta Acustica united with Acustica*, currently in revision:
559 <https://hal.archives-ouvertes.fr/hal-01245846>.

560 [von Helmholtz, 1877] von Helmholtz, H. (1877). *On the Sensations of Tone*. Dover Publications
561 Inc., New York, 608 pages.

562 [Wolfe and Smith, 2008] Wolfe, J. and Smith, J. (2008). Acoustical coupling between lip valves
563 and vocal folds. *Acoustics Australia*, 36(1):23–27.

564 [Wright and Campbell, 1998] Wright, H. and Campbell, M. (1998). Analysis of the sound of
565 Chilean pifilca flutes. *The Galpin Society Journal*, 51:51–63.

566 [Yoshikawa, 1995] Yoshikawa, S. (1995). Acoustical behavior of brass player’s lips. *J. Acoust.*
567 *Soc. Am.*, 97(3):1929–1939.

568 Figure 1: Cadenza from C.M. von Weber’s concertino for horn, including multiphonic sounds.

Figure 2: (color online) Sketch of the trombone positioning $p_m(t)$, $p(t)$ and $p_{ext}(t)$. Some variables
569 of the model defined in Section II.B are written in green.

Figure 3: (color online) Spectrum of glottis signal when playing an $F_3 - C_4$ multiphonic. Vertical
lines indicate the playing frequency f_{trb} (black), the singing frequency f_s (red) and f_{trb} first
harmonic (black, dash-dotted). Hanning window of width 0.2 s, zero-padding of the signal until
570 frequency precision is under 1 Hz.

Figure 4: (color online) Experiment: spectrograms of the pressures in the mouth p_m (a), in
the mouthpiece p (b) and radiated p_{ext} (c) measured *in vivo*. Hanning window of width 0.2 s,
95% overlap, zero-padding of the signal until frequency precision is under 1 Hz. The musician
successively sings C_4 (2.5 – 6.5s), plays F_3 (7 – 11s), then performs an $F_3 - C_4$ multiphonic
(12 – 21s). During multiphonic, the p and p_{ext} spectrograms exhibit spectral components which
571 do not belong to either the sung or the played note: they are designated with arrows in (b).

Figure 5: (color online) Experiment: spectra of the mouthpiece pressure $p(t)$ from the same performance: playing (a), singing (b), multiphonic (c). Spectra taken from the spectrograms in Fig. 4b). Peaks appear in the multiphonic spectrum, whose frequencies match neither the oscillation frequency $f_{trb} = 173.6Hz$ nor the singing frequency $f_s = 259.8Hz$ nor their harmonics. f_{trb} and f_s are represented as vertical plain lines (fundamentals) and dash-dotted lines (harmonics).

Figure 6: (color online) Simulation: spectrograms of the simulated pressures in the mouth p_m (a), in the mouthpiece p (b) and radiated p_{ext} (c). Hanning window of width 0.2 s, overlapping of 95%, zero-padding of the signal until frequency precision is under 1 Hz. The blowing pressure p_m has a stationary component ($p_m^0 = 4500$ Pa) and, after $t = 3$ s, an oscillating component (amplitude $p_m^1 = 1350$ Pa, frequency $f_s = 282.7$ Hz). The transient occurs at $t = 1.3s$, the forcing signal is added after $t = 3s$. As in the experiment, spectral components other than harmonics of the playing and the singing frequencies appear.

Figure 7: (color online) Simulation: spectrum of the mouthpiece pressure $p(t)$ from the simulation of the multiphonics. Spectrum is taken from spectrogram in Fig. 6b). $f_{trb} = 189$ Hz, $f_s = 282.7Hz$ and their harmonics are shown. Some frequencies are neither harmonic of f_s nor of f_{trb} but are integer combinations of those.

Figure 8: (color online) Experiment: Spectrograms of p_m (a), p (b) and p_{ext} (c) measured during a multiphonic. Hanning window of width 0.2 s, overlapping of 95%, zero-padding of the signal until frequency precision is under 1 Hz. The singing frequency is constant at $f_s = 255Hz$ (note C_4) for 5 seconds, then decreases towards $f_s = 185Hz$ (slightly above f_{trb} , note F_3) and remains at this value after $t = 10s$. The playing frequency remains as constant as possible, with $f_{trb} \in [167.2 : 173.4]$.

Figure 9: (color online) Experiment: Spectra taken from instants of Fig. 8b) each second between approximately $t = 3s$ and $t = 10s$, when f_s decreases. The plain lines represent f_{trb} (black) and f_s (red), the dash-dotted lines represent their respective harmonics.

Figure 10: (color online) Simulation: Spectrograms of the simulated p_m (a), p (b) and p_{ext} (c). Hanning window of width 0.2 s, overlapping of 95%, zero-padding of the signal until frequency precision is under 1 Hz. The forcing term appears after $t = 4$ seconds at $f_s = 282.7Hz$ (a fifth above f_{trb}), then is steady until $t = 6$ s; then decreases linearly towards $f_s = 206$ Hz (155 cents above f_{trb}) for 6 seconds: then it stays at this frequency for 2 seconds. The playing frequency remains stable at $f_{trb} = 188.7$ Hz all along the simulation. Vertical dash-dotted lines in (b) highlight the periodic oscillation regime at $38.5Hz$.

Figure 11: (color online) Simulation: Spectra of the simulated p , taken from the spectrogram in Fig. 10b) each second, between approximately $t = 5s$ and $t = 12s$ where f_s decreases. The plain vertical lines represent f_{trb} and f_s , the dash-dotted lines represent their respective harmonics.

Figure 12: (color online) Plot of f_s (red), f_{trb} (black) and some integer combinations (plain lines) along with the frequencies of the five first components of experimental (a) and simulation (b) mouthpiece pressure (marks). These marks correspond to the peak frequencies in Fig. 9 and 11. The recorded values very closely match the integer combinations.

Figure 13: (color online) Comparison of spectra of internal pressures simulated either with a sinusoidal forcing component, or a 3-harmonic forcing component. Hanning window of width 0.2 s, zero-padding of the signal until frequency precision is under 1 Hz. Frequency decreases as in Fig. 10. Fig. (a) is the spectra of p at the beginning of the simulations, Fig. (b) at the end. Differences in terms of amplitude are small in (a) and reach 6 dB in (b). Vertical lines indicate f_{trb} and f_s (plain) and their harmonics (dash-dotted).

Figure 14: (color online) Simulation: Spectrograms of p_m , p and p_{ext} simulated with a one-mode resonator. Hanning window of width 0.2 s, 95% overlapping, zero-padding of the signal until frequency precision is under 1 Hz. $f_{trb} = 180.5$ Hz, f_s from 270 Hz to 196.4 Hz. The results are comparable with those presented in Fig. 10 with weaker components at high frequencies.