Closed-loop identification and tracking control of Lagrangian systems under input constraints

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***Abstract*— In this paper global tracking and parameter convergence are proven for an adaptive control algorithm with independently saturated proportional and derivative terms through a persistency of excitation condition, while guaranteeing the inputs to never reach their natural saturation limit. As far as the authors are aware this is the first work that achieves globally the motion control objective and system identification, avoiding actuator saturation.**

I. INTRODUCTION

Adaptive tracking control of mechanical systems has been widely studied in an unbounded input setting (see for instance [1]), however, in a more realistic framework, when the natural limitations of real life actuators are taken into account, the number of results is limited due to the complexity in the design and analysis of bounded control schemes.

To the best of the authors knowledge, the only works addressing this problem are those appearing in [2], [3], [4]. To achieve the tracking objective, the adaptive control scheme developed in [2], considers the proportional and derivative correction terms bounded through the hyperbolic tangent function, and involving a term of adaptive desired compensation of the manipulator dynamics with parameter estimators. The adaptation algorithm is defined in terms of a discontinuous auxiliary dynamics by means of which the parameter estimators are prevented to take values beyond some prespecified limits, which consequently keeps the adaptive gravity term bounded. Semiglobal asymptotic tracking was concluded provided that the minimum eigenvalue of the derivative control gain was sufficiently large. In [3], a controller with proportional and derivative correction terms similarly structured –to the latter described scheme– was developed. This scheme involved only adaptive gravity compensation, bounded through a discontinuous adaptation algorithm, analog to that used in [2]. However, it is not clear how the desired trajectory was ensured to be a solution of the closed loop system. Recently, a generalized adaptive scheme giving rise to a family of bounded adaptive tracking controllers was developed in [4]. The proposed approach allows different saturating structures and a wide range of saturating functions, including the hyperbolic tangent as a particular case, while assuring the adaptive tracking objective for any initial condition (globally), avoiding discontinuities throughout the scheme, preventing the inputs to reach their natural saturation bounds, and imposing no saturation avoidance restriction on the control gains. However, only boundedness of the parametric error variable was proven in the previously described control schemes.

It is widely known that a sufficient and necessary condition to conclude parametric convergence is for the regression matrix to satisfy a persistency of excitation (PE) property, nonetheless, because of its complexity –since its general dependence on time and system states– it turns out to be a difficult task to analytically prove the PE condition. Motivated by the results presented in [5], where uniform global asymptotical stability was established through the satisfaction of a persistency of excitation property, we solve the parameter identification problem in the case where the inputs are limited by the actuator capabilities. Moreover, our necessary and sufficient condition is imposed on a function evaluated on the reference trajectories, and hence it may be verified online. In addition to system identification, proving parametric convergence is a key step to conclude *uniform global asymptotic stability*, which grants robustness with respect to small perturbations.

Given that the generalized control scheme developed in [4] proves global asymptotical stability of the closed-loop system, the parametric convergence of the particular case where the proportional and derivative correction terms are bounded individually (SP-SD case) –encompassed within the flexible structure of the algorithm– is studied in this work.

The rest of the document is structured as follows, in Section II definitions and results useful in the development of the work are presented. In Section III the structure of the considered SP-SD controller is described and the main objective stated, the proof is detailed in Section IV . Finally, conclusions are given in Section VI.

II. PRELIMINARIES

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout the work, X_{ij} denotes the element of X at its i^{th} row and j^{th} column, X_i stands for the i^{th} row of X, and y_i corresponds to i^{th} element of y. 0_n represents the origin of $\mathbb{R}^n$, I_n the $n \times n$ identity matrix, and $\mathbb{R}_+ = [0, \infty)$. $\|\cdot\|$ the Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=0}^{n} y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}(X^TX)}$, $\lambda_{\max}(X^TX)$ the maximum eigenvalue of X^TX. We denote $\mathcal{B}_r = \{x \in \mathbb{R}^n : \|x\| \leq r\} \subset \mathbb{R}^n$. For a dynamic/time variable υ, $\dot{\upsilon}$ and $\ddot{\upsilon}$ respectively denote its first- and second-order evolution/change rate. For a continuous scalar function $\phi : \mathbb{R} \to \mathbb{R}$, ϕ' denotes its derivative, when differentiable, and ϕ^{-1} its inverse, when invertible.

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A. Persistency of excitation

As the result is formulated through a persistency of excitation condition, let us introduce the following definitions.

Definition 1: [6] The locally integrable function $\Phi : \mathbb{R}_+ \to \mathbb{R}^{n\times p}$ is said to be ***persistently exciting*** (PE) if there exist $\mu > 0$ and $T > 0$ such that

$$\int_t^{t+T} \Phi(\tau)\Phi^T(\tau)d\tau \geq \mu I, \quad \forall t \in \mathbb{R}_+ \tag{1}$$

◁

Let $x \in \mathbb{R}^{r+q}$ be partitioned as $x \triangleq [x_1, x_2]^T$, where $x_1 \in \mathbb{R}^r$ and $x_2 \in \mathbb{R}^q$, and let $\phi : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^p$ be such that $t \mapsto \phi(t, x)$ is locally integrable.

