

# The study of the correlation between winding geometry and copper losses by means of analytic and finite element calculations

M. Al Eit, F. Bouillault, C. Marchand, and G. Krebs

GeePs, UMR CNRS 8507, CentraleSupélec, Univ Paris-Sud, Sorbonne Universités and UPMC Univ Paris 6  
3, 11 rue Joliot-Curie, Plateau de Moulon, 91192 Gif-sur-Yvette CEDEX, France, moustafa.aleit@supelec.fr

The strong correlation between the level of eddy current losses in copper conductors and the winding geometry shows the necessity to pay attention to the manner of disposition of coils in machine slots. Finite-element modeling of electric machines includes moving band technique to perform the rotor motion and Newton-Raphson iterations to take into consideration the non-linearity of magnetic circuits. In order to calculate the copper losses, finite element method leads then to a substantial calculation time and hinders any process of conception and optimization of winding geometries. An analytic calculation is proposed, it presents clear interests in repetitive analysis.

*Index Terms*— Finite element analysis, eddy currents, analytic calculation, switched reluctance machine.

## I. INTRODUCTION

The copper losses are subdivided into classical ohmic DC losses and additional eddy current losses. The latter exists due to the strong electromagnetic coupling between the current density and the time varying magnetic fields penetrating the copper conductors. Due to the fact that this interaction between electric and magnetic variables cannot be solved easily, then finite-element (FE) methods can be used to give a numerical solution. A strong correlation exists between the level of eddy current losses and the disposition of coils in machine slots [1]. Since, the choice of the optimized geometry distribution of the winding turns using the FE resolution leads to a substantial calculation time and requires large storage capacity. Therefore, this article proposes an analytic copper losses calculation which provides clear advantages in repetitive analyses for optimization problems in terms of copper losses. The test problem is a switched reluctance machine (SRM) wound with different winding configurations. The analytic values of copper losses are compared to those calculated by the 2D FE modeling method.

## II. FE METHOD FORMULATION

At low frequencies the Maxwell's equations in an electromagnetic system and the associated constitutive medium relationships are given by:

$$\overline{\text{curl}}(\vec{H}) = \vec{j}, \quad \text{div}(\vec{B}) = 0, \quad \overline{\text{curl}}(\vec{E}) = -\partial\vec{B}/\partial t \quad (1a-b-c)$$

$$\vec{H} = \nu\vec{B}, \quad \vec{j} = \sigma\vec{E} \quad (2a-b)$$

An easy coupling with the electric circuit leads to use in a privileged way the potentials for example the magnetic vector potential  $\vec{A}$  and the electric scalar potential  $V$ . The resultant vector potential formulation from (1) and (2) is given by:

$$\overline{\text{curl}}(\nu\overline{\text{curl}}(\vec{A})) = -\sigma\partial\vec{A}/\partial t - \sigma\overline{\text{grad}}(V) \quad (3)$$

A combined circuit equation reflects that the current feeding the conductor of cross section  $S$  is  $I(t)$ :

$$I(t) = \iint_S \vec{j} \cdot \vec{ds} = \iint_S (-\sigma\partial\vec{A}/\partial t - \sigma\overline{\text{grad}}(V)) \cdot \vec{ds} \quad (4)$$

The 2D FE formulation of (3), as well as considering (4) and the backward Euler method for time discretization, leads to the following matrix system [2]:

$$\begin{pmatrix} [S] + \frac{[T]}{\Delta t} & [D] \\ [D]^t & \Delta t[G] \end{pmatrix} \begin{bmatrix} [A_t] \\ [\Delta V] \end{bmatrix} = \begin{bmatrix} \frac{[T]}{\Delta t} [A_{t-\Delta t}] \\ \Delta t[I(t)] + [D]^t [A_{t-\Delta t}] \end{bmatrix} \quad (5)$$

Where  $\Delta V$  is the electric potential drop (voltage) per unit length and  $A_t$  is the z-component at the instant  $t$  of the magnetic vector potential.

