



HAL
open science

Revisiting the transitional dynamics of business-cycle phases with mixed frequency data

Marie Bessec

► **To cite this version:**

Marie Bessec. Revisiting the transitional dynamics of business-cycle phases with mixed frequency data. 2016. hal-01358595

HAL Id: hal-01358595

<https://hal.science/hal-01358595>

Preprint submitted on 1 Sep 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Revisiting the transitional dynamics of business-cycle phases with mixed frequency data

MARIE BESSEC*

THIS VERSION: August 29, 2016

Abstract

This paper introduces a Markov-switching model in which transition probabilities depend on higher frequency indicators and their lags through polynomial weighting schemes. The MSV-MIDAS model is estimated via maximum likelihood (ML) methods. The estimation relies on a slightly modified version of Hamilton's recursive filter. We use Monte Carlo simulations to assess the robustness of the estimation procedure and related test statistics. The results show that ML provides accurate estimates, but they suggest some caution in interpreting the tests of the parameters involved in the transition probabilities. We apply this new model to the detection and forecasting of business cycle turning points in the United States. We properly detect recessions by exploiting the link between GDP growth and higher frequency variables from financial and energy markets. The spread term is a particularly useful indicator to predict recessions in the United States. The empirical evidence also supports the use of functional polynomial weights in the MIDAS specification of the transition probabilities.

JEL Classification: C22, E32, E37

Keywords: Markov-switching, mixed frequency data, business cycles

*Université Paris Dauphine, PSL Research University, LEDa, email: marie.bessec@dauphine.fr. I am grateful to Denisa Banulescu, Yoosoon Chang, Gorkem Celik, Frédéric Karamé, Siem Jan Koopman, Yannick Le Pen and Thomas Raffinot for their useful comments and suggestions. I also thank the participants of the annual meeting of the European Meeting of the Econometric Society (ESEM) in Geneva 2016, the International Association for Applied Econometrics (IAAE) annual conference in Milan 2016, the International Conference on Computational Financial Econometrics (CFE) in London 2015, Labex MME-DII 3 Journée d'Econométrie Appliquée à la Macroéconomie (JEAM) in Paris 2015, the French Economic Association (AFSE) in Rennes 2015 and the 8th Financial Risks International Forum in Paris 2015. The usual disclaimers apply.

1 Introduction

The failure to detect downturns in economic activity is a major source of error in macroeconomic forecasting. At the onset of the great recession, practitioners in the United States surveyed by the *Survey of Professional Forecasters* in November 2007 believed that there was an approximately 20 percent chance of negative growth in each quarter of 2008 and believed that US activity would grow by 2.5 percent in 2008.¹

This paper introduces a new specification that could be useful for monitoring and predicting business cycles. We consider a Markov-switching model in which transition probabilities depend on higher frequency indicators (MSV-MIDAS model). As done in Diebold, Lee and Weinbach [1994] and Filardo [1994], the parameters of the model depend on an unobserved state variable following a first-order Markov chain with time-varying transition probabilities.² The innovation of this paper lies in specifying the transition probabilities such that they depend on a set of exogenous indicators sampled at a higher frequency. To address the discrepancy in the frequencies, we apply the MIDAS (mixed-data sampling) approach developed by Ghysels, Santa-Clara and Valkanov [2004] and Ghysels, Sinko and Valkanov [2007]. Instead of converting the indicator involved in the probabilities to the low frequency with an arbitrary weighting scheme, the optimal weights are estimated from the data. A parsimonious parameterization of the lagged coefficients of the high-frequency variable is obtained through the use of functional polynomial weights.

The MSV-MIDAS specification can incorporate the signals produced by a wide range of indicators of the current and future state of the economy into the transition mechanism of the system. In particular, there is extensive literature showing that financial indicators can be used to predict business cycle turning points. The yield curve holds a prominent place among these variables (see Estrella and Mishkin [1998], Kauppi and Saikkonen [2008], Rudebusch and Williams [2009] and Croushore and Marsten [2015] among many others), but practitioners also follow other indicators such as stock and commodity prices to predict business cycle troughs and peaks (see Hamilton [2003], Hamilton [2011] and Kilian and Vigfusson [2013] on the specific role of oil prices). In this context, the MIDAS structure is useful, as these indicators are available at a higher frequency than are macroeconomic variables. In this specification, it is not necessary to aggregate the financial indicators at a lower frequency in the transition probabilities, which could lead to a loss of a potential

¹According to the NBER, the US recession began in December 2007 and ended in June 2009. US real GDP fell by 0.3% in 2008 and 2.8% in 2009 (BEA, June 2015).

²Markov-switching models with time-varying probabilities have been recently reconsidered by Kim, Piger and Startz [2008], Bazzi, Blasques, Koopman and Lucas [2014] and Chang, Choi and Park [2015].

useful information and, therefore, to inefficient and/or biased estimates (Andreou, Ghysels and Kourtellos [2010]).

This paper is related to the literature using MIDAS regressions to show that financial variables are useful predictors of GDP growth. Andreou, Ghysels and Kourtellos [2013], Galvão [2013] and Ferrara, Marsilli and Ortega [2014] find a statistically significant improvement in GDP forecast accuracy in the euro area, UK and US when using models incorporating the forward-looking information contained in high-frequency financial data. Moreover, Guérin and Marcellino [2013], Bessec and Bouabdallah [2015] and Barsoum and Stankiewicz [2015] use a MIDAS approach to show that financial variables help to predict turning points in the United Kingdom and in the United States. From a methodological perspective, the present paper also contributes to the recent literature introducing time variation into MIDAS models. In the class of regime-switching models, Galvão [2013] includes a smooth transition model with high-frequency variables among the regressors and the threshold variable, while Guérin and Marcellino [2013] incorporate high-frequency regressors in a Markov-switching model with invariant transition probabilities. More recently, Schumacher [2014] considers MIDAS regressions with time-varying parameters, estimated with a particle filter.

The MSV-MIDAS model introduced in this paper is estimated via maximum likelihood methods. The estimation relies on a slightly modified version of the filter in Hamilton [1989]. Because the MSV-MIDAS model has never been considered in the literature, we use Monte Carlo simulations to investigate the small sample properties of the maximum likelihood estimators of the parameters as well as related test statistics. The simulations are conducted for various parameterizations and sample sizes. The Monte Carlo evidence shows that maximum likelihood provides accurate estimates. The average bias of the estimates and their volatilities are small and decrease with the size of the sample. However, as shown by Psaradakis, Sola, Spagnolo and Spagnolo [2010] in Markov-switching models with variables sampled at the same frequency, the t-statistics of the parameters involved in the transition probabilities may not be reliable. The significance tests of these parameters may lack power in small samples, especially in the shorter regime.

We apply the MSV-MIDAS model to US data. As leading indicators for the inference of the future state, we consider monthly indicators from financial and energy markets: the interest rate, term spread, stock returns and oil prices. These variables, widely recognized as business cycle predictors, are available without any publication lags and are not subject to revisions. The evaluation of the model is based both on an in-sample and

an out-of-sample analysis. We compare the detection of the business cycle turning points by the new specification and various benchmarks: several models with fixed transition probabilities, as well as MSV-MIDAS models with unrestricted lag polynomials. The new specification appears to provide better signals of economic downturn and recovery than the usual models with constant probabilities. The results are also supportive of the use of distributed lag functions in the MIDAS specification. Among the economic indicators used to improve the transition mechanism, the slope of the yield curve is a good candidate for the United States, which is in line with the previous literature. These results hold both in sample and out of sample.

The remainder of this paper proceeds as follows. In section 2, we present the MSV-MIDAS specification and describe the estimation techniques. In section 3, we use Monte Carlo simulations to assess the robustness of the estimation procedure and related test statistics to make inferences. Section 4 is devoted to the empirical application to US data. The final section offers some concluding remarks.

2 The MSV-MIDAS model

Let y_t be a variable with dynamics that differ according to the state of the economy. The unobserved state follows a first-order Markov chain, the transition probabilities of which depend on a higher frequency indicator $z_t^{(m)}$. In the following, the time index t denotes the time unit of the low-frequency variable y_t (a quarter in our application). The high-frequency indicator $z_t^{(m)}$ is sampled m times between two time units of y , e.g., t and $t - 1$ ($m = 3$ for monthly indicators, as in our application). The lag operator $L^{1/m}$ operates at the higher frequency, e.g., $L^{s/m} z_t^{(m)} = z_{t-s/m}^{(m)}$ is variable z observed s months before quarter t .

The low-frequency variable y_t follows an AR(p) process with a switching mean, as motivated by Hamilton [1989]. The dynamics of an MSM(M)-AR(p) model are described by the following equation:

$$y_t = \mu_{s_t} + \phi_1(y_{t-1} - \mu_{s_{t-1}}) + \dots + \phi_p(y_{t-p} - \mu_{s_{t-p}}) + \sigma \varepsilon_t \quad (1)$$

where μ_{s_t} and σ represent the mean and standard deviation of y_t , ϕ_i with $i = \{1, \dots, p\}$ are unknown autoregressive parameters and $\varepsilon_t \rightarrow NID(0, 1)$. The variable $s_t = \{1, 2, \dots, M\}$ denotes the unobserved state of the process at time t . The mean value μ_{s_t} varies according to the realized value of the state variable.

Following Diebold et al. [1994] and Filardo [1994], the variable s_t is assumed to follow a first-order Markov chain defined with time-varying transition probabilities. In the case of two regimes ($M = 2$), the four transition probabilities are expressed as follows:

$$\begin{aligned}
P(s_t = 1|s_{t-1} = 1, z_{t-1}^{(m)}) &= \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta) z_{t-1}^{(m)}] \\
P(s_t = 2|s_{t-1} = 2, z_{t-1}^{(m)}) &= \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta) z_{t-1}^{(m)}] \\
P(s_t = 2|s_{t-1} = 1, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta) z_{t-1}^{(m)}] \\
P(s_t = 1|s_{t-1} = 2, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta) z_{t-1}^{(m)}]
\end{aligned} \tag{2}$$

where Γ is the logistic function $\Gamma(x) = 1/(1 + \exp(-x))$, α_i and β_i are unknown parameters for regime $s_t = i$ and $z_{t-1}^{(m)}$ is an exogenous variable. In this model, the transition probabilities are not time invariant. Instead, they depend on an exogenous variable and its lags. When β_{s_t} is positive (negative), an increase in $z_{t-1}^{(m)}$ increases (decreases) the probability of staying in regime s_t . If $\beta_1 = \beta_2 = 0$, the specification simplifies to the usual model with constant transition probabilities.

The exogenous variable $z_t^{(m)}$ is sampled at a higher frequency. To keep the specification parsimonious, functional lag polynomials are employed. The function $B(L^{1/m}, \Theta)$ is the exponential Almon lag³ with:

$$B(L^{1/m}, \Theta) = \sum_{j=1}^K b(j, \Theta) L^{(j-1)/m}, \quad b(j, \Theta) = \frac{\exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=1}^K \exp(\theta_1 j + \theta_2 j^2)} \tag{3}$$

with $s_t = \{1, 2\}$ in the case of two states. The weights defined by $b(j, \Theta)$ are positive and sum to one. The coefficient $\Theta = \{\theta_1, \theta_2\}$ defines the lag structure in the two regimes, and the coefficient β_{s_t} in equation (2) gives the overall impact of the weighted past values of z on the probability of staying in regime s_t . If $\theta_2 < 0$, the weight decreases with lag j . In the particular case in which $\Theta = \{0, 0\}$, we obtain the standard equal weighting aggregation scheme (the high-frequency variable is simply aggregated to the low frequency with an arithmetic average). As suggested by Andreou et al. [2010], the null hypothesis for equal weights can be tested with a standard LR test.

The lag function $B(L^{1/m}, \Theta)$ allows a parsimonious specification because only two coefficients are needed for the K lags. This is particularly interesting in regime-switching models, in which the number of coefficients is large.⁴ As indicated by Ghysels et al. [2007],

³Other possible specifications of the MIDAS polynomials are based on beta or step functions. See Ghysels et al. [2007] for a presentation of the various parameterizations of $B(L^{1/m}, \Theta)$.

