



HAL
open science

Determination of an equivalent elastic system to a multilayer viscoelastic structure: application to the case of thick flexible pavement

Olivier Chupin, Armelle Chabot, Didier Bodin, Jean-Michel Piau

► **To cite this version:**

Olivier Chupin, Armelle Chabot, Didier Bodin, Jean-Michel Piau. Determination of an equivalent elastic system to a multilayer viscoelastic structure: application to the case of thick flexible pavement. 12th ISAP Conference on Asphalt Pavements, Jun 2014, RALEIGH, United States. pp.797-804, <10.1201/b17219-100>. <hal-01358108>

HAL Id: hal-01358108

<https://hal.science/hal-01358108v1>

Submitted on 15 Mar 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Determination of an equivalent elastic system to a multilayer viscoelastic structure: application to the case of thick flexible pavement

Chupin Olivier¹, Chabot Armelle¹, Bodin Didier², Piau Jean-Michel¹

(¹ LUNAM Université, IFSTTAR, CS4, F-44344 Bouguenais Cedex, France,
olivier.chupin@ifsttar.fr)

(² ARRB Group Ltd, 500 Burwood Highway, Vermont South VIC 3133, Australia,
didier.bodin@arrb.com.au)

ABSTRACT

Bituminous pavements are known to exhibit a thermo-viscoelastic behavior which is strongly dependent on temperature and load velocity. However, for many applications (pavement design, FWG analysis...) it is more convenient to deal with elastic calculations which facilitate parametric studies. But to be accurate such an approximation requires rules defining the right choice of elastic data sets. In this context, this paper presents a method to derive an equivalent elastic system to a multilayer viscoelastic pavement under given conditions of temperature and subject to loads moving at constant speed. Thus, the tool proposed hereafter should ease the elaboration of such rules. This paper explains the proposed method and illustrates its use on the case of a thick flexible pavement. By the way, we reexamine the “10Hz rule” of the French pavement design method within this framework. We recall that this rule assumes that the equivalent modulus of a bituminous layer is approximately equal to the norm of its complex modulus computed at 10Hz and at the temperature of the layer.

Keywords: thick flexible pavement, viscoelasticity, equivalent elastic system

1. INTRODUCTION

It is well known that the resilient behavior of asphalt concrete under traffic load is viscoelastic and highly sensitive to temperature, these features being inherited from those of bitumen binders. To take account of these features numerical programs such as ViscoRoute© 2.0 were developed more or less recently [1-5]. The high accuracy potential of these tools was demonstrated by way of comparison with experimental measurements stemming from instrumented pavements under simulated traffic, when varying amplitude and velocity of the loads, the pavement geometry or the profiles of temperature. Nevertheless for many applications such as pavement design or survey and back analysis, the use of elastic-based programs still remains at first step a very easy, fast and useful mean of modeling, which is also capable of providing an accurate quantitative description of pavement behavior. However the main condition for this is to make the right choice of the “equivalent elastic moduli” to be affected to the viscoelastic layers of the structure. In this context, the present paper aims at developing a methodology based on viscoelastic simulations and back calculation for determining these moduli. In addition, the results obtained from the method developed are eventually compared to the choice of modulus recommended by the French design method [6].

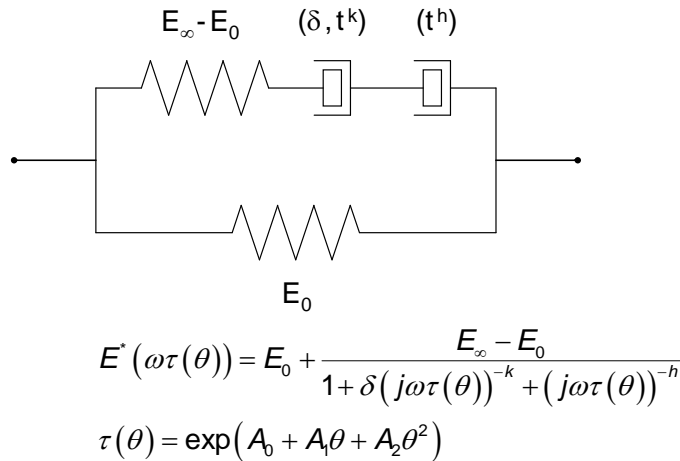
This paper, presented at ISAP2014 North Carolina (USA), is published in Kim, Y.R. (Ed.). (2014). Asphalt Pavements (1st ed.). CRC Press. <https://doi.org/10.1201/b17219>. The original publication is available at the publisher's web site: <https://www.taylorfrancis.com/chapters/edit/10.1201/b17219-100/determination-equivalent-elastic-system-multilayer-viscoelastic-structure-application-case-thick-flexible-pavement-olivier-armelle-didier-jean-michel?context=ubx&refId=3d7fb9c5-c1cf-4e06-a42c-b2d02a06e107>

