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A Frequency-domain test for long range dependence.

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Abstract

A new frequency-domain test statistic is introduced to test for short memory versus long memory. We provide its asymptotic distribution under the null hypothesis and show that it is consistent under any long memory alternative. Some simulation studies show that this test is more robust than various standard tests in terms of empirical size when the normality of observed process is lost.

Keywords : Long memory, dependence, time series, limit theorem, hypothesis test.

1 Introduction

Let us consider a stationary process (X_t) with a spectral density of some semi-parametric form:

$$f(\lambda) = |\lambda|^{-2d}g(\lambda) \tag{1}$$

with $0 \leq d < 1/2$ and g is an even, positive, continuous function on $[-\pi, \pi]$. We say that the process (X_t) is of short memory if $d = 0$ (i.e. the spectral density f is continuous over $[-\pi, \pi]$), and we say that the process is of long memory if $0 < d < 1/2$ (i.e. the spectral density is unbounded at zero). In time-domain approach this translates by saying that the covariance function is summable under short memory (i.e. covariance function $\gamma(h)$ goes to zero fast as the lag increases) and is not summable (i.e. the covariance function goes to zero slowly as the lag increases) under long memory. Literature is full of real world examples of both types of processes. Since the so-called R/S statistic was introduced by Hurst (1951), many authors have come with several time domain statistics for testing long memory, such as modified R/S, KPSS, V/S (See Mandelbrot (1969), Kwiatkowski *et al.* (1992), Lee and Schmidt (1996), Giraitis *et al.* (2003), Giraitis *et al.* (2006)). According to the simulation results provided in Giraitis *et al.* (2003), V/S statistic produces testing procedure with better empirical size and power as compared to the other time domain statistics. A detailed description of V/S statistic is given in Section 3. One common drawback in these statistics is the difficulty to calibrate the window parameters in order to ensure the right empirical size (in particular for the small sample) which comes with the heavy price of a very weak power. Also, their limiting distributions often involve approximating stochastic integrals. In this

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paper, we address the problem of high empirical size by introducing a very promising frequency domain statistic, for testing long memory versus short memory. This turns out to be very convenient and yields very simple limiting distributions. Our test statistic can be viewed as a contribution to the frequency-domain approach initiated by Lobato and Robinson (1998 (see Section 3 for a detailed description)). However, our approach is different in many respects, as will be explained later. The main interest of our approach is to preserve a good empirical size even the underlying distribution is not Gaussian. From a sample X_1, \dots, X_n of the process (X_t) , we are interested in building a testing procedure to discriminate between short and long range dependence. The proposed statistic is inspired from the well-known fact that, for a large class of weakly dependent stationary processes, at any Fourier frequency, the normalized periodogram is asymptotically exponentially distributed. That is, for $\lambda_j = 2\pi j/n$,

$$\frac{I_n(\lambda_j)}{f(\lambda_j)} \xrightarrow{d} E, \quad (2)$$

where E has exponential distribution with mean 1, and where I_n is the periodogram

$$I_n(\lambda) = \frac{1}{n} \left| \sum_{j=1}^n X_j e^{ij\lambda} \right|^2.$$

Moreover, for different Fourier frequencies, the variables on the left hand side of (2) are asymptotically independent. Since under short memory, the periodogram is asymptotically unbiased, we estimate $f(\lambda_j)$ by a sample average of periodograms. The procedure is as follows: we split our initial sample X_1, \dots, X_n into m blocks (or epochs), each of size ℓ , and we construct the periodogram $I_{n,i}$ on the i th block $X_{(i-1)\ell+1}, \dots, X_{i\ell}$. The construction of this averaging of periodograms is well known as Bartlett method to estimate the spectral density. Then we define our test statistic as follows:

$$Q_{n,m}(s) = \sum_{j=1}^s \frac{I_n(\lambda_j)}{\frac{1}{m} \sum_{i=1}^m I_{n,i}(\lambda_j)}. \quad (3)$$

where s is the number of Fourier frequencies we want to include in the test.

We assume that $m = m(n)$, and $\ell = \ell(n) \rightarrow \infty$ as $n \rightarrow \infty$. It is very important to emphasize the fact that m and ℓ increase with n and are not constant.

We are simply using notation m and ℓ rather than $m(n)$ and $\ell(n)$ only for the sake of simplicity. We should mention that our context is different from that of Reisen *et al* (2010), where the number of epochs is fixed.

2 Main results

In this section we give the asymptotic distribution of $Q_{n,m}(s)$ under the null hypothesis "the process has a short memory" and the alternative of "the process has a long memory".