Definition 2: [5] The function ϕ is said to be ***uniformly δ-persistently exciting*** (UδPE) with respect to x_1 if for each $x \in \mathcal{D}_1 \triangleq \{x \in \mathbb{R} | x_1 \neq 0\}$, there exist $\delta > 0$, $T > 0$, and $\mu > 0$ such that

$$\|z - x\| \leq \delta \Rightarrow \int_t^{t+T} \|\phi(\tau, z)\|d\tau \geq \mu, \quad \forall t \in \mathbb{R}_+ \tag{2}$$

◁

The definition of Uδ-PE means that for each *fixed* $x \neq 0$, $\Phi(t) \triangleq \phi(t, x)$ is PE, and μ and T are the same for every point in the neighborhood of x. For uniformly continuous functions, the following result is satisfied.

Lemma 1: [5] If $\phi(t, \cdot)$ is uniformly continuous in t then, $\phi(t, x)$ is Uδ-PE with respect to x_1 if and only if for each $x \in \mathcal{D}_1$ there exist $T > 0$, and $\mu > 0$ such that

$$\int_t^{t+T} \|\phi(\tau, x)\|d\tau \geq \mu, \quad \forall t \in \mathbb{R}_+ \tag{3}$$

◁

Remark 1: [5] Observe, from Lemma 1, that $\phi(t, x) \triangleq \Phi^T(t)x$ is Uδ-PE with respect to x if and only if Φ is PE. ◁

The latter becomes a useful tool for the analysis of systems whose regression matrix is linear in the parameters.

B. The dynamic model and its properties

Considering viscous friction, Euler-Lagrange systems may be represented by:

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{4}$$

where q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are, respectively, the position, velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q, \dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F = \mathrm{diag}[f_1, \ldots, f_n]$ where $f_i > 0$, $i = 1, \ldots, n$, are the viscous friction coefficients[1]. This work is addressed to Euler-Lagrange systems whose dynamic model satisfies the following properties[2].

Property 1: The inertia matrix $H(q)$ is positive definite, symmetric, and bounded, *i.e.* $\mu_m I_n \leq H(q) \leq \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some constants $0 < \mu_m \leq \mu_M$. ◁

[1]The terms in the left-hand side of (4) involve a set of parameters $\theta \in \mathbb{R}^p$. Subsequently, whenever convenient, such a parametric dependence will be explicitly denoted as $H(q, \theta)$, $C(q, \dot{q}, \theta)$, $F(\theta)$, and $g(q, \theta)$.

[2]Notice for instance, that Properties 4 and 6 are satisfied only by systems in which the terms involved in G(q) are bounded.

Property 2: The Coriolis matrix satisfies:
2.1: $y^T \left[\frac{1}{2}\dot{H}(x, y) - C(x, y)\right] y = 0$,
2.2: $\dot{H}(x, y) = C(x, y) + C^T(x, y)$,
2.3: $C(w, x + y)z = C(w, x)z + C(w, y)z$,
2.4: $C(x, y)z = C(x, z)y$,
2.5: $\|C(x, y)z\| = k_C\|y\|\|z\|$, for some $k_C \geq 0$.
$\forall w, x, y, z \in \mathbb{R}^n$. ◁

Property 3: The viscous friction coefficient matrix satisfies $f_m\|x\|^2 \leq x^T Fx \leq f_M\|x\|^2$, $\forall x \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \leq \max_i\{f_i\} \triangleq f_M$. ◁

Property 4: Every element of the gravity vector, $g_i(q)$, satisfies $|g_i(q)| \leq B_{gi}$, $\forall q \in \mathbb{R}^n$, for some positive constants B_{gi}, $i = 1, \ldots, n$. ◁

Property 5: The left-hand side of (4) can be rewritten as

$$H(q, \psi)\ddot{q} + C(q, \dot{q}, \psi)\dot{q} + F(\psi)\dot{q} + g(q, \psi) = Y(q, \dot{q}, \ddot{q})\psi \tag{5}$$

where $\psi \in \mathbb{R}^\rho$ is a constant vector whose elements depend only on system parameters, and $Y(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n\times\rho}$ –the regression matrix– is a continuous matrix function whose elements depend only on position, velocity and acceleration variables and do not involve any of the system parameters. Similarly, each term of the left-hand side of (5) can be rewritten as $H(q, \psi)\ddot{q} = Y_H(q, \ddot{q})\psi$, $C(q, \dot{q}, \psi)\dot{q} = Y_C(q, \dot{q})\psi$, $F(\psi)\dot{q} = Y_F(\dot{q})\psi$, and $g(q, \psi) = Y_g(q)\psi$, and actually $Y(q, \dot{q}, \ddot{q}) = Y_H(q, \ddot{q}) + Y_C(q, \dot{q}) + Y_F(\dot{q}) + Y_g(q)$. ◁

Property 6: Consider the dynamics (5), let ψ_{Mj} be an upper bound of ψ_j, such that $\psi_j \leq \psi_{Mj}$, $\forall j \in \{1, \ldots, \rho\}$, and let $\psi_M \triangleq \left(\psi_{M1}, \ldots, \psi_{M\rho}\right)^T$ and $\Psi \triangleq [-\psi_{M1}, \psi_{M1}] \times \cdots \times [-\psi_{M\rho}, \psi_{M\rho}]$.

