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Inflatable membrane system simulation used for load ejection

F. Muttin*

Abstract

The aim of this paper is to present the different mathematical tools used for the development of a novative technology. It concerns a rapid load ejection system based on the inflation of a membrane structure. A one-dimensional physical model has been used for exploratory simulations and rapid prototyping. A three-dimensional model has permitted to compute the membrane behavior and the material stress. At last, data analysis techniques have permitted a more extensive use of experimental datas.

Key words: Membrane Theory, Compressible Fluid Flow, Finite-Element Method, Principal Components Analysis, Regression Tree.

AMS subject classifications: 73K10, 76N10, 73V05, 62H25, 62H30.

1 Introduction

Load ejection systems, based on the pulse given by the inflation of a membrane, are very challenging engineering problems. Applications can be found in glass window ejection for emergency exit in railway passenger trucks and also in aeronautics for droopings. The difficulty is to deliver the high energy of a compressed gas, through a lightweight membrane structure, for giving a prescribed velocity at a mass. For that purpose mathematical tools are usefull to achieve the faisability, and the definition of the system. At first, a one-dimensional model has been used for the faisability task. The physical model is a set of non-linear second order differential equations which are integrated numerically. This simple approach permits to find a coarse range for the system definition parameters, for example; the membrane size, the gas tank pressure, and the pipe diameter. This model is the object of the first chapter. In a second stage, (definition task), a three-dimensional time dependant finite-element model has been used to solve the dynamic nonlinear membrane equilibrium problem. It permits to focus on the membrane behavior from its folding stage until its full deployment. The definition of the local reinforcement of the fabric has been obtained. This model is described in the second chapter. In the last stage, we have explored the experimental test results which have been realized. We will focus on the principal components analysis and on the regression tree methods. The first one permits to detect the test having a significative experimental measure error. The second may give an explanation to understand the difference between experimental measures and numerical results. These statistical treatments are the object of the third chapter.

2 One-dimensional model

The load to be ejected is considered as a ponctual mass M . We denote by $x(t)$ the co-ordinate of the load during the time t . The inflatable membrane is represented as a piston of constant section S . It is filled with a gas, coming from a tank, and flowing through a nozzle. As general information, the gas may be dry nitrogen or may be generated by pyrotechnic propergol burning.

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The mass flow rate of the filling gas is denoted $\frac{dm(t)}{dt}$. The flow is considered to be supersonic in the admission pipe. The gas source is considered to be uniform, during time, in term of gas temperature T_0 . The position of the mass under the pulse f provided by the piston is given by the following theorem.

Theorem 2.1

The co-ordinate $x(t)$ of the mass and the temperature $T(t)$ of the gas in the piston are the solutions of the following set of differential equations under the assumption that $\frac{dm(t)}{dt} = \dot{m}$ is constant during time.

$$(1) \quad M \frac{d^2 x(t)}{dt^2} = m R \frac{T(t)}{x(t)} - p_a S, \quad x(0) = x_0, \quad \frac{dx(0)}{dt} = 0,$$

$$(2) \quad c_v m \frac{dT(t)}{dt} + (c_v \dot{m} + m R \frac{d \log(x(t))}{dt}) T(t) = c_p T_0 \dot{m}, \quad T(0) = T_0.$$

Proof The equations 1 & 2 are obtained on one hand according to the dynamic fundamental principle:

$$(3) \quad M \frac{d^2 x(t)}{dt^2} = f(t),$$

and on the other hand according to the energy balance equation of the gas:

$$(4) \quad c_v \frac{d(m(t) T(t))}{dt} = c_p T_0 \dot{m} - p(t) \frac{dv(t)}{dt},$$

where c_v and c_p are physical constants and where $m(t)$ denotes the mass of the gas now in the piston. The proof consists in a few calculations using the three following definitions. The piston force $f(t)$ is defined by:

$$(5) \quad f(t) = (p(t) - p_a) S,$$

where p_a is the atmospheric pressure. The perfect gas state equation is given by:

$$(6) \quad p(t) v(t) = m(t) R T(t),$$

where R is the perfect gas constant. And finally, the volume of the piston is defined by:

$$(7) \quad v(t) = x(t) S.$$

■

Remark 2.1 *The right term of the assumption 4, can be interpreted as the energy given by the gas source, minus the energy loss by the gas due to the piston expansion.*

Remark 2.2 *The Van der Waals equation, may be used, instead of assumption 6, when interactive molecular actions must be take into account.*

