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# Gaussian Beam Shooting Algorithm based on Iterative Frame Decomposition

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**Abstract**—Adaptive beam re-shooting is proposed as a solution to overcome essential limitations of the Gaussian Beam Shooting technique. The proposed algorithm is based on iterative frame decompositions of beam fields in situations where usual paraxial formulas fail to give accurate enough results, such as interactions with finite obstacle edges. Collimated beam fields are successively re-expanded on narrow and wide window frames, allowing for re-shooting and further propagation of collimated beams. Closed-form formulas used in the course of these re-expansions are presented, as well as first numerical tests of accuracy.

## I. INTRODUCTION

Gaussian Beam Shooting (GBS) has been proposed as an alternative to ray based methods to perform field computations in the context of large problems involving multiple interactions with the environment and/or propagation in non homogeneous media. The main advantages of GBS as compared to ray based methods are the absence of caustics, the relatively moderate number of beams to launch by comparison with the number of rays in full 3D problems, the localization of beam fields around the beam axis.

Frame theory offers a rigorous mathematical ground to the decomposition of source fields on a set of Gaussian windows radiating paraxial Gaussian beams (GB) [1]–[3]. In this communication we propose to use this technique iteratively in the course of a GBS algorithm, to overcome two major deficiencies of the GBS technique: firstly, the lack of accuracy of GB paraxial formulas for large propagating distances and far away from the beam axis; secondly, the fact that paraxial GB reflection and refraction formulas do not account for abrupt transitions occurring in the presence of edges. The proposed algorithm is based on re-shooting beams from new frame decompositions performed either on virtual planes or on planar or smoothly curved surfaces of finite extension. To this end, beam fields are first decomposed on a frame of spatially narrow Gaussian windows. The frame coefficients on a frame of spatially wide (i.e. spectrally narrow) windows are then obtained by a frame change performed by a matrix-vector product. The use of these two successive frames makes it possible to take into account both abrupt transitions in space (via the spatially narrow windows) and paraxial transformations (via the spectrally narrow windows).

The final decomposition on spatially wide windows is used to launch a new set of well collimated paraxial beams, which propagate the fields after their interaction with the encountered obstacle surface.

It must be stressed that the re-expansion algorithm relies on the assumption that the field distribution on the diffracting obstacle is known. In the following, a Physical Optics (PO) like assumption will be used, taking truncated incident fields as the field distribution on the surface of the obstacle or in an aperture. Applying re-expansion to transformed fields, obtained by coupling to an exact method, would increase the computational cost, but could be found interesting in the case of very large problems where only a small proportion of launched beams are diffracted. The basic PO-like algorithm presented here could undoubtedly benefit from complementing its “diffracted” fields with Gaussian beam summation representations of half plane diffraction and line source radiation [4], [5] and with Complex Source Point formulations for beam diffraction [6].

In Section II, frame decomposition and paraxial GBS are shortly outlined (II-A) before presenting the formulation of beam re-expansion of propagated beam fields impinging on the edge of an obstacle with plane interface of limited size (II-B). Numerical results will validate this approach and give indications about its numerical implementation in Section III.

## II. DESCRIPTION OF THE ALGORITHM

### A. Frame decomposition and Gaussian beam shooting

In the  $L_2(\mathbb{R})$  Hilbert space, the set of Gaussian functions

$$\psi_{\boldsymbol{\mu}}(x) = \psi(x - m\bar{x})e^{in\bar{k}_x(x - m\bar{x})}, \quad \boldsymbol{\mu} = (m, n) \in \mathbb{Z}^2,$$

with  $\psi(x) = \sqrt{\frac{\nu}{L}}e^{-\pi\frac{x^2}{L^2}}$ , is a frame if and only if  $\bar{x}\bar{k}_x = 2\pi\nu$  with  $\nu < 1$  (oversampling factor) [7].  $\bar{x}$  and  $\bar{k}_x$  are respectively the spatial and spectral domain translation step. In the following, we shall use frames in  $L_2(\mathbb{R}^2)$  defined by the product of two frames in  $L_2(\mathbb{R})$ , to represent field distributions in planes. Unless specified, the same frame parameters will be used along both variables in  $\mathbb{R}^2$ , and the same oversampling factor will be used in spatial and spectral domains (“balanced” frames),

hence  $\bar{x} = L\sqrt{\nu}$ . For the sake of simplicity, we shall present scalar formulations, valid for field components.