a. By Properties 4 and 5, there exist positive constants $B_{gi}^{\psi_M} \geq B_{gi}$, such that $|g_i(x, y)| = |Y_{gi}(x)y| \leq B_{gi}^{\psi_M}$, $\forall x \in \mathbb{R}^n$, $\forall y \in \Psi$; and B_{G_i} such that $\|Y_{gi}(x)\| \leq B_{G_i}$ $\forall x \in \mathbb{R}^n$, $i = 1, \ldots, n$.
b. Let $\mathcal{X}$ and $\mathcal{Y}$ be compact subsets of $\mathbb{R}^n$. By Properties 1, 2.5, 3, 5, and 6a., there exist positive constants $B_{D_i}^{\psi_M}$, such that $|Y_i(w, x, y)z| \leq B_{Di}^{\psi_M}$, $\forall (w, x, y, z) \in \mathbb{R}^n \times \mathcal{X} \times \mathcal{Y} \times \Psi$; and B_{Y_i} such that $\|Y_i(w, x, y)\| \leq B_{Y_i}$ for all $(w, x, y) \in \mathbb{R}^n \times \mathcal{X} \times \mathcal{Y}$, $i = 1, \ldots n$.

◁

C. Saturation Functions.

In order to keep the control scheme bounded, the work developed in [4] defines and makes use of a general class of saturation functions, however we make use of the hyperbolic tangent since it is a individual case of such mentioned definition. Given that the hyperbolic tangent is a globally Lipschitz, strictly increasing sigmoidal function, it can be proved (see [7]) that it satisfies the following properties

Lemma 2: For a given positive constant k, the hyperbolic tangent function, satisfies

1. $\varsigma \tanh(\varsigma) > 0$ for all $\varsigma \neq 0$;
2. $|\tanh(\varsigma)| < 1$ for all $\varsigma \in \mathbb{R}$.
3. $0 \leq \frac{d}{d\varsigma}\tanh(\varsigma) \leq 1$, $\forall \varsigma \in \mathbb{R}$;
4. $|\tanh(k\varsigma + \eta) - \tanh(\eta)| < k|\varsigma|$, $\forall \varsigma, \eta \in \mathbb{R}$;
5. $|\tanh(k\varsigma)| < k|\varsigma|$, $\forall \varsigma \in \mathbb{R}$;

6. $\frac{\tanh^2(k\varsigma)}{2k} \leq \int_0^{\varsigma} \tanh(kr)dr \leq \frac{k\varsigma^2}{2}$, $\forall \varsigma \in \mathbb{R}$;
7. $\int_0^{\varsigma} \tanh(kr)dr > 0$, $\forall \varsigma \neq 0$;
8. $\int_0^{\varsigma} \tanh(kr)dr \to \infty$ as $|\varsigma| \to \infty$;
9. $\varsigma[\tanh(\varsigma+\eta) - \tanh(\eta)] > 0$, $\forall \varsigma \neq 0$, $\forall \eta \in \mathbb{R}$;

◁

III. Main Result

Let us begin by recalling the controller developed in [4], which achieves the tracking objective under the consideration of the following assumptions

Assumption 1: $T_i > B_{gi}$, $\forall i \in \{1, \ldots, n\}$. ◁

Assumption 2: The desired trajectory belongs to the set

$$\mathcal{Q}_d \triangleq \left\{ q_d \in \mathcal{C}^2(\mathbb{R}_+; \mathbb{R}^n) : \|\dot{q}_d(t)\| \leq B_{dv}, \|\ddot{q}_d(t)\| \leq B_{da} \right\}$$

for some positive constants B_{da} and $B_{dv} < \frac{f_m}{k_C}$. ◁

Assumption 1 deals with the fact that, for the system to be stabilizable at any desired configuration, the torque inputs must be able to overcome the conservative forces actuating on the system. And, given the bounded nature of the controllers, there exist constrains on the tractable trajectories, Assumption 2 explicitly shows such limits.

To prevent the inputs to reach their saturation bounds, the absolute value of each input element τ_i is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.* $|\tau_i| \leq T_i$, $i = 1, \ldots, n$. In other words, being u_i the controller output relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \text{sat}\left(\frac{u_i}{T_i}\right), \qquad \forall i \in \{1, \ldots, n\} \tag{6}$$

The exact parametric value is not required but an upper limit on each parameter is assumed to be known, *i.e.*

$$\psi_j < M_{aj}, \qquad \forall j \in \{1, \ldots, \rho\} \tag{7a}$$

with M_{aj} being positive constants, and such that

$$B_{gi}^{M_a} < T_i, \qquad \forall i \in \{1, \ldots, n\} \tag{7b}$$

where $B_{gi}^{M_a}$ are positive constants as stated in Property 6a., and consider small enough bound on the desired trajectories B_{dv} and B_{da} (in accordance to Assumption 2) such that

$$|Y_i(q, \dot{q}_d(t), \ddot{q}_d(t))\vartheta| \leq B_{Di}^{Ma} < T_i \tag{7c}$$

$i = 1, \ldots, n$, where B_{Di}^{Ma}, are positive constants as described in Property 6b. making use of the hyperbolic tangent, the adaptive control scheme proposed in [4], takes the form:

$$\begin{aligned} u(t, q, \dot{q}, \hat{\psi}) = & -M_P \mathrm{T_h}(K_P \bar{q}) - M_D \mathrm{T_h}(K_D \dot{\bar{q}}) \\ & + Y(q, \dot{q}_d(t), \ddot{q}_d(t))\hat{\psi} \end{aligned} \tag{8}$$

where $\bar{q} = q - q_d$, for any suitable (desired trajectory) vector function $q_d(t) \in \mathbb{R}^n$.

