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A self-healing control method for satellite attitude tracking based on simultaneous fault estimation and control design

Meng Zhou¹, Zhenhua Wang¹, Didier Theilliol², Yi Shen¹, Mickael Rodrigues³

Abstract—This paper proposes a novel self-healing control method for satellite attitude tracking based on simultaneous fault estimation and control design. The proposed method integrates the fault estimation and fault-tolerant control units in a dynamic system, which is less complex and more reliable than the separately designed self-healing architectures. In this paper, the model reference approach is used to obtain a tracking error dynamic equation. Following this, an augmented error system is constructed by taking the fault as an auxiliary vector. Based on the augmented error system, a fault estimator/controller is designed to achieve robust fault-tolerant control and robust fault estimation simultaneously. The design conditions for the proposed fault estimator/controller are transformed as a set of linear matrix inequalities, which can be easily solved. Finally, numerical simulation results are given to demonstrate the effectiveness of proposed method.

I. INTRODUCTION

Although the components in the satellite are designed and selected to be highly reliable, failures cannot be completely avoided in the orbit operation. Therefore, it is important to take necessary measures to guarantee the performance and safety of satellite. During the past decades, many related methods have been proposed to deal with the faults and ensure the satellite attitude control performance. [1] proposes an observer-based fault diagnosis method and a backstepping-based fault tolerant control law to deal with the loss of effectiveness fault. In [2], an adaptive failure compensation scheme is first developed to guarantee microsatellite system stability and asymptotic tracking properties, and then an adaptive failure identification scheme is designed by using multiple estimators. [3] proposes an indirect approach for fault diagnosis and fault-tolerant control in the satellite attitude control system with sampled-data measurements. In [4], a trajectory planning/re-planning method based is designed on attainable efforts analysis is applied for satellite system in rendezvous mission. However, it is noted that most of the existing results consider the fault diagnosis module and fault-tolerant control scheme separately. This may bring time delay in real application [5]. In order to deal with the

drawbacks, Liu et al. investigated integrated design of fault diagnosis and fault-tolerant control [6], [7]. However, the fault diagnosis module and fault-tolerant control module in the literatures are still designed separately.

Recently, the concept of self-healing control has been proposed to include the fault diagnosis and fault-tolerant control methods in a unified frame [8], [9]. Self-healing control methods are a class of control architectures which make the system counteract faults actively and maintain a certain level of control performance [8]. Simultaneous Fault Detection and Control (SFDC) provides an effective solution to self-healing control design and has attracted much attention in recent years [10], [11], [12], [13], [14]. The basic idea of SFDC is to integrate the control module and diagnostic module into one system unit such that control objective and fault detection objective can be satisfied simultaneously [15]. In [11], the SFDC problem is formulated as a mixed optimization problem and its solution is presented in terms of two coupled Riccati equations. A dynamic observer is presented to handle the SFDC problem for switched system in [12]. In [13] a robust SFDC approach is applied to the non-linear longitudinal dynamics of Boeing 747-100/200 aircraft. The SFDC problem is investigated in [14] for continuous-time linear system and the proposed method is applied to a satellite system. However, the output of satellite system is only regulated to zero by the the proposed method in [14], while in most situations, satellite controller must have the capacity to regulate the output asymptotically track a reference attitude, especially for an on-orbit satellite. Besides, it should be noted that only a few results considers the problem of simultaneous fault estimation and fault tolerant control. In [16], an integrated fault estimation and control system reconfiguration is designed, in which the fault estimator is based on a modified reduced-/full- order unknown input observer and the fault tolerant controller is based on a sliding mode control using state/output feedback. In this paper, we mainly focuses on the self-healing attitude tracking control design based on simultaneous fault estimation and control method for the linear satellite attitude tracking system.

This paper proposes a novel self-healing control method for satellite attitude tracking control. The main contribution of this paper lies in two-fold. First, the fault estimation objective and fault-tolerant attitude tracking control performance are satisfied simultaneously by designing a robust fault estimator/controller for an error system between the satellite system and reference model. Furthermore, the proposed fault estimator/controller design conditions are transformed into a set of Linear Matrix Inequalities (LMIs).