Remark 2.3 *The membrane structure here is approximated due to the consistency of the cross-section of the piston. The principal assumption of this model neglects the radial deformation of the membrane due to the elastic deformation of the material induce by the external actions.*

A mass flow rate definition can be used to complete the model [1]. The hypothesis of an isentropic uniform chocked flow can be done in the exhaust. For a pipe of section A , with a uniform gas source pressure P_0 , the mass flow rate is defined by:

$$(8) \quad \frac{dm(t)}{dt} = a A \frac{P_0}{T_0} \left(\frac{M_m \gamma}{R} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \right)^{\frac{1}{2}}, \quad m(0) = m_0,$$

where a is the jet restriction coefficient ($a \approx 0.4$), M_m is the gas molar weight, and γ is the ratio $\frac{c_p}{c_v}$, ($\gamma \approx 1.4$).

3 Three-dimensional model

When we let $\omega(t)$ be the midsurface of the membrane at time t then this surface is composed of four subsurfaces:

- #1: $\omega_b(t)$, the blocked part of the membrane, which is fixed on a rigid support,
- #2: $\omega_f(t)$, the free part, on which the gas pressure and the atmospheric pressure are the external actions,
- #3: $\omega_c(t)$, the part of the membrane in contact with the ejected load,
- #4: $\omega_s(t)$, the part of the membrane in contact with itself.

The presence of this last part is a consequence of the bag folding.

When we let $\Omega(t)$ be the volume of the body occupied by the load at time t then we note that $\Omega(t)$ can be decomposed in two parts; a free part, and a part representing the contact with the bag. This last subsurface is equal to $\omega_c(t)$.

We use the Lagrangian description to define the mechanical problem. The mechanical state of the membrane is described by means of the Piola Kirchoff stress tensor of second kind $\sigma(t)$, and the Green strain tensor $\epsilon(t)$. The constitutive material behavior law, between $\sigma(t)$ and $\epsilon(t)$, is supposed to be nonlinear and reversible. It will be not described here. The load is supposed to be a rigid body.

Starting from the inner volume of the bag limited by $\omega(t)$, the gas pressure in the membrane is simply computed by using the equations 4 and 6, which have been previously used in the one-dimensional model.

The motions of the membrane and of the load are given by the following theorem.

Theorem 3.1 *Let $x(t)$ be the displacement field of the membrane at time t , and $X(t)$, $\Theta(t)$ respectively the displacement and the rotation vector of the ejected load.*

I) The displacement $x(t)$ is solution of the following problem:

$$(9) \quad \frac{d}{dx} e(x) \cdot y = 0, \quad \forall y \text{ admissible}$$

with respect to all kinematically admissible membrane velocity fields y at time t , where the total membrane energy is defined by (ρ is the density of the fabric, tr is the trace operator, $f(x, X)$ is the force field applied by the inner gas, by the ejected load, and by the membrane with itself by friction):

$$(10) \quad e(x) = \int_{\omega(0)} \rho \frac{d^2 x}{dt^2} x + \int_{\omega(0)} tr(\sigma(x) \epsilon(x)) + \int_{\omega(0)} f(x, X) x,$$

and with the following constraints, on the contact force $F_{contact}$ on the interfaces (\vec{n} is the unit normal at $\omega(t)$):

$$(11) \quad F_{contact}(\sigma, x) \cdot \vec{n} \geq 0, \quad \text{on } \omega_c \cup \omega_s,$$

and on the displacement in the contact zones:

$$(12) \quad \forall p \in \omega_s, \exists p' \in \omega_s, p' \neq p \quad | \quad x \cdot \vec{n} |_p = x \cdot \vec{n} |_{p'}$$

$$(13) \quad x \cdot \vec{n} = X \cdot \vec{n}, \quad \text{on } \omega_c.$$

II) The displacement $X(t)$ and the rotation vector $\Theta(t)$ are solutions of the following differential equations:

$$(14) \quad M \frac{d^2 X(t)}{dt^2} = \vec{F}(x, X)$$

where \vec{F} is the force applied by the membrane on the ejected load,

$$(15) \quad I \frac{d^2 \Theta(t)}{dt^2} = \vec{M}(x, X)$$

where I is the inertia vector of the load, and \vec{M} is the moment of \vec{F} relative to the gravity center of the load.