Theorem 1 Assume that (X_t) is a short memory process in the sense that f satisfies (1) with $d = 0$, and (X_t) admits a linear representation of the form

$$X_t = \sum_{j=0}^{\infty} a_j \epsilon_{t-j}$$

where (ϵ_j) are iid random variables with zero mean and finite fourth moments, and where $\sum_{j=0}^{\infty} a_j^2 < \infty$.

If $\frac{1}{m} + \frac{m}{n} \rightarrow 0$ then we have

$$Q_{n,m}(s) \xrightarrow{d} Q,$$

where Q has Gamma distribution with mean s and variance s .

According to this convergence result, the critical region of the testing procedure at significance level α is of the form $\{Q_{n,m}(s) > q_\alpha(s)\}$ where $q_\alpha(s)$ is the quantile of order $1 - \alpha$ of Q .

Theorem 2 Assume that (X_t) is a long memory process with spectral density f of the form (1) with $0 < d < 1/2$, and admits a linear representation with same assumptions as in Theorem 1. If $\frac{1}{m} + \frac{m}{n} \rightarrow 0$ then we have

$$Q_{n,m}(s) \xrightarrow{P} \infty.$$

Under the condition of Theorem 2, our testing procedure is consistent, i.e. the power converges to 1 under any long memory alternative.

Proof of Theorem 1. From (3) we get

$$Q_{n,m}(s) = \sum_{j=1}^s \frac{I_n(\lambda_j)/f(\lambda_j)}{\frac{1}{m} \sum_{i=1}^m I_{n,i}(\lambda_j)/f(\lambda_j)}.$$

Applying Giraitis *et al.* Th 5.3.1 (2012), we have

$$\left(\frac{I_n(\lambda_1)}{f(\lambda_1)}, \dots, \frac{I_n(\lambda_s)}{f(\lambda_s)} \right) \xrightarrow{d} (E_1, \dots, E_s)$$

where E_1, \dots, E_s are iid exponentially distributed with mean 1. Using Slutsky Lemma, it will then be enough to show that, for each j , as $n \rightarrow \infty$,

$$\frac{1}{m} \sum_{i=1}^m \frac{I_{n,i}(\lambda_j)}{f(\lambda_j)} \xrightarrow{P} 1,$$

or equivalently (since f is continuous, and $\lambda_j = \frac{2\pi j}{n}$, then $f(\lambda_j) \rightarrow f(0)$),

$$\frac{1}{m} \sum_{i=1}^m I_{n,i}(\lambda_j) \xrightarrow{P} f(0) = \gamma(0) + 2 \sum_{k=1}^{\infty} \gamma(k). \quad (4)$$

with $\gamma(h) = \text{Cov}(X_i, X_{i+h})$.

Denoting

$$\hat{\gamma}_{n,i}(h) = \frac{1}{\ell} \sum_{k=1+(i-1)\ell}^{i\ell-|h|} X_k X_{k+h},$$

we can write

$$\begin{aligned} I_{n,i}(\lambda) &= \frac{1}{\ell} \left| \sum_{k=1+(i-1)\ell}^{i\ell} X_k e^{i\lambda k} \right|^2 = \sum_{s,t=1+(i-1)\ell}^{i\ell} X_s X_t e^{i(s-t)\lambda} = \sum_{|h|<\ell} e^{-ih\lambda} \hat{\gamma}_{n,i}(h) \\ &= \hat{\gamma}_{n,i}(0) + 2 \sum_{h=1}^{\ell-1} \cos(\lambda h) \hat{\gamma}_{n,i}(h). \end{aligned}$$

Hence the LHS of (4) is made of two parts. Firstly,

$$\frac{1}{m} \sum_{i=1}^m \hat{\gamma}_{n,i}(0) = \frac{1}{m} \frac{1}{\ell} \sum_{i=1}^m \sum_{k=1+(i-1)\ell}^{i\ell} X_k^2 = \frac{1}{n} \sum_{u=1}^n X_u^2 \xrightarrow{a.s.} \sigma^2 = \gamma(0), \text{ as } n \rightarrow \infty \quad (5)$$

by virtue of ergodic theorem. Secondly, we prove the following L^2 convergence

$$\frac{2}{m} \sum_{i=1}^m \sum_{h=1}^{\ell-1} \hat{\gamma}_{n,i}(h) \cos(\lambda_j h) \xrightarrow{L^2} 2 \sum_{k=1}^{\infty} \gamma(k). \quad (6)$$