⁴In a linear context, Foroni, Marcellino and Schumacher [2015] compare MIDAS models with func-

the use of distributed lag polynomials also avoids lag-length selection for the variable in the probabilities. The decay rate of the weights estimated from the data determines the number of lags of the high frequency indicator in the transition probabilities. Hence, the more or less persistent impact of $z_t^{(m)}$ can be captured according to the shape of the function. This feature is attractive in our model because the inference on the transition parameters is fragile, as shown by Psaradakis et al. [2010] in the case of data sampled at the same frequency. The next section of this study will confirm this fragility for models involving data sampled at different frequencies.

The model is estimated by maximum likelihood.⁵ The likelihood is derived in a modified version of Hamilton’s filter to account for the variation in the transition probabilities. In the first step of the filter, the fixed transition probabilities are replaced with time-varying probabilities related to the high-frequency variable $z_t^{(m)}$, as specified in equations (2) and (3). The rest of the estimation procedure is similar.⁶ Newton’s search method is applied to find the vector of parameters maximizing the function. The estimation algorithm is initialized with several sets of parameters to avoid local optima. A smoothing algorithm is then applied to obtain a better estimation of the states, as described in Kim [1994]. The standard errors of the parameters are obtained from the inverse of the information matrix at the optimum. In the estimation procedure, the parameter θ_2 of the Almon function is constrained to be negative, which guarantees, in both regimes, a declining weight of $z_t^{(m)}$ as the lag length increases.

3 Monte Carlo simulations

In this section, we describe several Monte Carlo experiments to assess the robustness of the estimation procedure and explore the reliability of the usual test statistics for conducting inference on the model parameters. A similar exercise is conducted by Psaradakis and Sola [1998] in MSM models with constant probabilities and by Psaradakis et al. [2010] in MSM models with time-varying probabilities. We extend their analysis to the case of mixed-frequency data.

tional distributed lags to MIDAS models with unconstrained weights estimated by least squares. The unconstrained specification performs well for small differences in sampling frequencies. In our model, this alternative consists to replace $\beta_{s_t} B(L^{1/m}, \Theta)$ in the transition probabilities (2) with $\sum_{j=1}^K b_{j,s_t} L^{(j-1)/m}$. As seen in the empirical section, this option is less attractive given the high number of parameters already involved in regime-switching models. The transition probabilities of the two-state model contains $2 + 2K$ parameters instead of 6, e.g. 26 parameters instead of 6 for $K = 12$ as studied later.

⁵We use Matlab for all simulations and estimations.

⁶See the Appendix for a presentation of the filter and the derivation of the log likelihood in the MSV-MIDAS model.

3.1 Design of the Monte Carlo study

We use Monte Carlo experiments to investigate the small-sample properties of the maximum likelihood (ML) estimators and related test statistics.

In the Monte Carlo study, we generate many realizations of the MSV-MIDAS process. The experiment involves the following steps. First, we simulate the high-frequency variable $z_\tau^{(m)}$ according to an autoregressive process:

$$z_\tau^{(m)} = c + \rho z_{\tau-1}^{(m)} + \omega u_\tau^{(m)} \quad \tau = 1, \dots, T \times m \quad (4)$$

As a second step, we generate a first-order Markov chain s_t , $t = 1 \dots, T$ with time-varying transition probabilities as defined in equations (2) and (3). We consider $K = 12$ lags in the polynomial $B(L^{1/m}, \Theta)$. Finally, we simulate the low-frequency variable y_t , $t = 1 \dots, T$ as a first-order autoregressive process subject to Markov shifts in mean as described in equation (1). The residuals u_t and ε_t are i.i.d. standard normal and independent. They are generated via a pseudo-random number generator. The first 100 simulated observations of s_t and y_t and the first $100 \times m$ observations of $z_\tau^{(m)}$ are discarded to reduce the effect of the initial conditions. We assume that $m = 3$, which corresponds to a model mixing quarterly and monthly data. We consider various sample sizes $T = \{200, 400, 800\}$, and we use 1,000 Monte Carlo replications for each experiment.

[INSERT TABLE 1 HERE]

The values of the parameters in equations (1)-(4) are given in Table 1. The benchmark configuration (DGP1) is close to the empirical setting obtained for US data in section 4. The low-frequency variable follows an AR(1) process with a switching mean. The mean parameter is negative in the least persistent regime. The high-frequency indicator positively affects the transition probability of the favorable state and is negatively related to the probability of staying in the recession state. In DGP1, the dynamics of the high-frequency indicator is state-independent. Alternatively, in DGP2, we allow a switch in the intercept and in the variance of equation (4).⁷ Allowing a change in the dynamics of the high-frequency indicator is relevant since the leading indicators used to perform business cycle inferences typically depend on the business cycle. In DGP3, the high-frequency indicator z_t is less persistent. In DGP4, the impact of the high-frequency variable on

⁷The intercept and the variance parameters vary according to the value taken by a two-state first-order Markov chain. The probability of staying in the high-growth state is equal to 0.9 and the probability of staying in the low-growth state is equal to 0.8.

the transition probabilities is weaker (lower β_1 and β_2). In DGP5, the difference between the mean parameters is smaller across the two regimes, which may adversely affect the classification of the observations in the two regimes. Finally, DGP6 is used to investigate the sensitivity of the results to the shape of the weighting function. In this last DGP, the profile is flatter, with lower values of θ_1 and θ_2 , i.e., more uniform weights are assigned to the K past values of z .

3.2 Robustness of the ML estimates

In a first step, we explore the finite sample performance of the ML estimator for the data generating processes considered in Table 1.

The parameters of the models are estimated via a numerical optimization of the log-likelihood of the model. As starting values, we use the true vector of the considered parameters to generate the data, plus random values drawn from a normal distribution with a standard deviation equal to 0.1.⁸ To gauge the robustness of ML estimates, we examine the average bias of the estimated coefficients of the model and the standard deviations of the estimates in the 1,000 replications. To measure the quality of the estimated parameters involved in the transition probabilities, we report additional criteria. For parameters $\Theta = \{\theta_1, \theta_2\}$, we provide an average measure of the error in the weights given by:

$$err_b_j = \frac{\sum_{j=1}^K [b(j, \hat{\Theta}) - b(j, \Theta)]^2}{\sum_{j=1}^K b(j, \Theta)^2} \quad (5)$$

Second, we compare the simulated transition probabilities with the estimated ones using mean absolute error statistics:⁹

$$\begin{aligned} err_p_{11} &= \frac{1}{T} \sum_{t=1}^T \left| \Gamma(\alpha_1 + \beta_1 B(L^{1/m}, \Theta) z_{t-1}^{(m)}) - \Gamma(\hat{\alpha}_1 + \hat{\beta}_1 B(L^{1/m}, \hat{\Theta}) z_{t-1}^{(m)}) \right| \\ err_p_{22} &= \frac{1}{T} \sum_{t=1}^T \left| \Gamma(\alpha_2 + \beta_2 B(L^{1/m}, \Theta) z_{t-1}^{(m)}) - \Gamma(\hat{\alpha}_2 + \hat{\beta}_2 B(L^{1/m}, \hat{\Theta}) z_{t-1}^{(m)}) \right| \end{aligned} \quad (6)$$

This last criterion measures the effect of the estimation error in parameters α , β and θ on the time-varying transition probabilities. It gives the overall impact of errors in the set

⁸We do not assess the effect of possible model misspecifications on the estimation accuracy. The model is estimated with the true number of autoregressive parameters and with the same parameters subject to changes in regime.

⁹The mean squared errors of the probabilities are qualitatively similar. The results are available from the author upon request.

of parameters in the transition probabilities on the identification of the state. We report the average values of these criteria in the 1,000 Monte Carlo simulations.

[INSERT TABLE 2 HERE]

The results in Table 2 show that the estimation procedure provides accurate estimates of the parameters present in the equation for y_t . The average bias is generally very close to zero, and the dispersion is low. The bias is slightly larger for ϕ in small samples. The estimated parameters of the transition probabilities α_i and β_i , $i = \{1, 2\}$, are less accurate, especially for small values of T , and the estimates of these four parameters show a higher dispersion. The error in the weights is also larger for small samples, although it is rather limited, as shown by the relatively low values of err_b_j . However, the mean absolute error in the probabilities remains moderate, even for small values of T (less than 5 points for $p_{11,t}$ and 11 points for $p_{22,t}$ when $T = 200$ and less than 3 and 6 points, respectively, in the largest sample). Hence, the error in parameters α , β and θ has a limited impact on the classification of the observations in the two regimes.

Comparing results across DGPs, the quality of parameter estimates in the transition probabilities increases with less persistent dynamics of z_t (DGP3) or with more uniformly distributed weights (DGP6). The average bias and the standard deviation of the estimated α_i and β_i , $i = \{1, 2\}$ is more limited. In DGP6, the average error in the weights is smaller too for small samples. By contrast, considering a high-frequency indicator also subject to changes in regime (DGP2) or decreasing the difference between the parameters of the two states (DGP5) has an adverse effect on estimation accuracy. In particular, the parameters entering the probabilities show a higher bias for $T = 200$ (e.g. in DGP2 and DGP5, the bias in β_1 is twice that in the reference model) and are more volatile. The error in the weights is also larger for smaller coefficients β (DGP4). However, in all cases, the mean absolute errors in the probabilities are close to those in DGP1. Hence, the impact on the identification of the regimes is rather limited relative to the benchmark.

3.3 Robustness of the tests

We now turn to the reliability of the t-statistics related to the parameters of the model.

The t-statistics associated with the estimated parameters of the model are expected to be approximately distributed as $N(0, 1)$. Francq and Roussignol [1998] and Douc, Moulines and Ryden [2004] provide results concerning the consistency and asymptotic normality of the ML estimator in Markov-switching autoregressive models with fixed

probabilities. As indicated in Psaradakis et al. [2010], no equivalent results are available on the distribution for the parameters in Markov-switching models with time-varying probabilities. However, practitioners generally rely on the normal distribution when they conduct significance tests in these models. To assess this property, Table 3 reports some characteristics of the sampling distribution of the t-statistics of the parameters obtained in the Monte Carlo simulations: the mean, the standard deviation, the skewness, the excess kurtosis, and the p-value of the Jarque-Bera test for normality for the 1,000 simulated t-statistics. The t-statistics are computed as the ratio of the estimation error to the estimated standard error. The estimated standard errors are based on the Hessian matrix of the estimated log-likelihood function.

The results indicate some departure from normality in the distribution of the t-statistics. The standard deviations of the 1,000 simulated t-statistics are generally close to one. However, the average t-statistics associated with $\hat{\phi}$ in the equation for y_t and with $\hat{\alpha}_i$ and $\hat{\beta}_i$ for $i = \{1, 2\}$ in the transition probabilities depart from zero¹⁰. Moreover, the skewness coefficient shows some asymmetry in the distributions of the t-statistics for $\hat{\alpha}_i$ and $\hat{\beta}_i$, $i = \{1, 2\}$. The distributions of $\hat{\beta}_1$ and $\hat{\beta}_2$ are also highly leptokurtic for small values of T . As a consequence, the null of normality is strongly rejected by the Jarque-Bera test for $\hat{\alpha}_i$ and $\hat{\beta}_i$, even in large samples. The comparison across DGPs shows that the deviation from normality is lower with less persistent dynamics of z_t (DGP3) or with more uniformly distributed weights (DGP6). By contrast, the normality is more rejected when the high-frequency indicator is subject to changes in regime, for smaller β_1 and β_2 and closer μ_1 and μ_2 (DGP2, 4 and 5).

[INSERT TABLE 3 HERE]

To assess the potential effect of non-normality on the inference, we investigate the performance of the t-statistics over the 1,000 Monte Carlo simulations when we use standard normal critical values. Figure 1 provides the empirical size of the two-sided tests of the equality of each parameter to its true value, as well as the empirical power of the significance tests for each parameter at the 5% significance level.¹¹ The empirical sizes are found close to the nominal level (5%).¹² However, as in Psaradakis et al. [2010], we

¹⁰Psaradakis and Sola [1998] obtain similar results in Markov-switching models with fixed transition probabilities, as do Psaradakis et al. [2010] in an MS model with time-varying probabilities.

