



HAL
open science

Dynamic sensitivity analysis of a suspension model

Sabra Hamza, Floriane Anstett-Collin, Qiaochu Li, Lilianne Denis-Vidal,
Abderazik Birouche, Michel Basset

► **To cite this version:**

Sabra Hamza, Floriane Anstett-Collin, Qiaochu Li, Lilianne Denis-Vidal, Abderazik Birouche, et al..
Dynamic sensitivity analysis of a suspension model. 13th International Symposium on Advanced
Vehicle Control, AVEC'16, Sep 2016, Munich, Germany. hal-01361082

HAL Id: hal-01361082

<https://hal.science/hal-01361082>

Submitted on 6 Sep 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Dynamic sensitivity analysis of a suspension model

S. Hamza & F. Anstett-Collin

*Centre de Recherche en Automatique de Nancy (UMR 7039)
Université de Lorraine, France*

Q. Li & L. Denis-Vidal

*Laboratoire de Mathématiques Appliquées de Compiègne (EA 2222)
Sorbonne Universités, Université de Technologie de Compiègne, France*

A. Birouche & M. Basset

*Laboratoire Modélisation, Intelligence, Processus, Systèmes (EA 2332)
Université de Haute-Alsace, France.*

ABSTRACT: A sensitivity analysis of a suspension model has been performed in order to highlight the most influential parameters on the sprung mass displacement. To analyse this dynamical model, a new global and bounded dynamic method is investigated. This method, based on the interval analysis, consists in determining lower and upper bounds including the dynamic sensitivity indices. It requires only the knowledge of the parameter variation ranges and not the joint probability density function of the parameters which is hard to estimate. The advantage of the proposed approach is that it takes into account the recursive behavior of the system dynamics.

1 INTRODUCTION

Suspension plays an important role in vehicle safety and road holding. In general, the suspension system behavior is described by dynamical models depending on parameters that are subject to uncertainty due to insufficient knowledge, measurement error or imprecision, etc. Sensitivity analysis can help to evaluate the impact of this lack of knowledge on the model response, which here is the displacement of the sprung mass.

Numerous studies have focused on the sensitivity analysis for static models (Saltelli, Ratto, Andres, Campolongo, Cariboni, Gatelli, Saisana, & Tarantola 2008). In the case of dynamical models, local approaches based on partial derivatives are often used. However, in the automotive field, it can be of great importance to consider the entire uncertainty range of parameters since they can vary within large intervals depending on their meaning. Few global approaches have been proposed for dynamical models. In general, these methods are statistical and are based on the analysis of the output variance. In the case of dynamical model, they consist in computing the sensitivity indices at each time instant (Haro Sandoval, Anstett-Collin,

& Basset 2012). This can lead to an important amount of informations, not easy to analyse. Moreover, these approaches require the knowledge of the joint probability density function of the parameters which is hard to estimate. In this work, a new sensitivity analysis method based on interval analysis is provided.

The key idea is to determine upper and lower bounds including the sensitivity functions, based on the knowledge of the parameter variation ranges. If they exist, these bounds are guaranteed (Lin & Stadtherr 2008). Unlike the statistical methods, the proposed approach does not require the knowledge of the joint probability density function of the parameter. Furthermore, the approach takes into account the recursive behavior of the system dynamics. This paper is organized as follows: in section 2 problem statement is introduced. In section 3, partial derivative based sensitivity analysis method is described. In section 4, the principle of interval analysis technique is introduced. An illustrative example, is presented in the end of section 3 and 4. In section 5, an application of the method on a quarter vehicle model is presented. Conclusion is given in section 6.

2 PROBLEM STATEMENT

Consider a linear dynamic model represented in the state space form as:

$$\begin{cases} \dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) \\ y(t) = C(\theta)x(t) + D(\theta)u(t) \\ x(t_0) = x_0 \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^l$ is the output vector, $A(\cdot) \in \mathbb{R}^{n \times n}$, $B(\cdot) \in \mathbb{R}^{n \times m}$, $C(\cdot) \in \mathbb{R}^{l \times n}$ and $D(\cdot) \in \mathbb{R}^{l \times m}$. $\theta = [\theta_1, \dots, \theta_p]$ is the uncertain parameter vector, where p is the parameters number, $\theta_i \in \mathbb{R}$.

