

A NEW DESIGN PARADIGM FOR THE ANALYSIS AND OPTIMISATION OF COMPOSITE STRUCTURES

M. Montemurro, Arts et Métiers ParisTech, I2M CNRS UMR 5295

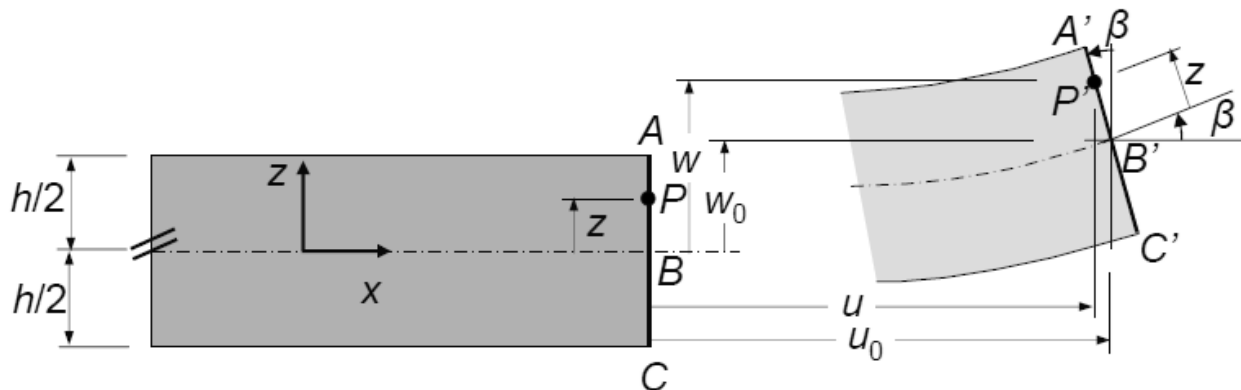
Table of contents

- The classical laminate theory
- The polar analysis of the First-order Shear Deformation Theory
- The polar analysis of the Third-order Shear Deformation Theory
- Designing the elastic symmetries of a laminate: a unified approach
- Example: simultaneous shape and material optimisation of sandwich panels with honeycomb core for additive manufacturing
- Example: multi-scale design and optimisation of variable angle tow laminates

The classical laminate theory (CLT)

Hypotheses (Kirchhoff-Love model)

- straight lines normal to the mid-surface remain normal to the mid-surface after deformation
- straight lines normal to the mid-surface remain straight after deformation
- the thickness of the plate does not change during the deformation



Drawbacks

1. The effect of the plate transverse shear stiffness is not taken into account
2. Not suited for moderately thick or thick plates

The polar analysis of the First-order Shear Deformation Theory (FSDT)

Hypotheses

- ~~straight lines normal to the mid-surface remain normal to the mid-surface after deformation~~
- straight lines normal to the mid-surface remain straight after deformation
- the thickness of the plate does not change during the deformation

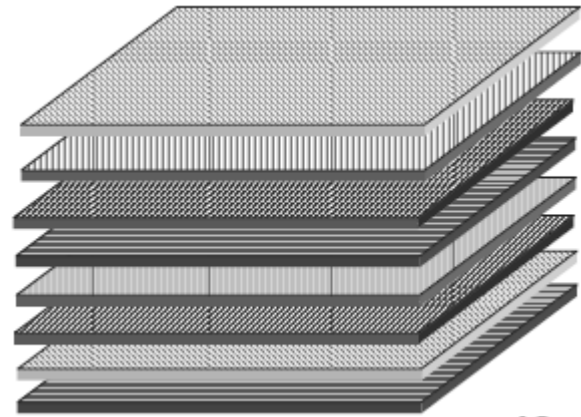
Displacement field of the laminate

$$u(x, y, z) = u_0(x, y) + \phi_x(x, y)z$$

$$v(x, y, z) = v_0(x, y) + \phi_y(x, y)z$$

$$w(x, y, z) = w_0(x, y)$$

5 kinematic unknowns: $u_0, v_0, w_0, \phi_x, \phi_y$



M. Montemurro. An extension of the polar method to the First-order Shear Deformation Theory of laminates. *Composite Structures* 127: 328-339, 2015.

M. Montemurro. Corrigendum to "An extension of the polar method to the First-order Shear Deformation Theory of laminates"[*Compos. Struct.* 127 (2015) 328–339]. *Composite Structures* 131: 1143-1144, 2015.