¹¹In contrast to the linear case, the particular test of the nullity of β_1 in $P(s_t = 1 | s_{t-1} = 1, z_{t-1}^{(m)})$ in a model with time-invariant weights does not involve non-identified parameters under the null hypothesis, as the vector Θ is still present in the other transition probability $P(s_t = 2 | s_{t-1} = 2, z_{t-1}^{(m)})$. The same applies to β_2 .

¹²The conclusions are similar at the 10% significance level. The results are available from the author upon request.

observe that the parameters entering the transition probabilities are more likely to be insignificant in small samples. Indeed, the frequency of rejecting the nullity of α_2 and β_2 in the shorter state is lower for $T = 200$ and $T = 400$. For instance in DGP3, the nullity of α_2 is rejected in 35% of the cases and that of β_2 in 53% of the cases for $T = 200$. Considering lower values for β coefficients in DGP4 leads to even smaller rejection rates for α_2 and β_2 (28% and 47%), while the frequency of rejection is approximately 100% for the other parameters. In sum, the t-statistics of the parameters α_i and β_i should be interpreted with caution, especially in the shorter regime.

4 Application to US GDP

We now illustrate the empirical relevance of the MSV-MIDAS model through a business cycle analysis of the United States.

4.1 Data and specifications

The database consists of the quarterly growth rate of real GDP and a set of monthly financial indicators for the United States. The dataset was collected in July 2014.

The data on GDP cover the period from 1959Q1 to 2013Q4 (220 quarters). This sample includes 8 recessions. The business cycle chronology is taken from the NBER. To account for data revisions in the out-of-sample evaluation, we use vintages of output growth from the real-time datasets constructed by Croushore and Stark [2001] and available on the website of the Federal Reserve Bank of Philadelphia.¹³ Our real time data-set for US GDP growth consists of 291 vintages, released from January 1990 to March 2014. It might be more challenging to identify a recession using the GDP data available at the time due the revision of GDP data, especially during recessionary periods. Over the period 1990-2010, the quarterly growth rate of US GDP was revised by an average of 0.26 points three years after its first publication. This revision reaches up to 0.37 points for the recessionary quarters, as opposed to 0.21 points during expansionary quarters.

We will assess whether monthly financial indicators can help to detect, in real time, the recessions for this country when they are incorporated into the transition probabilities of the MSV-MIDAS model. The set of monthly indicators includes a short-term interest rate, the term spread, and stock and oil prices. Interest rates are considered in differences and the term spread in levels. Stock and oil prices are in log differences. US interest rates

¹³<http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/>

are released by the Federal Reserve Bank of Saint Louis.¹⁴ We consider the effective federal funds rate and the slope of the yield curve measured as the difference between the 10-Year Treasury bond and the 3-month Treasury bill. The stock market index SP500 is provided by Yahoo Finance. Finally, we consider the Brent oil price in London (datastream). We assume that the financial variables are not revised.

We consider two-state MSV-MIDAS models for GDP growth in which the transition probabilities depend on one of the four monthly financial variables. To ensure exogeneity with respect to the dependent variable, we lag the financial indicators by one quarter in the transition probabilities. We retain $K = 12$ lags in the Almon function, and hence the probabilities may depend on the monthly indicators over the entire past year. To select the number of autoregressive terms, we use the AIC in the linear specification. Tests for omitted autocorrelation are implemented to determine whether these autoregressive orders are sufficient. We apply Ljung-Box tests either to the standardized generalized residuals (Gourieroux, Monfort, Renault and Trognon [1987]) or to standard-normal residuals constructed with the Rosenblatt transformation (Smith [2008]).¹⁵ To check the gain due to the use of mixed frequency data, we also conduct a test for the flat aggregation scheme, as suggested by Andreou et al. [2010] in the linear case. When $\Theta = \{0, 0\}$, the high-frequency indicator is converted to low-frequency data with a simple average. The relevance of these restrictions is tested with a standard LR test.

We compare the performance of the MSV-MIDAS specification with those of several models. First, to assess the gain due to the inclusion of time-variation in the transition probabilities, the MSV-MIDAS model is compared with several benchmarks with fixed transition probabilities (FTP). At this level, we consider the two-state autoregressive specification with a switching mean as in Hamilton [1989] (MSM2) and a two-state model with a switching mean and a switching variance (MSMH2). The shift in the variance might be useful to capture the reduction in the volatility of business cycle fluctuations starting in the mid-1980s. These specifications are constrained versions of the model presented in section 2 when β_1 and β_2 are set to zero. We also consider three-state models

¹⁴<http://www.research.stlouisfed.org/fred2/>

¹⁵In a MSM(M)-AR(p) model with $p = 1$ lag, the standardized generalized residuals are obtained as: $\sum_{i=1}^M \sum_{j=1}^M P(s_t = i, s_{t-1} = j \mid I_{t-1}; \Theta) \times \sigma^{-1}(\varepsilon_t - \mu_i - \phi(y_{t-1} - \mu_j))$, while the Rosenblatt residuals

are defined as: $\Phi^{-1}\left(\sum_{i=1}^M \sum_{j=1}^M P(s_t = i, s_{t-1} = j \mid I_{t-1}; \Theta) \times \Phi[\sigma^{-1}(\varepsilon_t - \mu_i - \phi(y_{t-1} - \mu_j))]\right)$, with Φ denoting the cumulative distribution function of the standard normal distribution and I_{t-1} the observed information on y and z available at time $t - 1$. Using Monte Carlo experiments, Smith [2008] shows that the test applied to the Rosenblatt transformation of standardized residuals performs well in detecting autocorrelation in Markov-switching models.

with switching mean and/or variance (MSM3 and MSMH3). The four models include two autoregressive terms. Diagnostic tests do not show any sign of remaining autocorrelation either in the generalized residuals or in the standard-normal residuals constructed with the Rosenblatt transformation as explained above.

Second, we include MSV-MIDAS model with unrestricted distributed lags (MSV-UMIDAS) in our comparison. The MIDAS polynomial in the time-varying transition probabilities may impose a tight structure on the data in that there is a high degree of non-linearity to estimate the MIDAS parameters (i.e., two ratios of exponential functions to ensure that both the transition probabilities and the MIDAS weights are bounded between 0 and 1). To circumvent the use of MIDAS polynomial and simplify the dynamics of the transition probabilities, we use the unrestricted MIDAS approach of Forni et al. [2015]. This approach does not resort to functional lag polynomials, as the Almon lag polynomial presented in section 2. Instead, the transition probabilities are directly related to the monthly lags of the high frequency indicator with a linear unrestricted lag polynomial. This specification offers more flexibility and may be easier to estimate. Nevertheless, it is also far less parsimonious when the impact of high-frequency indicator on the transitions is persistent. This may be problematic in regime-switching models where the number of parameters is already high. To specify the lag order, we use the AIC criterion with a maximal number equal to 12. We retain 6 lags for stock returns and term spread, 7 lags for federal fund rate and 1 lag for oil prices.

4.2 Estimation results

In a first step, we investigate the in-sample performance of the MSV-MIDAS models in tracking US GDP dynamics and identify the business cycle turning points in the entire sample.

We estimate the MSV-MIDAS models using the full sample, from 1959 to 2013. The estimation of the models is performed for a large set of initial conditions. Table 4 provides the models' estimates and significance tests. The flat aggregation and the residual diagnostic tests are given in the second part of the table.

[INSERT TABLE 4 HERE]

The models are estimated with two lags, as found in the linear specification. The Ljung-Box tests applied to generalized or Rosenblatt residuals support the assumption of no remaining autocorrelation. We also observe a gain from incorporating monthly

indicators into the probabilities rather than converting them to the quarterly frequency via the simple average. The LR test reported in the second part of the table (line LR flat) shows that, at the 5% or 10% significance level, the likelihood of the model is significantly improved relative to a model estimated with $\Theta = \{0, 0\}$. Hence, the usual weighting scheme relying on equal weights is empirically rejected which is a first evidence in favour of our approach. GDP exhibits a positive mean growth rate in the first regime and declines in the second state. The coefficients β_1 and β_2 generally have opposite signs, showing that a variation in z_t leads to movements of $p_{11,t}$ and $p_{22,t}$ in the opposite direction. Even when the two coefficients have the same sign, the size of the coefficients is clearly different. In the shorter regime, the coefficient β_2 is often non-significant. However, the t-statistics must be interpreted with caution given the possible lack of power in the significance tests, as shown in the previous section. This caveat is particularly notable for the shorter regime.

The estimated parameters in the transition probabilities have the expected signs. Lower stock returns increase the risk of recession ($\beta_1 > 0$) while making a recovery less likely ($\beta_2 < 0$). A similar pattern holds for term spread. The positive coefficient in the expansion probability is consistent with the sharp decline in the slope of the yield curve and, in some instances, the inversion of the yield curve observed before economic downturns. The coefficients for the central bank rates are negative in the expansion probability and positive in the recession probability. In particular, policy tightening increases the probability of switching to a recessionary state ($\beta_1 < 0$). Finally, the impact of oil price is ambiguous. A rise in oil price increases the probability of entering a recession ($\beta_1 < 0$). As noted in Hamilton [2013], a majority of US recessions have been preceded by a sharp rise in the price of crude petroleum.¹⁶ However, the impact on the probability of remaining in recession is also negative ($\beta_2 < 0$).

Table 5 presents the in-sample comparison of the MSV-MIDAS models with the two sets of benchmarks. First, we use the usual Akaike and Hannan-Quinn information criteria to confront the goodness-of-fit of the MSV-MIDAS models with the one of the models with fixed transition probabilities (FTP). We also compare the models estimated with exponential Almon lags (MSV-MIDAS) and with unrestricted lags (MSV-UMIDAS).

Several results are worth commenting on. The models with time-varying probabilities exhibit better fit to GDP growth than do the FTP models with regime-independent variance (MSM2 and MSM3). The information criteria reach their lowest values in the

¹⁶The only exception among the postwar recessions in the United States is the economic crisis of 1960.

two models allowing for regime-dependent heteroscedasticity (MSMH2 and MSMH3) but we will see below that allowing a change in the variance has a detrimental effect on the detection of turning points. Relative to MSM2 and MSM3, the improvement of the log-likelihood is particularly strong when the probabilities are related to stock returns and federal funds rate. When taking into account the number of parameters, the two MSV-MIDAS models still provide substantially smaller AIC and HQC values than in MSM2 and MSM3. This is also the case of the model including term spread, even though the AIC value is closer to that in MSM3. On the other side, the MSV-MIDAS model incorporating oil prices is outperformed by all specifications. Turning to the comparison of MSV-MIDAS and MSV-UMIDAS models, the models with unrestricted lags generally show much higher information criteria than their constrained counterpart. This result is particularly strong with the Hannan-Quinn criterion imposing a stronger penalty on the number of parameters (the MSV-UMIDAS models including the first three financial indicators contain nearly twice as many parameters than the specifications with restricted lags).

[INSERT TABLE 5 HERE]

Table 5 also reports criteria assessing the quality of the inference on the state: the quadratic probability score (QPS) and the area under the roc curve (AUC). The quadratic probability score is defined as $\frac{2}{T} \sum_{t=1}^T (P(s_t = i | I_T; \Theta) - r_t)^2$ with r_t a dummy variable equal to one if the regime i is the true regime in t and zero otherwise. The QPS value lies in $[0,2]$. The lower the QPS, the better the state is estimated. The roc curve is created by plotting the true positive rate called sensitivity (that is the proportion of recessions that are correctly identified as such) against the false positive rate or 1-specificity (that is the proportions of false signals) at various threshold settings (see Figure 2 for a representation of the roc curves for all the models under consideration). The area under the roc curve (AUC, also referred to AUROC) takes value between 0.5 and 1. An AUC equal to 1 indicates a perfect classification of the states, while a value of 0.5 corresponds to a random ranking.¹⁷

According to the QPS and AUC criteria, high-frequency information in the transition probabilities helps to track the state of the US economy. Among the FTP models, the

¹⁷Developed during World War II to assess radar signals and then traditionally used in medicine, radiology and biometrics, the receiver operating characteristic (ROC) analysis was introduced by Berge and Jorda [2011] for the business cycle analysis. Candelon, Dumitrescu and Hurlin [2012] provide a review of the ROC curve methodology, as well as other accuracy measures of probabilistic forecasts. A toolbox is provided by the authors to compute these criteria.

two models with a constant variance (MSM2 and MSM3) are good competitors, while the heteroskedastic models (MSMH2 and MSHM3) do not perform well. The shift in the variance captures the great moderation starting in the mid-1980s but deteriorates the detection of turning points, especially in the two-state model. The models with fixed transition probabilities are outperformed by MSV-MIDAS models. The information provided by stock price and interest rates is particularly helpful. The MSV-MIDAS model with the term spread yields a lower QPS (0.121 versus 0.161 in the best FTP model MSM2) and a higher AUC (0.970 against 0.939). The results for the federal rate are also supportive of the new specification (QPS equal to 0.117 and AUC to 0.956).¹⁸ On the other side, the model with oil prices does not provide better signals of economic downturn and recovery than the usual models with constant probabilities. The QPS criterion is very close to the one in MSM2 and the AUC is much lower than in the models with regime-independent variance (MSM2 and MSM3).