In order to study parameters variation impact on the output model, classically, local sensitivity analysis is applied. This method consists of computing parameter effect around a nominal value. The influence of parameter θ_i is defined by the partial derivative of the output $y(t)$ with respect to the parameter θ_i and is given by:

$$S_i(t) = \frac{\partial y(t)}{\partial \theta_i} \quad (2)$$

Assuming that $\theta_i \in [\theta_{i_{min}}, \theta_{i_{max}}]$, local sensitivity analysis application can lead to erroneous results and fault index interpretation. Consequently, it cannot be efficient to measure the influence of each parameter on the model output.

To overcome this problem and those ones given in the introduction, a new methodology is presented. The method consists of determining an upper and lower bound, ensuring the existence of sensitivity function inside, using interval analysis technique. In the next section, the sensitivity analysis method based on partial derivative, is explained.

3 PARTIAL DERIVATIVE-BASED SENSITIVITY ANALYSIS

Consider the model representation given by (1). The objective of this section is to determine a system of sensitivity functions describing the dynamic behavior of sensitivity functions which measure the influence of each model parameter. This system is linear and can be written in a state space form (see figure 1). It is determined using available informations such as: the output measurements y and the model input u .

The objective of the following paragraph is to determine the system of sensitivity functions.

3.1 Sensitivity functions representation

The partial derivative of the state vector $x(t)$ given by (1) is:

$$\frac{\partial \dot{x}(t)}{\partial \theta_i} = A(\theta) \frac{\partial x(t)}{\partial \theta_i} + \frac{\partial A(\theta)}{\partial \theta_i} x(t) + \frac{\partial B(\theta)}{\partial \theta_i} u(t) + B(\theta) \frac{\partial u(t)}{\partial \theta_i} \quad (3)$$

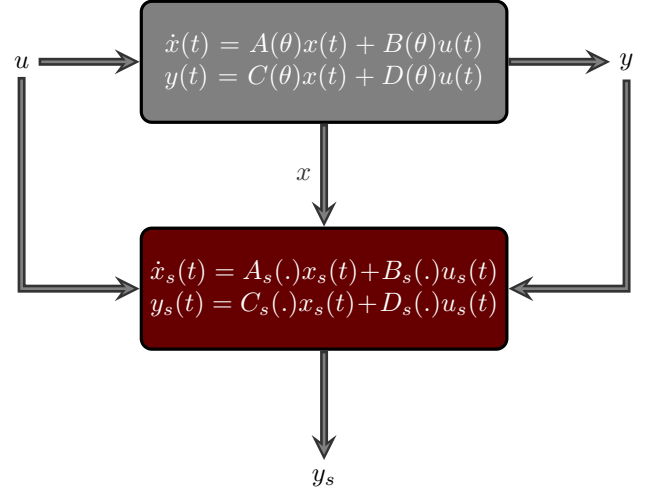


Figure 1: Principle of sensitivity functions system

Assuming u is a function of time which does not depend on parameter θ_i , $\frac{\partial}{\partial \theta_i} \left(\frac{\partial x(t)}{\partial t} \right) = \frac{\partial}{\partial t} \left(\frac{\partial x(t)}{\partial \theta_i} \right)$ as θ_i is a constant, not varying with time. We assume also there is no dependence between these parameters: $\frac{\partial \theta_i}{\partial \theta_j} = 0$ for $i \neq j$

Let define two sensitivity matrices of the model output $S_y(t)$ and of the state vector $S_x(t)$ given by:

$$S_y(t) = \frac{\partial y(t)}{\partial \theta} \quad S_x(t) = \frac{\partial x(t)}{\partial \theta} \quad (4)$$

For each parameter θ_i , the following subsystem is defined:

$$\begin{cases} \dot{S}_x^i(t) = A(\theta) S_x^i(t) + \frac{\partial A(\theta)}{\partial \theta_i} x(t) + \frac{\partial B(\theta(t))}{\partial \theta_i} u(t) \\ S_y^i(t) = C(\theta) S_x^i(t) + \frac{\partial C(\theta)}{\partial \theta_i} x(t) + \frac{\partial D(\theta)}{\partial \theta_i} u(t) \\ S_x(t_0) = S_{x_0} \end{cases} \quad (5)$$

Grouped all subsystems together, the sensitivity of the state vector is defined by $x_s(t) = [S_x^1(t), S_x^2(t), \dots, S_x^p(t)]^T$ which contains the sensitivity functions and the sensitivity functions of the output model define $y_s(t) = [S_y^1(t), S_y^2(t), \dots, S_y^p(t)]^T$.