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II. PROBLEM FORMULATION

A. Satellite model formulation

Based on the Euler moment equation, the attitude dynamic of a rigid satellite is obtained as

$$J\dot{\omega} = -\omega^\times J\omega + T_u + T_v \quad (1)$$

where $J \in \mathbb{R}^{3 \times 3}$ denotes the momentum inertia matrix, $\omega = [\omega_x, \omega_y, \omega_z]^T$ represents the angular velocity of the body frame with respect to the inertial frame, $T_u = [T_{ux}, T_{uy}, T_{uz}]^T$ denotes the torques provided by the actuator and $T_v = [T_{vx}, T_{vy}, T_{vz}]^T$ represents the external disturbance torques. ω^\times denotes a skew-symmetric matrix defined as

$$\omega^\times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad (2)$$

In this paper, it is assumed that $J = \text{diag}([J_x, J_y, J_z])$, where J_x, J_y, J_z are the principal moments of inertia. Then the system (1) can be written as

$$\begin{cases} J_x \dot{\omega}_x - (J_y - J_z)\omega_y\omega_z = T_{ux} + T_{vx} \\ J_y \dot{\omega}_y - (J_z - J_x)\omega_z\omega_x = T_{uy} + T_{vy} \\ J_z \dot{\omega}_z - (J_x - J_y)\omega_x\omega_y = T_{uz} + T_{vz} \end{cases} \quad (3)$$

Under a small attitude angle condition [17], [18], the attitude kinematic equation of the satellite is expressed as

$$\begin{cases} \omega_x = \dot{\phi} - \omega_0\psi \\ \omega_y = \dot{\theta} - \omega_0 \\ \omega_z = \dot{\psi} + \omega_0\phi \end{cases} \quad (4)$$

where ω_0 is the orbital rate of the satellite and ϕ, θ, ψ are roll, pitch and yaw angle, respectively.

The external disturbance T_v mainly consisted with gravity gradient torque T_g and the other environmental disturbance T_d . The gravity-gradient torque is approximated by

$$\begin{cases} T_{gx} = 3\omega_0^2(J_z - J_y)\phi \\ T_{gy} = 3\omega_0^2(J_z - J_x)\theta \\ T_{gz} = 0 \end{cases} \quad (5)$$

Then substitute (4) and (5) into (3), then (3) can be approximated by

$$\begin{cases} J_x \ddot{\phi} - \omega_0(J_x - J_y + J_z)\dot{\psi} + 4\omega_0^2(J_y - J_z)\phi = T_{ux} + T_{dx} \\ J_y \ddot{\theta} + 3\omega_0^2(J_x - J_z)\theta = T_{uy} + T_{dy} \\ J_z \ddot{\psi} + \omega_0(J_x - J_y + J_z)\dot{\phi} + \omega_0^2(J_y - J_x)\psi = T_{uz} + T_{dz} \end{cases} \quad (6)$$

Define $x = [\phi \ \theta \ \psi \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$ as the state vector and consider the additive actuator fault T_f , the system (6) is rewritten as

$$\dot{x}(t) = A_c x(t) + B_c(T_u(t) + T_f(t)) + B_{cw}T_d(t) \quad (7)$$

where the system matrices are

$$A_c = \begin{bmatrix} 0 & I_3 \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0 \\ M^{-1}I_3 \end{bmatrix}, B_{cw} = \begin{bmatrix} 0 \\ M^{-1}I_3 \end{bmatrix}$$

with

$$M = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix}$$

$$K = \begin{bmatrix} 4\omega_0^2(J_y - J_z) & 0 & 0 \\ 0 & 3\omega_0^2(J_x - J_z) & 0 \\ 0 & 0 & \omega_0^2(J_y - J_x) \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & -\omega_0(J_x + J_z - J_y) \\ 0 & 0 & 0 \\ \omega_0(J_x + J_z - J_y) & 0 & 0 \end{bmatrix}$$

In this paper, it is assumed that only the Euler angles are measured. i.e.

$$y(t) = Cx(t) \quad (8)$$

where $C = [I_3 \ 0]$.

In this paper, the continuous-time model (7) is approximated by using the Euler approximation method. For a given sampling period t_s , the discrete-time approximation of the satellite is obtained as follows

$$\begin{cases} x(k+1) = Ax(k) + BT_u(k) + B_f T_f(k) + B_w T_d(k) \\ y(k) = Cx(k) \end{cases} \quad (9)$$

where $A = I_6 + A_c t_s$, $B = B_f = B_c t_s$, $B_w = B_{cw} t_s$.