The QPS and AUC criteria also support the use of functional lag polynomials in the MIDAS specification. In the MSV-UMIDAS models, the two criteria are never improved with respect to their constrained counterparts (e.g. for the federal rate, QPS and AUC stand at 0.159 and 0.802 in the MSV-UMIDAS model against 0.117 and 0.956 in the MSV-MIDAS model). This low performance is illustrated in Figure 2. This figure shows the ROC curves obtained in the FTP, MSV-MIDAS and MSV-UMIDAS specifications. The closer the ROC curve follows the left-hand border and then the top border, the more accurate the model. With the exception of oil prices, the reference model outperforms the FTP models, while the signals provided by the unconstrained specification are clearly less accurate. The outperformance of the constrained MSV-MIDAS models over the unconstrained specifications is particularly striking for the federal rate. This is not surprising since this indicator has the most persistent impact on the transition probabilities, which requires a high number of parameters to be estimated in the unconstrained version of the model (21 parameters against 11 in the model with the exponential Almon function).

[INSERT FIGURE 2 HERE]

The good performance of the constrained MSV-MIDAS model in identifying past recessions in the United States is also evident from Figures 3-4. These figures display the smoothed probabilities of being in the low-growth state obtained in the FTP and in

¹⁸The MSV-MIDAS specifications are also superior to a naive forecasting model assuming that economy is always in expansion regime. The naive model provides a QPS equal to 0.275 superior to that obtained in the MSV-MIDAS models.

the MSV-MIDAS models (Figure 3) and in the MSV-UMIDAS and MSV-MIDAS models (Figure 4), together with the NBER recession periods. We focus on the results obtained with the best performing model in each group in terms of AUC criterion: the two-state model with a regime-switching mean MSM2 among the FTP models, the MSV-MIDAS model including term spread among the restricted models and the MSV-UMIDAS model including term spread among the unconstrained specifications.¹⁹ Interestingly in Figure 3, spread improve the signals of the last three recessions, which were driven by financial factors (see Ng and Wright [2013]). In particular, the dot-com bubble in 2000-01 is well detected by the new specification, while the signal was almost nonexistent in the FTP model. As for the great recession, the probability of recession in the MSV-MIDAS models increases in summer 2007, as the first signs of distress appear in the financial markets. The signals obtained for the four episodes in the 1970s and 1980s are also much clearer, and a false signal in the fourth quarter of 1977 disappears. In Figure 4, the signals of recession provided by the MSV-UMIDAS model are stronger but the unconstrained model gives a false signal of recession in the second and third quarters of 1967 and a false signal of recovery in the second quarter of 1982. Moreover, the recessions at the beginning of the 1980s and the 1990s as well as the great recession are announced with a significant advance in the unconstrained specification.

[INSERT FIGURES 3-4 HERE]

4.3 Real time business cycle forecasting

Incorporating monthly indicators into the transition probabilities might help to improve signals of future recessions. In this last section, we assess the ability of the MSV-MIDAS model to infer, in real-time, the current and future state of the economy.

We conduct an out-of-sample study with a recursive window scheme. The last observations of the sample are discarded for the forecasting exercise. The forecasting window spans from 1990Q1 to 2013Q4 and includes three recessions. The forecasted chain is sampled at a quarterly frequency, but the forecast can be updated every month after the release of the monthly indicator included in the transition probabilities. By contrast, the update of the forecasts in the models with constant probabilities only reflects the monthly revisions of GDP data.²⁰ In this study, we focus on the forecast of each quarter made from

¹⁹The results obtained with the other models are available from the author upon request.

²⁰The Bureau of Economic Analysis publishes an ‘advance’ estimate of the quarterly GDP about one month after the end of the reference quarter. A ‘second’ estimate, including more complete product data

6 months to a few days before the GDP release, approximately one month after the end of the reference quarter. We recursively expand the estimation period. The parameters of the models are estimated using the only information available at the time of the forecast. The evaluation is conducted in real-time conditions. The models are estimated from the observations available at the time of the forecast. At this level, we use the vintages of output growth provided by the Federal Reserve Bank for the US (the financial variables are assumed not to be subject to data revisions).

Table 6 shows the QPS and AUC criteria for the forecast states. Again, the results are reported for the MSV-MIDAS models, as well as the models with constant probabilities and the unrestricted MSV-UMIDAS models.

[INSERT TABLE 6 HERE]

The MSV-MIDAS models estimated with restricted lags of term spread and stock returns give the best forecasts of the recessionary state in the United States. They outperform clearly the models with fixed transition probabilities (among the FTP models, the Hamilton specification MSM2 performs better as well as the MSMH3 model at some horizons). The QPS and AUC criteria are nearly always better in the two MSV-MIDAS models. One quarter ahead, the MSV-MIDAS model with term spread provides better signals of upcoming recessions than all FTP models with QPS and AUC equal to 0.134 and 0.923 respectively (versus 0.191 and 0.849 in the best FTP model MSM2). At the one-month horizon, the QPS criterion is equal to 0.104 with stock returns and the AUC criterion stands at 0.979 against 0.181 and 0.836 in the same benchmark.²¹

The out-of-sample results also favor the use of restricted lag polynomials. The QPS and AUC criteria deteriorate in the unrestricted version of the model. The only exception is the case of stock returns. The unrestricted model estimated with this indicator displays lower QPS values at intermediate horizons (e.g. 0.168 against 0.209 four months before the publication of GDP) and a higher AUC value at the shortest one (0.973 versus 0.938). On the other side, the worst performance is shown by the unconstrained model estimated with the federal rate (QPS at 0.192 and AUC at 0.715 for the forecasts made a few

and the first estimates of corporate profits is available at the end of the second month and a ‘third’ estimate based on more complete source data is disclosed at the end of the third month.

²¹The two MSV-MIDAS models perform better than other benchmarks. A naive model forecasting expansion regime at all quarters provides a QPS equal to 0.229, whereas the two MSV-MIDAS models yield lower QPS values at all horizons. The performance at horizon 2/3 is also better than that of the anxious index (the probability of a decline in real GDP, as reported in the Survey of Professional Forecasters in the second month of each quarter). On this basis, the QPS computed for this indicator over 1990Q1-2013Q4 is equal to 0.152.

days before the GDP publication against 0.143 and 0.952 respectively in the constrained model). The model is even outperformed by the two-state model with fixed transition probabilities MSM2.

Overall it appears both in the in-sample and out-of-sample evaluation that the curse of dimensionality due to the use of unrestricted lags is detrimental to the performance of the MSV-MIDAS specification. Unconstrained MIDAS models require the estimation of many parameters (twice as much in the model including the federal rate), which introduces some uncertainty in the model analysis and leads to less accurate forecasts. This result might be due to the fragility of the inference on the parameters involved in the transition probabilities, as shown in the Monte Carlo simulations. This contrasts with the more favorable findings of Foroni et al. (2015) to the unconstrained MIDAS model in a linear context, where the increase in the number of parameters is more limited and where the weights can be estimated with ordinary least squares.

5 Concluding remarks

In this paper, we introduce the MSV-MIDAS model. This specification incorporates higher frequency information in the transition mechanism of Markov-switching models.

The MSV-MIDAS model is estimated via ML methods. Monte Carlo evidence suggests that our estimation procedure provides robust estimates of the parameters of the model. The Monte Carlo experiments also show that the t-statistics associated with the coefficients in the time-varying probabilities should be used with caution. In the empirical application, the new specification is applied to the detection and forecasting of business cycle turning points. We find that the MSV-MIDAS model detects recessions more successfully than the specification with invariant transition probabilities in the United States. The slope of the yield curve provides particularly useful signals for the identification and forecasting of economic downturns and recoveries. The empirical results also support the use of parsimonious lag functions in the time-varying transition probabilities of the models. These findings hold both in sample and out of sample.

There are a number of potential extensions to this paper. We could incorporate several leading indicators in the transition probabilities. This could help for signaling oncoming recessions, given the different sources and characteristics of recessions. It would also be interesting to include high-frequency regressors in the equation for GDP. Exploiting the information provided by weekly or daily data is also on our research agenda. Finally, this

model could be applied to other areas of macroeconomics and finance.

References

- Andreou, E., E. Ghysels, and A. Kourtellos, Regression models with mixed sampling frequencies, *Journal of Econometrics*, 2010, *158*, 246–261.
- , —, and —, Should macroeconomic forecasters use daily financial data and how?, *Journal of Business and Economic Statistics*, 2013, *31* (2), 240–251.
- Barsoum, F. and S. Stankiewicz, Forecasting GDP growth using mixed-frequency models with switching regimes, *International Journal of Forecasting*, 2015, *31*, 33–50.
- Bazzi, M., F. Blasques, S.J. Koopman, and A. Lucas, *Time Varying Transition Probabilities for Markov Regime Switching Models*, Tinbergen Institute Discussion Paper 072 2014.
- Berge, T.J. and O. Jorda, Evaluating the Classification of Economic Activity into Recessions and Expansions, *American Economic Journal: Macroeconomics*, 2011, *3* (2), 246–277.
- Bessec, M. and O. Bouabdallah, Forecasting GDP over the business cycle in a multi-frequency and data-rich environment, *Oxford Bulletin of Economics and Statistics*, 2015, *77* (3), 360–84.
- Candelon, B., E. Dumitrescu, and C. Hurlin, How to Evaluate an Early Warning System? Towards a Unified Statistical Framework for Assessing Financial Crises Forecasting Methods, *IMF Economic Review*, 2012, *60* (1), 75–113.
- Chang, Y., Y. Choi, and J.Y. Park, *Regime Switching Model with Endogenous Autoregressive Latent Factor*, mimeo 2015.
- Croushore, D. and K. Marsten, Reassessing the relative power of the yield spread in forecasting recessions, *Journal of Applied Econometrics*, 2015.
- and T. Stark, A real-time data set for macroeconomists, *Journal of Econometrics*, 2001, *105* (1), 111–130.
- Diebold, F.X., J.H. Lee, and G.C. Weinbach, Regime switching with time varying transition probabilities, in C.P. Hargreaves, editor, *Nonstationary Time Series Analysis and Cointegration*, Oxford University Press: Oxford (UK), 1994, pp. 283–302.