1 **2. CALCULATION METHOD**

2 The calculation method relies on the use of software ViscoRoute© 2.0 dedicated to the
 3 computation of the dynamic response (3D) under moving loads of pavements incorporating
 4 viscoelastic material layers. The software program is based on a semi-analytical solution
 5 technique described in other references to which the reader is referred for a comprehensive
 6 description [3-5]. The viscoelastic model is considered as the reference to simulate the pavement
 7 response at a given temperature and loading speed. The same tool is then used to calculate the
 8 response of the associated elastic structure of same geometry and subject to similar loading. The
 9 Young moduli of the different layers will be determined in order to obtain, in “some sense”,
 10 comparable responses for the two structures. For this purpose, we define a mathematical function
 11 $\phi(\mathbf{E})$ to minimize which expresses the “distance” between the two models:
 12

13
$$\phi(\mathbf{E}) = \sqrt{\sum_{i=1}^n \left(\frac{c_{i,\max}^{ve}(V, \theta, z_i) - c_{i,\max}^e(\mathbf{E}, z_i)}{c_{i,\max}^{ve}(V, \theta, z_i)} \right)^2}$$
 (1)

14
 15 c_{\max}^{ve} and c_{\max}^e denote the selected quantities (e.g. deflection, strain, etc.) that we wish to be close
 16 in the viscoelastic and the elastic cases, respectively. The “equivalence criterion” ϕ is defined
 17 using n quantities which may be calculated at different depths z in the structure. c_{\max}^{ve} represents
 18 the reference computed using ViscoRoute© 2.0 in which asphalt layers are modeled through the
 19 Huet-Sayegh model [7-9] (Figure 1). c_{\max}^{ve} is thus a function of speed V and temperature θ so the
 20 minimization technique is run for given values of these parameters. c_{\max}^e depends on \mathbf{E} which is
 21 an array of size nve (number of viscoelastic layers) composed of the unknown elastic moduli.
 22
 23



24
 25
 26 **FIGURE 1 Schematic of the Huet-Sayegh rheological model and its complex modulus as a**
 27 **function of pulsation and temperature ($h > k > 0$; $j^2 = -1$)**
 28
 29

1 The minimization is performed according to a gradient method with respect to \mathbf{E} for which an
 2 increment of modulus, $d\mathbf{E}$, is computed at each iteration as follows:

$$3$$

$$4 \quad d\mathbf{E} = -\lambda \frac{\partial \phi(\mathbf{E})}{\partial \mathbf{E}} \quad \text{with} \quad \lambda = \frac{\alpha \phi(\mathbf{E})}{|\partial \phi(\mathbf{E}) / \partial \mathbf{E}|^2} \quad (2)$$

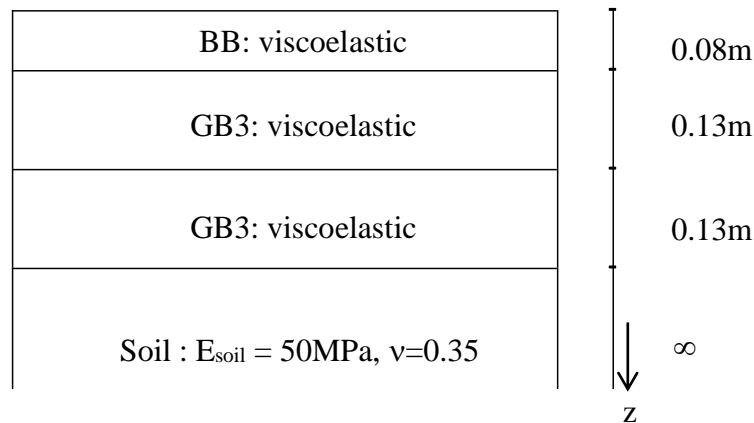
5
 6 The minimization process is stopped when $|\nabla \phi(\mathbf{E})| \leq \varepsilon$. α and ε are small positive quantities.

7 For pavement design applications, the important issue is to obtain comparable true and
 8 equivalent critical strains or stresses governing the material damage criteria. Thus here we define
 9 ϕ as a function of the maximal strain and stress at the bottom of the asphalt layers, considered as
 10 the most relevant quantities for asphalt fatigue prediction. Note that this function can be
 11 extended to other quantities. For instance, if the application is focused on deflections or
 12 curvature then the whole deflection bowl could be used in the ϕ function.

