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# RECENT ADVANCES IN PARAMETRIC NONLINEAR MODEL ORDER REDUCTION: TREATMENT OF SHOCKS, CONTACT AND INTERFACES, STRUCTURE-PRESERVING HYPER REDUCTION, ACCELERATION OF MULTISCALE FORMULATIONS, AND APPLICATION TO DESIGN OPTIMIZATION

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## ABSTRACT

Keywords: ECSW, Hyper reduction, Model order reduction, Multiscale modeling, Reducedorder model, Structure-preserving

Parametric, projection-based, Model Order Reduction (MOR) is a mathematical tool for constructing a parametric Reduced-Order Model (ROM) by projecting a given parametric High Dimensional Model (HDM) onto a Reduced-Order Basis (ROB). It is rapidly becoming indispensable for a large number of applications including, among others, computational-based design and optimization, multiscale analysis, statistical analysis, uncertainty quantification, and model predictive control. It is also essential for scenarios where real-time simulation responses are desired. During the last two decades, linear, projection-based, parametric MOR has matured and made a major impact in many fields of engineering including electrical engineering, acoustics, and structural acoustics, to name only a few. By comparison, nonlinear, projection-based, parametric MOR remains somehow in its infancy. Nevertheless, giant strides have been recently achieved in many of its theoretical, algorithmic, and offline/online organizational aspects. The main purpose of this lecture is to highlight some of these advances, discuss their mathematical and computer science underpinnings, and report on their impact for an important class of problems in aerodynamics, fluid mechanics, nonlinear solid mechanics and structural dynamics, failure analysis, multiscale analysis, uncertainty quantification, and design optimization.

To this effect, nonlinear, projection-based, parametric MOR will be first interpreted as a constrained semidiscretization on a subset of a compact Stiefel manifold, using a low-dimensional basis of global shape functions constructed a posteriori — that is, after some knowledge about the response of the system of interest has been developed. Usually, such a knowledge is gathered using the given parametric HDM and an offline training procedure where the model parameters are sampled with a greedy strategy based on a cost-effective ROM error indicator. Specifically, a set of problems related to the parametric problem of interest

are solved at the sampled parameter points using the given HDM, and the computed solution snapshots are compressed to obtain the desired global ROB. Depending on the mathematical type of the governing equations underlying the given HDM, a dual ROB is also constructed and the sought-after nonlinear parametric ROM is constructed by Galerkin (or Petrov-Galerkin) projection of the HDM onto the global ROB (and its dual counterpart).

Unfortunately, despite its low dimension, the resulting nonlinear ROM does not necessarily guarantee computational feasibility. This is because the construction process described so far does not scale only with the size of the target ROM, but also with that of the underlying HDM. This caveat is remedied by equipping the nonlinear MOR tool with a rigorous scheme for approximating the resulting reduced operators whose computational complexity scales only with the small size of the ROM. Such a procedure is often referred to in the literature as Hyper Reduction (HR). Here, the Energy Conserving Sampling and Weighting (ECSW) technique introduced in [1] and analyzed in [2] will be discussed, and its distinctive structure-preserving and numerical stability properties will be highlighted. In particular, different implementations of ECSW will be contrasted and discussed with respect to their potential for an online implementation.

For highly nonlinear problems characterized by different physical regimes (such as, for example, pre-shock, shock, and after-shock; or pre-buckling, buckling, and post-buckling), keeping the dimension of the global ROB sufficiently small while maintaining accuracy is challenging. For such problems, a concept of local ROB where locality refers a priori neither to space nor to time, but to the region of the manifold where the solution lies at a given time, was recently introduced in [3]. In this lecture, the implementation of this concept using a preferred clustering technique, its coupling with the structure-preserving HR method ECSW, and its merit for nonlinear MOR will be discussed and demonstrated with the real-time solution of aerodynamics problems with shocks, and the real-time simulation of the collapse of metallic structures. Enabling the latter family of applications requires the reduction of contact constraints. To this effect, the approach recently developed in [4] for reducing the dimension of nonlinear contact models when the contact constraints are enforced by Lagrange multipliers will be first outlined. Then, new results pertaining to the stability analysis and performance of the HR of the Lagrange multiplier field in the context of local ROB will be presented.

For highly nonlinear fluid-structure interaction problems with arbitrarily evolving domains and interfaces, HDMs are often based on an embedded (or immersed) boundary method for CFD (Computational Fluid Dynamics). In this case, the computational fluid domain typically contains both real and ghost regions, which complicates the collection and compression of solution snapshots. In [5], this issue was addressed by formulating the snapshot compression problem as a weighted low-rank approximation problem where the binary weighting identifies the evolving component of the individual computational snapshots. This simple approach will be revisited here with particular attention of computational efficiency in the presence of very large-scale HDMs.

For multidisciplinary optimization problems, a recently developed [6] adaptive approach for using ROMs as surrogate models will be also presented and illustrated. This approach distinguishes itself from alternatives by the fact that it breaks the traditional offline/online framework of MOR. It formulates a high-dimensional PDE (Partial Differential Equation)-constrained optimization problem as a sequence of reduced optimization problems constrained by an evolving ROM, and trains this ROM at a given instance of the sequence by sampling the parametric HDM at the solution points of some of the previous reduced optimization problems in the sequence. This adaptive approach also equips the reduced optimization problems with a nonlinear trust-region based on a residual error indicator in order to keep the optimization trajectory in a region of the parameter space where the ROM is accurate. The net result is a ROM that is progressively constructed along the trajectory of the optimization procedure, rather than in a global parameter domain sampled by a design of experiments approach. For a given objective function (or a small set of them), this approach delivers a superior computational efficiency.

All aforementioned ideas, concepts, and mathematical techniques have been integrated into a computational framework for nonlinear MOR and HR. The versatility of this framework will be illustrated with its application to multiscale HDMs constructed using the multilevel finite element method FE2 [7]. In particular, it will be shown that for such multilevel models, MOR is computationally feasible even for a

single analysis at a single parameter point. Finally, the potential of this framework for practical engineering operations will be also demonstrated for challenging applications pertaining to underbody blast, underwater implosion, and aircraft shape design [8]. In all cases, speedup factors ranging between two and five orders of magnitude will be demonstrated, depending on the problem complexity and its setup.

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