The whole system of sensitivity functions is given finally as:

$$\begin{cases} \dot{x}_s(t) = \begin{pmatrix} A(\theta) S_x^1(t) + \frac{\partial A(\theta)}{\partial \theta_1} x(t) + \frac{\partial B(\theta)}{\partial \theta_1} u(t) \\ A(\theta) S_x^2(t) + \frac{\partial A(\theta)}{\partial \theta_2} x(t) + \frac{\partial B(\theta)}{\partial \theta_2} u(t) \\ \vdots \\ A(\theta) S_x^p(t) + \frac{\partial A(\theta)}{\partial \theta_p} x(t) + \frac{\partial B(\theta)}{\partial \theta_p} u(t) \end{pmatrix} \\ y_s(t) = \begin{pmatrix} C(\theta) S_x^1(t) + \frac{\partial C(\theta)}{\partial \theta_1} x(t) + \frac{\partial D(\theta)}{\partial \theta_1} u(t) \\ C(\theta) S_x^2(t) + \frac{\partial C(\theta)}{\partial \theta_2} x(t) + \frac{\partial D(\theta)}{\partial \theta_2} u(t) \\ \vdots \\ C(\theta) S_x^p(t) + \frac{\partial C(\theta)}{\partial \theta_p} x(t) + \frac{\partial D(\theta)}{\partial \theta_p} u(t) \end{pmatrix} \end{cases}$$

(6)

Thus, the sensitivity system can be written in a state space system form as:

$$\begin{cases} \dot{x}_s(t) = A_s(\theta) x_s(t) + B_s(\theta) u_s(t) \\ y_s(t) = C_s(\theta) x_s(t) + D_s(\theta) u_s(t) \end{cases} \quad (7)$$

where:

$$A_s(\theta) = \begin{pmatrix} A(\theta) & 0_{n \times n} & \cdots & 0_{n \times n} \\ 0_{n \times n} & A(\theta) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{n \times n} \\ 0_{n \times n} & \cdots & 0_{n \times n} & A(\theta) \end{pmatrix} \quad (8)$$

$$B_s(\theta) = \begin{pmatrix} \frac{\partial A(\theta)}{\partial \theta_1} & \frac{\partial B(\theta)}{\partial \theta_1} \\ \frac{\partial A(\theta)}{\partial \theta_2} & \frac{\partial B(\theta)}{\partial \theta_2} \\ \vdots & \vdots \\ \frac{\partial A(\theta)}{\partial \theta_p} & \frac{\partial B(\theta)}{\partial \theta_p} \end{pmatrix} \quad (9)$$

$$C_s(\theta) = \begin{pmatrix} C(\theta) & 0_{l \times n} & \cdots & 0_{l \times n} \\ 0_{l \times n} & C(\theta) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{l \times n} \\ 0_{l \times n} & \cdots & 0_{l \times n} & C(\theta) \end{pmatrix} \quad (10)$$

$$D_s(\theta) = \begin{pmatrix} \frac{\partial C(\theta)}{\partial \theta_1} & \frac{\partial D(\theta)}{\partial \theta_1} \\ \frac{\partial C(\theta)}{\partial \theta_2} & \frac{\partial D(\theta)}{\partial \theta_2} \\ \vdots & \vdots \\ \frac{\partial C(\theta)}{\partial \theta_p} & \frac{\partial D(\theta)}{\partial \theta_p} \end{pmatrix} \quad (11)$$

$$u_s(t) = \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \quad (12)$$

with $x_s(t_0) = 0_{n \times 1}$. $0_{l \times k}$ is a $l \times k$ zero matrix. In the next paragraph, a test example is considered and sensitivity analysis functions, computed using method previous described, are determined.

3.2 Illustrative example

3.2.1 System description

Consider a mass-spring-damper system given in figure 2.