Frames are (over)-complete sets of functions in the relevant Hilbert space, thus any field distribution can be represented as a sum of weighted Gaussian frame windows. The weights or “frame coefficients” are obtained by projecting the field distribution on the “dual frame” windows. The “dual frame” windows can be approximated by the initial frame windows multiplied by a constant factor [3]. The more overcomplete the initial frame, i.e. the smaller the oversampling factor  $\nu$ , the more accurate this approximation is.

GBS is relatively easy to perform from such frame decompositions: each Gaussian window radiates in the form of a paraxial Gaussian beam, in as much as its parameter  $L$ , characterizing its spatial width, is large enough to ensure sufficient collimation [8]. Such beam fields can be expressed in the form of complex rays:

$$\mathbf{B}_{\boldsymbol{\mu}}^i(\mathbf{x}, z) \simeq \left( \frac{\det \Gamma(z_b)}{\det \Gamma(0)} \right)^{1/2} e^{ikz_b} e^{i\frac{k}{2}\mathbf{x}_b^T \Gamma(z_b)\mathbf{x}_b} \quad (1)$$

In this expression  $\mathbf{x} = (x, y)$  is the position vector of a point in the plane of decomposition, in the cartesian system of coordinates  $(x, y, z)$ ;  $\boldsymbol{\mu} = (m, n, p, q)$  is a composite index in  $\mathbb{Z}^4$ , with  $m$  (resp.  $p$ ) the translation index in space along  $x$  (resp.  $y$ ), and  $n$  (resp.  $q$ ) the translation index in spectral domain along  $k_x$  (resp.  $k_y$ ).  $(\mathbf{x}_b, z_b)$  is a system of coordinates associated to the beam: the  $z_b$  axis is the “beam axis”, and it is directed along the wavevector with projection  $(nk_x, qk_y)$  in the  $(x, y)$  plane. The beam complex curvature matrix  $\Gamma(z_b)$  (hence the denomination of “complex ray”) accounts both for Gaussian decrease of the magnitude transversely to the beam axis, and to equiphase surface curvature.

### B. Re-expansion algorithm

In environments comprising a number of obstacles with planar or smoothly curved interfaces, beams impinging on a given interface are transformed into reflected or transmitted GB through paraxial formulas if:

- 1) their previous propagating distance is not “too large” as compared to their collimating distance,
- 2) the incident field on edges can be considered negligible.

Otherwise, paraxial reflexion, transmission or further propagation is not valid, and we propose then to apply a re-expansion algorithm to the incident beam field [9]. We present the algorithm with a view toward ground based Radar applications, hence in the context of outdoor semi-urban environments. Planar building walls can be considered as the most common finite size obstacles encountered by propagating beams in that context. The re-expansion algorithm is well suited to such cases. It will be performed in two steps:

- 1) The incident field is decomposed on a frame of narrow Gaussian windows in the plane of the wall, using closed form expressions previously established via paraxial approximation of a spectral integral (see II-B). To account for the “partial” beam incidence on the wall, the

coefficients of narrow windows centered outside the wall are put to zero. In this way, beam fields are roughly truncated along the edge.

- 2) A change of frame is performed, via a matrix-vector product (see II-B), yielding the vector of coefficients for the “truncated” incident field decomposition on a frame of wide Gaussian windows. Each wide window radiates in the form of a collimated Gaussian beam, and its reflected or refracted fields are known in the form of paraxial Gaussian beams. The total field reflected or refracted by the wall is thus finally expressed as a weighted summation of collimated Gaussian beams.

Importantly, the same frames are used for all the beams incident on a given surface, so that the final weighting coefficients of the beams issued from this decomposition are obtained as the sum of those obtained for all incident beams. The number of “new” beams to be launched after re-expansion does not depend on the number of beams impinging on edges, but on the number of wide frame windows significantly excited by these beams and radiating non negligible fields into the regions of interest. In Radar retro-diffusion simulations for instance, only the beams reaching the target, and propagating back to the radar antenna have to be taken into account. As a consequence, the number of frame coefficients to compute at each re-expansion step is expected to remain moderate. The same frames, and consequently a unique frame change matrix, can be used for all the re-expansions required in a given environment, allowing for the pre-computation of the frame change matrix coefficients.