The third term in the right-hand side of (8) is a compensation term that involves online position measurements and desired velocities and accelerations, where $Y(\cdot, \cdot, \cdot)$ is the regression matrix characterizing the system open-loop structure, *i.e.* such that

$$\begin{aligned} Y\left(q, \dot{q}_d(t), \ddot{q}_d(t)\right)\hat{\psi} = & \; H(q, \hat{\psi})\ddot{q}_d(t) + C\left(q, \dot{q}_d(t), \hat{\psi}\right)\dot{q}_d(t) \\ & + F(\hat{\psi})\dot{q}_d(t) + g(q, \hat{\psi}) \end{aligned} \tag{9}$$

and $\hat{\psi}$ (the vector of estimated parameters) being the output variable of an auxiliary dynamic subsystem defined as

$$\dot{\phi} = -\Gamma Y^T(q, \dot{q}_d(t), \ddot{q}_d(t))\left[\dot{\tilde{q}} + \varepsilon M_P \mathrm{T_h}(K_P \tilde{q})\right] \tag{10a}$$

$$\hat{\psi} = M_a \mathrm{T_h}(\phi) \tag{10b}$$

where ϕ is the state of the dynamics in (10a); $M_a \mathrm{T_h}(x) = \left(M_{a1}\tanh(x_1), \ldots, M_{a\rho}\tanh(x_\rho)\right)^T$, with bounds M_{aj} satisfying (7); $\Gamma \in \mathbb{R}^{\rho \times \rho}$ is a positive definite diagonal constant matrix, and ε is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2\} \tag{11}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\mu_m}{\mu_M^2 \beta_P}} \quad \text{and} \quad \varepsilon_2 \triangleq \frac{f_m - k_C B_{dv}}{\beta_M + \left(k_C B_{dv} + \frac{f_M + \beta_D}{2}\right)^2}$$

(satisfaction of Assumption 2 ensures positivity of ε_2) with

$$\beta_P \triangleq \max_i\{M_{Pi} k_{Pi}\} \quad , \quad \beta_D \triangleq \max_i\{M_{Di} k_{Di}\}$$

$$B_P \triangleq \sqrt{\sum_{i=0}^{n} M_{Pi}^2} \quad , \quad \beta_M \triangleq k_C B_P + \mu_M \beta_P$$

with μ_m, μ_M, k_C, f_m, and f_M as defined in Properties 1–3.

Observe that under the consideration of Assumption 2 and by means of suitable strict bounds on the elements of θ, in accordance to Property 6, the third term the right-hand side of (8) is guaranteed to be bounded.

The first and second terms in the right-hand side of (8) correspond, respectively, to a position error correction term and, to a motion dissipation term where $K_P, K_D \in \mathbb{R}^{n \times n}$ are positive definite diagonal matrices, $M_P \mathrm{T_h}(x) = \left(M_{P1}\tanh(x_1), \ldots, M_{Pn}\tanh(x_n)\right)^T$ and $M_D \mathrm{T_h}(x) = \left(M_{D1}\tanh(x_1), \ldots, M_{Dn}\tanh(x_n)\right)^T$, with bounds M_{Pi} and M_{Di}, respectively, and such that

$$M_{Pi} + M_{Di} \leq T_i - B_{Di}^{Ma}$$

Proposition 1: For the system (4)-(6) in closed-loop with the controller defined through (8), (9), fulfilling Assumptions 1 and 2, the solution $\{q = q_d(t), \dot{q} = \dot{q}_d(t), \hat{\psi} = \psi\}$ is globally uniformly asymptotically stable, provided that $Y^T(q_d(t), \dot{q}_d(t), \ddot{q}_d(t))$ is persistently exciting. ◁

IV. Proof of the main result

Consider system (4)-(6) taking $u = u(t, q, \dot{q}, \hat{\psi})$ as defined through (8)-(10). Observe that, under Assumption 1 and the consideration in (6), the fulfillment of (7) guarantees that

$$T_i > \left|u_i\left(t, q, \dot{q}, s_a(\phi)\right)\right| = |u_i| = |\tau_i| \;, \; i = 1, \ldots, n \tag{12}$$

$\forall(t,q,\dot{\tilde{q}},\phi)\in\mathbb{R}_+\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^\rho$. Thus, considering Property 6a., the closed-loop system takes the form

$$H(q)\ddot{\tilde{q}}+\big[C(q,\dot{q})+C(q,\dot{q}_d(t))\big]\dot{\tilde{q}}+F\dot{\tilde{q}}= \\ -M_P\mathrm{T_h}(K_P\bar{q})-M_D\mathrm{T_h}(K_D\dot{\tilde{q}})+Y(q,\dot{q}_d(t),\ddot{q}_d(t))\bar{s}_a(\bar{\phi}) \tag{13a}$$

$$\dot{\bar{\phi}}=-\Gamma Y^T(q,\dot{q}_d(t),\ddot{q}_d(t))\big[\dot{\tilde{q}}+\varepsilon M_P\mathrm{T_h}(K_P\bar{q})\big] \tag{13b}$$