13 3. STUDY OF A THICK FLEXIBLE PAVEMENT

14 The calculation method is applied to the study of a typical thick flexible pavement
 15 modeled with either two or three viscoelastic layers. In the first case, only two unknown moduli
 16 must be determined. This case will be used to illustrate the shape of the ϕ function and evaluate
 17 the calculation method. On the other hand, the case with three viscoelastic layers aims at
 18 illustrating a complicated situation involving thermal gradient in the structure. For both cases, we
 19 also examine the validity of the “10 Hz rule” used in the French pavement design method [6] to
 20 assess the elastic stiffness of bituminous layers (see further).

21



22

23 **FIGURE 2 Cross-section of the pavement structure considered**

24

25 A schematic representation of the pavement structure is shown in Figure 2. It is
 26 composed of viscoelastic layers supported by a semi-infinite elastic foundation. For the analysis
 27 of the two-viscoelastic-layer case we consider that the two GB3 layers are melded. Table 1 gives
 28 the values of the Huet-Sayegh parameters of the different viscoelastic layers.

29

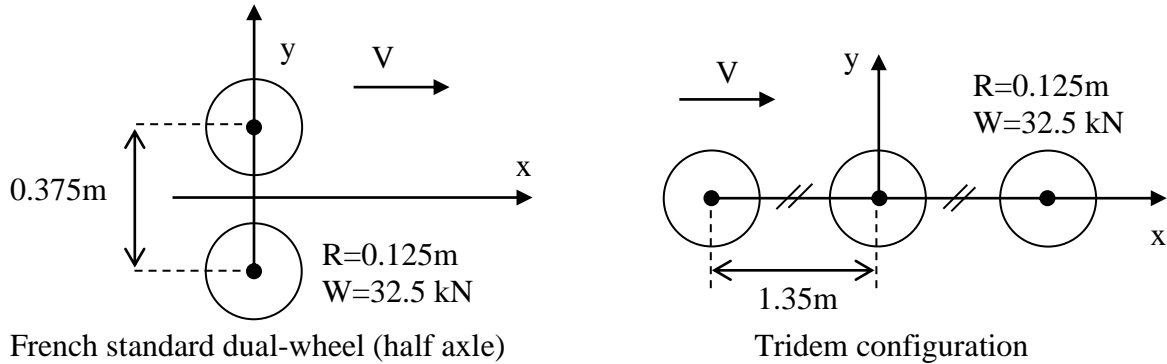
1

TABLE 1 Values of the Huet-Sayegh parameters

	E_0 (MPa)	E_∞ (MPa)	δ	k	h	A0	A1	A2
GB3	11	28000	2.0	0.18	0.5	2.8	-0.4	1.7E-3
BB	11	18000	2.0	0.18	0.45	2.3	-0.4	1.7E-3

2

3 Two loading configurations are considered further. The imprints of these configurations are
 4 represented in Figure 3. These are composed of two or three identical circular loads.
 5



6

FIGURE 3 Top views of the loading configurations applied to the pavement

7

3.1 Case of a Two-Viscoelastic-Layer Structure

8

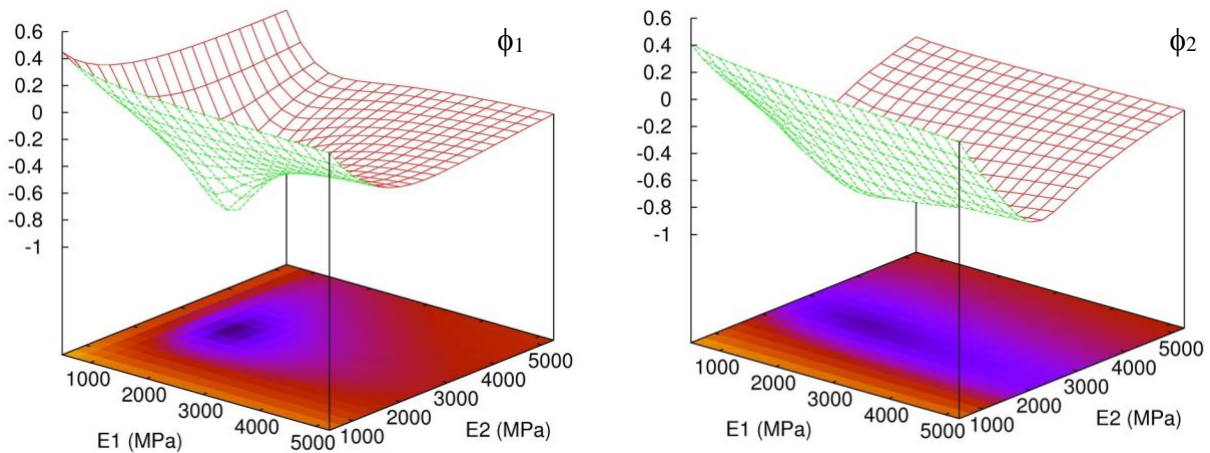
9 In this section, the pavement is subjected to the French standard dual-wheel moving at
 10 constant speed $V=72km/h=20m/s$. The weight of each wheel (32,5kN) is assumed to be
 11 uniformly distributed at the surface of the pavement over a disk of radius 0.125m (Figure 3).