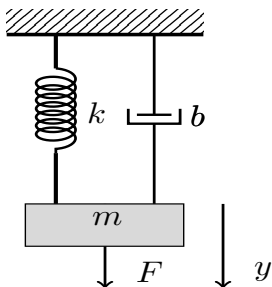


Figure 2: Schematic representation of a mass-spring-damper model

As a mechanical system, the dynamics of a mass-spring-damper system can be described by the following 2nd-order differential equation:

$$m\ddot{y}(t) + b\dot{y}(t) + ky(t) = F(t) \quad (13)$$

where m is the mass, b is the damping coefficient, k is the stiffness coefficient and F is the force acting on the mass.

We introduce the state vector $\dot{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$.

where x_1 and x_2 are respectively the mass displacement and velocity.

The system (13) can be then written in a state space form as follows:

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t) \\ y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{cases} \quad (14)$$

The objective is to study the influence of parameters variation of m , b and k on the mass position x_1 .

3.2.2 Partial derivative-based sensitivity analysis

Firstly, let us study the influence of parameters m , b and k around their nominal values which are fixed to 1. A constant integration step size is chosen with $h = 0.1m$ and the step signal is applied on F . figure 3 represents the sensitivity function of each parameter. These functions have been determined using (7).

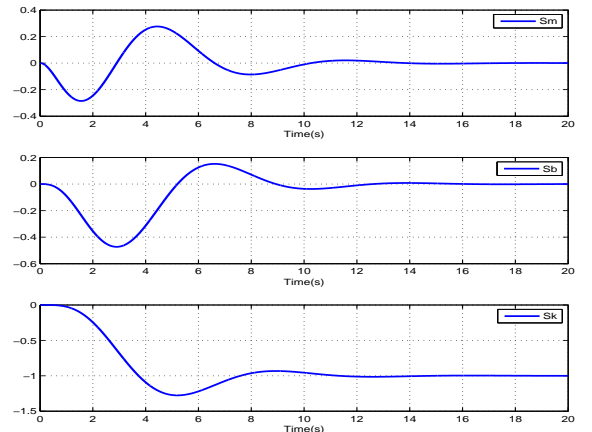


Figure 3: Sensitivity analysis functions

We can see that all parameters are influent on the mass displacement until $t = 10s$. After this transient time, only the stiffness coefficient k dominates the mass position variation. In fact, the mass and the damping coefficient depend on the velocity

and the acceleration, thus, they are influent in the transient region. When the velocity and the acceleration tend to zero, only the stiffness coefficient is influent. Its influence can be seen in the steady state. In the next section, interval analysis method is introduced.

4 INTERVAL ANALYSIS METHOD

Here, the principle of interval analysis for linear and nonlinear model is explained. The objective is to find a validated enclosure of all solutions of parametric autonomous system:

$$\dot{x}_s(t) = f(x_s(t), \theta), \quad x_s(t_0) = x_{s_0} \in X_{s_0}, \quad \theta \in \Theta. \quad (15)$$

where $t \in [t_0, t_m]$. θ is a p -dimensional parameter vector, x_s is the n -dimensional vector of state variables and x_{s_0} is the n -dimensional vector of initial values. The interval vectors Θ and X_{s_0} represent enclosures of the uncertainties in θ and x_{s_0} , respectively.

$f : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$ is $(k - 1)$ times continuously differentiable with respect to the parameter θ on \mathbb{R}^n .

VSPODE tool (Lin & Stadtherr 2007) is used to compute a rigorously guaranteed enclosure on the trajectories of a linear or nonlinear ODE system with interval-valued initial values or parameters. This enclosure is computed by using an interval Taylor series method, combined with Taylor models to overcome the dependency of each uncertain quantities and wrapping effect. The use of Taylor models leads to a considerable large reduction of the overestimation which is often associated with interval methods. Before describing how this method works, a convenient notation for Taylor Coefficients (TC) should be introduced.

$$\begin{aligned} f^{[0]}(x_s) &= x_s \\ f^{[1]}(x_s) &= f \\ f^{[i]}(x_s) &= \frac{1}{i} \left(\frac{\partial f^{[i-1]}}{\partial x_s} f \right) (x_s) \quad \text{for } i \geq 2 \end{aligned} \quad (16)$$

This interval method consists of two algorithms applied at each integration step. In the first step, existence and uniqueness of the solution are proved using the Picard-Lindelöf operator. In the second step, a tighter enclosure of the solution is computed.