- Douc, R., E. Moulines, and T. Ryden, Asymptotic properties of the maximum likelihood estimator in autoregressive models with Markov regime, *The Annals of Statistics*, 2004, *32* (5), 2254–2304.
- Estrella, A. and F.S. Mishkin, Predicting U.S. Recessions: Financial Variables as Leading Indicators, *Review of Economics and Statistics*, 1998, *80*, 45–61.
- Ferrara, L., C. Marsilli, and J.P. Ortega, Forecasting growth during the Great Recession: is financial volatility the missing ingredient?, *Economic Modelling*, 2014, *36*, 44–50.
- Filardo, A.J., Business-cycle phase and their transitional dynamics, *Journal of Business and Economic Statistics*, 1994, *12* (3), 299–308.
- Forni, C., M. Marcellino, and C. Schumacher, Unrestricted mixed data sampling (MIDAS): MIDAS regressions with unrestricted lag polynomials, *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 2015, *178* (1), 57–82.
- Francq, C. and M. Roussignol, Ergodicity of Autoregressive Processes with Markov-Switching and Consistency of the Maximum-Likelihood Estimator, *Statistics: A Journal of Theoretical and Applied Statistics*, 1998, *32* (2), 151–173.
- Galvão, A.B., Changes in predictive ability with mixed frequency data, *International Journal of forecasting*, 2013, *29*, 395–410.
- Ghysels, E., A. Sinko, and R. Valkanov, MIDAS regressions: Further results and new directions, *Econometric Reviews*, 2007, *26* (1), 53–90.
- _____, P. Santa-Clara, and R. Valkanov, *The MIDAS touch: mixed data sampling regression models*, Cirano working paper 2004.
- Gourieroux, C., A. Monfort, E. Renault, and A/ Trognon, Generalised residuals, *Journal of Econometrics*, 1987, *34* (1-2), 5–32.
- Guérin, P. and M. Marcellino, Markov-Switching MIDAS models, *Journal of Business and Economic Statistics*, 2013, *31* (1), 45–56.
- Hamilton, J.D., A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica*, 1989, *57*, 357–384.
- _____, What is an Oil Shock?, *Journal of Econometrics*, 2003, *113*, 363–398.

- , Nonlinearities and the Macroeconomic Effects of Oil Prices, *Macroeconomic Dynamics*, 2011, *15*, 364–378.
- , Historical Oil Shocks, in R.E. Parker and R. Whaples, editors, *Routledge Handbook of Major Events in Economic History*, New York: Routledge Taylor and Francis Group, 2013, pp. 239–365.
- Kauppi, H. and P. Saikkonen, Predicting U.S. recessions with dynamic binary response models, *Review of Economics and Statistics*, 2008, *90* (4), 777–791.
- Kilian, L. and R.J. Vigfusson, Do Oil Prices Help Forecast U.S. Real GDP? The Role of Nonlinearities and Asymmetries, *Journal of Business and Economic Statistics*, 2013, *31* (1), 78–93.
- Kim, C.J., Dynamic linear models with Markov-switching, *Journal of Econometrics*, 1994, *60* (1-2), 1–22.
- , J. Piger, and R. Startz, Estimation of Markov regime-switching regression models with endogenous switching, *Journal of Econometrics*, 2008, *143*, 263–273.
- Ng, S. and J.H. Wright, *Facts and Challenges from the Great Recession for Forecasting and Macroeconomic Modeling*, NBER Working Paper 19469 2013.
- Psaradakis, Z. and M. Sola, Finite-Sample Properties of the Maximum Likelihood Estimator in Autoregressive Models with Markov Switching, *Journal of Econometrics*, 1998, *86*, 369–386.
- , — , F. Spagnolo, and N. Spagnolo, *Some cautionary results concerning Markov-switching models with time-varying transition probabilities*, mimeo 2010.
- Rudebusch, G.D. and J.C. Williams, Forecasting Recessions: The Puzzle of the Enduring Power of the Yield Curve, *Journal of Business and Economic Statistics*, 2009, *27* (4), 492–503.
- Schumacher, C., *MIDAS regressions with time-varying parameters: An application to corporate bond spreads and GDP in the Euro area*, mimeo 2014.
- Smith, D.R., Evaluating specification tests for Markov-Switching time-series models, *Journal of time series analysis*, 2008, *29* (4), 629–652.

APPENDIX

Filter and derivation of the log-likelihood in the MSV-MIDAS model

Let $\{y_t\}_{t=1}^T$ be a time series following an MSM(M)-AR(p) process with transition probabilities depending on a high-frequency indicator $z_t^{(m)}$, as described in section 2. The conditional log-likelihood function of the observed data is given by:

$$L(\theta) = \sum_{t=p+1}^T \ln f(y_t | \underline{y}_{t-1}, \underline{z}_{t-1}^{(m)}; \lambda)$$

with $\underline{y}_{t-1} = \{y_{t-1}, \dots, y_1\}$, $\underline{z}_{t-1}^{(m)} = \{z_{t-1}^{(m)}, \dots, z_1^{(m)}\}$, the past of y_t and $z_t^{(m)}$, and λ representing the vector of parameters of the model.

The conditional log-likelihood function is derived from the following computations iterated for $t = p + 1, \dots, T$. In a first step, we derive the joint probability:

$$\begin{aligned} P(s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y}_{t-1}, \underline{z}_{t-1}^{(m)}; \lambda) = \\ P(s_t = i | s_{t-1} = j, z_{t-1}^{(m)}) \times P(s_{t-1} = j, \dots, s_{t-p} = k | \underline{y}_{t-1}, \underline{z}_{t-2}^{(m)}; \lambda) \end{aligned}$$

with $i, j, k = \{1, 2, \dots, M\}$ and where the time-varying transition probabilities are expressed as follows in the case of two regimes ($M = 2$):

$$\begin{aligned} P(s_t = 1 | s_{t-1} = 1, z_{t-1}^{(m)}) &= \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta_1) z_{t-1}^{(m)}] \\ P(s_t = 2 | s_{t-1} = 2, z_{t-1}^{(m)}) &= \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta_2) z_{t-1}^{(m)}] \\ P(s_t = 2 | s_{t-1} = 1, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta_1) z_{t-1}^{(m)}] \\ P(s_t = 1 | s_{t-1} = 2, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta_2) z_{t-1}^{(m)}] \end{aligned}$$

with the function $B(L^{1/m}, \Theta)$ specified as:

$$B(L^{1/m}, \Theta) = \sum_{j=1}^K b(j, \Theta) L^{(j-1)/m}, \quad b(j, \Theta) = \frac{\exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=1}^K \exp(\theta_1 j + \theta_2 j^2)}$$

In a second step, the joint density is derived as follows:

$$\begin{aligned} f(y_t, s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y}_{t-1}, \underline{z}_{t-1}^{(m)}; \lambda) = \\ f(y_t | s_t = i, s_{t-1} = j, \dots, s_{t-p} = k, y_{t-1}, \dots, y_{t-p}, \underline{z}_{t-1}^{(m)}; \lambda) \times P(s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y}_{t-1}, \underline{z}_{t-1}^{(m)}; \lambda) \end{aligned}$$

where the conditional density of y_t given the past and current states $s_t, s_{t-1}, \dots, s_{t-p}$ and the

past observations of y and $z^{(m)}$ is given by:

$$f(y_t | s_t, s_{t-1}, \dots, s_{t-p}, y_{t-1}, \dots, y_{t-p}, \underline{z_{t-1}^{(m)}}; \lambda) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ \frac{y_t - \mu_{s_t} - \phi_1(y_{t-1} - \mu_{s_{t-1}}) - \dots - \phi_p(y_{t-p} - \mu_{s_{t-p}})}{2\sigma^2} \right\}$$

In a third step, the conditional density $f(y_t | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)$ is derived by summing over all possible state sequences:

$$f(y_t | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda) = \sum_{i=1}^M \sum_{j=1}^M \dots \sum_{k=1}^M f(y_t, s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)$$

Finally, we derive the joint probability of the p states conditional upon $\underline{y_t}$ and $\underline{z_{t-1}^{(m)}}$ from:

$$\begin{aligned} P(s_t = j, \dots, s_{t-p+1} = k | \underline{y_t}, \underline{z_{t-1}^{(m)}}; \lambda) &= \sum_{k=1}^M P(s_t = j, \dots, s_{t-p+1} = k, s_{t-p} = l | \underline{y_t}, \underline{z_{t-1}^{(m)}}; \lambda) \\ &= \sum_{k=1}^M \frac{f(y_t, s_t = i, s_{t-1} = j, \dots, s_{t-p+1} = k, s_{t-p} = l | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)}{f(y_t | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)} \end{aligned}$$

The initialization of the filter relies on the ergodic probabilities of the state in the FTP model.

Table 1: Monte Carlo experiment - DGP

	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
c	0.1	0.5/-0.2	0.1	0.1	0.1	0.1
ρ	0.8	0.8	0.5	0.8	0.8	0.8
ω	1.0	0.5/1.0	1.0	1.0	1.0	1.0
μ_1	1.0	1.0	1.0	1.0	0.5	1.0
μ_2	-0.8	-0.8	-0.8	-0.8	-0.4	-0.8
ϕ	0.5	0.5	0.5	0.5	0.5	0.5
σ	0.3	0.3	0.3	0.3	0.3	0.3
α_1	2.0	2.0	2.0	2.0	2.0	2.0
α_2	0.5	0.5	0.5	0.5	0.5	0.5
β_1	2.0	2.0	2.0	1.0	2.0	2.0
β_2	-1.0	-1.0	-1.0	-0.5	-1.0	-1.0
θ_1	2.0	2.0	2.0	2.0	2.0	0.2
θ_2	-0.15	-0.15	-0.15	-0.15	-0.15	-0.015

Note: This table details the parameterizations of the MSV-MIDAS models used to simulate the Monte Carlo samples.

Table 2: Monte Carlo results - Estimate accuracy of the MSV-MIDAS model

	T	μ_1	μ_2	ϕ	σ	α_1	α_2	β_1	β_2	err_bj	err_p11	err_p22
DGP1	200	0.000 (0.05)	0.001 (0.06)	-0.017 (0.06)	-0.003 (0.02)	0.115 (0.62)	-0.044 (0.44)	0.309 (0.75)	-0.214 (0.56)	0.20	0.04	0.09
	400	0.000 (0.03)	0.000 (0.04)	-0.010 (0.04)	0.000 (0.01)	0.060 (0.33)	-0.010 (0.28)	0.140 (0.40)	-0.070 (0.29)	0.09	0.03	0.06
	800	0.000 (0.02)	0.000 (0.03)	0.000 (0.03)	0.000 (0.01)	0.030 (0.21)	-0.010 (0.18)	0.060 (0.26)	-0.050 (0.18)	0.03	0.02	0.04
DGP2	200	-0.001 (0.04)	0.001 (0.09)	-0.013 (0.06)	-0.002 (0.02)	0.159 (0.81)	-0.092 (1.99)	0.615 (1.06)	-0.631 (4.93)	0.21	0.03	0.11
	400	0.000 (0.03)	-0.001 (0.06)	-0.006 (0.04)	-0.002 (0.01)	0.064 (0.48)	0.014 (0.44)	0.218 (0.54)	-0.197 (0.58)	0.14	0.02	0.08
	800	0.000 (0.02)	-0.002 (0.04)	-0.004 (0.03)	0.000 (0.01)	0.037 (0.29)	-0.013 (0.29)	0.093 (0.33)	-0.083 (0.27)	0.07	0.01	0.06
DGP3	200	-0.002 (0.05)	-0.002 (0.06)	-0.016 (0.06)	-0.003 (0.02)	0.090 (0.36)	-0.012 (0.38)	0.248 (0.71)	-0.215 (0.71)	0.21	0.04	0.09
	400	0.000 (0.03)	0.000 (0.04)	-0.005 (0.04)	-0.001 (0.01)	0.044 (0.24)	-0.006 (0.24)	0.107 (0.45)	-0.064 (0.41)	0.09	0.03	0.06
	800	0.001 (0.02)	0.001 (0.03)	-0.003 (0.03)	-0.001 (0.01)	0.017 (0.16)	-0.001 (0.16)	0.042 (0.29)	-0.029 (0.27)	0.04	0.02	0.04
DGP4	200	0.000 (0.04)	0.002 (0.06)	-0.011 (0.07)	-0.003 (0.02)	0.068 (0.36)	-0.062 (0.40)	0.170 (0.36)	-0.143 (0.41)	0.45	0.04	0.10
	400	0.001 (0.03)	0.003 (0.04)	-0.009 (0.04)	-0.001 (0.01)	0.031 (0.23)	-0.031 (0.25)	0.058 (0.22)	-0.063 (0.24)	0.27	0.03	0.07
	800	0.000 (0.02)	0.001 (0.03)	-0.003 (0.03)	0.000 (0.01)	0.020 (0.16)	-0.007 (0.17)	0.028 (0.15)	-0.012 (0.15)	0.13	0.02	0.05
DGP5	200	0.000 (0.05)	0.001 (0.07)	-0.023 (0.08)	-0.002 (0.02)	0.272 (0.99)	-0.097 (0.87)	0.636 (1.32)	-0.380 (1.18)	0.25	0.05	0.11
	400	0.001 (0.03)	0.002 (0.05)	-0.007 (0.05)	-0.001 (0.01)	0.096 (0.44)	-0.006 (0.34)	0.177 (0.53)	-0.114 (0.39)	0.13	0.04	0.07
	800	0.000 (0.02)	-0.001 (0.03)	-0.005 (0.04)	-0.001 (0.01)	0.052 (0.27)	-0.015 (0.22)	0.095 (0.33)	-0.054 (0.24)	0.05	0.03	0.05
DGP6	200	-0.001 (0.05)	0.000 (0.07)	-0.016 (0.06)	-0.003 (0.01)	0.133 (0.48)	-0.044 (0.41)	0.246 (0.67)	-0.150 (0.57)	0.14	0.04	0.09
	400	0.001 (0.03)	-0.001 (0.05)	-0.008 (0.04)	-0.001 (0.01)	0.053 (0.28)	-0.044 (0.29)	0.099 (0.40)	-0.097 (0.35)	0.06	0.03	0.07
	800	0.001 (0.02)	0.001 (0.03)	-0.004 (0.03)	-0.001 (0.01)	0.033 (0.19)	-0.007 (0.20)	0.068 (0.26)	-0.038 (0.23)	0.03	0.02	0.05