*1) Closed form expressions for beam re-expansion on a narrow window frame:* In this subsection we give the expression of the narrow window frame coefficients  $A'_{\boldsymbol{\mu}'}$ , obtained by paraxial approximation of the projection integral of an incident beam field on the narrow frame windows  $\{\psi'_{\boldsymbol{\mu}'}\}$  defined in the  $P'(O', x', y')$  plane. The incident beam is well collimated, and radiated into the  $z > 0$  half space by a  $\psi_{\boldsymbol{\mu}}(\mathbf{x})$  frame window defined in the  $P(O, x, y)$  plane. We introduce the notation  $\sigma_x = \sigma_y = L/\sqrt{2\pi}$  and  $\sigma = \sigma_x\sigma_y$ .  $\mathbf{k}_{\mathbf{x}} = (k_x, k_y)$  (resp.  $\mathbf{k}_{\mathbf{x}'} = (k_{x'}, k_{y'})$ ) are the spectral variables related to the space variables  $\mathbf{x} = (x, y)$  (resp.  $\mathbf{x}' = (x', y')$ ). The spectrum of the incident beam field is given by:

$$\tilde{\psi}'_{\boldsymbol{\mu}'}(\mathbf{k}_{\mathbf{x}'}) = \tilde{\psi}_{\boldsymbol{\mu}}(\mathbf{k}_{\mathbf{x}}) \frac{k_z}{k_{z'}} e^{i\mathbf{k} \cdot \overrightarrow{OO'}} = \sum_{\boldsymbol{\mu}'} A'_{\boldsymbol{\mu}'} \tilde{\psi}'_{\boldsymbol{\mu}'}(\mathbf{k}_{\mathbf{x}'}) \quad (2)$$

Projecting this spectrum on the spectra of the approximate dual narrow windows and using a paraxial approximation yields:

$$A'_{\boldsymbol{\mu}'} = 2\pi c_0 f_{\boldsymbol{\mu}'}(\mathbf{k}_{\mathbf{x}'}) e^{ig_{nq}} \frac{\exp\left(-\frac{i}{2}\mathbf{g}'_{nqT}[\mathbf{g}''_{nq}]^{-1}\mathbf{g}'_{nq}\right)}{\sqrt{-\det[\mathbf{g}''_{nq}]}} \quad (3)$$

with:

- $c_0 = 4\pi\sqrt{\sigma\sigma'}\frac{\nu'_x\nu'_y}{|\psi'|^2}$

- $f_{\mu'}(\mathbf{k}_{x'}) = \frac{k_z}{k_{z'}} \exp\left[-\frac{\sigma_x'^2}{2}(k_{x'} - n'\bar{k}_{x'})^2 - \frac{\sigma_y'^2}{2}(k_{y'} - q'\bar{k}_{y'})^2\right]$
- $g(\mathbf{k}_{x'}) = 0.5i[\sigma_x^2(k_x - n\bar{k}_x)^2 + \sigma_y^2(k_y - q\bar{k}_y)^2] - [(k_x - n\bar{k}_x)m\bar{x} + (k_y - q\bar{k}_y)p\bar{y} - \mathbf{k} \cdot \overrightarrow{OO'} - (k_{x'} - n'\bar{k}_{x'})m'\bar{x}' - (k_{y'} - q'\bar{k}_{y'})p'\bar{y}']$

$g_{nq}$ ,  $\mathbf{g}_{nq}'$  and  $\mathbf{g}_{nq}''$  are the values of the function  $g$  defined above, of the vector of its first partial derivatives and of the matrix of its second partial derivatives, respectively, for the  $\mathbf{k}_{x'}$  spectral variable corresponding to  $\mathbf{k}_x = (n\bar{k}_x, q\bar{k}_y)$ .

2) *Frame change*: We outline here the way we perform the computation of frame coefficients in a given frame, knowing the frame coefficients of the same distribution in a first frame. This “frame change” is easily performed by a matrix-vector product, with a matrix whose elements are available in closed form.

Let us denote  $A'_{\mu'}$  (resp.  $A_{\mu}$ ) the frame coefficients of the  $U(\mathbf{x})$  distribution on a first (resp. second) frame. The first frame windows are denoted by  $\psi'_{\mu'}$  and the dual frame windows for the second frame are denoted by  $\varphi_{\mu}$ .  $\times$  denotes the complex conjugate. According to frame theory, we can write:

$$\begin{aligned} A_{\mu} &= \int_{-\infty}^{\infty} U(\mathbf{x}) \varphi_{\mu}^{\times}(\mathbf{x}) d\mathbf{x} \\ &= \sum_{\mu'} A'_{\mu'} \int_{-\infty}^{\infty} \psi'_{\mu'}(\mathbf{x}) \varphi_{\mu}^{\times}(\mathbf{x}) d\mathbf{x} \\ &= \sum_{\mu'} C_{\mu}^{\mu'} A'_{\mu'} \quad \text{with } C_{\mu}^{\mu'} = \int_{-\infty}^{\infty} \psi'_{\mu'}(\mathbf{x}) \varphi_{\mu}^{\times}(\mathbf{x}) d\mathbf{x} \end{aligned} \quad (4)$$