The polar analysis of the FSDT

In-plane reduced stiffness matrix of the constitutive lamina

$$Q_{xx} = T_0 + 2T_1 + R_0 \cos 4(\Phi_0 + \delta_k) + 4R_1 \cos 2(\Phi_1 + \delta_k),$$

$$Q_{xy} = -T_0 + 2T_1 - R_0 \cos 4(\Phi_0 + \delta_k),$$

$$Q_{xs} = R_0 \sin 4(\Phi_0 + \delta_k) + 2R_1 \sin 2(\Phi_1 + \delta_k),$$

$$Q_{yy} = T_0 + 2T_1 + R_0 \cos 4(\Phi_0 + \delta_k) - 4R_1 \cos 2(\Phi_1 + \delta_k),$$

$$Q_{ys} = -R_0 \sin 4(\Phi_0 + \delta_k) + 2R_1 \sin 2(\Phi_1 + \delta_k),$$

$$Q_{ss} = T_0 - R_0 \cos 4(\Phi_0 + \delta_k),$$

Transverse shear stiffness matrix of the constitutive lamina

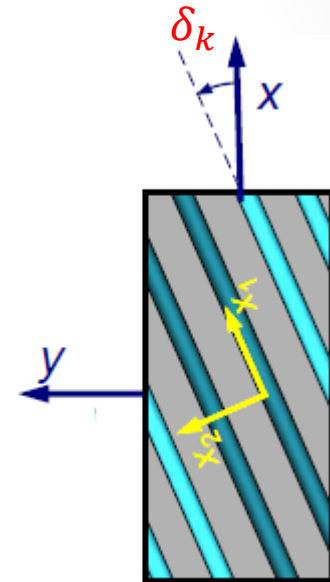
$$\widehat{Q}_{qq} = T + R \cos 2(\Phi - \delta_k),$$

$$\widehat{Q}_{qr} = R \sin 2(\Phi - \delta_k),$$

$$\widehat{Q}_{rr} = T - R \cos 2(\Phi - \delta_k).$$

Remarks

- $[Q] \rightarrow$ fourth-rank elasticity-like plane tensor
- $[\widehat{Q}] \rightarrow$ second-rank symmetric plane tensor with the local frame turned *clockwise* by an angle δ_k



The polar analysis of the FSDT

Laminate constitutive equation

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\epsilon_0\} \\ \{\chi_0\} \end{Bmatrix},$$

$$\{F\} = [H] \{\gamma_0\},$$

h : overall thickness of the laminate

n : number of layers

Laminate with identical plies

$$[A] = \frac{h}{n} \sum_{k=1}^n [Q(\delta_k)],$$

$$[B] = \frac{1}{2} \left(\frac{h}{n}\right)^2 \sum_{k=1}^n b_k [Q(\delta_k)],$$

$$[D] = \frac{1}{12} \left(\frac{h}{n}\right)^3 \sum_{k=1}^n d_k [Q(\delta_k)],$$

$$[H] = \begin{cases} \frac{h}{n} \sum_{k=1}^n [\hat{Q}(\delta_k)] & \text{(basic),} \\ \frac{5h}{12n^3} \sum_{k=1}^n (3n^2 - d_k) [\hat{Q}(\delta_k)] & \text{(modified).} \end{cases}$$

Remarks

- Basic form of $[H]$: shear stresses constant through the thickness of the ply
- Modified form of $[H]$: a) parabolic variation of the shear stresses; b) the shear stresses vanish on the top and bottom face of the laminate

The polar analysis of the FSDT

Laminate homogenised stiffness matrices

$$[A^*] = \frac{1}{h} [A],$$

$$[B^*] = \frac{2}{h^2} [B],$$

$$[D^*] = \frac{12}{h^3} [D],$$

$$[H^*] = \begin{cases} \frac{1}{h} [H] & \text{(basic),} \\ \frac{12}{5h} [H] & \text{(modified).} \end{cases}$$

$$[C_1^*] = [A^*] - [D^*],$$

Polar parameters of $[H^*]$

$$T_{H^*} = \begin{cases} T & \text{(basic),} \\ 2T & \text{(modified),} \end{cases}$$

$$R_{H^*} e^{i2\Phi_{H^*}} = \begin{cases} \frac{1}{n} R e^{i2\Phi} \sum_{k=1}^n e^{-i2\delta_k} & \text{(basic),} \\ \frac{1}{n^3} R e^{i2\Phi} \sum_{k=1}^n (3n^2 - d_k) e^{-i2\delta_k} & \text{(modified),} \end{cases}$$