Proof The minimal energy assumption of the membrane during time comes from the virtual work principle. The functional e , defined by 10, is the summation of three terms. It give respectively the kinetic energy of the membrane, its internal elastic deformation energy and the loss of the potential energy due to a displacement field x [2]. The constraint 11 comes from the assumption of a positive contact force between the bodies. The non interpenetration constraint 12 of the membrane with itself, means that two adjacent material points of ω_s have the same location in

the space and must have the same normal displacement relative to ω_s . The normal components of the displacement of the membrane and of the load must be equal in each point of ω_c . It gives the constraint 13. The dynamic fundamental principle gives the equations of motion 14 & 15. ■

From the numerical point of view, the textile structure is represented by a mesh and composed of a set of quadrilateral bilinear element surfaces. The mesh, as the real membrane, must be folded in its initial state. The equations of the previous theorem is solved using a numerical time integration scheme. Figure (1) shows the evolution of a square inflatable airbag acting on a plane load guided vertically.

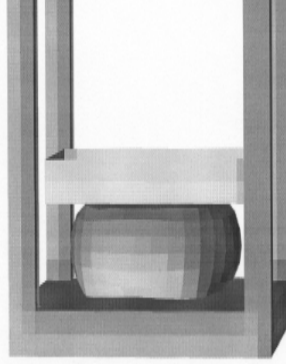


Figure 1: The finite-element simulation of a load ejection system.

4 Data analysis methods

Two data analysis methods have been used to operate on experimental results. The use of the principal components analysis is described first. Secondly, the regression tree method is discussed.

4.1 Principal components analysis

Frequently, during a test, some experimental measures are entatched of an accidental error. It is a consequence of technological or human deficiencies. As a consequence, some parts of the datas must be avoided before being used extensively. We use the principal components analysis [3] to tract the tests where errors have been traced.

Let X be a matrix containing experimental and numerical measures. The row indice i corresponds to the test labels. The colon indice j corresponds to the different measures, for example; the maximal pressure in the membrane, the maximal acceleration of the load, the maximal pulse force and the ejection distance. At first, we give the definition of the principal components analysis applied to the matrix X .

Definition 4.1 The matrix denoted X_p , which contains the principal components co-ordinates of the tests, is defined in term of X by the following set of equations:

$$(16) \quad X_s = \frac{X - \bar{X}}{\sigma_X},$$

which defines the scaled matrix X_s , in term of the mean measure values \bar{X} , and of the standard deviation of the measure values σ_X ,

$$(17) \quad X_s^t X_s = P D P^t,$$

where D is the diagonal matrix containing the variance of each principal component and P the matrix containing the principal basis vectors components,

$$(18) \quad X_p = P^t X_s. \quad \blacksquare$$

The searched tests are identified graphically by means of the representation of the whole tests in their principal variance plane. By this way, we detect a test, only when it contains a significative part of the variance with respect to the global variance. From the practical point of view, it is sufficient to get a valuable help.

Let us take the following example. The matrix X contains the maximal measured and computed pressures and the maximal pulse force on the load. Figure (2) shows the tests in their principal variance plane.

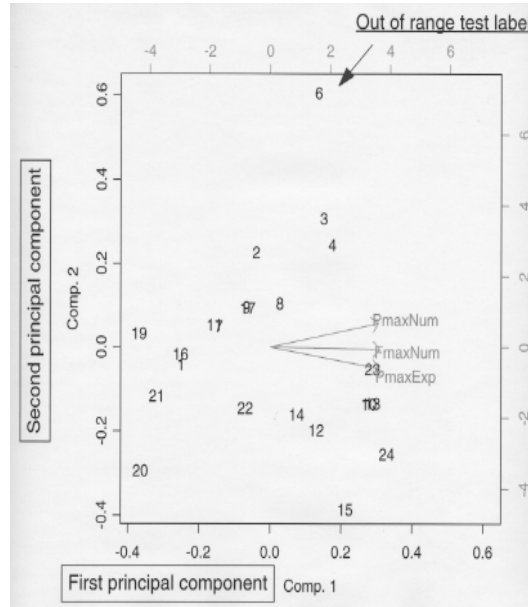


Figure 2: The measures of tests in their principal components plane.

In the top of figure (2), the test out of range, labeled 6, is clearly distinct from the others. Its principal variance contribution comes from its maximal measured pressure.