where $\bar{\phi}=\phi-\phi^*$ and

$$\bar{s}_a(\bar{\phi})=M_a\mathrm{T_h}(\bar{\phi}+\phi^*)-M_a\mathrm{T_h}(\phi^*) \tag{14}$$

with $\phi^*=\big(\phi_1^*,\ldots,\phi_\rho^*\big)^T$ such that $M_a\mathrm{T_h}(\phi^*)=\psi$, or equivalently, $\phi_j^*=\mathrm{arctanh}(\frac{\psi_j}{M_{aj}})$, $j=1,\ldots,\rho$. Note that due to the strictly increasin character of $tanh(\cdot)$, the elements of $\bar{s}_a(\bar{\phi})$ in (14), *i.e.*

$$\bar{\sigma}_{aj}(\bar{\phi}_j)=M_{aj}\big[\tanh(\bar{\phi}_j+\phi_j^*)-\tanh(\phi_j^*)\big]$$

$j=1,\ldots,\rho$, satisfy Lemma 2.

Redefining the state vector as $x\triangleq[x_1,x_2]^T$, with $x_1\triangleq[x_{11},x_{12}]^T$, and $x_2\triangleq\bar{\phi}$ where $x_{11}=\bar{q}$ and $x_{12}=\dot{\tilde{q}}$, the closed-loop dynamics (13), takes the form

$$\dot{x}=F(t,x)\triangleq\begin{pmatrix}A(t,x_1)+B(t,x)\\ M(t,x)\end{pmatrix} \tag{15}$$

where[3]

$$A(t,x_1)=\begin{pmatrix}x_{12}\\ H^{-1}(q)\Big[-M_P\mathrm{T_h}(K_Px_{11})-M_D\mathrm{T_h}(K_Dx_{12})\\ \qquad -Fx_{12}-\big[C(q,\dot{q})+C(q,\dot{q}_d(t))\big]x_{12}\Big]\end{pmatrix}$$

$$B(t,x)=\begin{pmatrix}0\\ H^{-1}(q)Y(q,\dot{q}_d(t),\ddot{q}_d(t))\bar{s}_a(x_2)\end{pmatrix}$$

$$M(t,x_1)=-\Gamma Y^T(q,\dot{q}_d(t),\ddot{q}_d(t))\big[x_{12}+\varepsilon M_P\mathrm{T_h}(K_Px_{11})\big]$$

Notice that the closed-loop form in (13) is equivalent to the one studied in [5] and such that $F(t,0)=0$. The following tools developed in [5] help us achieve the desired objective. Let us define

$$B_o(t,x_2)\triangleq B(t,x)|_{x_1=0} \tag{16}$$

observe that $B_o(t,0)\equiv 0$.

Assumption 3: [5] There exists a continuously differentiable function $V:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}_+$ which is positive definite, decrescent, radially unbounded and has a negative semidefinite time derivative. More precisely, assume that there exist functions $\alpha_1,\alpha_2\in\mathcal{K}_\infty$ and $U:\mathbb{R}^q\to\mathbb{R}_+$ continuous positive definite, such that

$$\alpha_1(\|x\|)\le V(t,x)\le\alpha_2(\|x\|) \tag{17}$$

$$\dot{V}(t,x)\le -U(x_1) \tag{18}$$

for all $(t,x)\in\mathbb{R}_+\times\mathbb{R}^n$. ◁

Assumption 4: The function $B(t,x)$ is continuously differentiable and uniformly bounded in t on each compact set of the state. More precisely, for each $\Delta>0$ there exist $b_M>0$ and continuous non-decreasing functions $\rho_i:\mathbb{R}_+\to\mathbb{R}_+$ with $i=1,2$ such that $\rho_i(0)=0$; $\forall t\in\mathbb{R}_+$, $\forall x\in\mathbb{R}^n$

$$\max_{|x_2|\le\Delta}\left\{|B_o(t,x_2)|,\left|\frac{\partial B_o}{\partial t}\right|,\left|\frac{\partial B_o}{\partial x_2}\right|\right\}\le b_M \tag{19}$$

$$\max_{|x_2|\le\Delta}|B(t,x)-B_o(t,x_2)|\le\rho_1(\|x_1\|) \tag{20}$$

$$\max_{|x_2|\le\Delta}\{|A(t,x_1)|,|M(t,x)|\}\le\rho_2(\|x_1\|) \tag{21}$$

◁

Theorem 1: [6] System (15), under Assumptions 3 and 4 is *uniformly global asymptotically stable* if and only if (Assumption 5) $B_o(t,x_2)$ is UδPE with respect to x_2. ◁

To accomplish the objective stated in Proposition 1, we need to verify that Assumptions 3, 4, and 5, are satisfied.