12

3.1.1 Choice of ϕ and Validation of the Calculation Method

13

14 In the present case, function ϕ depends on two elastic moduli and its minimum can be
 graphically determined in a simple way. To validate the calculation method these moduli can be



15

FIGURE 4 Plot of function $\phi_1(E_1, E_2)$ and $\phi_2(E_1, E_2)$ for the two-viscoelastic-layer pavement

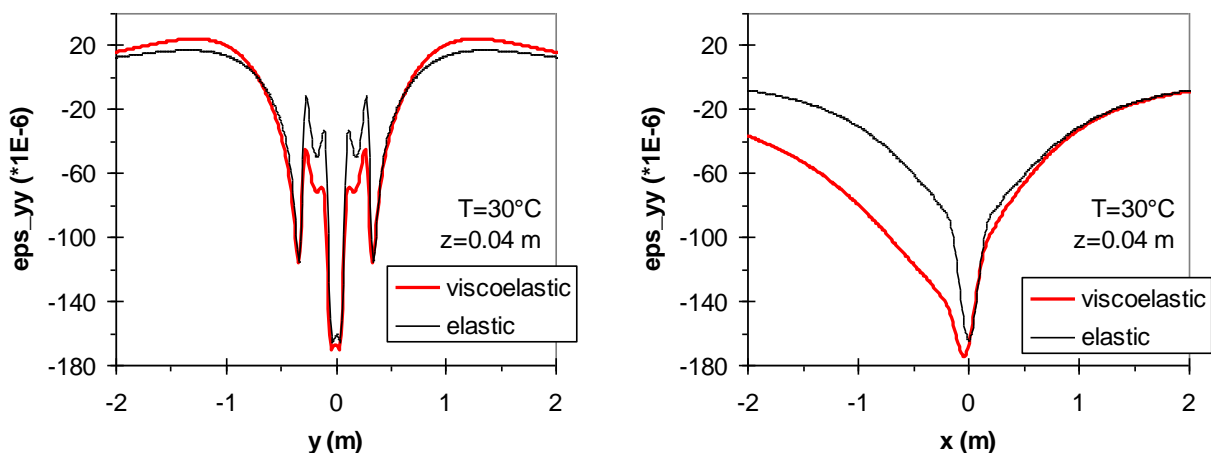
1 compared to those obtained by minimization. To evaluate the impact of the selected function ϕ
 2 on the back-calculated moduli, two functions composed of three quantities ($n=3$) are used below.

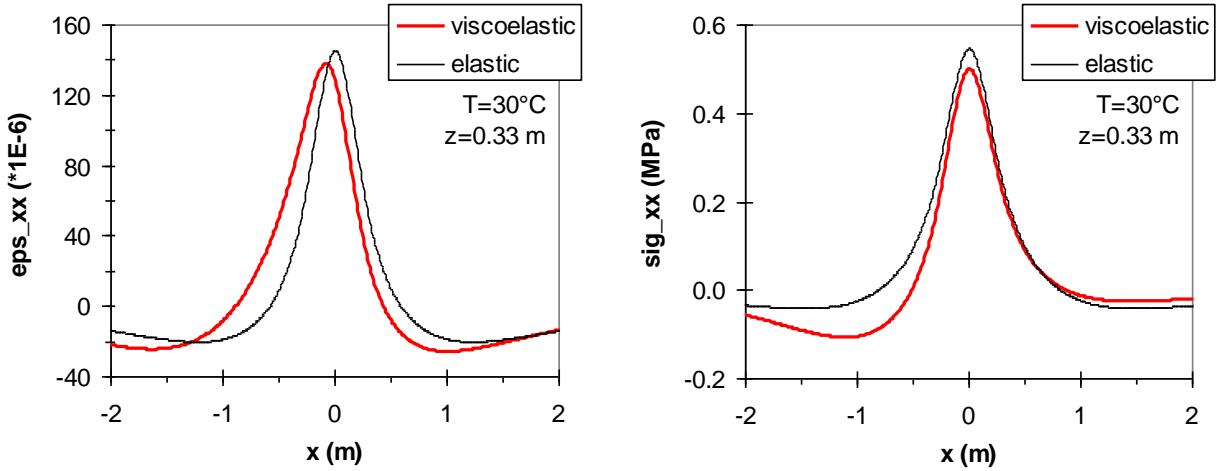
3 The first function ϕ_1 includes ε_T and σ_T computed at the bottom ($z=0.33m$) of the
 4 pavement as well as ε_T computed at the top ($z=0.04m$) (see Figure 2). ε_T denotes the maximum
 5 between the absolute values of the longitudinal strains ε_{xx} and ε_{yy} ; σ_T is the equivalent for stress.
 6 In the second function denoted ϕ_2 , ε_T in $z=0.04m$ is replaced by ε_{zz} calculated at the top of the
 7 soil foundation ($z=0.35m$). Figure 4 shows a plot of the logarithm (\log_{10}) of these two functions.
 8 In this figure, E_1 and E_2 are respectively the values of the Young modulus of layer 1 and 2
 9 attributed for elastic structure. Besides the viscoelastic computation is performed for $V=20m/s$
 10 and $\theta=30^\circ C$ (temperature is uniform in both layers).