4.1 Validating existence and uniqueness

To get a priori bounds of an ordinary differential equation, we use an interval evaluated Taylor series with respect to time. Suppose that at t_j we

have an enclosure $[y_{s_j}]$ of $y_s(t; t_0, [y_{s_0}], \theta)$. By using the Picard-Lindelöf operator and the Banach fixed-point theorem, one can show that if a stepsize h_j and a priori enclosure $[\tilde{y}_j]$ satisfy:

$$[y_{s_j}] + [0, h_j] f([\tilde{y}_{s_j}], \theta) \subseteq [\tilde{y}_{s_j}] \quad (17)$$

then (15) has a unique solution $y_s(t; t_j, y_{s_j}, \theta) \in [\tilde{y}_{s_j}]$

4.2 Computing a tighter enclosure

A basic instinct to obtain the tighter enclosure $[y_{s_{j+1}}]$ will be by using the interval Taylor series with the following form:

$$[y_{s_{j+1}}] = [y_{s_j}] + \sum_{i=1}^{k-1} h_j^i f^{[i]}(y_{s_j}, \theta) + h_j^k f^{[k]}(y_s; t_j, t_{j+1}, \theta) \quad (18)$$

This basic first order interval evaluation form will lead sooner the sets $[y_{s_{j+1}}]$ be overestimated as the intrinsic wrapping problem. Without applying the Taylor model directly on the inclusion function f , VSPODE using a mean-value form to reduce the propagation of wrapping effect which results big overestimation, the limitation of the use of this approach has been discussed by Neumaier (Neumaier 2002). By projecting the original domain sets to an orthogonal domain with the help of parallelepiped or QR-factorization methods (Nedialkov, Jackson, & Corliss 1999), the wrapping effect in the polynomial part of Taylor model could be relatively reduced but it still propagates in the reminder part due to the intrinsic dependency problem of interval analysis.

In the next paragraph, method described above is applied on mass-spring-damper system considered on previous section .

4.3 Illustrative example

Now, let us consider the parameters m , b and k varying within the range of 0.9 to 1.1. Like the previous case, a constant integration step size is chosen with $h = 0.1m$ and the step signal is applied on F.

Consider the initial state of $x(0) = [0, 0]^T$, we use VSPODE to determine a verified state enclosure for $t \in [0; 20]$. The order of the Taylor model and the interval Taylor series were chosen as 17 and 20, respectively, which are aimed to avoid the overestimation during calculation.

figure 4 shows the curves representing the upper and lower bounds and the sensitivity functions corresponding on the influence of m , b and k on the

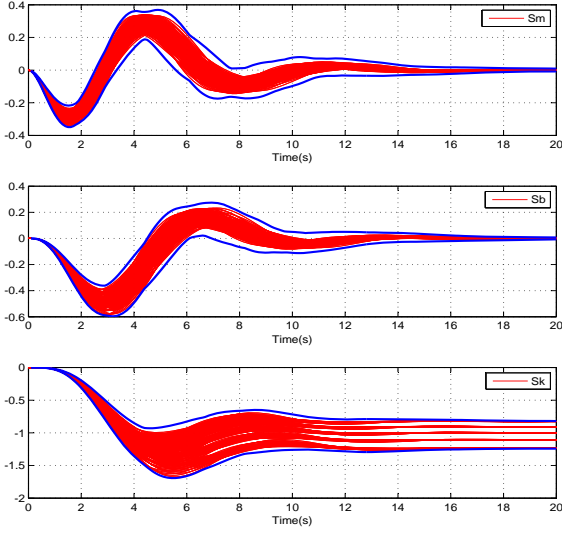


Figure 4: Interval enclosure for sensitivity functions

mass displacement.

Bounds are obtained using VSPODE. Several points are chosen from parameters interval and sensitivity functions have been determined by a validated integrator VNODE (Validated Numerical ODE). Sensitivity functions are guaranteed within the bounds. A large interval has been obtained in the transient regime until $t = 12s$. m , b and k are influential on the mass position. However, in the static regime we can observe that the enclosure is tightest so for any value of parameter in this regime and only k is influential. In order to show the effectiveness of this method, it is applied to study the sensitivity analysis of quarter vehicle model parameters.

5 SENSITIVITY ANALYSIS OF A SUSPENSION SYSTEM

5.1 System description

Consider a simple quarter vehicle model given in figure 5. This model is often used in studies dealing with suspensions.