Hence  $\mathbf{A} = \mathbf{C}\mathbf{A}'$  where  $\mathbf{A}$  (resp.  $\mathbf{A}'$ ) is the vector of frame coefficients for the decomposition on the wide (resp. narrow) window frame and  $\mathbf{C}$  can be viewed as a frame change matrix. The integrals involved in the calculation of the  $\mathbf{C}$  matrix elements are easily evaluated in closed form. It must be noted that this matrix exhibits specific features favorable to fast computation of the matrix vector product (periodic block matrix).

### III. NUMERICAL TESTS: RE-EXPANSION OF TRUNCATED GAUSSIAN BEAM FIELDS

For first tests of our re-expansion algorithm, we address the typical problem of a diffracting aperture. Let us consider one beam among the set of beams shot from a frame decomposition representing a directive antenna radiation. The frame is defined in the  $(O, x, y)$  plane. This source frame parameters are the same along the variables  $x$  and  $y$ :  $L = 10\lambda$ ,  $\nu = 0.09$ . Hence the collimating distance for beams with axis along  $z$ :  $b_o = L^2/\lambda = 100\lambda$ . With a view towards UHF ground surface radar application in semi-urban environments, the working frequency is taken as  $f = 430\text{MHz}$ , i.e.  $\lambda = 0.7\text{m}$ . The beam collimating distance is then equal to 70m. The beam field is linearly polarized along the  $y$  axis.

The frames used at steps (1) and (2) of the re-expansion algorithm are defined as follows:

- 1) narrow window frame:  $L = 0.2\lambda = 0.14\text{m}$ ,  $\nu = 0.09$ , hence  $\bar{x} = \bar{y} = 0.06\lambda = 0.042\text{m}$ ,  $\bar{k}_x = \bar{k}_y = 1.5k_o$  with  $k_o$  the free space wavenumber.
- 2) wide window frame:  $L = 10\lambda = 7\text{m}$ ,  $\nu = 0.09$ , hence  $\bar{x} = \bar{y} = 3\lambda = 2.1\text{m}$ ,  $\bar{k}_x = \bar{k}_y = 0.03k_o$ .

Let us propagate the beam radiated by the  $\psi_{\mu}$  frame window with  $\mu = (0, 0, 0)$ . The beam axis is then along the  $z$  axis, and the beam system of coordinates is  $(O, x, y, z)$ . To assess the accuracy of the paraxial approximation used to compute the narrow window frame coefficients in our re-expansion algorithm, we consider an absorbing screen parallel to the source plane, at distance  $z = 0.5b_o = 35\text{m}$ , with a centered square aperture of lateral dimensions  $5\lambda = 3.5\text{m}$ .

Fig. 1 presents a comparison between:

- a reference plane wave spectrum integral computation of the beam fields truncated by the aperture;
- a synthesis of the truncated beam fields based on the narrow window frame expansion, taking into account only windows centered inside the aperture;
- a synthesis of the truncated beam fields based on the wide window frame re-expansion obtained by frame change from the previous narrow window frame expansion.

The field magnitude is presented along the  $y$  axis. The field value calculated with paraxial beam expressions is not presented on that curve, for it is not visibly different from the reference, owing to the proximity of the aperture points to the beam axis and to the short propagation distance (less than  $b_o$ ). The synthesis with the narrow window frame appears to be extremely accurate. The computed maximum normalized error is  $5.10^{-4}$ , i.e. -66dB, even slightly smaller than the paraxial approximation error (-65dB). The synthesis with the wide window frame is visibly less accurate, with oscillations caused by spectral truncation, resulting in a  $4.8.10^{-2}$  (-26dB) maximum error. It is however well established that abruptly truncated field synthesis in the plane of truncation requires very large numbers of spectral domain coefficients for windows centered outside of visible spectrum, while these coefficients are useless to further propagate fields by GBS.