Note: This table provides the average bias and, in brackets, the standard deviation of the parameter estimates of the MSV-MIDAS models for sample sizes $T = \{200, 400, 800\}$ over 1,000 replications. The last three columns show the error measures for the weights (err_bj) and the transition probabilities (err_p11 and err_p22). We report the average values of these three criteria in the 1,000 Monte Carlo simulations.

Table 3: Monte Carlo results - Distribution of the t-statistics

	T	μ_1	μ_2	ϕ	α_1	α_2	β_1	β_2	μ_1	μ_2	ϕ	α_1	α_2	β_1	β_2
					DGP1							DGP2			
mean	200	-0.01	0.00	-0.22	0.03	-0.05	0.23	-0.20	-0.03	0.01	-0.17	0.03	-0.08	0.37	-0.21
	400	0.02	0.01	-0.12	0.08	-0.02	0.21	-0.10	0.01	-0.01	-0.10	0.04	0.01	0.20	-0.19
	800	-0.04	-0.04	-0.12	0.07	-0.05	0.14	-0.16	-0.02	-0.06	-0.12	0.06	-0.06	0.14	-0.16
std	200	1.08	1.01	1.02	0.98	0.98	0.95	0.96	1.03	1.03	1.03	0.95	0.91	0.84	0.92
	400	0.99	0.99	0.96	1.00	1.01	0.98	0.98	1.02	0.97	1.03	0.97	0.99	0.95	0.95
	800	1.00	1.00	1.02	0.99	0.99	0.98	0.95	1.02	1.00	1.01	0.97	1.01	0.94	0.94
skew	200	-0.02	-0.04	-0.05	-0.30	0.05	-0.44	0.42	-0.12	0.03	0.04	-0.39	0.00	-0.78	0.74
	400	0.05	-0.05	0.04	-0.15	0.05	-0.37	0.33	-0.05	0.04	0.03	-0.15	-0.02	-0.45	0.51
	800	-0.09	0.04	0.06	-0.20	0.02	-0.26	0.40	0.11	-0.11	-0.14	-0.15	-0.11	-0.20	0.27
kurt	200	0.10	0.19	0.22	-0.16	-0.43	-0.13	0.06	0.24	-0.08	0.16	-0.42	-0.58	0.41	0.33
	400	-0.02	0.09	0.46	0.10	-0.26	0.41	0.21	0.32	0.02	-0.03	-0.35	-0.54	-0.11	0.10
	800	-0.03	0.10	-0.10	-0.3	-0.24	-0.08	0.28	0.34	0.05	-0.15	0.02	-0.27	-0.06	0.02
JB	200	0.78	0.41	0.29	0.00	0.02	0.00	0.00	0.08	0.82	0.50	0.00	0.00	0.00	0.00
	400	0.83	0.67	0.01	0.11	0.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	800	0.54	0.69	0.61	0.00	0.30	0.00	0.00	0.03	0.38	0.11	0.14	0.08	0.04	0.00
					DGP3							DGP4			
mean	200	-0.05	-0.04	-0.21	0.13	-0.02	0.20	-0.14	0.00	0.03	-0.14	0.08	-0.12	0.32	-0.21
	400	-0.01	0.00	-0.07	0.10	-0.01	0.11	-0.03	0.04	0.06	-0.18	0.06	-0.09	0.15	-0.14
	800	0.05	0.03	-0.07	0.05	0.00	0.07	-0.01	0.01	0.04	-0.07	0.07	-0.02	0.10	0.01
std	200	1.06	1.05	0.98	0.98	1.01	1.02	1.03	1.03	1.01	1.04	0.99	0.97	0.97	0.95
	400	1.05	1.03	1.02	1.00	1.00	1.05	1.03	1.03	1.00	1.00	0.98	0.97	1.00	
	800	1.02	1.03	1.05	0.99	0.99	0.96	1.00	0.99	0.97	1.03	0.99	0.99	0.98	1.00
skew	200	0.04	-0.03	-0.08	-0.30	-0.05	-0.50	0.54	0.08	0.06	-0.15	-0.21	0.03	-0.68	0.46
	400	0.01	0.03	-0.04	-0.06	-0.20	-0.29	0.34	0.09	0.16	0.11	-0.18	0.03	-0.40	0.17
	800	0.11	0.01	-0.06	-0.12	0.02	-0.07	0.26	-0.16	-0.01	-0.07	-0.17	0.05	-0.09	0.08
kurt	200	0.32	0.25	0.06	-0.19	-0.24	0.58	0.27	0.44	0.11	0.36	-0.33	-0.31	1.37	0.27
	400	0.08	-0.11	-0.09	-0.18	-0.06	0.20	-0.07	0.16	0.42	-0.07	0.07	0.06	0.88	0.01
	800	0.45	-0.02	-0.17	-0.02	-0.24	-0.10	0.05	0.03	-0.05	0.10	0.16	0.06	0.04	-0.25
JB	200	0.10	0.25	0.53	0.00	0.24	0.00	0.00	0.01	0.55	0.01	0.00	0.13	0.00	0.00
	400	0.86	0.70	0.75	0.36	0.04	0.00	0.00	0.28	0.00	0.34	0.05	0.84	0.00	0.08
	800	0.01	0.98	0.39	0.29	0.51	0.00	0.12	0.95	0.52	0.05	0.77	0.52	0.17	
					DGP5							DGP6			
mean	200	0.01	0.01	-0.25	0.08	-0.06	0.28	-0.17	-0.03	0.01	-0.21	0.13	-0.04	0.18	-0.07
	400	0.05	0.03	-0.11	0.08	0.03	0.12	-0.08	0.02	-0.01	-0.15	0.08	-0.10	0.10	-0.13
	800	0.01	-0.04	-0.11	0.09	-0.03	0.13	-0.08	0.06	0.04	-0.12	0.10	0.00	0.15	-0.05
std	200	1.04	1.07	1.06	1.01	0.94	0.99	1.01	1.07	1.04	0.99	0.98	0.90	0.94	0.98
	400	1.03	1.00	0.94	1.00	0.99	0.98	1.02	0.97	1.06	1.00	0.97	1.00	0.96	1.01
	800	0.99	1.00	1.03	1.00	0.99	0.98	1.03	1.04	0.96	1.05	1.00	1.03	0.97	1.05
skew	200	0.14	0.18	0.05	-0.56	0.14	-0.71	0.76	-0.05	0.01	-0.09	-0.32	-0.05	-0.64	0.52
	400	0.00	0.05	0.03	-0.25	0.19	-0.60	0.52	-0.01	0.00	-0.04	-0.17	-0.05	-0.32	0.25
	800	-0.04	0.15	0.04	-0.36	-0.10	-0.40	0.41	0.02	-0.04	0.04	-0.16	-0.06	-0.32	0.28
kurt	200	0.16	-0.07	-0.03	0.15	-0.49	0.15	0.57	0.31	-0.29	-0.04	-0.26	-0.28	0.60	0.33
	400	0.03	0.04	0.07	-0.23	-0.15	0.24	0.06	-0.04	-0.12	-0.07	-0.25	-0.17	0.29	-0.20
	800	-0.15	-0.07	-0.06	0.00	-0.25	0.18	0.08	-0.07	0.03	-0.08	0.11	-0.06	0.08	-0.10
JB	200	0.10	0.06	0.78	0.00	0.00	0.00	0.00	0.10	0.18	0.53	0.00	0.16	0.00	0.00
	400	0.98	0.78	0.83	0.00	0.03	0.00	0.00	0.97	0.75	0.80	0.03	0.47	0.00	0.00
	800	0.57	0.12	0.81	0.00	0.11	0.00	0.00	0.87	0.86	0.76	0.09	0.66	0.00	0.00

Notes: This table reports various results for the t-statistics of the estimated coefficients of the MSV-MIDAS models in the Monte Carlo simulations. The t-statistics are computed as the ratio of the estimation error to the estimated standard error. We report the mean of the 1,000 simulated t-statistics (line *mean*), the standard deviation (line *std*), the skewness (line *skew*), the excess kurtosis (line *kurt*) and the p-value of the Jarque-Bera test for normality (line *JB*).

Table 4: Estimation results of the MSV-MIDAS models on US data (1959-2013)

	STOCK	SPREAD	RATE	POIL
μ_1	1.045*** (0.07)	0.999*** (0.084)	0.978*** (0.096)	0.914*** (0.089)
μ_2	-0.093 (0.119)	-0.232 (0.236)	-0.785*** (0.188)	-0.804*** (0.216)
ϕ_1	0.106 (0.068)	0.138 (0.085)	0.261*** (0.071)	0.273*** (0.083)
ϕ_2	0.127* (0.069)	0.098 (0.085)	0.277*** (0.072)	0.194** (0.082)
σ	0.685*** (0.034)	0.692*** (0.039)	0.625*** (0.034)	0.647*** (0.043)
α_1	8.125** (3.837)	1.453 (1.058)	3.964*** (0.648)	3.047*** (0.452)
α_2	4.380** (1.765)	3.085* (1.761)	1.231* (0.732)	-1.653 (1.517)
β_1	3.401** (1.693)	2.969* (1.689)	-5.768*** (1.871)	-0.11* (0.059)
β_2	-1.600*** (0.621)	-1.367 (0.921)	2.729* (1.435)	-2.325 (1.815)
θ_1	7.159 (2.124)	4.964 (5.794)	14.344 (12.554)	12.381 (11.599)
θ_2	-1.040 (0.294)	-0.608 (0.638)	-1.2756 (1.1095)	-1.090 (1.006)
LR flat	0.01	0.10	0.00	0.06
LB1(4)	0.13	0.70	0.26	0.75
LB1(12)	0.10	0.43	0.65	0.12
LB1(20)	0.03	0.21	0.37	0.13
LB2(4)	0.15	0.67	0.60	0.71
LB2(12)	0.12	0.31	0.77	0.16
LB2(20)	0.03	0.13	0.50	0.12

Notes: This table shows the estimation results of the MSV-MIDAS models including stock returns (column STOCK), term spread (SPREAD), central bank rate (RATE) and oil prices (POIL) for the US over the period 1959Q1-2013Q4. The first part gives the parameter estimations and the associated standard errors in brackets. Significance levels: *** if the coefficient is significant at a 1%, ** at a 5%, * at a 10% level. The second part of the table shows the p-value of the LR test for the null hypothesis of equal weights (line LR flat), the p-values of the Ljung-Box test for omitted autocorrelation of order 1 to p in the generalized residuals (lines LB1(p)) and in the Rosenblatt's residuals (lines LB2(p)).