11 These two functions exhibit a single minimum which is more apparent for ϕ_1 in contrast
 12 with ϕ_2 . Consequently, function ϕ_1 is probably more conducive to determining numerically the
 13 equivalent moduli by means of gradient descent. Note that depending on its definition, function ϕ
 14 may have several local minima and the choice of the quantities included in ϕ should be made
 15 with caution. For instance, ϕ_2 has not a well defined minimum probably because the value of ε_{zz}
 16 in $z=0.35m$ is more sensitive to the mechanical properties of the soil foundation than to E_1 and
 17 E_2 . In the rest of the paper, we use function ϕ_1 ($\phi=\phi_1$). Under the present conditions, this function
 18 leads to values of the equivalent moduli obtained by minimization of 1625 and 2712 MPa for E_1
 19 and E_2 , respectively. These values match those of the minimum shown in Figure 4 thus
 20 validating the calculation method.

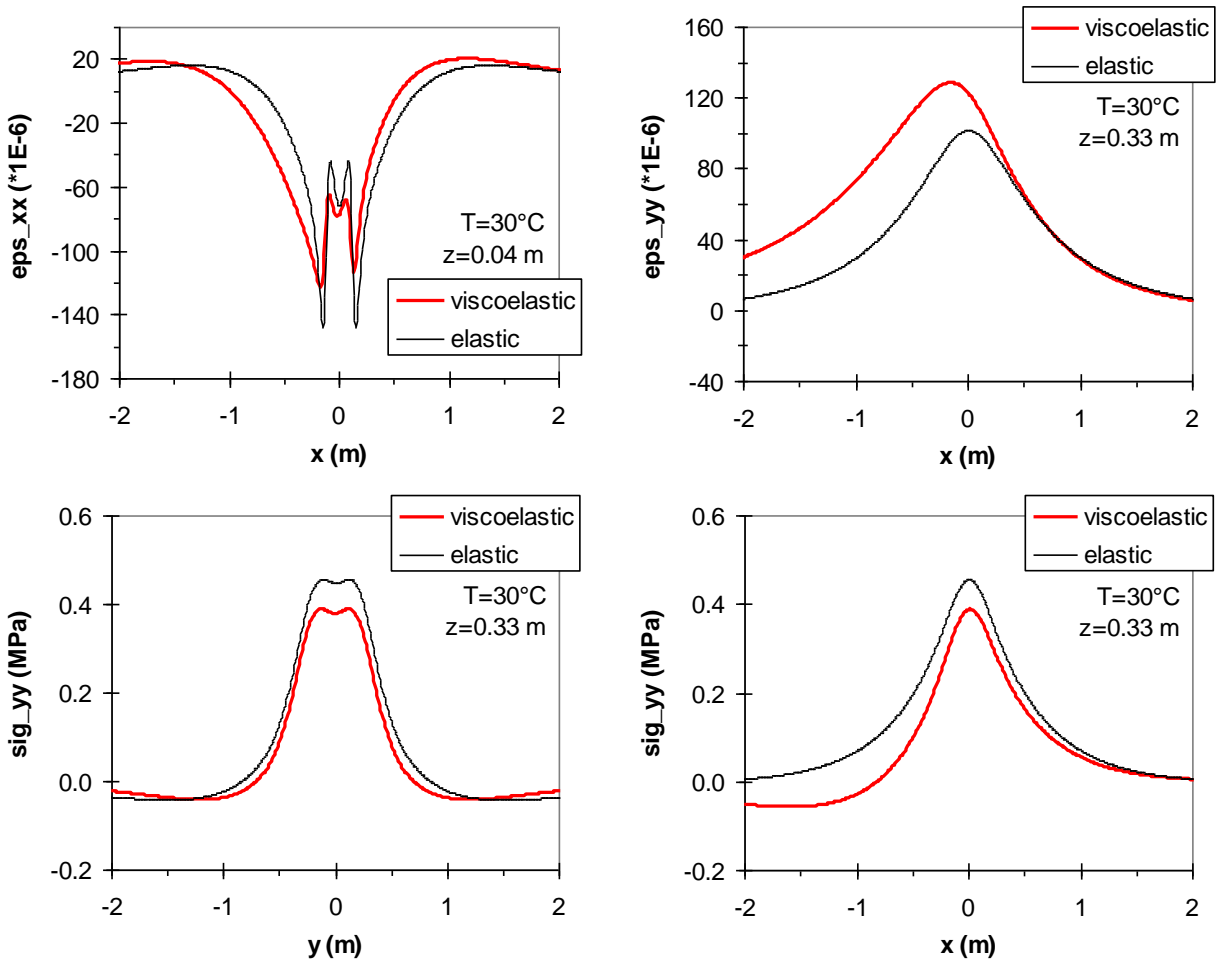
21 3.1.2 Detailed Comparison Between the Viscoelastic and Elastic Equivalent Solutions ($\theta=30^\circ C$)