A. Proving Assumption 3

Proposition 2: Under the satisfaction of Assumptions 1 and 2, for any positive definite diagonal matrices K_P, K_D, and Γ, and any ε satisfying (11), the closed-loop system in (15) satisfies the conditions stated through Assumption 3; with $|\tau_i(t)|=|u_i(t)|<T_i$, $i=1,\ldots,n$, $\forall t\ge t_0$. ◁

Proof: By (12) and under the latter adaptive scheme, input saturation is avoided. In order to develop the stability/convergence analysis, let us define the scalar function

$$V(t,x)=\frac{1}{2}x_{12}^TH(q)x_{12}+\sum_{i=1}^n k_{Pi}^{-1}M_{Pi}\ln\big(\cosh(k_{Pi}x_{11i})\big) \\ +\varepsilon x_{12}^TH(q)[M_P\mathrm{T_h}(K_Px_{11})]+\int_{0_p}^{x_2}\bar{s}_a^T(r)\Gamma^{-1}dr \tag{22}$$

with

$$\int_{0_\rho}^{x_2}\bar{s}_a^T(r)\Gamma^{-1}dr= \\ \sum_{j=1}^{\rho}M_{aj}\gamma_j^{-1}\big[\ln\big(\cosh(\bar{\phi}_j+\phi_j^*)\big)-\ln\big(\cosh(\phi_j^*)\big)\big]$$

Observe that from Property 1, items 5, 6, 7, 8, and 9 of Lemma 2, $V(t,x)$ can be bounded above and below by

$$W_1(x)\le V(t,x)\le W_2(x)$$

where

$$W_1(x)=W_{11}(x_1)+\int_{0_p}^{x_2}\bar{s}_a^T(r)\Gamma^{-1}dr \\ +(1-\alpha)\sum_{i=1}^n k_{Pi}^{-1}M_{Pi}\ln\big(\cosh(k_{Pi}x_{11i})\big)$$

$$W_2(x)=W_{12}(x_1)+\int_{0_p}^{x_2}\bar{s}_a^T(r)\Gamma^{-1}dr$$

with

$$W_{11}(x_1)=\frac{1}{2}\begin{pmatrix}\|M_P\mathrm{T_h}(K_Px_{11})\|\\ \|x_{12}\|\end{pmatrix}^TQ_{11}\begin{pmatrix}\|M_P\mathrm{T_h}(K_Px_{11})\|\\ \|x_{12}\|\end{pmatrix}$$

$$W_{12}(x_1)=\frac{1}{2}\begin{pmatrix}\|x_{11}\|\\ \|x_{12}\|\end{pmatrix}^TQ_{12}\begin{pmatrix}\|x_{11}\|\\ \|x_{12}\|\end{pmatrix}$$

with Q_{11} and Q_{12} given by

$$Q_{11}=\begin{pmatrix}\frac{\alpha}{\beta_P} & -\varepsilon\mu_M\\ -\varepsilon\mu_M & \mu_m\end{pmatrix}\quad,\quad Q_{12}=\begin{pmatrix}\beta_P & \varepsilon\mu_M\beta_P\\ \varepsilon\mu_M\beta_P & \mu_M\end{pmatrix}$$

[3]Notice that in the error variable space $x_{11}=q-q_d(t)$, $x_{12}=\dot{q}-\dot{q}_d(t)$, however, for ease of reading, q and $\dot{q}$ will be written instead of $x_{11}+q_d(t)$, $x_{12}+\dot{q}_d(t)$, respectively.

and α being a positive constant satisfying

$$\frac{\varepsilon^2}{\varepsilon_1^2} < \alpha < 1 \tag{23}$$

Notice that, by (23), $W_1(x)$ and $W_2(x)$ are positive definite Further observe that $W_1(0_n, x_{12}) \to \infty$ as $\|x_{12}\| \to \infty$. From this, inequality (23) (whence $1-\alpha > 0$), and items 7 and 8 of Lemma 2, $V(t,x)$ proves to be positive definite, radially unbounded, and decrescent, which fulfills condition (17) of Assumption 3. After some basic computations and making use of Assumption 2, Properties 1–3, items 2 and 3 of Lemma 2, and the positive definite character of K_P, we have that

$$\dot{V}(t,x) \leq -W_3(x_1)$$

where

$$W_3(x_1) = \begin{pmatrix} \|M_P \mathrm{T_h}(K_P x_{11})\| \\ \|x_{12}\| \end{pmatrix}^T Q_3 \begin{pmatrix} \|M_P \mathrm{T_h}(K_P x_{11})\| \\ \|x_{12}\| \end{pmatrix}$$

with

$$Q_3 = \begin{pmatrix} \varepsilon & -\varepsilon\left(\frac{f_M+\beta_D}{2} + k_C B_{dv}\right) \\ -\varepsilon\left(\frac{f_M+\beta_D}{2} + k_C B_{dv}\right) & f_m - k_C B_{dv} - \varepsilon\beta_M \end{pmatrix}$$

Note that, from the satisfaction of (11), $W_3(x_1)$ is positive definite.Thus, we have $\dot{V}(t,x) \leq 0$, $\forall (t, x_{11}, x_{12}, x_2) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^\rho$, with $\dot{V}(t,x) = 0 \iff (x_{11}, x_{12}) = (0_n, 0_n)$ (see [8]). Therefore condition (18) of Assumption 3 is fulfilled, concluding the proof.

■

B. Proving Assumption 4

From the definition of $B(t,x)$ in (15) and $B_o(t,x_2)$ as defined in (16), we have that

$$B_o(t,x_2) = \begin{pmatrix} 0 \\ H^{-1}\big(q_d(t)\big) Y\big(q_d(t), \dot{q}_d(t), \ddot{q}_d(t)\big) \bar{s}_a(x_2) \end{pmatrix}$$

Notice that, under the consideration made in Assumption 2, each term in the definition of $B_o(t,x_2)$ is bounded (see item 5 of Lemma 2, Properties 1 and 6) and hence inequality (19) is satisfied.