Table 5: In-sample evaluation on US data (1959-2013)

	k	LL	AIC	HQC	QPS	AUC
<i>MSV-MIDAS models (exponential Almon lag)</i>						
STOCK	11	-238.489	498.977	514.015	0.208	0.951
SPREAD	11	-242.712	507.423	522.462	0.121	0.970
RATE	11	-235.870	493.739	508.777	0.117	0.956
POIL	11	-247.742	517.484	532.521	0.159	0.808
<i>Models with fixed transition probabilities (FTP)</i>						
MSM2	7	-253.074	520.149	529.718	0.161	0.939
MSMH2	8	-235.986	487.971	498.908	0.721	0.787
MSM3	12	-242.011	508.022	524.427	0.175	0.930
MSMH3	14	-231.835	491.669	510.808	0.208	0.689
<i>MSV-UMIDAS models (unconstrained lags)</i>						
STOCK	19	-230.943	499.885	525.859	0.291	0.924
SPREAD	19	-234.110	506.220	532.194	0.183	0.934
RATE	21	-231.676	505.352	534.060	0.159	0.802
POIL	9	-250.015	518.030	530.334	0.189	0.773

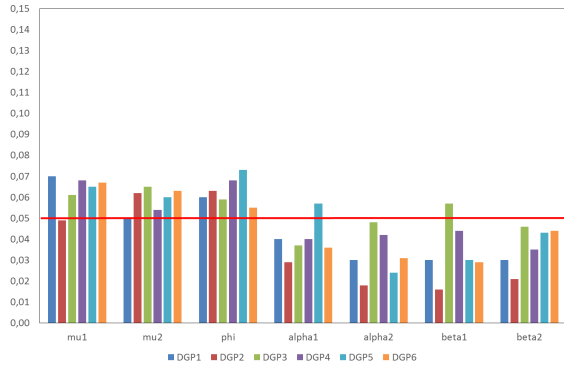
Notes: This table provides for each model the number of estimated parameters (k), the estimated log-likelihood (LL), the Akaike and Hannan-Quinn information criteria (AIC and HQC), the QPS and AUC criteria for the estimated states in the US over the period 1959Q1-2013Q4. The results are reported for the model with fixed transition probabilities (lines FTP) and the MSV-MIDAS models estimated with restricted (MSV-MIDAS) and unrestricted lags (MSV-UMIDAS) of stock returns (lines STOCK), term spread (SPREAD), central bank rate (RATE) and oil prices (POIL). For each criterion and each group of models, entries in bold indicate the best performing model.

Table 6: Out-of-sample evaluation on US data (1990-2013)

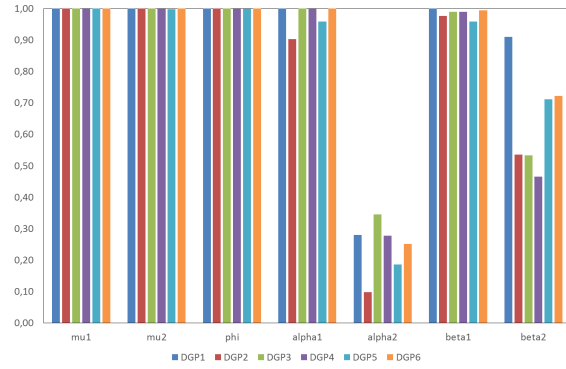
h		2	5/3	4/3	1	2/3	1/3	0
<i>MSV-MIDAS models (exponential Almon lag)</i>								
QPS	STOCK	0.194	0.243	0.209	0.159	0.121	0.104	0.079
	SPREAD	0.184	0.183	0.175	0.134	0.141	0.152	0.043
	RATE	0.230	0.236	0.271	0.190	0.191	0.206	0.143
	POIL	0.211	0.202	0.196	0.167	0.167	0.149	0.068
AUC	STOCK	0.898	0.832	0.838	0.923	0.967	0.979	0.938
	SPREAD	0.793	0.803	0.791	0.911	0.910	0.851	0.990
	RATE	0.677	0.595	0.629	0.839	0.702	0.740	0.952
	POIL	0.631	0.702	0.642	0.738	0.840	0.802	0.996
<i>Models with fixed transition probabilities (FTP)</i>								
QPS	MSM2	0.217	0.211	0.212	0.191	0.180	0.181	0.118
	MSMH2	1.193	1.122	1.105	1.137	1.066	1.053	1.076
	MSM3	1.431	1.464	1.433	0.343	0.353	0.347	0.150
	MSMH3	0.289	0.260	0.290	0.195	0.198	0.184	0.075
AUC	MSM2	0.633	0.654	0.657	0.849	0.838	0.836	0.944
	MSMH2	0.511	0.638	0.649	0.587	0.716	0.737	0.647
	MSM3	0.591	0.576	0.573	0.682	0.727	0.725	0.927
	MSMH3	0.723	0.774	0.730	0.796	0.764	0.809	0.917
<i>MSV-UMIDAS models (unconstrained lags)</i>								
QPS	STOCK	0.213	0.190	0.168	0.079	0.084	0.068	0.086
	SPREAD	0.251	0.236	0.247	0.218	0.159	0.165	0.066
	RATE	0.262	0.277	0.262	0.234	0.258	0.233	0.192
	POIL	0.223	0.231	0.228	0.199	0.197	0.193	0.145
AUC	STOCK	0.710	0.708	0.717	0.909	0.861	0.864	0.973
	SPREAD	0.562	0.608	0.604	0.705	0.713	0.712	0.941
	RATE	0.595	0.600	0.549	0.650	0.610	0.659	0.715
	POIL	0.702	0.555	0.594	0.765	0.613	0.714	0.952

Notes: This table shows the QPS and AUC criteria for the forecast states in the US over the period 1990Q1-2013Q4. Forecasts are made at horizons h of 2 quarters to a few days before the GDP release. The results are reported for the MSV-MIDAS models, the four models with fixed transition probabilities (MSM2, MSM3, MSMH2, MSMH3) and the MSV-MIDAS models estimated with restricted (MSV-MIDAS) and unrestricted lags (MSV-UMIDAS) of stock returns (lines STOCK), term spread (SPREAD), central bank rate (RATE) and oil prices (POIL). For each criterion and each group of models, entries in bold indicate the best performing model.

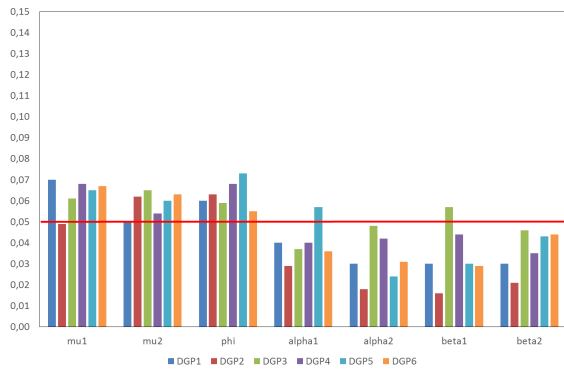
Figure 1: Size and power of the t-statistics (for a level of significance = 5%)



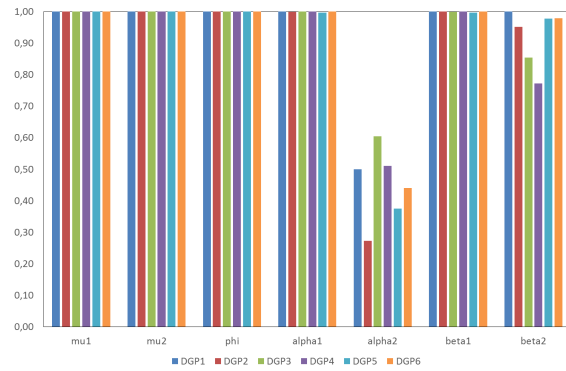
(a) Size (T=200)



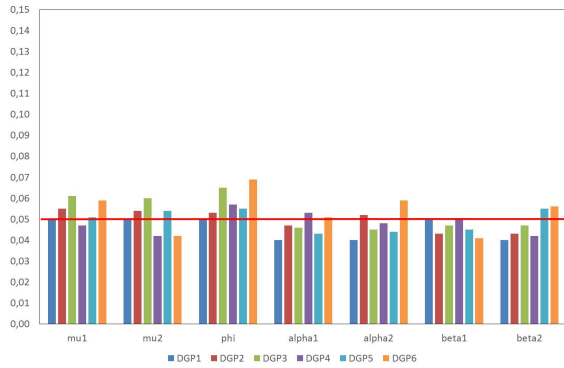
(b) Power (T=200)



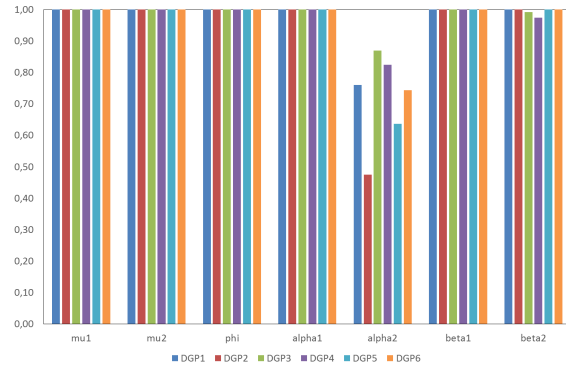
(c) Size (T=400)



(d) Power (T=400)



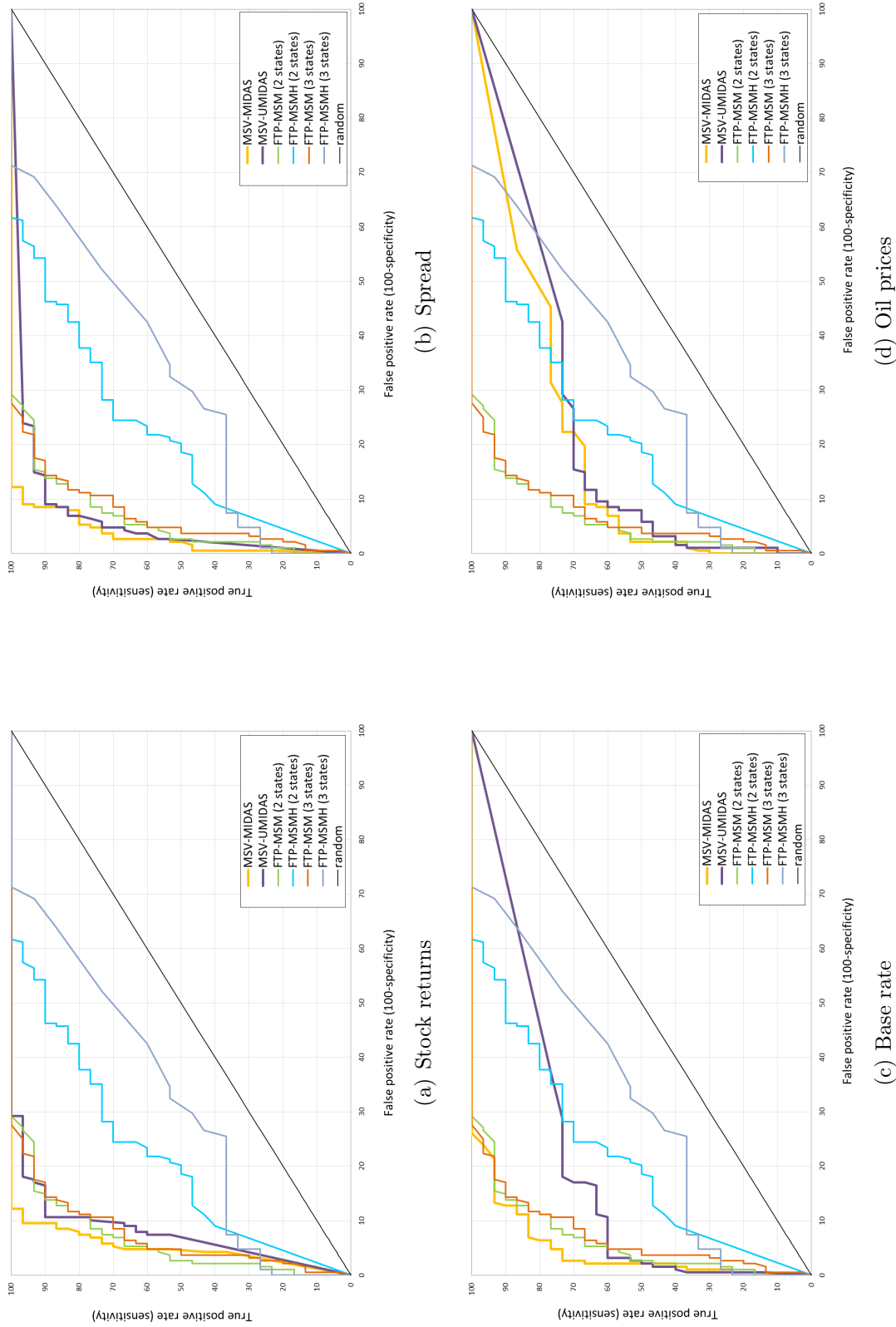
(e) Size (T=800)