Polar parameters of $[A^*]$, $[B^*]$, $[D^*]$

$$T_{0A^*} = T_0,$$

$$T_{1A^*} = T_1,$$

$$R_{0A^*} e^{i4\Phi_{0A^*}} = \frac{1}{n} R_0 e^{i4\Phi_0} \sum_{k=1}^n e^{i4\delta_k},$$

$$R_{1A^*} e^{i2\Phi_{1A^*}} = \frac{1}{n} R_1 e^{i2\Phi_1} \sum_{k=1}^n e^{i2\delta_k},$$

$$T_{0B^*} = 0,$$

$$T_{1B^*} = 0,$$

$$R_{0B^*} e^{i4\Phi_{0B^*}} = \frac{1}{n^2} R_0 e^{i4\Phi_0} \sum_{k=1}^n b_k e^{i4\delta_k},$$

$$R_{1B^*} e^{i2\Phi_{1B^*}} = \frac{1}{n^2} R_1 e^{i2\Phi_1} \sum_{k=1}^n b_k e^{i2\delta_k},$$

$$T_{0D^*} = T_0,$$

$$T_{1D^*} = T_1,$$

$$R_{0D^*} e^{i4\Phi_{0D^*}} = \frac{1}{n^3} R_0 e^{i4\Phi_0} \sum_{k=1}^n d_k e^{i4\delta_k},$$

$$R_{1D^*} e^{i2\Phi_{1D^*}} = \frac{1}{n^3} R_1 e^{i2\Phi_1} \sum_{k=1}^n d_k e^{i2\delta_k},$$

The polar analysis of the FSDT

Major analytical results

$$R_{H^*} e^{i2\phi_{H^*}} = \begin{cases} R_{1A^*} \frac{R}{R_1} e^{i2(\phi + \phi_1 - \phi_{1A^*})} & \text{(basic),} \\ \frac{R}{R_1} e^{i2(\phi + \phi_1)} (3R_{1A^*} e^{-i2\phi_{1A^*}} - R_{1D^*} e^{-i2\phi_{1D^*}}) & \text{(modified),} \end{cases}$$

1. The overall mechanical response of the laminate depends only on the anisotropic polar moduli and the related polar angles of matrices $[A^*]$, $[B^*]$, $[D^*]$ (12 polar parameters)
2. After choosing the material of the constitutive ply, the number of polar parameters to be designed remains unchanged when passing from the CLT to the FSDT
3. Basic formulation: $[H^*]$ is isotropic if and only if $[A^*]$ has a square symmetric behaviour $R_{H^*} = 0 \Leftrightarrow R_{1A^*} = 0$
4. Modified formulation: a sufficient condition ensuring an isotropic behaviour for $[H^*]$ is that $R_{1A^*} = R_{1D^*} = 0$ (the converse relationship is not necessarily true)
5. Modified formulation: if $[C_1^*] = [O]$ then $R_{H^*} = 0 \Leftrightarrow R_{1A^*} = R_{1D^*} = 0$

The polar analysis of the Third-order Shear Deformation Theory (TSDT)

Hypotheses

- straight lines normal to the mid-surface remain normal to the mid-surface after deformation
- straight lines normal to the mid-surface remain straight after deformation
- the thickness of the plate does not change during the deformation

Displacement field of the laminate

$$u(x, y, z) = u_0(x, y) + \phi_x(x, y) \left(z - \frac{4z^3}{3h^2} \right) - \frac{4z^3}{3h^2} \frac{\partial w_0}{\partial x},$$

$$v(x, y, z) = v_0(x, y) + \phi_y(x, y) \left(z - \frac{4z^3}{3h^2} \right) - \frac{4z^3}{3h^2} \frac{\partial w_0}{\partial y},$$

$$w(x, y, z) = w_0(x, y),$$

(valid for traction-free BCs on the top and bottom faces of the laminate)

5 kinematic unknowns: $u_0, v_0, w_0, \phi_x, \phi_y$

M. Montemurro. The polar analysis of the Third-order Shear Deformation Theory of laminates. *Composite Structures* 131: 775-789, 2015.

The polar analysis of the TSDT

Laminate constitutive equation (small strains and moderate rotations)

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{P\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [E] \\ [B] & [D] & [F] \\ [E] & [F] & [H] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^{(0)}\} \\ \{\varepsilon^{(1)}\} \\ \{\varepsilon^{(3)}\} \end{Bmatrix},$$

$$\begin{Bmatrix} \{Q\} \\ \{R\} \end{Bmatrix} = \begin{bmatrix} [\hat{A}] & [\hat{D}] \\ [\hat{D}] & [\hat{F}] \end{bmatrix} \begin{Bmatrix} \{\gamma^{(0)}\} \\ \{\gamma^{(2)}\} \end{Bmatrix}.$$

Coupling matrices (basic and higher-order terms)