## III. ANALYTIC COPPER LOSSES CALCULATION

We consider the case of  $n$  individual conductors, placed in a symmetric slot with parallel edges and each fed by the imposed current  $I(t)$ . We start from the assumptions that the permeability of the iron is very high then the magnetic field in the lamination material is negligible and that the magnetic and electric fields are independent of  $x$  and  $z$  (Fig. 1) and have one component  $H_x(y) = H(y)$  and  $E_z(y) = E(y)$  respectively [3].

Applying Ampere's law and Faraday's law to a conductor section of height  $dy$ , length along  $z$ -axis  $l$  and width  $l_c$  placed in a slot of width  $l_s$ , results in the following differential equations of  $H(y)$  and  $E(y)$ :

$$\frac{d(H(y))}{dy} = \frac{\sigma l_c}{l_s} E(y), \quad \frac{d(E(y))}{dy} = \mu \partial H(y) / \partial t \quad (6a-b)$$

Where  $\mu$  and  $\sigma$  are the copper permeability and conductivity respectively. Assuming sinusoidal quantities and combining (6a) and (6b) yield a second order differential equation:

$$\frac{d^2(H(y))}{dy^2} - j\omega\mu \frac{\sigma l_c}{l_s} H(y) = 0 \quad (7)$$

with the solving approach:

$$H(y) = c_1 e^{ry} + c_2 e^{-ry}; \quad r = \sqrt{j\omega\mu\sigma l_c / l_s} \quad (8)$$

To determine the constants  $c_1$  and  $c_2$ , the boundary conditions of the magnetic field of the regarded conductor can be utilized. The origin of the coordinate system is considered located at the lower edge of the regarded conductor. A parameter  $p$  identifies the conductor layer within the slot with  $1 \leq p \leq n$ , where  $n$  is the total number of slot conductors placed on top of each other. The line of magnetic flux passing

through the lower edge of the conductor at  $y=0$  surrounds  $(p-1)$  times the current  $I(t)$  and that at  $y=h$  surrounds  $p$  times the current  $I(t)$ . Where  $h$  is the height of the conductor layer. Determining  $c_1$  and  $c_2$  gives the magnetic field distribution along the conductor. The expression of the electric field can be deduced from (6a). By multiplying the electric field with  $\sigma$ , the current density valid for layer  $p$  is calculated to:

$$J(y) = \frac{rI}{l_c \sinh rh} [p \cosh ry - (p-1) \cosh(ry - rh)] \quad (9)$$

The total copper losses of conductor  $p$  can be calculated according to the following equation :

$$P = l_c l / \sigma \int_0^h |J(y)|^2 dy \quad (10)$$

$$P = \frac{l}{\sigma h l_c} I^2 [p(p-1)\psi(x) + \varphi(x)]; \quad x = hr/\sqrt{2j} \quad (11)$$

$$\psi(x) = 2x \frac{(\sinh x - \sin x)}{\cosh x + \cos x} \quad (12)$$

$$\varphi(x) = x \frac{(\sinh 2x + \sin 2x)}{\cosh 2x - \cos 2x} \quad (13)$$

Where  $I$  is the rms value of the sinusoidal current  $I(t)$ . In case of periodic non sinusoidal current waveforms, the total losses can be calculated by summing the losses of each Fourier spectral component.

#### IV. APPLICATION EXAMPLE

We start the study of two test case examples. The first one is a skin effect example, it is about one conductor ( $n=1$ ) of cross section  $S$  placed in a symmetric slot with parallel edges and fed by a typical SRM phase current  $I(t)$  at the frequency of 1000 Hz. The second is a proximity effect example, it is about 18 conductors ( $3 \times 6$ ) fed each one by the current  $I(t)/18$ . ( $3 \times 6$ ) configuration means that we have three horizontal layers with 6 vertical conductors by layer. Each conductor has a cross section of  $S/18$ . Due to symmetry reasons, the current distribution inside the six conductors of each layer must be equal since all conductors are fed by the same current and are penetrated by the same magnetic field  $H_x(y)$ . Hence, the six conductors can be merged into one solid conductor fed by the current  $I(t)/3$ . The proximity effect example is then about 3 individual conductors which can be interpreted as three-layer winding ( $n=3$ ) (Fig. 1).