22 The viscoelastic response of the pavement is compared to that obtained in the elastic case
 23 using the equivalent moduli computed before (i.e. for $V=20m/s$ and $\theta=30^\circ C$). For that purpose,
 24 Figure 5 shows horizontal profiles in the x- or y-direction (at a given depth z) of the mechanical
 25 fields used to define the equivalence criterion, i.e. to calculate function ϕ . We observe that these
 26 profiles provide as expected a good estimate of the maximal values ε_T and σ_T . Nonetheless, the
 27 shapes of the profiles are somehow different since creep/relaxation inherent to viscoelasticity
 28 leads to non-symmetric profiles in the moving direction and to a shifting of the maximum
 29 positions.
 30





1 **FIGURE 5 Profiles in the x- or y-direction of the fields used in the minimization process:**
 2 **comparison between elastic and viscoelastic computations**
 3

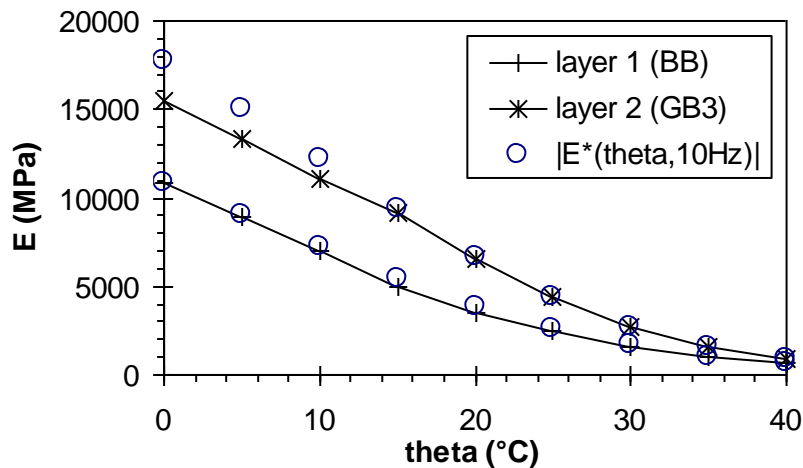


4 **FIGURE 6 Profiles of fields not incorporated in function ϕ : comparison between elastic**
 5 **and viscoelastic computations**
 6

1 Besides, Figure 6 shows profiles of other signals which were not incorporated in the function ϕ .
 2 For these fields, in addition to some differences in shape between the elastic and the viscoelastic
 3 response non negligible differences are also observed at peak values. This could be expected
 4 since no constraint is imposed to match these components of the viscoelastic response.
 5 Nevertheless, the maximum of these fields remain lower than ε_T and σ_T and in the present
 6 example this difference would not impact design based on maximum strain or stress. Globally,
 7 the elastic calculation leads to quite acceptable results.

8 3.1.3 Equivalent Moduli as a Function of Temperature ($V=20m/s$) – The “10 Hz Rule”

9 The variation of the equivalent moduli with respect to temperature is now studied. The
 10 minimization is run for different temperatures which are assumed to be homogeneous with the
 11 same value in both viscoelastic layers. The temperature is varied successively from 0 to 40°C.
 12 Figure 7 shows the results obtained for the BB and the GB3 layers. As expected the equivalent
 13 moduli decrease with temperature ($V=20m/s$), i.e. as the viscoelastic materials soften. At $\theta=30^\circ C$
 14 the equivalent moduli are those already computed in the previous section ($E_1=1625MPa$ and
 15 $E_2=2712MPa$). For the different temperatures, the equivalent moduli are also compared to the
 16 magnitude of the complex modulus of the corresponding layer computed at temperature θ and
 17 frequency $f=\omega/2\pi=10Hz$, which is the rule applied in the design of pavements in France to
 18 account for variations of temperature. The values of the complex modulus for each layer are
 19 plotted using circle markers in Figure 7. These values are in good agreement with the equivalent
 20 back-calculated moduli indicating that the rule under consideration performs well in the present
 21 case.