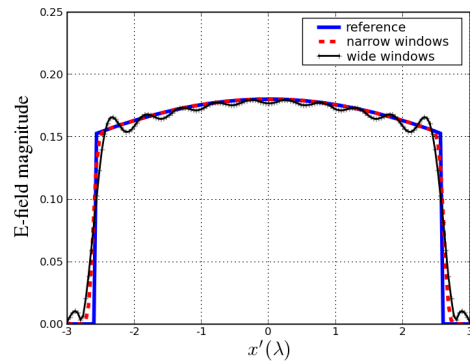


Fig. 1. Field magnitudes obtained by a reference solution and by frame synthesis, after paraxial re-expansion of an incident beam field truncated by a rectangular aperture

This frame re-expansion is then used to shoot beams and compute fields, after “diffraction” by the aperture. Fig. 2 presents a map of these diffracted fields, in dB, in the plane  $z = 0.7b_o$  i.e. at a distance  $0.2b_o = 20\text{m}$  from the diffracting aperture. This close distance between the aperture and the observation plane allows to check the ability of GBS to represent fields in regions not situated in the far field of obstacles or sources. Fig. 3 presents a cut of this map along the  $x$  axis, together with a “reference” diffracted field, obtained by plane wave spectrum integral of the truncated reference field in the plane of the aperture. Finally, Fig. 4 shows a map of the normalized error, in dB, for the fields presented in Fig. 2. From all these results, it appears that the frame re-expansion algorithm leads to accurate results. Subsequent work will evaluate the ability of this algorithm to yield accurate results in more general configurations. It was already shown that in the case of free space propagation, the results obtained by tilted beam re-expansion were more accurate than paraxial beam formulas.

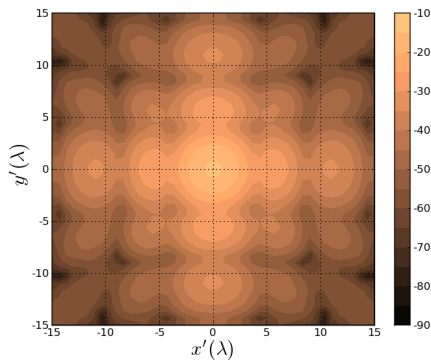


Fig. 2. Diffracted Gaussian beam field at a distance  $20\lambda$  ( $0.2b_o$ ) behind a  $(5\lambda)^2$  aperture, obtained by GB re-shooting from a frame re-expansion in the aperture

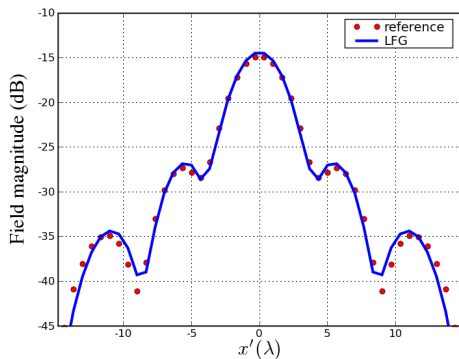


Fig. 3. Comparison of diffracted field magnitudes (in dB) obtained by GBS after re-expansion and by reference plane wave spectrum integral - same case as in Fig. 2

#### IV. CONCLUSION

A re-expansion algorithm is proposed to overcome the limitations of paraxial GBS algorithms in terms of propagation

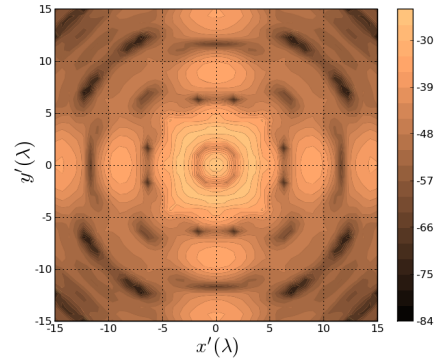


Fig. 4. Normalized error (in dB) of the field obtained with GBS after re-expansion - same case as in Fig. 2

path lengths and diffraction by limited size obstacles. The re-expansion scheme is based on an initial frame decomposition of collimated beam fields on narrow window frames. Closed form expressions for the frame coefficients of such decompositions have been established through paraxial approximations. The frame change matrix used in the second step of the re-expansion scheme to re-expand the beam fields on spatially wide windows is analytically pre-computed. First numerical checks with this algorithm have lead to accurate results. Future work will consist in testing the ability of the algorithm to address more general scenarii: lateral obstacles, grazing incidence... Some of the constraints on oversampling and on narrow window widths will be relaxed in future tests, in order to keep the computational burden as low as possible.

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