(f) Power (T=800)

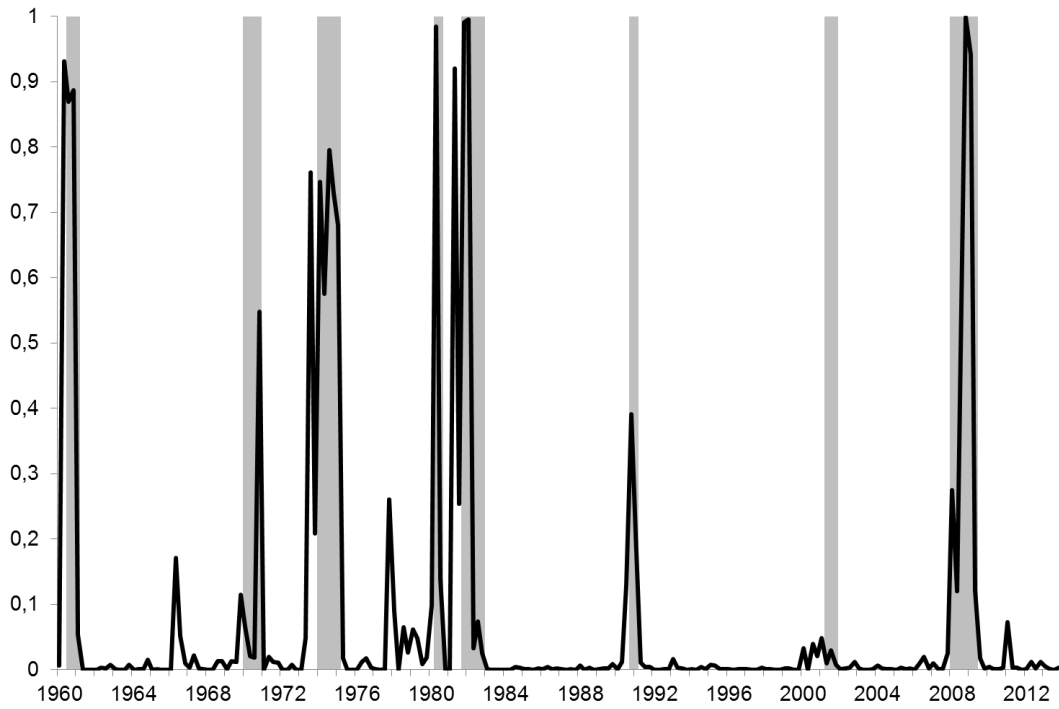
Notes: The graphs on the left plot the frequency of rejection of the equality of each coefficient to its true value at the 5 percent significance level among the 1,000 simulations. The horizontal line gives the nominal level of the tests. The graphs on the right depict the frequency of rejection of the nullity of each coefficient at the 5 percent significance level with the 1,000 simulated t-statistics.

Figure 2: The ROC curve

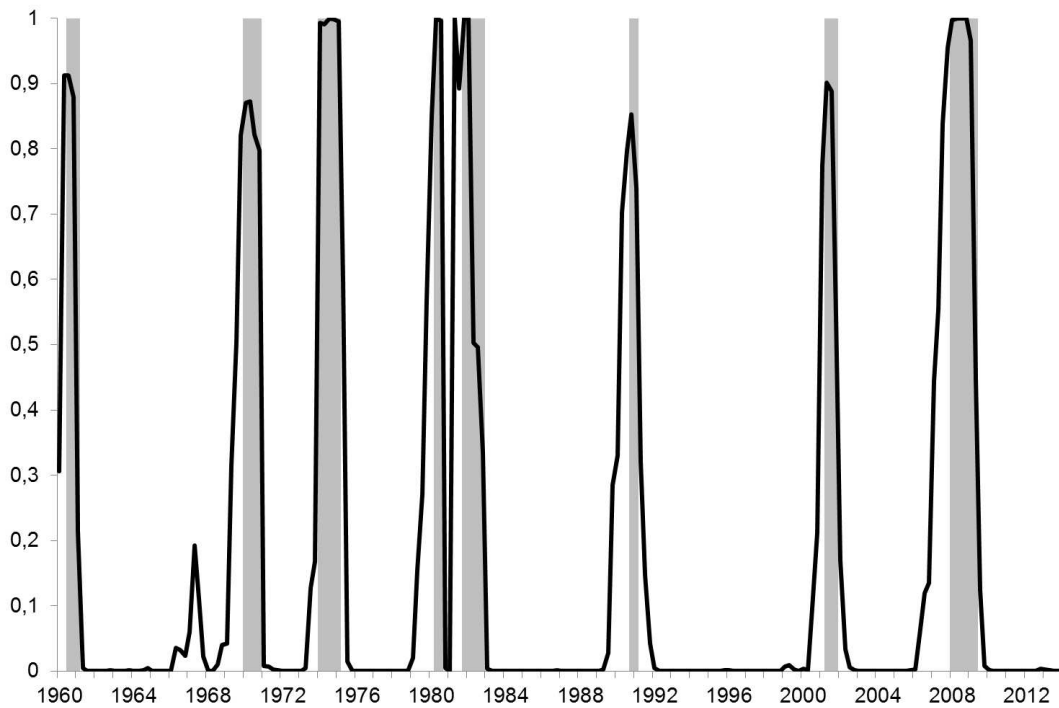


Notes: This figure depicts the ROC curves of the MSV-MIDAS models with constrained (MSV-MIDAS) and unrestricted weights (MSV-UMIDAS) and the four models with fixed transition probabilities (FTP): the two-state and three-state models with a regime-switching mean (MSM2 and MSM3) and the two-state and three-state models with a regime-switching mean and variance (MSMH2 and MSMH3). The best performing model has the largest area under the curve. An area of 100% represents the perfect classification.

Figure 3: Smoothed US recession probabilities (1959-2013)



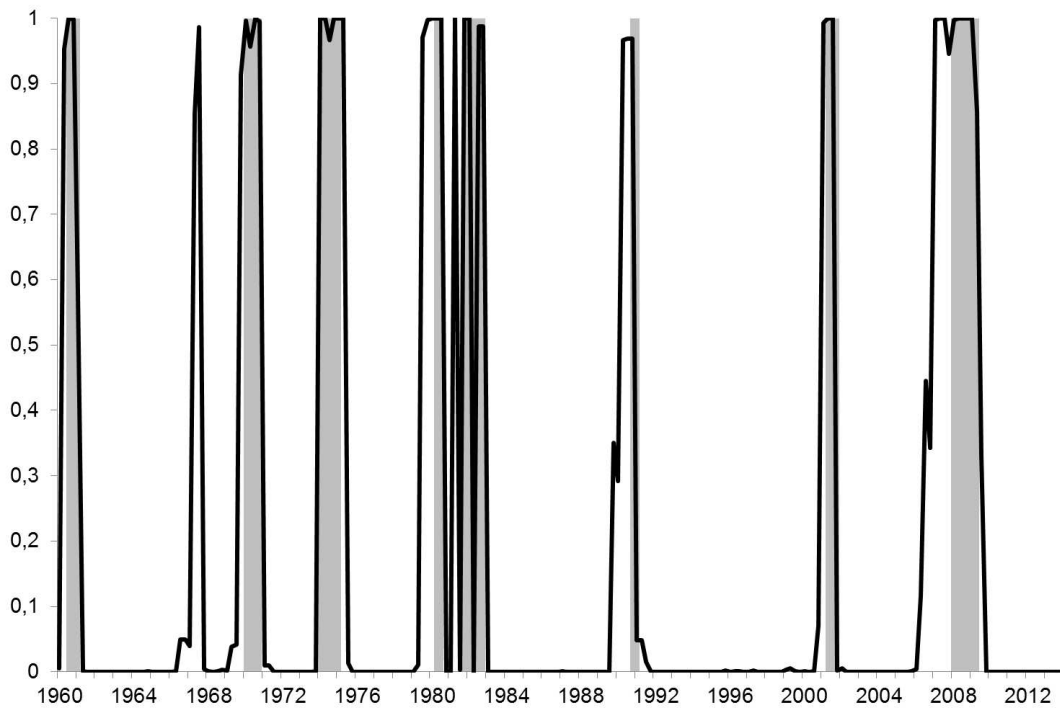
(a) Model with fixed transition probabilities



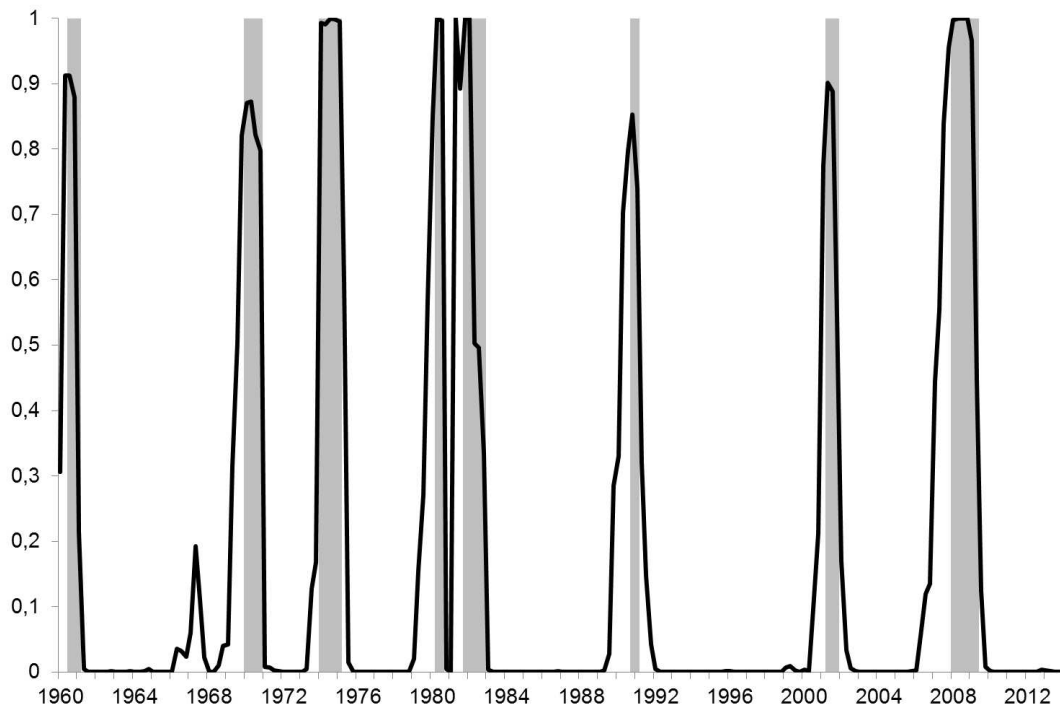
(b) MSV-MIDAS model (exponential almon lag)

Notes: This figure shows the smoothed recession probabilities in the United States taken from the Hamilton model with fixed transition probabilities (MSM2) and the MSV-MIDAS model including spread term. The shaded areas represent recessions according to the NBER business cycle classification.

Figure 4: Smoothed US recession probabilities (1959-2013)



(a) MSV-UMIDAS model (unconstrained lags)



(b) MSV-MIDAS model (exponential almon lag)

Notes: This figure shows the smoothed recession probabilities in the United States taken from the MSV-UMIDAS and MSV-MIDAS models including spread term. The shaded areas represent recessions according to the NBER business cycle classification.