Observing that $(\hat{\gamma}_{n,1}(h), \dots, \hat{\gamma}_{n,m}(h))$ being identically distributed),

$$\begin{aligned} \mathbb{E} \left(\frac{2}{m} \sum_{i=1}^m \sum_{h=1}^{\ell-1} \hat{\gamma}_{n,i}(h) \cos(\lambda_j h) \right) &= 2 \sum_{h=1}^{\ell-1} \mathbb{E}(\hat{\gamma}_{n,1}(h)) \cos(\lambda_j h) \\ &= 2 \sum_{h=1}^{\ell-1} \gamma(h) (1 - h/\ell) \cos(\lambda_j h) \\ &\xrightarrow{n \rightarrow \infty} 2 \sum_{k=1}^{\infty} \gamma(k), \end{aligned}$$

by the dominated convergence theorem, it suffices to prove that, as $n \rightarrow \infty$,

$$\text{Var} \left(\frac{1}{m} \sum_{i=1}^m \sum_{h=1}^{\ell-1} \hat{\gamma}_{n,i}(h) \cos(\lambda_j h) \right) \rightarrow 0. \quad (7)$$

The LHS of (7) can be written as

$$\begin{aligned} &\frac{1}{m^2} \sum_{s=1}^m \sum_{t=1}^m \text{Cov} \left(\sum_{h=1}^{\ell-1} \hat{\gamma}_{n,s}(h) \cos(\lambda_j h), \sum_{h=1}^{\ell-1} \hat{\gamma}_{n,t}(h) \cos(\lambda_j h) \right) \\ &= \frac{1}{m} \sum_{u=-m}^m \left(1 - \frac{|u|}{m} \right) \text{Cov} \left(\sum_{h=1}^{\ell-1} \hat{\gamma}_{n,1}(h) \cos(\lambda_j h), \sum_{h=1}^{\ell-1} \hat{\gamma}_{n,1+|u|}(h) \cos(\lambda_j h) \right) \\ &= \frac{1}{m} \sum_{u=-m}^m \left(1 - \frac{|u|}{m} \right) \sum_{p=1}^{\ell-1} \sum_{q=1}^{\ell-1} \cos(p\lambda_j) \cos(q\lambda_j) \text{Cov} \left(\hat{\gamma}_{n,1}(p), \hat{\gamma}_{n,1+|u|}(q) \right) \end{aligned}$$

It will be enough to show that for large values of u ,

$$\sum_{p=1}^{\ell} \sum_{q=1}^{\ell} \text{Cov} \left(\hat{\gamma}_{n,1}(p), \hat{\gamma}_{n,1+u}(q) \right) \rightarrow 0, \quad \text{as } n, \text{ (or equivalently } \ell) \rightarrow \infty. \quad (8)$$

We have

$$\begin{aligned}
\text{Cov}\left(\hat{\gamma}_{n,1}(p), \hat{\gamma}_{n,1+u}(q)\right) &= \sum_{s=1}^{\ell-p} \sum_{t=1+\ell u}^{\ell(u+1)-q} \mathbb{E}[X_t X_{t+p} X_s X_{s+q}] - \frac{(\ell-p)(\ell-q)}{\ell^2} \gamma(p)\gamma(q) \\
&= \frac{1}{\ell^2} \sum_{s=1}^{\ell-p} \sum_{t=1}^{\ell-q} \mathbb{E}[X_t X_{t+p} X_{t+s-t+\ell u-p+p} X_{t+s-t+\ell u-p+p+q}] \\
&= \frac{1}{\ell^2} \sum_{s=1}^{\ell-p} \sum_{t=1}^{\ell-q} \left[\gamma(p)\gamma(q) + \gamma(s-t+\ell u)\gamma(s-t+\ell u+q-p) \right. \\
&\quad \left. + \gamma(s-t+\ell u+q)\gamma(s-t+\ell u-p) \right. \\
&\quad \left. + (\eta-3)\sigma^4 \sum_{i=-\infty}^{\infty} a_i a_{i+p} a_{i+s-t+\ell u} a_{i+s-t+\ell u+q} \right] - \frac{(\ell-p)(\ell-q)}{\ell^2} \gamma(p)\gamma(q) \\
&= \frac{1}{\ell} \sum_{h=1+q-\ell}^{\ell-p+1} \left(1 - \frac{|h|}{\ell}\right) T_u(h, p, q),
\end{aligned}$$

where

$$T_u(h, p, q) = T_u^{(1)}(h, p, q) + T_u^{(2)}(h, p, q) + T_u^{(3)}(h, p, q),$$

with

$$T_u^{(1)}(h, p, q) = \gamma(\ell u - h)\gamma(\ell u - h + q - p),$$

$$T_u^{(2)}(h, p, q) = \gamma(\ell u - h + q)\gamma(\ell u - h - p),$$

and

$$T_u^{(3)}(h, p, q) = (\eta-3)\sigma^4 \sum_{i=-\infty}^{\infty} a_i a_{i+p} a_{i+h+\ell u} a_{i+h+\ell u+q}.$$