Higher-order flexural stiffness matrices

Higher-order shear stiffness matrices

Laminate with identical plies

$$[A] = \frac{h}{n} \sum_{k=1}^n [Q(\delta_k)], \quad [B] = \frac{1}{2} \left(\frac{h}{n}\right)^2 \sum_{k=1}^n b_k [Q(\delta_k)],$$

$$[D] = \frac{1}{12} \left(\frac{h}{n}\right)^3 \sum_{k=1}^n d_k [Q(\delta_k)], \quad [E] = \frac{1}{8} \left(\frac{h}{n}\right)^4 \sum_{k=1}^n e_k [Q(\delta_k)],$$

$$[F] = \frac{1}{80} \left(\frac{h}{n}\right)^5 \sum_{k=1}^n f_k [Q(\delta_k)], \quad [H] = \frac{1}{448} \left(\frac{h}{n}\right)^7 \sum_{k=1}^n h_k [Q(\delta_k)],$$

$$[\hat{A}] = \frac{h}{n} \sum_{k=1}^n [\hat{Q}(\delta_k)], \quad [\hat{D}] = \frac{1}{12} \left(\frac{h}{n}\right)^3 \sum_{k=1}^n d_k [\hat{Q}(\delta_k)],$$

$$[\hat{F}] = \frac{1}{80} \left(\frac{h}{n}\right)^5 \sum_{k=1}^n f_k [\hat{Q}(\delta_k)],$$

The polar analysis of the TSDT

Laminate homogenised stiffness matrices

$$[A^*] = \frac{1}{h} [A], \quad [B^*] = \frac{2}{h^2} [B], \quad [D^*] = \frac{12}{h^3} [D],$$

$$[E^*] = \frac{8}{h^4} [E], \quad [F^*] = \frac{80}{h^5} [F], \quad [H^*] = \frac{448}{h^7} [H],$$

$$[\hat{A}^*] = \frac{1}{h} [\hat{A}], \quad [\hat{D}^*] = \frac{12}{h^3} [\hat{D}], \quad [\hat{F}^*] = \frac{80}{h^5} [\hat{F}].$$

The laminate homogeneity matrices

$$[C_1^*] = [A^*] - [D^*], \quad [C_2^*] = [D^*] - [F^*], \quad [C_3^*] = [F^*] - [H^*].$$

The polar analysis of the TSDT

Polar parameters of $[\mathbf{E}^*]$, $[\mathbf{F}^*]$, $[\mathbf{H}^*]$

$$T_{0E^*} = 0,$$

$$T_{1E^*} = 0,$$

$$R_{0E^*} e^{i4\Phi_{0E^*}} = \frac{1}{n^4} R_0 e^{i4\Phi_0} \sum_{k=1}^n e_k e^{i4\delta_k},$$

$$R_{1E^*} e^{i2\Phi_{1E^*}} = \frac{1}{n^4} R_1 e^{i2\Phi_1} \sum_{k=1}^n e_k e^{i2\delta_k},$$

$$T_{0F^*} = T_0,$$

$$T_{1F^*} = T_1,$$

$$R_{0F^*} e^{i4\Phi_{0F^*}} = \frac{1}{n^5} R_0 e^{i4\Phi_0} \sum_{k=1}^n f_k e^{i4\delta_k},$$

$$R_{1F^*} e^{i2\Phi_{1F^*}} = \frac{1}{n^5} R_1 e^{i2\Phi_1} \sum_{k=1}^n f_k e^{i2\delta_k},$$

$$T_{0H^*} = T_0,$$

$$T_{1H^*} = T_1,$$

$$R_{0H^*} e^{i4\Phi_{0H^*}} = \frac{1}{n^7} R_0 e^{i4\Phi_0} \sum_{k=1}^n h_k e^{i4\delta_k},$$

$$R_{1H^*} e^{i2\Phi_{1H^*}} = \frac{1}{n^7} R_1 e^{i2\Phi_1} \sum_{k=1}^n h_k e^{i2\delta_k},$$

Polar parameters of $[\hat{\mathbf{A}}^*]$, $[\hat{\mathbf{D}}^*]$, $[\hat{\mathbf{F}}^*]$

$$T_{\hat{A}^*} = T,$$

$$R_{\hat{A}^*} e^{i2\Phi_{\hat{A}^*}} = \frac{1}{n} R e^{i2\Phi} \sum_{k=1}^n e^{-i2\delta_k},$$

$$T_{\hat{D}^*} = T,$$

$$R_{\hat{D}^*} e^{i2\Phi_{\hat{D}^*}} = \frac{1}{n^3} R e^{i2\Phi} \sum_{k=1}^n d_k e^{-i2\delta_k},$$

$$T_{\hat{F}^*} = T,$$

$$R_{\hat{F}^*} e^{i2\Phi_{\hat{F}^*}} = \frac{1}{n^5} R e^{i2\Phi} \sum_{k=1}^n f_k e^{-i2\delta_k}.$$