22
 23 **FIGURE 7 Variation of the equivalent moduli with respect to temperature and comparison**
 24 **with the norm of the complex modulus computed for θ and 10Hz**

25 3.2 Case of a Three-Viscoelastic-Layer Structure

26 We consider in this section a three-viscoelastic-layer structure subjected to a tridem
 27 loading configuration. A thermal gradient is imposed in the asphalt concrete part of the structure.
 28 The temperature in the viscoelastic materials is assumed uniform per layer and equal to 35, 25
 29 and 15°C for layer 1 (BB), 2 (GB3) and 3 (GB3), respectively. The same function ϕ as
 30 previously is used in the minimization. Figure 8 shows profiles of the selected fields used to
 31 compute function ϕ . The comparison between the viscoelastic and the elastic equivalent
 32

TABLE 2 Back calculated moduli for the different layers and design moduli from the “10Hz rule”

	Layer 1 – BB (35°C)	Layer 2 – GB3 (25°C)	Layer 3 – GB3 (15°C)
Eeq (Mpa)	929	2695	6379
E(theta,10Hz) (MPa)	1029	4461	9375

responses leads to similar conclusions as evoked for the two-viscoelastic-layer structure, i.e. a good match of the maximum values with some differences in the shape of the profiles. However, the back calculated moduli obtained and summarized in Table 2 are much lower than the current design moduli stemming from the “10Hz rule”. In terms of mechanical response shown in Figure 8, this yields underestimation of ε_T in $z=0.04m$ and $0.33m$, and overestimation of σ_T in $z=0.33m$ when considering the design rule. The application of this rule is thus not well adapted to the present case whereas the elastic calculation still performs well provided that the equivalent elastic moduli are correctly chosen.