B. Attitude tracking error model

In this paper, the model reference method is used to design tracking controller for system (9). The reference model is given by

$$\begin{cases} x_r(k+1) = Ax_r(k) + BT_{ur}(k) \\ y_r(k) = Cx_r(k) \end{cases} \quad (10)$$

where $x_r \in \mathbb{R}^6$, $T_{ur} \in \mathbb{R}^3$, and $y_r \in \mathbb{R}^3$ are the state, control input and output vectors of the reference model, respectively. The output of the reference system is the desired trajectory. Therefore, the following control law is presented for the reference model (10)

$$T_{ur}(k) = K_x x_r(k) + K_r r(k) \quad (11)$$

where $K_x \in \mathbb{R}^{3 \times 6}$ is the state feedback control matrix which is designed such that $A + BK_x$ have to be stable and $K_r \in \mathbb{R}^3$ is the feedforward control matrix which regulates the output of the reference model $y_r(k)$ to track the reference attitude $r(k)$ [9].

To address the attitude tracking problem, attitude tracking error is defined as follows

$$\Delta x(k) = x(k) - x_r(k) \quad (12)$$

Then, by subtracting (9) from (10), the error dynamic equation is obtained as

$$\begin{cases} \Delta x(k+1) = A\Delta x(k) + B\Delta T_u(k) + B_f T_f(k) \\ \quad + B_w T_d(k) \\ \Delta y(k) = C\Delta x(k) \end{cases} \quad (13)$$

where

$$\Delta T_u(k) = T_u(k) - T_{ur}(k) \quad (14)$$

$$\Delta y(k) = y(k) - y_r(k) \quad (15)$$

It can be seen that if a suitable controller $\Delta T_u(k)$ is designed such that the attitude error converge asymptotically to zero, then the attitude of the satellite will track the desired attitude.

C. The SFEC problem

Motivated by [19], [20], this paper assumes that the fault dynamics is described as

$$T_f(k+1) = A_f T_f(k) + \Delta T_f(k) \quad (16)$$

where $\Delta T_f(k)$ is the error of fault model between two consecutive time interval, A_f is defined by $A_f = \text{diag}[A_{f1}, A_{f2}, A_{f3}]$. Note that A_f represents the additional degree of freedom in SFEC design which is obtained to add another state to the new state equations. Besides, A_f should be fixed between $[0 \cdots 1]$.

In order to facilitate the SFEC design, the state augmentation approach is used in this paper. The actuator fault $T_f(k)$ is considered as an auxiliary state vector, i.e.

$$\eta(k) = [\Delta x^T(k) \quad T_f^T(k)]^T \quad (17)$$

Using (13) and (16), an augmented system is obtained as

$$\begin{cases} \eta(k+1) = \bar{A}\eta(k) + \bar{B}\Delta T_u(k) + \bar{B}_w w(k) \\ \Delta y(k) = \bar{C}\eta(k) \end{cases} \quad (18)$$

where

$$\bar{A} = \begin{bmatrix} A & B_f \\ 0 & A_f \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, w(k) = \begin{bmatrix} T_d(k) \\ \Delta T_f(k) \end{bmatrix}, \\ \bar{B}_w = \begin{bmatrix} B_d & 0 \\ 0 & I_3 \end{bmatrix}, \bar{C} = [C \quad 0]$$

Based on the augmented system (18), the proposed fault estimator/controller is given as follows

$$\begin{cases} \hat{\eta}(k+1) = A_F \hat{\eta}(k) + B_F \Delta y(k) \\ \Delta T_u(k) = K_F \hat{\eta}(k) \\ \hat{T}_f(k) = D_F \hat{\eta}(k) \end{cases} \quad (19)$$

where $\hat{\eta}(k) \in \mathbb{R}^9$ is the state vector of the proposed fault estimator/controller, and $A_F \in \mathbb{R}^{9 \times 9}$, $B_F \in \mathbb{R}^{9 \times 3}$, $K_F \in \mathbb{R}^{3 \times 9}$, $D_F \in \mathbb{R}^{3 \times 9}$ are the parameter matrices to be designed.