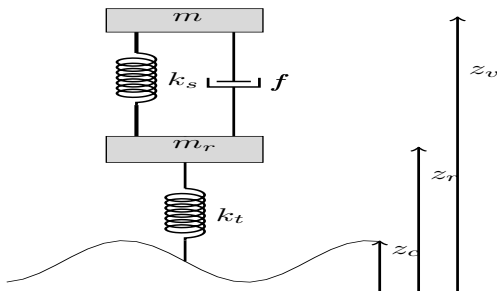


Figure 5: Model of quarter vehicle

The model is composed of a sprung mass m and

an unsprung mass m_r connected by a spring with the stiffness coefficient k_s and a damper with the damping coefficient f . The tire is modeled by a spring with the stiffness coefficient k_t . Distances z_c , z_r and z_v are, respectively, the road profile, the vertical displacements of unsprung and sprung masses.

The vertical dynamics of the quarter vehicle is governed by the following equations:

$$\begin{cases} m_s \ddot{z}_v(t) = -k_s (z_v(t) - z_r(t)) - b (\dot{z}_v(t) - \dot{z}_r(t)) \\ m_r \ddot{z}_r(t) = -k_s (z_r(t) - z_v(t)) - b (\dot{z}_r(t) - \dot{z}_v(t)) \\ -k_r (z_r(t) - z_c(t)) \end{cases} \quad (19)$$

The system (19) can be written in a state space form as follows:

$$\begin{bmatrix} \dot{z}_v(t) \\ \ddot{z}_v(t) \\ \dot{z}_r(t) \\ \ddot{z}_r(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{b}{m_s} & \frac{k_s}{m_s} & \frac{b}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_r} & \frac{b}{m_r} & -\frac{k_r - k_s}{m_r} & -\frac{b}{m_r} \end{bmatrix} \begin{bmatrix} z_v(t) \\ \dot{z}_v(t) \\ z_r(t) \\ \dot{z}_r(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_r}{m_r} \end{bmatrix} z_c(t)$$

$$y(t) = [1 \ 0 \ 0 \ 0] \begin{bmatrix} z_v(t) \\ \dot{z}_v(t) \\ z_r(t) \\ \dot{z}_r(t) \end{bmatrix} \quad (20)$$

The objective here is to study the influence of parameters k_s , m , f , m_r and k_t on the vertical displacement of the sprung mass. We determine then sensitivity functions as defined in (7).

5.2 Results

Firstly, sensitivity functions have been determined, using (7), around the nominal values of parameters k_s , m , f , m_r and k_t which are fixed respectively to $29500N.m^{-1}$, $450kg$, $2000Ns.m^{-1}$, $40kg$ and $210000N.m^{-1}$. figure 6 shows the sensitivity functions, considering a step road profile (the amplitude is $0.1m$).

One can observe that in the transient region, the most important parameter is the unsprung mass m_r . When the system has passed its transient phase, all sensitivity functions converge to zero. Thus parameters become not influent. In fact, this result depends on road profile applied which corresponds to a situation where a vehicle goes up on a sidewalk. When the road profile changes, the sprung mass is the most influent. Once the vehicle is on the sidewalk, the vertical displacement of

