Abstract: Effective representation of the diffusion signal’s dependence on diffusion time is a sought-after, yet still unsolved challenge in diffusion MRI. We propose a functional basis approach that is specifically designed to represent the dMRI signal in this four-dimensional space - that we call the multi-spherical space. We provide regularization tools to drastically reduce the number of measurements we need to probe the properties of this multi-spherical space.

1 The Multi-Spherical Space

Diffusion restriction occurs when water diffusion is obstructed by tissue boundaries. The amount of restriction is time-dependent, meaning that the observed diffusion coefficient will change for varying diffusion times [1].

Multi-Spherical MRI [2] describes diffusion restriction by fitting the diffusion signals over varying:
- Gradient strength (G)
- Gradient direction (g)
- Diffusion time (τ)

We call this four-dimensional space the Multi-Spherical Space. We sampled this space on 35 different "shells", varying only G, for different G ranging from [50-490] mT/m and τ ranging from [9.1-18.3] ms.

2 Modeling the Multi-Spherical Space

Multi-Spherical MRI uses a separable Fourier Basis to reconstruct diffusion propagator \( P(r, \tau, c) \) from signal attenuation \( E(q, \tau, c) \), represented in coefficients \( c \).

\[
\dot{E}(q, \tau, c) = \sum_{i} c_i \Phi_i(q) T_k(\tau) \quad \Rightarrow \quad \dot{P}(r, \tau, c) = \sum_{i} c_i \Psi_i(r) T_k(\tau)
\]

\( \Psi_i(r) = F \Phi_i(q) \): 3D Fourier basis over \( q \) and displacement \( r \) [3].

We constrain the fitting of \( c \) to respect boundary conditions of the signal and impose signal smoothness and sparsity:

\[
\arg\min_c \int \left[ \dot{E}(q, \tau, c) - E(q, \tau, c) \right]^2 dq d\tau + \int \left[ \nabla^2 \dot{E}(q, \tau, c) \right]^2 dq d\tau + \left[ c \right]_1
\]

Where smoothness is imposed using closed-form Laplacian regularization.

Once fitted, all q-space indices [3] can be estimated for any τ. As examples we show:
- Mean Squared Displacement (MSD), related to restriction
- Return-To-Origin Probability (RTOP), related to cellularity

3 In-Silico results

We study fitting performance under random subsampling by simulating the multi-spherical diffusion signal from gamma-distributed axons using Camino [5].

- Combined sparsity and Laplacian regularization produces the lowest fitting error (left).
- Time-dependent MSD and RTOP follow expected trends - MSD increasing and RTOP decreasing over time - down to about 200 DWIs (right two)

4 Application In-vivo Mouse Data

After eddy current correction, we chose an ROI of 173 voxels in Corpus Callosum. After subsampling we find:
- Stable fitting errors from 400 down to 200 DWIs
- Expected trends for time-dependent MSD and RTOP

5 Discussion and Conclusions

- Multi-Spherical MRI allows for the characterization of diffusion restriction through time-dependent q-space indices.
- Through signal sparsity and smoothness, our approach can represent the multi-spherical signal with less samples, allowing more realistic acquisition schemes.

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References