Next for $u \geq 2$, putting $k = q - p$,

$$\begin{aligned}
\frac{1}{\ell} \sum_{p=1}^{\ell} \sum_{q=1}^{\ell} \sum_{h=-\ell}^{\ell} |T_u^{(1)}(h, p, q)| &= \sum_{h=-\ell}^{\ell} \sum_{k=-\ell}^{\ell} \left(1 - \frac{|k|}{\ell}\right) |\gamma(\ell u - h)\gamma(\ell u - h + k)| \\
&\leq \sum_{h=-\ell}^{\ell} \sum_{k=-\ell}^{\ell} |\gamma(\ell u - h)\gamma(\ell u - h + k)| \\
&= \sum_{h=-\ell}^{\ell} |\gamma(\ell u - h)| \left(\sum_{k=-\ell}^{\ell} |\gamma(\ell u - h + k)| \right) \\
&\leq 2 \sum_{i=\ell}^{\infty} |\gamma(i)| \left(2 \sum_{j=0}^{\infty} |\gamma(j)| \right) \rightarrow 0 \quad \text{as } \ell \rightarrow \infty,
\end{aligned}$$

since the covariances are summable. Similar computation shows also that

$$\frac{1}{\ell} \sum_{p=1}^{\ell} \sum_{q=1}^{\ell} \sum_{h=-\ell}^{\ell} |T_u^{(2)}(h, p, q)| \rightarrow 0, \quad \text{as } \ell \rightarrow \infty.$$

As for the last term, we get similarly as in Brockwell and Davis (1991, pages 226-227), permuting the order of summation, as $\ell \rightarrow \infty$,

$$\sum_{h=-\ell}^{\ell} T_u^{(3)}(h, p, q) \rightarrow \sum_{i=-\infty}^{\infty} \sum_{h=-\infty}^{\infty} a_i a_{i+p} a_{i+h+\ell u} a_{i+h+\ell u+q} = \gamma(p)\gamma(q)$$

and therefore, as $\ell \rightarrow \infty$,

$$\frac{1}{\ell} \sum_{p=1}^{\ell} \sum_{q=1}^{\ell} \sum_{h=-\ell}^{\ell} T_u^{(3)}(h, p, q) \rightarrow 0,$$

which completes the proof of (8).

Proof of Theorem 2 As $Q_{n,m}(s)$ is the som of nonnegative terms, it will be enough to show that for each $j = 1, \dots, s$

$$\frac{I_n(\lambda_j)/f(\lambda_j)}{\frac{1}{m} \sum_{i=1}^m I_{n,i}(\lambda_j)/f(\lambda_j)} \xrightarrow{P} \infty. \quad (9)$$

Using Deo (1997), the numerator in (9) converges in distribution to a non degenerated positive random variable. Therefore it will be enough to show that

$$\frac{1}{m} \sum_{i=1}^m \frac{I_{n,i}(\lambda_j)}{f(\lambda_j)} \xrightarrow{P} 0 \quad \text{as } n \rightarrow \infty. \quad (10)$$

It is well known that if $Z_n \geq 0$ and $\mathbb{E}(Z_n) \rightarrow 0$ as $n \rightarrow 0$ then $Z_n \rightarrow 0$ in probability. Therefore, since the variables involved in (10) are nonnegative, it suffices to show that

$$\frac{1}{m} \mathbb{E} \left(\sum_{i=1}^m \frac{I_{n,i}(\lambda_j)}{f(\lambda_j)} \right) \rightarrow 0 \quad \text{as } n \rightarrow \infty. \quad (11)$$

But $I_{n,1}, \dots, I_{n,m}$ are identically distributed, the LHS of (11) is simply equal to

$$\mathbb{E} \left(\frac{I_{n,1}(\lambda_j)}{f(\lambda_j)} \right) = \frac{\ell^{-1} \sum_{s,t=1}^{\ell} \gamma(s-t) e^{i(s-t)\lambda_j}}{f(\lambda_j)} = \frac{\sum_{h=-\ell}^{\ell} \left(1 - \frac{|h|}{\ell}\right) \gamma(h) \cos(\lambda_j h)}{f(\lambda_j)}. \quad (12)$$