In this paper, the SFEC design is transformed into a multiple-objective H_∞ optimization problem stated as follows.

SFEC design: Given two scalars $\gamma_y > 0$ and $\gamma_e > 0$, determine the matrices A_F , B_F , K_F , D_F in (19) such that

- (i) The error system (13) is asymptotically stable such that the output of the faulty system can track the reference trajectory.
- (ii) The tracking error of controlled output $\Delta y(k)$ is robust to disturbance $w(k)$, i.e.

$$\|\Delta y\|_2 < \gamma_y \|w\|_2 \quad (20)$$

- (iii) The error of fault estimation $e_f(k)$ is robust to disturbance $w(k)$, i.e.

$$\|e_f\|_2 < \gamma_e \|w\|_2 \quad (21)$$

where $e_f(k) = T_f(k) - \hat{T}_f(k)$ denotes the fault estimation error.

III. SIMULTANEOUS FAULT ESTIMATION AND CONTROL DESIGN

In this section, the parameter matrices A_F , B_F , K_F , D_F in the fault estimator/controller (19) are designed to satisfy the conditions (i)-(iii).

Combing (18) and (19), the overall system is expressed as

$$\begin{cases} \eta(k+1) = \bar{A}\eta(k) + \bar{B}K_F \hat{\eta}(k) + \bar{B}_w w(k) \\ \hat{\eta}(k+1) = A_F \hat{\eta}(k) + B_F \bar{C}\eta(k) \\ \Delta y(k) = \bar{C}\eta(k) \\ e_f(k) = C_f \eta(k) - D_F \hat{\eta}(k) \end{cases} \quad (22)$$

Herein, the following relation is used

$$T_f(k) = C_f \eta(k) \quad (23)$$

where $C_f = [0 \quad I_3]$.

For the sake of brevity, we define

$$\xi(k) = [\eta(k) \quad \hat{\eta}(k)]^T \quad (24)$$

The overall system (22) is formulated as the following two subsystems

$$\begin{cases} \xi(k+1) = \tilde{A}\xi(k) + \tilde{B}w(k) \\ \Delta y(k) = \tilde{C}_y \xi(k) \\ e_f(k) = \tilde{C}_e \xi(k) \end{cases} \quad (25)$$

where

$$\tilde{A} = \begin{bmatrix} \bar{A} & \bar{B}K_F \\ B_F \bar{C} & A_F \end{bmatrix}, \tilde{B} = \begin{bmatrix} \bar{B}_w \\ 0 \end{bmatrix}, \\ \tilde{C}_y = [\bar{C} \quad 0], \tilde{C}_e = [C_f \quad -D_F] \quad (26)$$

Based on (25), the following theorem is proposed to design the parameter matrices of the estimator/controller (19) to satisfy the two above optimal problems.

Theorem 1. Given $\gamma_y > 0$ and $\gamma_e > 0$, there exists positive-definite matrices $X = X^T \in \mathbb{R}^{9 \times 9}$, $Y = Y^T \in \mathbb{R}^{9 \times 9}$ and matrices $\hat{A} \in \mathbb{R}^{9 \times 9}$, $\hat{B} \in \mathbb{R}^{9 \times 3}$, $\hat{C} \in \mathbb{R}^{3 \times 9}$, $\hat{D} \in \mathbb{R}^{3 \times 9}$ such that the following LMIs hold:

$$\begin{bmatrix} -X & * & * & * & * & * \\ -I_9 & -Y & * & * & * & * \\ 0 & 0 & -\gamma_y^2 I_6 & * & * & * \\ \bar{C}X & \bar{C} & 0 & -I_3 & * & * \\ \bar{A}X + \bar{B}\hat{C} & \bar{A} & \bar{B}_w & 0 & -X & * \\ \hat{A} & Y\bar{A} + \hat{B}\hat{C} & Y\bar{B}_w & 0 & -I_9 & -Y \end{bmatrix} < 0 \quad (27)$$