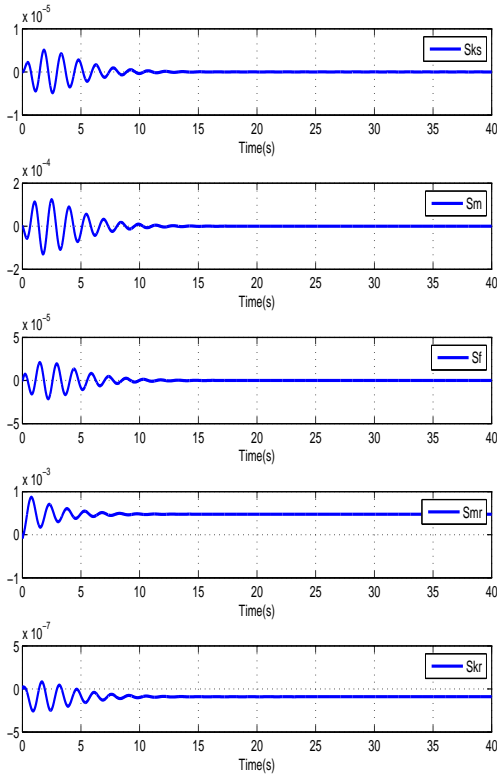


Figure 6: Sensitivity analysis functions

the unsprung mass z_v returns to its first state. We assume now that parameters k_s , m , f , m_r and k_r vary in the interval $[265500N.m^{-1}; 32450N.m^{-1}]$, $[260kg; 540kg]$, $[1800Ns.m^{-1}; 2200Ns.m^{-1}]$, $[36kg; 44kg]$ and $[189000N.m^{-1}; 231000N.m^{-1}]$.

Figure 7 shows the curves representing the upper and lower bounds obtained using VSPODE. Figure 7 gives also sensitivity functions computed around different values of parameters in their interval variation in order to verify the determined bounds.

It is clearly observed that upper and lower bounds have ensured the existence of sensitivity functions inside. In this interval of variation, unsprung mass m_r is the most influential. The sprung mass m follows. Other parameters appear of less impact, compared to m_r and m , on the displacement of the sprung mass.

6 CONCLUSION

In this paper, global and dynamic sensitivity analysis method have been presented. Sensitivity functions are firstly computed using the partial derivative then interval enclosure ensuring the existence of sensitivity function inside is determined. The upper and lower interval enclosure are computed using VSPODE solver. The effectiveness of the proposed approach has been demonstrated through an illustrative example and an application on a quarter vehicle model. Consistent results have been shown. However, it is necessary

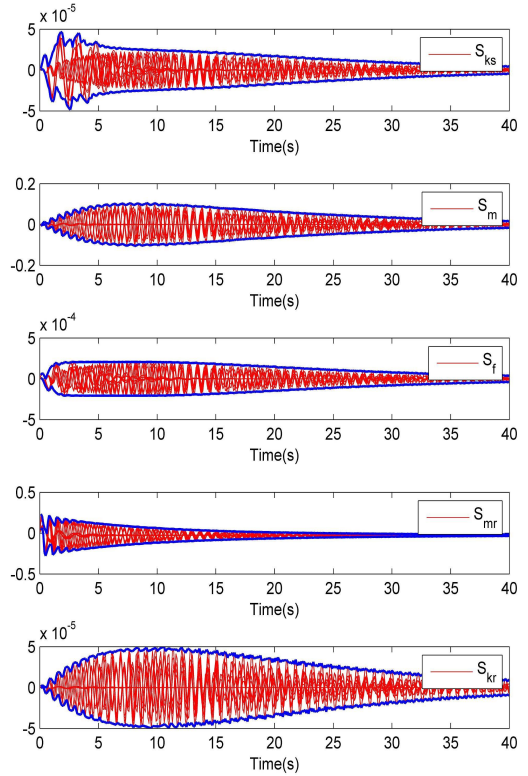


Figure 7: Interval enclosure for sensitivity functions

to note that interval enclosure depends on the initial condition of the states and the parameters. When dealing with large intervals, possibly, this interval approach produces the overestimation of the reachable sets. One can divide the initial sets with several small sets to reduce the overestimation, or try to take all the parameter around 1 will also release the pessimism. For near future, it will be interesting to consider a more complex model.

REFERENCES

- Haro Sandoval, E., F. Anstett-Collin, & M. Basset (2012). Sensitivity study of dynamic systems using polynomial chaos. *Reliability Engineering System Safety* 104(0), 15 – 26.
- Lin, Y. & M. Stadtherr (2007). Guaranteed state and parameter estimation for nonlinear continuous-time systems with bounded-error measurements. *Industrial and Engineering Chemistry Research*, 7198–7207.
- Lin, Y. & M. Stadtherr (2008). Validated solutions of initial value problems for parametric odes. *Applied Numerical Mathematics* 57(10), 1145 – 1162.
- Nedialkov, N., K. Jackson, & G. Corliss (1999). Validated solutions of initial value problems for ordinary differential equations. *Applied Mathematical Computing* 105, 21 – 68.
- Neumaier, A. (2002). Taylor forms - use and limits. *Reliable computing*.
- Saltelli, A., M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, & S. Tarantola (2008). *Global Sensitivity Analysis. The Primer*. Wiley.