Under (1), $f(\lambda) \sim c\lambda^{-2d}$ near zero ($0 < d < 1/2$), thus $f(\lambda_j) \sim cn^{2d}$, and the numerator above is bounded by

$$\sum_{h=-\ell}^{\ell} |\gamma(h)| \leq C_1 \sum_{h=1}^{\ell} h^{2d-1} \sim C_2 \ell^{2d}$$

for some positive constants c, C_1, C_2 . Hence the ratio (12) is $O((\ell/n)^{2d}) \rightarrow 0$ as $n \rightarrow \infty$ since $d > 0$ and $\frac{\ell}{n} = \frac{1}{m} \rightarrow 0$.

3 Simulation Studies

In this section we illustrate the performance of our test based on $Q_{n,m}$ statistic and we compare it with the well known spectral domain test proposed by Lobato and Robinson (1998) and the time domain one based on VS statistic, briefly introduced below.

Lobato Robinson test (LR test).

In a one-dimensional case, Lobato and Robinson test statistic is $LM = t^2$, where

$$t = \sqrt{m} \frac{\sum_{j=1}^m \nu_j \operatorname{Re}(I(\lambda_j))}{\sum_{j=1}^m \operatorname{Re}(I(\lambda_j))},$$

and

$$\nu_j = \log j - \frac{1}{m} \sum_{j=1}^m \log j.$$

Under certain conditions on the bandwidth m , if the spectral density is twice boundedly differentiable near 0 and $d = 0$, the statistic t^2 converges in distribution to χ^2 with 1 degree of freedom. Our approach is different since we use a blocking method rather than estimating $f(0)$ and we do not assume such smoothness conditions on the spectral density near zero, except the continuity of g in (1), or on the tuning parameters except that the epoch length, $\ell(n) = o(n)$. In the simulation (Figure 3) we select for m the optimal value given in Lobato and Robinson (1998).

V/S test.

This test is defined as the ratio $M_n = V_n/s_{n,q}^2$, where

$$V_n = \frac{1}{n^2} \left[\sum_{k=1}^n (S_k^*)^2 - \frac{1}{n} \left(\sum_{k=1}^n S_k^* \right)^2 \right],$$

with $S_k^* = \sum_{j=1}^k (X_j - \bar{X}_n)$, $\hat{s}_{n,q}^2 = \tilde{\gamma}_0 + 2 \sum_{j=1}^q \left(1 - \frac{j}{q}\right) \tilde{\gamma}_j$, and where $\tilde{\gamma}$ is the sample covariance

$$\tilde{\gamma}_j = n^{-1} \sum_{i=1}^{n-j} (X_i - \bar{X}_n)(X_{i+j} - \bar{X}_n), \quad 0 \leq j < n$$

The null hypothesis of V/S test consists in the summability of the auto-covariances and the convergence of partial sums normalized by \sqrt{n} towards the Brownian motion (essentially $d = 0$ in our context). Under some technical conditions on the bandwidth q and the fourth order cumulants (see Giraitis *et al.* (2003) for the exact hypotheses), M_n converges to non degenerated distribution with cdf $F_{VS}(x) = F_K(\pi\sqrt{x})$ where F_K is the asymptotic cdf of the standard Kolmogorov statistic. The authors suggest the use of $q = n^{1/3}$ for optimal results. This is the chosen value in our simulation (see Figure 3).

Choice of m in $Q_{n,m}$ defined in (3)

The choice of m is done to ensure that the empirical size will be close to the nominal size of the test. To do this we simulate independent copies of AR(1) and we evaluate for each sample the pvalue associated to $Q_{n,m}$ -test. Then we draw the empirical cumulative distribution function of pvalues. Under the null hypothesis this cdf should match the uniform distribution's.

Figure 3 represents these empirical cdfs for several Gaussian AR(1) models. We take $m = n^b$, $b = .3, .4, .5, .6$ and $n = 1000, 5000$ and 10000 .

Figure 3 shows a good fitting between the line $y = x$ and the empirical cdf associated with $m = \sqrt{n}$. Note that the importance of this choice tends to vanish as n increases. These numerical results are then in favour of this choice i.e. $m = \sqrt{n}$. We also note that moderately increasing s or deviating from normality of the innovation lead to the same findings.

