Multi-Spherical MRI: Breaking the Boundaries of Diffusion Time

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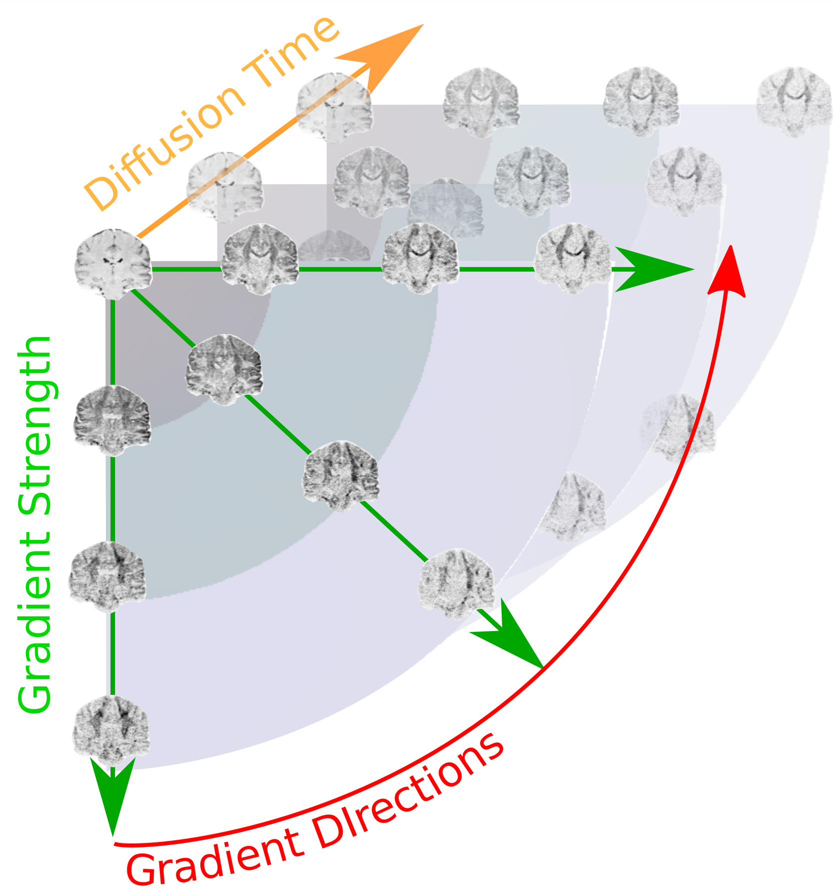
Abstract: Effective representation of the diffusion signal's dependence on diffusion time is a sought-after, yet still unsolved challenge in diffusion MRI. We propose a functional basis approach that is specifically designed to represent the dMRI signal in this four-dimensional space - that we call the multi-spherical space. We provide regularization tools to drastically reduce the number of measurements we need to probe the properties of this multi-spherical space.

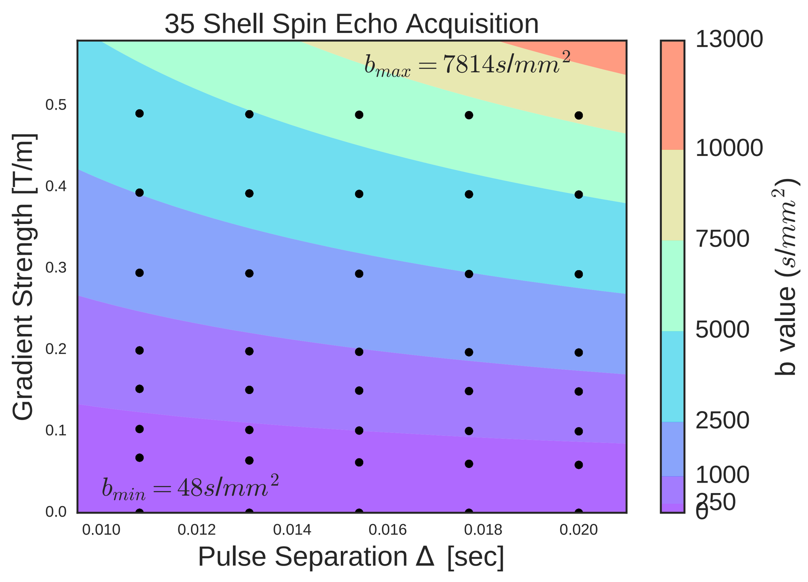
1 The Multi-Spherical Space



Diffusion restriction occurs when water diffusion is obstructed by tissue boundaries. The amount of restriction is **time-dependent**, meaning that the observed diffusion coefficient will change for varying diffusion times [1].

Multi-Spherical MRI [2] describes diffusion restriction by fitting the diffusion signals over varying:

- **Gradient strength (G)**
- **Gradient direction (g)**
- **Diffusion time (τ)**



We call this four-dimensional space the **Multi-Spherical Space**.