In order to verify that condition (20) is fulfilled observe that $B(t,x)$ is linear with respect to $\bar{s}_a(x_2)$, the difference $|B(t,x) - B_o(t,x_2)|$ can be rewritten as $|B(t,x) - B_o(t,x_2)| = |P(t,x_1)\bar{s}_a(x_2)|$, where $P(t,x_1)$ is uniformly bounded in t, and such that $P(t,0) = 0$.

To prove the satisfaction of (21), observe that, from item 1 of Lemma 2, that $A(t,0) = 0$ and $M(t,0) = 0$. And notice, from item 5 of Lemma 2 and Property 6b., that $A(t,x_1)$ and $M(t,x_1)$ are bounded.

C. Proving Assumption 5

Since $B_o(t,x_2)$ is uniformly continuous in t, Lemma 1 can be used to prove that $B_o(t,x_2)$ is UδPE with respect to x_2, *i.e.* we need to prove that for each $x_2 \neq 0$ there exist T and μ such that

$$\int_t^{t+T} \big[H^{-1}\big(q_d(\tau)\big) Y\big(q_d(\tau), \dot{q}_d(\tau), \ddot{q}_d(\tau)\big) \bar{s}_a(x_2)\big]^T \times \big[H^{-1}\big(q_d(\tau)\big) Y\big(q_d(\tau), \dot{q}_d(\tau), \ddot{q}_d(\tau)\big) \bar{s}_a(x_2)\big] d\tau \geq \mu \tag{24}$$

Observe that, since $H\big(q_d(t)\big)$ is full rank, (24) is satisfied if and only if $\phi(t,x_2) \triangleq Y\big(q_d(\tau), \dot{q}_d(\tau), \ddot{q}_d(\tau)\big)\bar{s}_a(x_2)$ is Uδ-PE with respect to $s_a(x_2)$. Using Property 1 we have that

$$\|B_o(t,x_2)\|^2 \geq \frac{1}{\mu_M^2} \|Y(q_d, \dot{q}_d(t), \ddot{q}_d(t))\bar{s}_a(x_2)\|^2$$

Notice that $Y(q_d, \dot{q}_d(t), \ddot{q}_d(t))\bar{s}_a(x_2)$ has the form described in Remark 1, $\Phi(t) = Y(q_d, \dot{q}_d(t), \ddot{q}_d(t))$, we conclude that B_o is UδPE if and only if $Y(q_d, \dot{q}_d(t), \ddot{q}_d(t))$ is PE.

And hence, by Theorem 1, the closed-loop system trivial solution (x_{11}, x_2) is concluded to be uniformly asymptotically stable, for any initial condition $\big(t_0, x_{11}(t_0), x_{12}, x_2(t_0),\big) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$.

V. SIMULATION RESULTS

In order to corroborate the effectiveness of the studied scheme, simulations where implemented using a two degree of freedom robot model appearing in [9], [10]. Using Property 5 the regression matrix and parameter vector of the considered dynamics can be written as

$$Y^T(q,\dot{q},\ddot{q}) = \begin{pmatrix} \ddot{q}_1 & 0 \\ (2\ddot{q}_1 + \ddot{q}_2)\cos(q_2) - \dot{q}_2(2\dot{q}_1 + \dot{q}_2)\sin(q_2) & \ddot{q}_1\cos(q_2) + \dot{q}_1^2\sin(q_2) \\ \ddot{q}_2 & \ddot{q}_1 + \ddot{q}_2 \\ \dot{q}_1 & 0 \\ 0 & \dot{q}_2 \\ \sin(q_1) & 0 \\ \sin(q_1+q_2) & \sin(q_1+q_2) \end{pmatrix}$$

$$\psi^T = \begin{pmatrix} 0.323 & 0.0127 & 0.0122 & 0.274 & 0.144 & 11.508 & 0.4596 \end{pmatrix}$$

and Properties 1-4 are satisfied with $\mu_m = 0.0974$ kg·m^2, $\mu_M = 0.7193$ kg·m^2, $k_C = 0.0487$ kg·m^2, $f_m = 0.144$ kg·m^2/s, $f_M = 0.274$ kg·m^2/s, $B_{g1} = 11.9674$ Nm, and $B_{g2} = 0.4596$ Nm. The input saturation bounds are $T_1 = 15$ Nm for the first link and $T_2 = 4$ Nm for the second one. Simulations were done with the following saturation values, $M_{P1} = M_{D1} = 0.9$, $M_{P2} = M_{D2} = 1.5$, and the parameter bounds $M_{aj} = [0.3387\,,\ 0.0133\,,\ 0.0128\,,\ 0.2877\,,\ 0.1512\,,\ 12.08\,,\ 0.4825]$, for $j = 1, \ldots, 7$. The initial link positions, velocities, and auxiliary states were taken as $q_i(0) = \dot{q}_i(0) = \phi_j(0) = 0$, $\forall i = 1, 2$, $j = 1, \ldots, 7$. The desired trajectory is given by

$$q_d(t) = \begin{pmatrix} q_{d1}(t) \\ q_{d2}(t) \end{pmatrix} = \begin{pmatrix} \frac{\pi}{2} + \frac{3}{\pi^2}\tanh\left(\frac{\pi}{5}t\right)\sin\left(\frac{\pi}{3}t\right) \\ \frac{\pi}{2} + \frac{pi}{4}\left(1 - e^{-\frac{t}{3}}\right)\sin\left(\frac{2}{\pi}t\right) \end{pmatrix}$$