$$\begin{bmatrix} -X & * & * & * & * & * \\ -I_9 & -Y & * & * & * & * \\ 0 & 0 & -\gamma_e^2 I_6 & * & * & * \\ C_f X - \hat{D} & C_f & 0 & -I_3 & * & * \\ \bar{A}X + \bar{B}\hat{C} & \bar{A} & \bar{B}_w & 0 & -X & * \\ \hat{A} & Y\bar{A} + \hat{B}\hat{C} & Y\bar{B}_w & 0 & -I_9 & -Y \end{bmatrix} < 0 \quad (28)$$

then the closed-loop system (22) is asymptotic stable and the designed estimator/controller (19) achieves the attitude tracking control performance (ii) and fault estimation objective (iii) simultaneously. Moreover, the parameter matrices A_F , B_F , K_F and D_F can be determined by

$$\begin{aligned} A_F &= N^{-1}(\hat{A} - \hat{B}\hat{C}X - Y\hat{B}\hat{C} - Y\hat{A}X)M^{-T} \\ B_F &= N^{-1}\hat{B}, K_F = \hat{B}M^{-T}, D_F = \hat{D}M^{-T} \end{aligned} \quad (29)$$

where

$$\begin{aligned} \hat{A} &= Y\hat{A}X + NB_F\hat{C}X + Y\hat{B}K_F M^T + NA_F M^T, \\ \hat{B} &= NB_F, \hat{C} = K_F M^T, \hat{D} = D_F M^T \end{aligned} \quad (30)$$

with $M \in \mathbb{R}^{9 \times 9}$ and $N \in \mathbb{R}^{9 \times 9}$ solved from

$$MN^T = I - XY \quad (31)$$

Proof. Applying the bounded real lemma [21] to system (25), it is known that conditions (i)-(iii) are satisfied if there exists a positive-definite matrix $P > 0$ such that

$$\begin{bmatrix} \tilde{A}^T P \tilde{A} - P + \tilde{C}_y^T \tilde{C}_y & \tilde{A}^T P \tilde{B} \\ \tilde{B}^T P \tilde{A} & \tilde{B}^T P \tilde{B} - \gamma_y^2 I_6 \end{bmatrix} < 0 \quad (32)$$

and

$$\begin{bmatrix} \tilde{A}^T P \tilde{A} - P + \tilde{C}_e^T \tilde{C}_e & \tilde{A}^T P \tilde{B} \\ \tilde{B}^T P \tilde{A} & \tilde{B}^T P \tilde{B} - \gamma_e^2 I_6 \end{bmatrix} < 0 \quad (33)$$

hold.

We first show that (32) holds if (27) is fulfilled. Using the Schur complement lemma [22], (32) is equivalent to

$$\begin{bmatrix} -P & 0 & \tilde{C}_y^T & \tilde{A}^T P \\ 0 & -\gamma_y^2 I_6 & 0 & \tilde{B}^T P \\ \tilde{C}_y & 0 & -I_3 & 0 \\ P\tilde{A} & P\tilde{B} & 0 & -P \end{bmatrix} < 0 \quad (34)$$

It should be noted that (34) is not a standard linear matrix inequality of the parameter matrices A_F , B_F , K_F , D_F . Therefore, further transformations are needed to make (34) solvable. To this end, according to [23], the matrices P and P^{-1} are partitioned as

$$P = \begin{bmatrix} Y & N \\ N^T & R \end{bmatrix}, P^{-1} = \begin{bmatrix} X & M \\ M^T & S \end{bmatrix} \quad (35)$$

where X and Y are symmetric matrices. It is noted that the matrices X , Y , M , N , R and S should fulfill $PP^{-1} = I_9$, which implies that (31) holds.

Constructing two matrices Π_1 and Π_2 as

$$\Pi_1 = \begin{bmatrix} X & I_9 \\ M^T & 0 \end{bmatrix}, \Pi_2 = \begin{bmatrix} I_9 & Y \\ 0 & N^T \end{bmatrix} \quad (36)$$

From (35) and (36), it can be easily derived that

$$P\Pi_1 = \Pi_2 \quad (37)$$

Pre- and post- multiplying (34) by $\text{diag}(\Pi_1^T, I, I, \Pi_1^T)$ and $\text{diag}(\Pi_1, I, I, \Pi_1)$, respectively, it comes

$$\begin{bmatrix} -\Pi_1^T P \Pi_1 & 0 & \Pi_1^T \tilde{C}_y^T & \Pi_1^T \tilde{A}^T P \Pi_1 \\ 0 & -\gamma_y^2 I_6 & 0 & \tilde{B}^T P \Pi_1 \\ \tilde{C}_y \Pi_1 & 0 & -I_3 & 0 \\ \Pi_1^T P \tilde{A} \Pi_1 & \Pi_1^T P \tilde{B} & 0 & -\Pi_1^T P \Pi_1 \end{bmatrix} < 0 \quad (38)$$