4.2 Regression trees

The tree method is based on both classification and regression techniques [4]. This method has been used, for example in medical diagnostics. Here, this method gives a diagnostic, in term of test parameters, for the differences observed between the experimental measures and the numerical results. It permits for instance, to elucidate the limitation of the one-dimensional model. As a consequence, intuitively speaking, a remedy can be found.

We define first, the observed variable, denoted y . It corresponds to one of the observed differences between experiments and computations. The predictors variables, denoted x , are secondly choosen in the set of the test parameters, for example; the mass M of the load, the maximal momentum of the load, or the maximal kinetic energy of the load. In the sequel, their ranges will become the set of the possible candidates to explain the range of the observed variable. Let us assume that for a set of tests we know for each test the x values and the y value. We suppose that x and y corresponds to continuous variables.

The regression tree method constructs a hierarchical set of splits in term , at each level, of one of the x variables. The choosen split permits, at each step, the maximal decrease of the global y variance, considered test case by test case. Each consecutive split can be seen as the solution of a minimization problem. The minimization problem is given by the following definition.

Definition 4.2 The following constrained minimization problem defines, for the predictors variables x , and an observed variable y , each split $s(x_i)$, $i \in \{1..N\}$, of a regression tree, where N is the number of predictor variables.

$$(19) \quad \text{Min } D(s), \quad \forall s(x_i), i \in \{1..N\}$$

where the total tree deviance $D(s)$ is defined by [5]:

$$(20) \quad D(s) = \sum_{\{cases j\}_s} (y_j - \mu_{[j]})^2,$$

where $\mu_{[j]}$ is the mean value of the y variable for the case number j , and with the constraints which are defined by:

$$(21) \quad D_{(case\ j)} \geq \epsilon, \quad \forall j,$$

$$(22) \quad card(case\ j) \geq N, \quad \forall j,$$

where ϵ is the minimal case deviance and N is the minimal case size. ■

In the following example y is the difference observed on the maximal pressure in the bag during load ejection. The variables x are the final kinetic energy of the load and the final momentum of the load. The regression tree, showed on figure (3), has been obtained with the values $N = 10$ and ϵ equals to 0.01 of the initial global deviance.

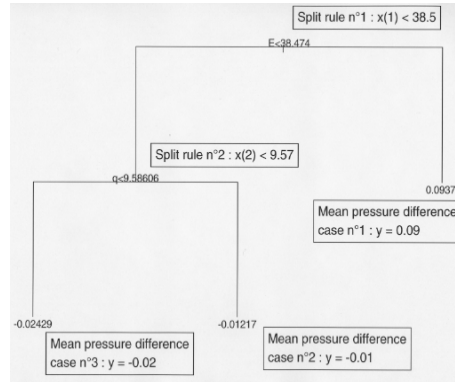


Figure 3: The regression tree on the difference in the maximal pressure between experiments and computations.

It can be observed that the level of y is explained first by the level of the final kinetic energy of the load. It has been concluded that a dynamic effect was not treated by the one-dimensional model. As a consequence, the effect of the mass of the membrane may be included in the one-dimensional model.

5 Conclusion

Using a one-dimensional model applied to a physical problem that includes a great number of parameters, we can process a large range of parameter values to identify the best technical compromises. The three-dimensional model is physically more complete although it is more costly to run (software and hardware). It allows to validate and improve the technical solutions previously identified as optimal.

The manufacturer which uses such external technical knowledge in applied mathematics obtains a great return on investment; new markets, a better commercial offer, cost reductions through reduced product development time and adequate know-how. By investing in R&D and taking deliberately chosen risk, the industrial company becomes more competitive on its own market.

Through industrial cooperation, the mathematician is provided with funding support and becomes involved in industrial targets. His skills and know-how are used to develop new industrial products.

References

- [1] R. Comolet, *Mécanique expérimentale des fluides*, Tome 1, 5th edition, Masson, Paris, 1990, pp. 173–183.
- [2] P. Germain, *Mécanique*, Tome 1, Ellipse, Ecole Polytechnique, 1986, pp. 329–389.
- [3] G. Saporta, *Probabilités, Analyse de données, Statistiques*, Editions Technip, Paris, 1990, pp. 165–171.
- [4] L. Breiman, J.H. Friedman, R.A. Olshen, C.J. Stone, *Classification and regression trees*, Chapman & Hall, New York, 1993, pp. 216–264.
- [5] W.N. Venables, B.D. Ripley, *Modern Applied Statistics with S-Plus*, Springer, New York, 1996, pp. 334–335.