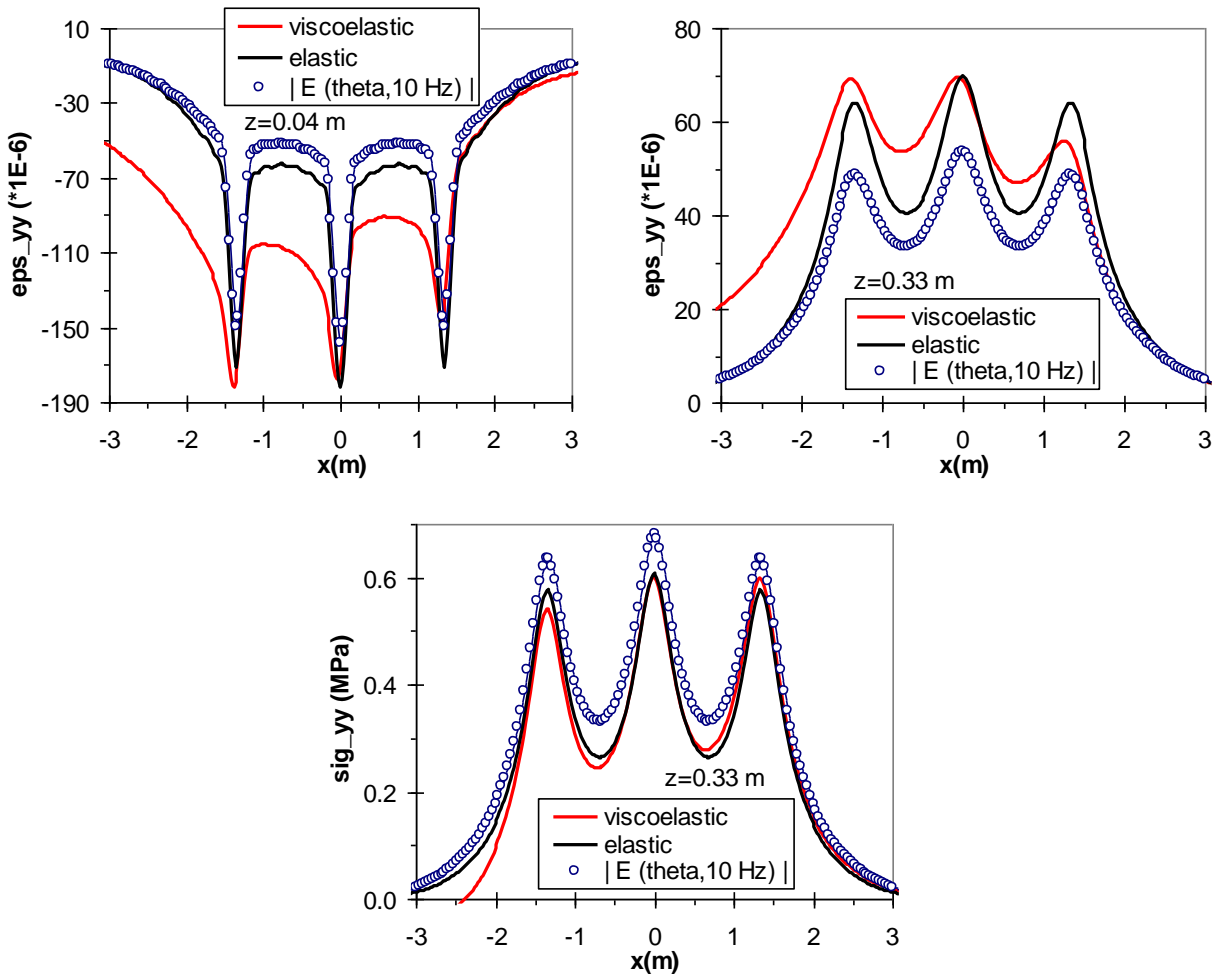


FIGURE 8 Profiles versus x of the fields used in the minimization process of the three-viscoelastic-layer structure: comparison between viscoelastic, elastic equivalent, and “10 Hz rule” computations

1 The differences observed could be attributed to a change of the appropriate loading
2 frequency (load pulse duration) to be selected at different depth [10] for the thick asphalt
3 pavements. As a remark, the loading frequencies corresponding to the back calculated moduli
4 (inversion of the Huet-sayegh law) are 7.6, 2.0 and 1.5 Hz for the layers 1, 2 and 3, respectively.

5 **4. CONCLUSION**

6 The aim of this paper was to present a method intended to help the setup of rules for
7 defining equivalence between viscoelastic and elastic calculations of pavements, depending on
8 the context of use and of a desired accuracy. In this framework, the proposed method was used to
9 compute the equivalent moduli to be affected to bituminous layers of a thick flexible pavement
10 subject to moving loads. This method has proven to provide accurate estimate of the equivalent
11 moduli for various conditions of loading and temperature. The back calculated moduli obtained
12 using the developed tool have also been compared to those recommended by the French
13 pavement design method. Depending on the situation under consideration, the difference
14 between these moduli was found more or less important. Nevertheless, to conclude on this aspect
15 it would be necessary to define an acceptable difference after integrating weighting of the results
16 to account for traffic, weather conditions, etc. which are elements considered in the design
17 method. Finally, the type of approach presented here could be used to extrapolate the pavement
18 design method to other contexts as done already for airfield pavements in France for which the
19 equivalent moduli are determined as a function of temperature, load speed and thickness of the
20 layers.

21 **5. REFERENCES**

- 22 [1] Hopman P.C. VEROAD: A Viscoelastic Multilayer Computer Program,
23 Transportation Research Record 1539, 72-80. 1996.
- 24 [2] Siddharthan R.V., Yao J., Sebaaly P.E. Pavement strain from moving dynamic 3D
25 load distribution, Journal of Transportation Engineering 124 (6) 557-566. 1998.
- 26 [3] Duhamel D., Chabot A., Tamagny P., Harfouche L. ViscoRoute: visco-elastic
27 modeling for asphalt pavements. Bulletin des Laboratoires des Ponts et Chaussées
28 (<http://www.lcpc.fr/en/sources/blpc/index.php>), 258-259:89-103. 2005.
- 29 [4] Chabot A., Chupin O., Deloffre L., Duhamel D. Viscoroute 2.0: a tool for the
30 simulation of moving load effects on asphalt pavement, RMPD Special Issue on Recent
31 Advances in Numerical Simulation of Pavements, 11(2):227–250. 2010.
- 32 [5] Chupin O., Chabot A., Piau J.-M., Duhamel D. Influence of sliding interfaces on the
33 response of a layered viscoelastic medium under a moving load, International Journal of Solids
34 and Structures, 47:3435-3446. 2010.
- 35 [6] Corte J.-F., Goux M.T. Design of pavement structures: the French technical guide,
36 Transport Research Report, 1539:116-124. 1996.
- 37 [7] Huet C. Etude par une méthode d'impédance du comportement viscoélastique des
38 matériaux hydrocarbonés, Université de Paris (France), Ph.D. thesis. 1963.
- 39 [8] Huet C. Coupled size and boundary-condition effects in viscoelastic heterogeneous
40 and composite bodies, Mechanics of Materials, 31:787–829. 1999.
- 41 [9] Sayegh G. Contribution à l'étude des propriétés viscoélastiques des bitumes purs et
42 des bétons bitumineux, Faculté des Sciences de Paris (France), Ph.D. thesis. 1965.

- 1 [10] Denneman E. Improved Design Procedures for Asphalt Pavements: Pavement
- 2 Temperature and Load Frequency Estimation, Report AP-T248-13, Austroads, Sydney. 2013.