Considering the definitions of \tilde{A} , \tilde{B} , \tilde{C}_y in (26), and using the equation (37), it can be derived that

$$\begin{aligned} \Pi_1^T P \Pi_1 &= \begin{bmatrix} X & I_9 \\ I_9 & Y \end{bmatrix}, \Pi_1^T P \tilde{A} \Pi_1 = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix}, \\ \Pi_1^T P \tilde{B} &= \begin{bmatrix} \tilde{B}_w \\ Y \tilde{B}_w \end{bmatrix}, \tilde{C}_y \Pi_1 = [\tilde{C}_z X \quad \tilde{C}_z] \end{aligned} \quad (39)$$

where

$$\begin{aligned} \mathcal{M}_{11} &= \tilde{A}X + \tilde{B}K_F M^T, \mathcal{M}_{12} = \tilde{A}, \\ \mathcal{M}_{21} &= Y\tilde{A}X + NB_F\tilde{C}X + Y\tilde{B}K_F M^T + NA_F M^T, \\ \mathcal{M}_{22} &= Y\tilde{A} + NB_F\tilde{C} \end{aligned}$$

Using the definitions of \hat{A} , \hat{B} and \hat{C} in (30), the term $\Pi_1^T P \tilde{A} \Pi_1$ is rewritten as

$$\Pi_1^T P \tilde{A} \Pi_1 = \begin{bmatrix} \tilde{A}X + \tilde{B}\hat{C} & \tilde{A} \\ \hat{A} & Y\tilde{A} + \tilde{B}\hat{C} \end{bmatrix} \quad (40)$$

Substituting (39),(40) into (38), the inequality (38) becomes (27). Therefore, it can be concluded that (27) is sufficient for the conditions (i) and (ii).

By using a similar procedure, it can also be shown that (28) is equivalent to the inequality (33). Herein, the details of this proof are omitted for the sake of brevity.

Once the matrices \hat{A} , \hat{B} , \hat{C} , \hat{D} , M , N are determined, the matrices A_F , B_F , K_F , D_F can be obtained by (29). ■

IV. SIMULATIONS

In this section, numerical simulations are used to verify the effectiveness of the proposed method. Table 1 gives the satellite parameters, which are borrowed from [14].

TABLE 1
SATELLITE PARAMETERS

J_x	12.49 kg.m ²	ω_0	0.0011 rad/s
J_y	13.85 kg.m ²	A_s	1.5×10^{-4} Nm
J_z	15.75 kg.m ²		

And the external disturbance is assumed to be as

$$\begin{cases} T_{dx} = A_s(3 \cos \omega_0 k + 1) \\ T_{dy} = A_s(3 \cos \omega_0 k + 1.5 \sin \omega_0 k) \\ T_{dz} = A_s(3 \sin \omega_0 k + 1) \end{cases} \quad (41)$$

where A_s is the amplitude of the disturbance.

Combined with the definition of the system matrices, it is easily to obtain that

$$A = \begin{bmatrix} 1.0000 & 0 & 0 & 0.1000 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0.1000 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0.1000 \\ 0 & 0 & 0 & 1.0000 & 0 & 0.0001 \\ 0 & 0.00 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & -0 & -0.0001 & 0 & 1.0000 \end{bmatrix}$$

$$B = B_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.08 & 0 & 0 \\ 0 & 0.0072 & 0 \\ 0 & 0 & 0.0063 \end{bmatrix}$$