We sampled this space on 35 different "shells", varying only **g**, for different **G** ranging from [50-490] mT/m and **τ** ranging from [9.1-18.3] ms.

2 Modeling the Multi-Spherical Space

Multi-Spherical MRI uses a separable **Fourier Basis** to reconstruct diffusion propagator P(**r**,τ;**c**) from signal attenuation E(**q**,τ;**c**), represented in coefficients **c**.

$$\hat{E}(\mathbf{q},\tau;\mathbf{c})=\sum_i^{N_\mathbf{q}}\sum_k^{N_\tau}\mathbf{c}_{ik}\,\Phi_i(\mathbf{q})\,T_k(\tau)\;\overset{\mathrm{FT}}{\Longleftrightarrow}\;\hat{P}(\mathbf{r},\tau;\mathbf{c})=\sum_i^{N_\mathbf{q}}\sum_k^{N_\tau}\mathbf{c}_{ik}\,\Psi_i(\mathbf{r})\,T_k(\tau)$$

$\Psi_i(\mathbf{r})=FT(\Phi_i(\mathbf{q}))$: 3D *Fourier* basis over **q** and displacement **r** [3].

$T_m(\tau)$: Exponential diffusion time basis over τ [4].

We constrain the fitting of **c** to respect boundary conditions of the signal and impose **signal smoothness and sparsity**:

$$\mathrm{argmin}_{\mathbf{c}}\overbrace{\iint\left[E(\mathbf{q},\tau)-\hat{E}(\mathbf{q},\tau;\mathbf{c})\right]^2 d\mathbf{q}d\tau}^{(1)\,\mathrm{Data\,Fidelity}}+\overbrace{\iint\left[\nabla^2\hat{E}(\mathbf{q},\tau;\mathbf{c})\right]^2 d\mathbf{q}d\tau}^{(2)\,\mathrm{Smoothness}}+\overbrace{\|\mathbf{c}\|_1}^{(3)\,\mathrm{Sparsity}}$$

Where smoothness is imposed using **closed-form Laplacian regularization**.

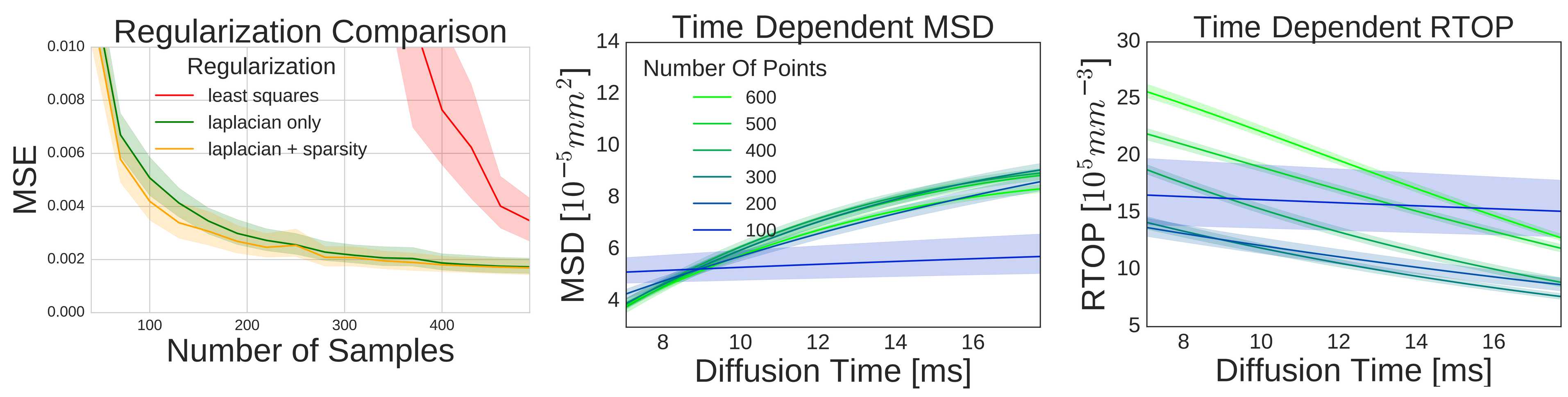
Once fitted, all q-space indices [3] can be estimated for any τ. As examples we show:

- Mean Squared Displacement (MSD), related to restriction
- Return-To-Origin Probability (RTOP), related to cellularity

3 In-Silico results

We study fitting performance under random subsampling by simulating the multi-spherical diffusion signal from gamma-distributed axons using Camino [5].

