Two New Reformulation Convexification Based Hierarchies for 0-1 MIPs

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Abstract Let n and m two nonnegative integers. Let $X \subseteq \mathbb{R}^{n+m}$ be the set of feasible solutions of a mixed integer linear problem (n binary variables and m continuous variables). Without lost of generality, we will assume that X is a subset of the hypercube $[0,1]^{n+m}$.

By *hierarchy of relaxations* (hierarchy for short) of the set X we mean a finite family of *continuous relaxations* indexed by an integer, called *rank*, such that : (i) the relaxation of rank 0 coincides with the continuous relaxation of X; (ii) for every integer d, the relaxation of rank d is always included in the relaxation of rank $d-1$ and (iii) the relaxation of rank n (the number of binary variables) coincides with the convex hull of the set X. We will say that an hierarchy $\mathcal{A}$ *dominates* another hierarchy $\mathcal{B}$ if, for avery integer d, the rank d relaxation of the hierarchy $\mathcal{A}$ is included in the rank d relaxation of the hierarchy $\mathcal{B}$. An hierarchy $\mathcal{A}$ is said to be *equivalent* to an hierarchy $\mathcal{B}$ if $\mathcal{A}$ and $\mathcal{B}$ dominate each other.

The hierarchies we will address in this work are all defined using four steps : reformulation, convexification, linearization and projection. The *reformulation* we will consider was introduced by Sherali and Adams (see [2]). The *linearization* step consists in replacing, using new variables, the nonlinear terms appearing in the description of the set obtained after the reformulation (or convexification) step. Different linearizations are possible and, as proved in [1], this gives rise to different hierarchies. The linear description obtained after the linearization step is called *extended linear description*. The *projection* step consists in projecting back the extended linear description onto the space of the original variables.

In this work, first, we introduce two new hierarchies called *RTC* and *RSC* for which the rank d continuous relaxations are denoted $\hat{P}^d_{RTC}$ et $\hat{P}^d_{RSC}$ respectively. These two hierarchies are obtained using two different convexification schemes : *term convexification* in the case of the *RTC* hierarchy and *standard convexification* in the case of the *RSC* hierarchy. Secondly, we compare the *strength* (see [1]) of these two hierarchies. We will prove that : (i) the hierarchy *RTC* dominates the hierarchy *RSC*; (ii) the hierarchy *RTC* is *equivalent* to the *RLT* hierarchy of Sherali-Adams and (iii) the hierarchy *RSC* is dominated by the Lift-and-Project hierarchy (see [1]). Finally, for every rank d, we will prove that $conv\left(\mathcal{T}^d \cap \mathcal{E}^d_t\right) \subseteq \hat{P}^d_{RTC} \subseteq \mathcal{T}^d$ and $conv\left(\mathcal{S}^d \cap \mathcal{E}^d_s\right) \subseteq \hat{P}^d_{RSC} \subseteq \mathcal{S}^d$ where : for every rank d, the sets $\mathcal{T}^d$ and $\mathcal{S}^d$ are convex; while $\mathcal{E}^d_t$ and $\mathcal{E}^d_s$ are two non convex sets with empty interior (all these sets depend on the convexification step). The first inclusions allow, in some cases, an explicit characterization of *RLT* relaxations.

References

[1] M. Minoux and H. Ouzia. DRL*: A strong decomposable hierarchy of linear relaxations based on Reformulation Linearization for 0-1 MIPs. *Discrete Applied Mathematics*, 158:2031–2048, 2010.

[2] H. D. Sherali and W. P. Adams. A hierarchy of relaxations between the continuous and convex hull representation for 0-1 programming problems. *SIAM J. Discrete Math.*, 3:411–430, 1990.