In the SFEC design, the fault system matrix is assumed as $A_f = 0.99I_3$ and the H_∞ performance indexes are chosen as $\gamma_y = 5$, $\gamma_e = 5$. By solving the LMIs (28) and (27) in Theorem 1, the matrices in the proposed fault estimator/controller gain matrices are obtained as

$$A_F = [A_{F1} \quad A_{F2}], B_F = \begin{bmatrix} B_{F1} \\ B_{F2} \end{bmatrix}$$

$$K_F = [K_{F1} \quad K_{F2}], D_F = \begin{bmatrix} D_{F1} \\ D_{F2} \end{bmatrix}$$

where

$$A_{F1} = \begin{bmatrix} 0.988 & 0 & 0 & -0.0002 & -0 \\ 0 & 0.9878 & 0 & -0 & -0.5490 \\ 0 & -0 & 0.9876 & 0.5261 & -0 \\ -0 & -0 & 0.0086 & -0.8857 & 0 \\ 0 & -0.0075 & -0 & -0 & -0.9154 \\ -0.0067 & -0 & -0 & -0 & 0 \\ 0 & 0 & -0.0012 & 0.2688 & 0 \\ -0 & 0.0012 & 0 & -0 & 0.2955 \\ -0.0011 & 0 & -0 & 0 & -0 \end{bmatrix}$$

$$A_{F2} = \begin{bmatrix} -0.5675 & -0.0013 & 0 & 2.4931 \\ 0 & 0 & -2.5646 & 0 \\ -0.0003 & 2.6519 & -0 & 0.0013 \\ 0 & -7.8000 & 0 & 0.0001 \\ 0 & 0 & -7.3184 & 0 \\ -0.9406 & 0.0001 & 0 & 6.9510 \\ 0 & 0 & -0.9508 & 0 \\ 0 & 0 & -0.9618 & 0 \\ -0.3193 & 0 & 0 & -0.9721 \end{bmatrix}$$

$$B_{F1} = \begin{bmatrix} -49.885 & -0 & -0.0151 & -0.0326 & -0 \\ -0 & -48.78 & -0 & -0 & -223.562 \\ 0.0125 & 0 & -47.1873 & 224.65 & -0 \end{bmatrix}^T$$

$$B_{F2} = \begin{bmatrix} -222.3986 & 0.0291 & -0 & -155.3724 \\ 0 & 0 & 152.4019 & -0 \\ -0.0271 & -148.7478 & 0 & -0.0286 \end{bmatrix}^T$$

$$K_{F1} = \begin{bmatrix} 38.9880 & 0 & 0.0206 & 0.0004 & -0 \\ -0 & 39.5307 & 0 & -0 & -18.0183 \\ -0.02 & -0 & 40.184 & 18.1938 & -0 \end{bmatrix}$$

$$K_{F2} = \begin{bmatrix} -17.8742 & -0.0054 & 0 & 50.6313 \\ 0 & -0 & -54.7466 & 0 \\ -0.0005 & 60.4595 & -0 & 0.0068 \end{bmatrix}$$

$$D_{F1} = \begin{bmatrix} -39.007 & -0 & -0.0206 & -0.0006 & 0 \\ 0 & -39.553 & -0 & -0 & 12.264 \\ 0.0200 & 0 & -40.2117 & -12.1145 & 0 \end{bmatrix}^T$$

$$D_{F2} = \begin{bmatrix} 12.3598 & 0.0016 & -0 & -7.6068 \\ -0 & 0 & 7.9253 & -0 \\ 0.0001 & -8.3446 & 0 & -0.0018 \end{bmatrix}^T$$

In the simulation, the reference attitude is assumed as $r(k) = [0.01 \quad 0.02 \quad 0.05]^T$ (rad). The matrices K_x and K_r are given as

$$K_x = \begin{bmatrix} -47.08 & 0 & 0 & -132.03 & 0 & -0.016 \\ 0 & -52.21 & 0 & 0 & -146.41 & 0 \\ 0 & 0 & -59.37 & 0.016 & 0 & -166.5 \end{bmatrix}$$

$$K_r = \begin{bmatrix} 47.0812 & 0 & 0 \\ 0 & 52.2078 & 0 \\ 0 & 0 & 59.3699 \end{bmatrix}$$

In the simulation, the modelling uncertainty is considered as $\Delta A = 10^{-3} A \sin(0.02k)$. In order to verify the effectiveness of the proposed method, the commonly encountered fault scenarios are considered in the simulation. First, an abrupt fault scenario is simulated. In the simulation, it is assumed that two abrupt faults simultaneously occur at $k = 200$ with the following characteristics

$$T_f(k) = \begin{cases} [0 \quad 0 \quad 0]^T & k < 200 \\ [-0.5 \quad 1.2 \quad 0]^T & 200 \leq k < 600 \end{cases} \quad (42)$$