For the chosen trajectory Assumption 2 is satisfied with $B_{dv} < 0.779 < \frac{f_m}{k_C} \approx 2.95$ and $B_{da} = 1.074$. The control parameter values are shown in Table I. Figures 1 and 2 show the tracking error evolution and the obtained control signals, observe that the algorithm avoid input saturation even when the chosen control gains are rather large. The parametric estimation is shown in Figure 3, notice that due to the small value of ε the convergence rate is slow. To verify the satisfaction of the PE condition, let us define $\Upsilon(t) \triangleq \int_t^T \Phi(\tau)\Phi^T(\tau)d\tau$, with $\Phi(t) = Y\big(q_d(t), \dot{q}_d(t), \ddot{q}_d(t)\big)$. Note from Figure 4 that the eigenvalues of $\Upsilon(t)$ are greater than

TABLE I

CONTROL PARAMETER VALUES

Parameter	Control Gains
diag$\{k_P\}$	[38 30]
diag$\{k_D\}$	[5 3.5]
diag$\{\Gamma\}$	[2.7 0.49 1.25 2.25 0.5 10 0.335]
ε	0.97

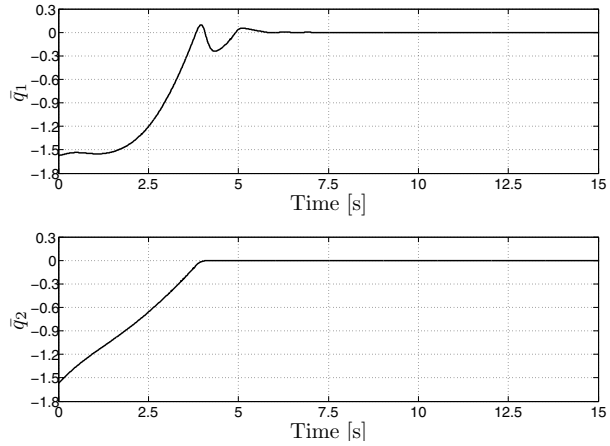

Fig. 1. Position errors

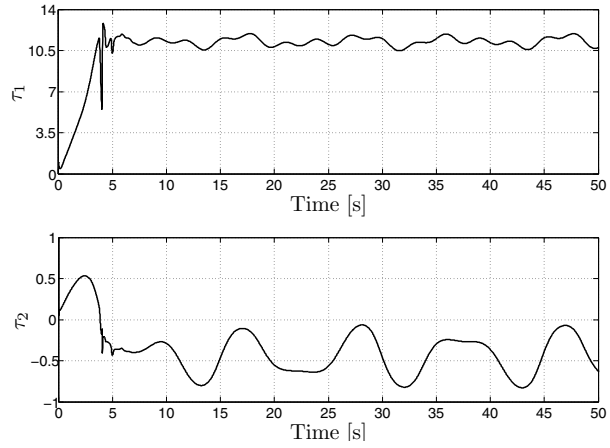

Fig. 2. Control inputs

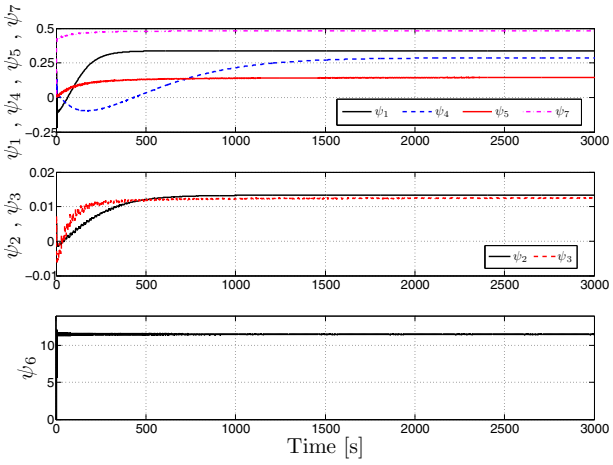

Fig. 3. Parameter estimation

zero for all simulated time, which makes it positive definite.

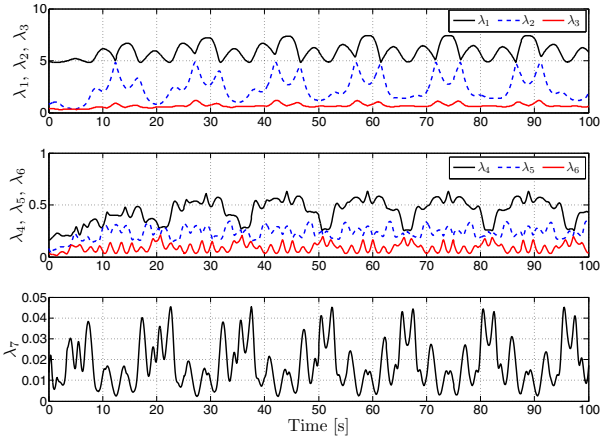

Fig. 4. Persistency of excitation condition

VI. CONCLUSIONS

Making use of a persistency of excitation condition, it was demonstrated that the described SP-SD adaptive control scheme not only achieves tracking of the desired trajectories but also parameter convergence for any initial condition while preventing input saturation.

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