Non Gaussian innovations

Simulations have shown that under normality assumption LR test tends to outperform all time domain tests such as VS, KPSS, RS etc. Here we compare our $Q_{n,m}$ test with LR test and VS test under the null hypothesis when the innovations of AR(1) are non Gaussian. We take centered exponential errors. Figure 3 shows a lack of robustness to non-normality of VS and LR tests in the sense that they seem to suffer from high empirical size. On the opposite, $Q_{n,m}$ test seems to have a more reasonable empirical size. Similar conclusions hold for centered log-normal errors as well.

Power of $Q_{n,m}$ test.

Figure 3 illustrates the behavior of the empirical power of this test when s varies. We only represent the graphs for the non-Gaussian case but there are no noticeable differences with the Gaussian case. LR and VS tests tend to be more powerful than $Q_{n,m}$ test especially in Gaussian case. Since their empirical sizes are quite high as compared to $Q_{n,m}$ test, this is very much anticipated. And so the power comparison at the same significance level is not well sound.

Our findings confirm that frequency-domain approach should be used more often when it comes to testing for long memory or unit root (as AR(1) coefficient tends to 1) in the time series analysis.

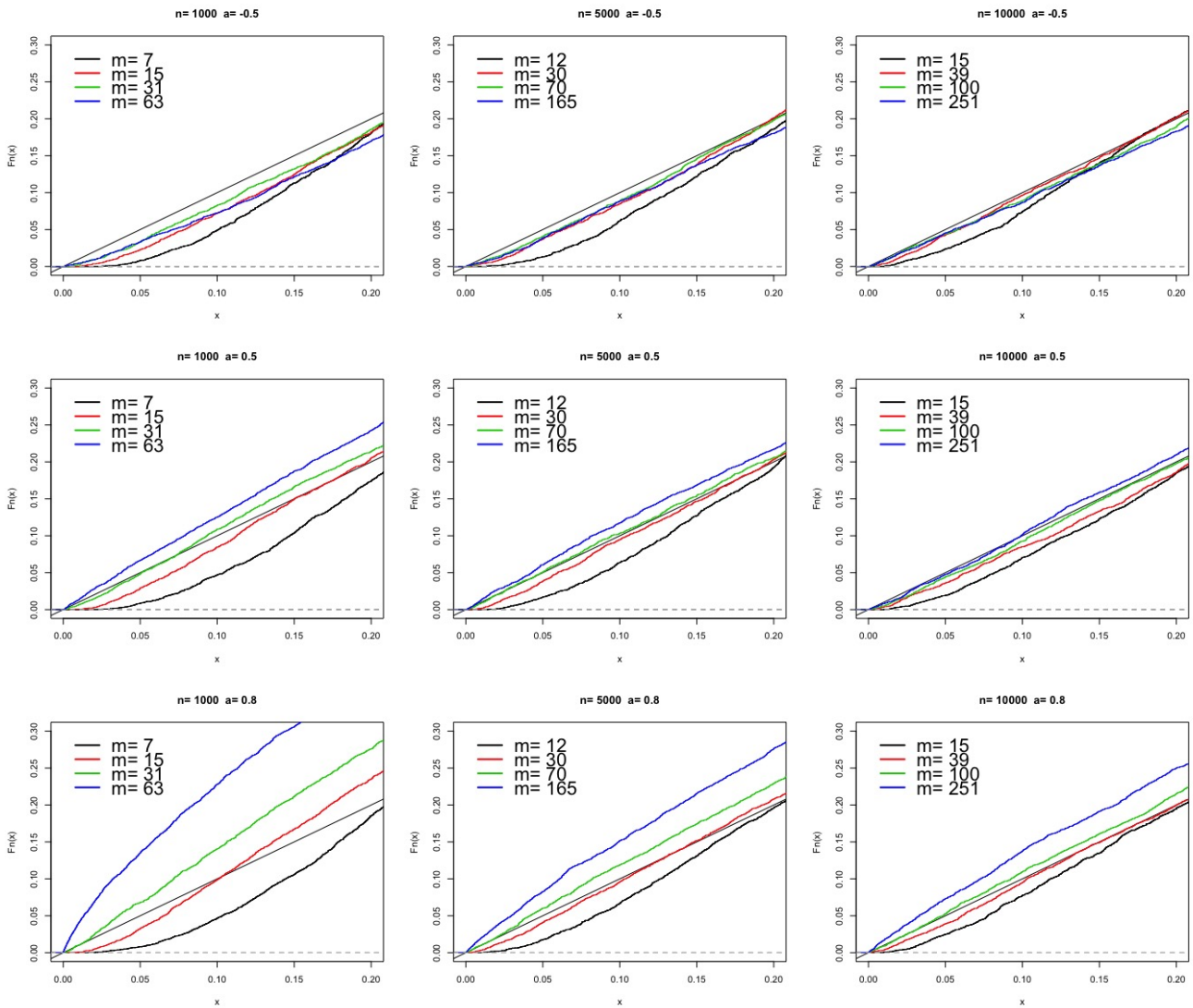


Figure 1: Empirical size: Empirical cdf of pvalues for various values of m, n and fixed $s = 2$: Results based on 5000 independent replications of AR(1) with coefficient $a = -0.5, 0.5, 0.8$ models with Gaussian innovations. The sample size $n = 10000$.

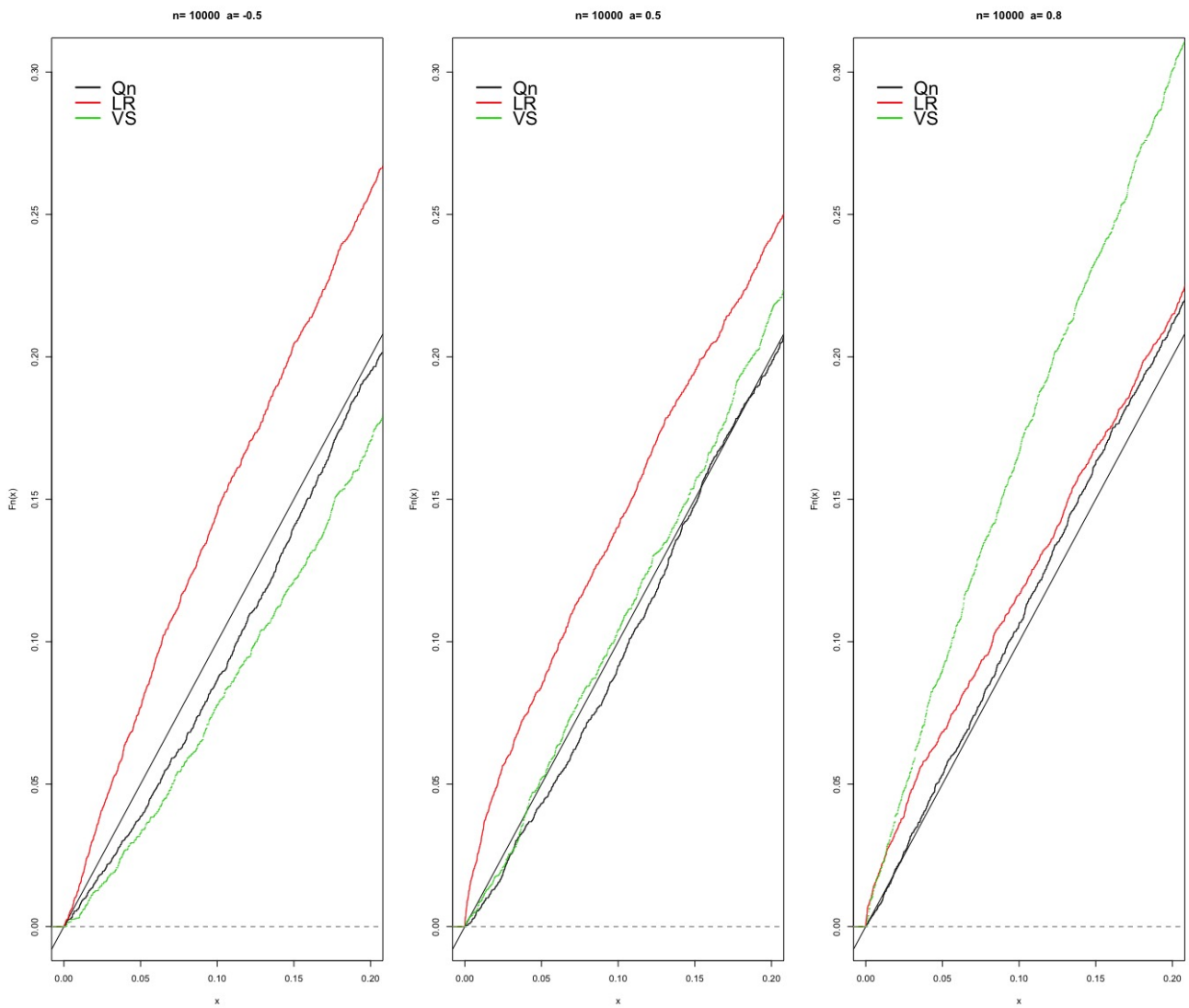


Figure 2: Empirical size : Empirical cdf of pvalues for the three testing procedures. Results based on 5000 independent replications of AR(1) with coefficient $a = -0.5, 0.5, 0.8$ models with non gaussian innovations