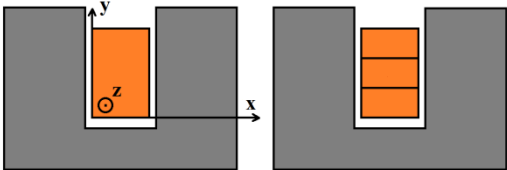


Fig.1 Left: Skin effect example ( $n=1$ ). Right: Proximity effect example ( $n=3$ ).

Although the chosen conductor dimensions and the excitation frequency do not match realistic conditions for a slot winding application, the examples illustrate the strong influence of the winding configuration on the ac losses. Both configurations have the same dc resistance and produce the same amount of magnetomotive force but still the ac losses of the second example are 1.5 times higher than those of the first example. The FE simulations copper losses are very close to analytic values with an error that does not exceed the 3.2% (Table 1).

Table 1. Comparison between the loss calculation methods for the examined test case examples.

Test Case	Copper Losses in W		Ohmic Losses in W
	Analytic Calculation	FE Model	
Example 1 ( $n=1$ )	87.81	89.43	29.18
Example 2 ( $n=3$ )	131.92	127.72	

An SRM 8/6 is taken as another example. Only one phase is fed, it consists of two coils connected in series; each of them has 18 turns. These turns can be distributed in the slot in multiple configurations. They can be placed horizontally ( $18 \times 1$ ), vertically ( $1 \times 18$ ) or as other different configurations such as ( $9 \times 2$ ), ( $6 \times 3$ ), ( $3 \times 6$ ) or ( $2 \times 9$ ). For analytic loss calculations according to (11) several geometry parameters  $l, l_c, l_s$  and  $h$  need to be determined. Since the slot edges in SRM are not parallel, it seems reasonable to calculate a mean value for  $l_s$ . Then we come across the geometry studied in the test case example. The analytic calculation in case of sinusoidal currents shows the correlation between the copper losses and the winding geometry configurations (Fig.2). Now, two real operating points at speeds 1100 and 5000 rev/min and three winding configurations are studied as an example (Table 2). The configuration ( $9 \times 2$ ) seems to be the optimal choice. Analytic calculation does not give very similar copper loss values to those calculated by FE method in case of SRM, but it can be used as an efficient mean to choose the optimized winding configuration for a given frequency operating range.

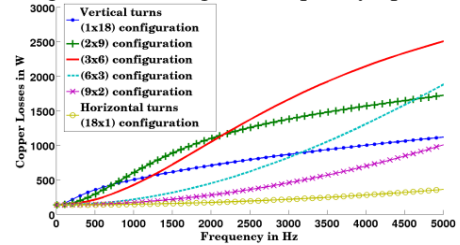


Fig.2 Correlation between the copper losses and the winding configurations.

Table 2. Comparison between the loss calculation methods for the SRM

Configuration Type	Copper Losses in W			
	Operating point 1		Operating point 2	
	Analytic	FE	Analytic	FE
( $2 \times 9$ )	79.9	85.3	14.4	22.2
( $3 \times 6$ )	78.3	81.1	11.9	18.1
( $9 \times 2$ )	77.1	77.9	9.1	10.9
Ohmic Losses in W	76.9		8.6	

#### V. CONCLUSION

Starting from several simplifying assumptions, the analytic model allows very fast calculation of winding copper losses. As a coarse model, it may be associated with the finite element modeling and can be used thus in optimization methodologies such as the output space mapping technique.

#### VI. REFERENCES

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