The fault-tolerant control and fault estimation results are depicted in Fig. 1 and Fig. 2, respectively. From Fig. 1, it is seen that the desired attitude are followed with high accuracy even in the presence of actuator failure and external disturbances. Moreover, the last three states illustrate that the proposed fault estimator/controller ensure the whole system stabilized. Fig. 2 shows that the proposed fault estimator/controller provides quick and accurate fault estimations.

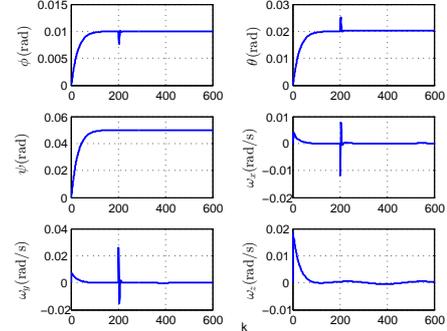


Fig. 1. Fault-tolerant control result in an abrupt actuator fault scenario

In the second scenario, it is assumed that a time-varying fault occurs with the following form

$$T_f(k) = \begin{cases} [0 \quad 0 \quad 0]^T & k < 100 \\ [0 \quad 0.03(k-100) \quad 0]^T & 100 \leq k < 300 \\ [0 \quad 0.6 - 0.03(k-100) \quad 0]^T & 300 \leq k < 500 \\ [0 \quad 0 \quad 0]^T & 500 \leq k < 600 \end{cases} \quad (43)$$

The fault-tolerant control and fault estimation results are depicted in Fig. 3 and Fig. 4, respectively. It can be seen that the proposed method also guarantee the attitude tracing performance and robust fault estimation performance in the presence of time-varying fault.

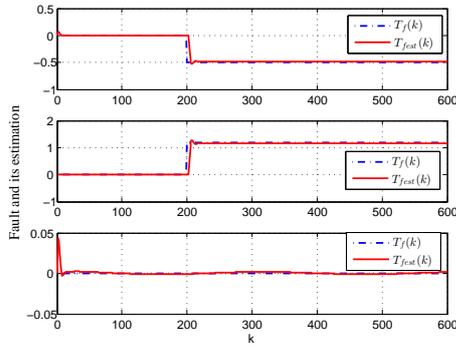


Fig. 2. Fault estimation result in an abrupt fault scenario

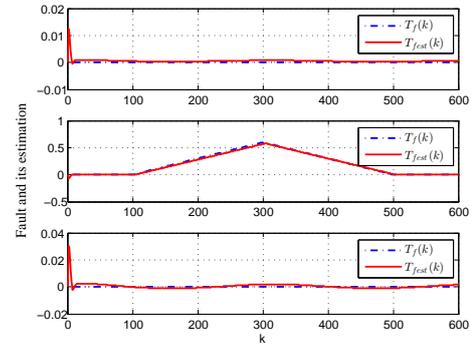


Fig. 4. Fault estimation result in a time-varying actuator fault scenario

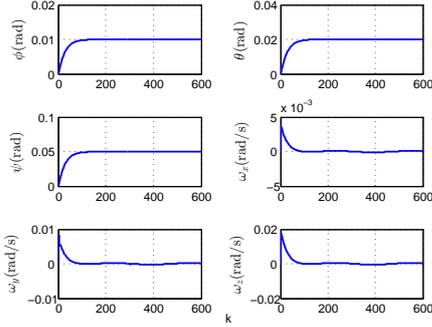


Fig. 3. Fault-tolerant control result in a time-varying actuator fault scenario

V. CONCLUSIONS

In this paper a novel self-healing attitude tracking control method based on simultaneous fault estimation and control design for satellite is proposed. The main idea of the proposed self-healing method is to construct an augmented error system and then design a fault estimator/controller based on the augmented system. In this paper, the design of the fault estimator/controller is formulated as a multi-objective H_∞ optimization problem and further transformed as a set of LMIs. Finally, numerical simulations are used to demonstrate the performance of the proposed method.

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