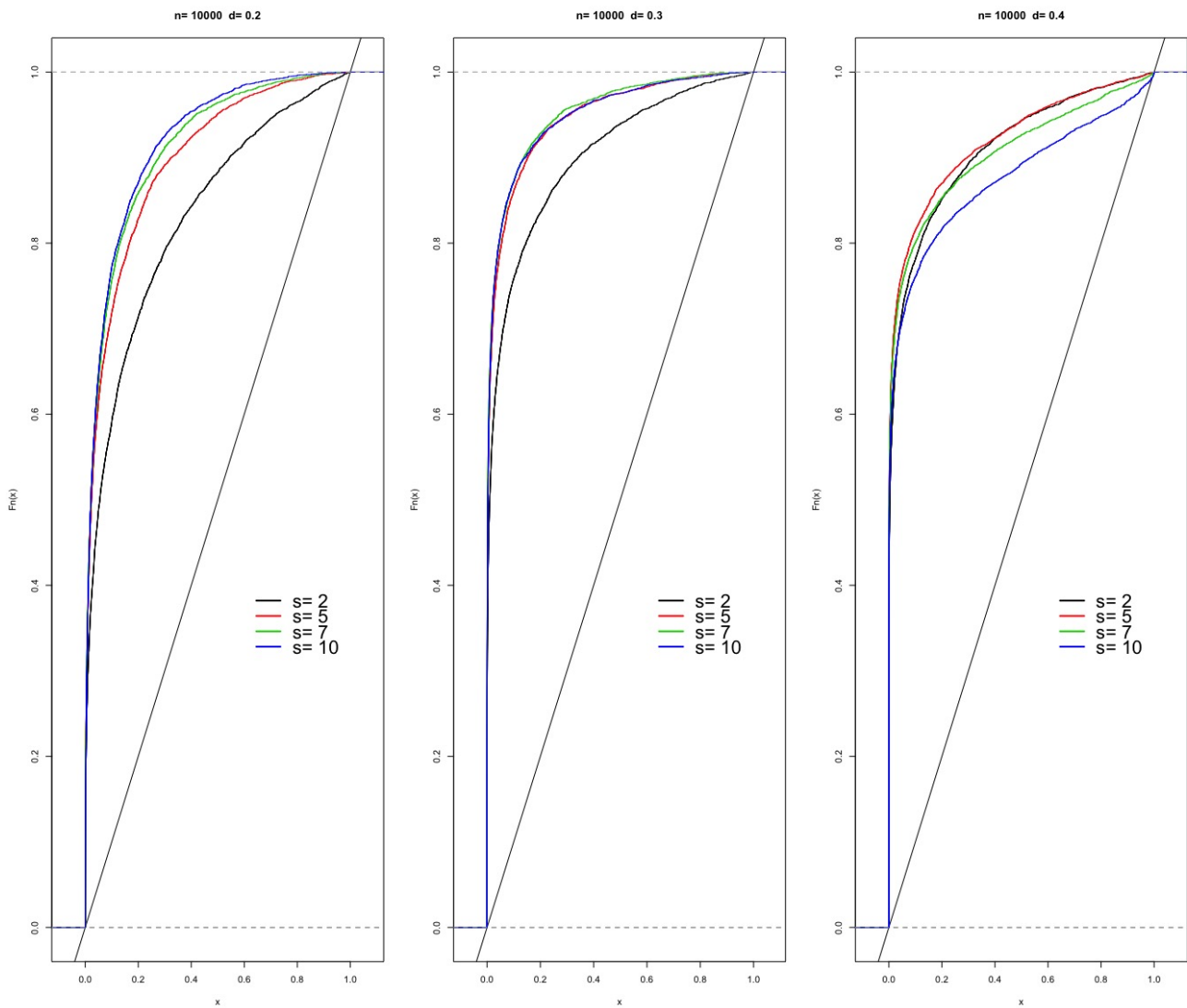


Figure 3: Power function : Empirical cdf of pvalues for various values of s and fixed $m = \sqrt{n}$ and $n = 10000$ Results based on 5000 independent replications of ARFIMA(0, d ,0) with long memory parameter $d = 0.2, 0.3, 0.4$ models with non gaussian innovations

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