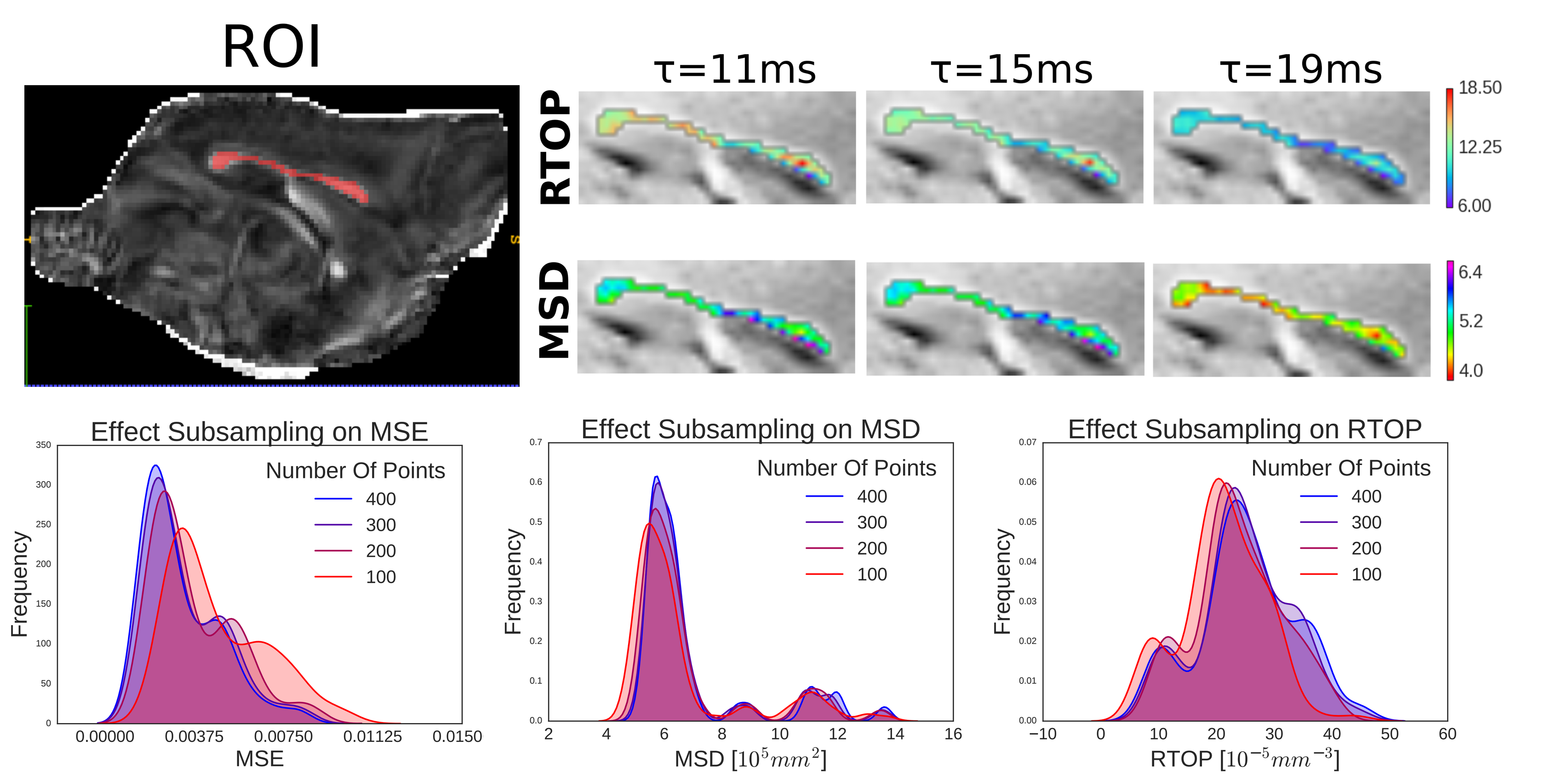
- Combined sparsity and Laplacian regularization produces the lowest fitting error (left).
- Time-dependent MSD and RTOP follow expected trends - MSD increasing and RTOP decreasing over time - down to about 200 DWIs (right two)



4 Application In-vivo Mouse Data

After eddy current correction, we chose an ROI of 173 voxels in Corpus Callosum. After subsampling we find

- Stable fitting errors from 400 down to 200 DWIs
- Expected trends for time-dependent MSD and RTOP



5 Discussion and Conclusions

- Multi-Spherical MRI allows for the characterization of diffusion restriction through time-dependent q-space indices.
- Through signal sparsity and smoothness, our approach can represent the multi-spherical signal with less samples, allowing more realistic acquisition schemes.
- Additional signal or propagator constraints can be conveniently included in the optimization.
- Through resampling, our approach could be used as a pre-processing for other methods studying properties of the multi-spherical space, e.g. axon packing [6].

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References [1] Fieremans et al. NeuroImage 129 (2016): 414-427. [2] Fick et al. CD-MRI 2016. [3] Özarslan et al. NeuroImage 78 (2013): 16-32. [4] Fick, Rutger, et al. IPMI 2015. [5] Cook et al. ISMRM, 2006. [6] Novikov et al. 111.14 (2014): 5088-5093.