The polar analysis of the TSDT

Major analytical results

$$\begin{aligned} R_{\hat{A}^*} e^{i2\Phi_{\hat{A}^*}} &= \frac{R}{R_1} R_{1A^*} e^{i2(\Phi + \Phi_1 - \Phi_{1A^*})}, \\ R_{\hat{D}^*} e^{i2\Phi_{\hat{D}^*}} &= \frac{R}{R_1} R_{1D^*} e^{i2(\Phi + \Phi_1 - \Phi_{1D^*})}, \\ R_{\hat{F}^*} e^{i2\Phi_{\hat{F}^*}} &= \frac{R}{R_1} R_{1F^*} e^{i2(\Phi + \Phi_1 - \Phi_{1F^*})}. \end{aligned}$$

1. The overall mechanical response of the laminate depends only on the anisotropic polar moduli and the related polar angles of matrices $[A^*]$, $[B^*]$, $[D^*]$, $[E^*]$, $[F^*]$, $[H^*]$ (24 polar parameters)
2. After choosing the material of the constitutive ply and **under some prescribed hypotheses**, the number of polar parameters to be designed remains unchanged when passing from the CLT to the TSDT
3. No need of defining a “shear correction factor” to overcome static inconsistencies

The polar analysis of the TSDT

Major analytical results

Fully uncoupled laminate

$$[\mathbf{B}^*] = [\mathbf{E}^*] = [\mathbf{0}];$$

Laminate homogeneous in bending

$$[\mathbf{C}_2^*] = [\mathbf{C}_3^*] = [\mathbf{0}];$$

Laminate homogeneous in membrane and bending

$$[\mathbf{C}_1^*] = [\mathbf{C}_2^*] = [\mathbf{C}_3^*] = [\mathbf{0}];$$

Fully uncoupled and homogeneous laminate

$$[\mathbf{B}^*] = [\mathbf{E}^*] = [\mathbf{C}_1^*] = [\mathbf{C}_2^*] = [\mathbf{C}_3^*] = [\mathbf{0}].$$

Remarks

- The homogeneity conditions apply also for the shear stiffness matrices
- If the laminate is fully uncoupled the number of polar parameters characterising the mechanical response reduces from 24 to 16
- If the laminate is fully uncoupled and homogeneous the number of polar parameters to be designed reduces from 24 to 4

Designing the elastic symmetries of a laminate: a unified approach

GOAL → find at least one stacking sequence meeting the elastic requirements for the multilayer plate (in terms of the elastic symmetries of the different stiffness matrices governing the laminate behaviour)

- In the framework of the polar formalism (applied to the TSDT), this problem can be stated in the form of an **unconstrained minimisation problem**

$$\min_{\{\delta_1, \dots, \delta_n\}} \Psi(\delta_1, \dots, \delta_n) = \{\mathbf{f}\}^T [\mathbf{W}] \{\mathbf{f}\},$$

- The objective function is
 - **positive semi-definite and convex in the space of the laminate polar parameters** (sum of convex functions)
 - **highly non-convex in the space of plies orientations** (the polar parameters depend upon circular functions of layers orientation angles)
- The **design variables** are the **plies orientations** $\{\delta_1, \dots, \delta_n\}$
- $\{\mathbf{f}\} \in \mathbb{R}^{37 \times 1}$: vector of the partial objective functions
- $[\mathbf{W}] \in \mathbb{R}^{37 \times 37}$: positive semi-definite diagonal weight matrix

Designing the elastic symmetries of a laminate: numerical examples

1. Fully uncoupled laminate with square symmetric membrane behaviour (with a prescribed orientation of the main axis of orthotropy)

$$\Psi = \left(\frac{\| \| [B^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [E^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{R_{1A^*}}{R_1} \right)^2 + \left(\frac{\Phi_{0A^*} - \widehat{\Phi}_{0A^*}}{\pi/4} \right)^2.$$

2. Fully uncoupled laminate homogeneous in bending with an orthotropic behaviour (with a prescribed orientation of the main axis of orthotropy)

$$\Psi = \left(\frac{\| \| [B^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [E^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [C_2^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [C_3^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\Phi_{0D^*} - \Phi_{1D^*} - \pi/4}{\pi/4} \right)^2 + \left(\frac{\Phi_{1D^*} - \widehat{\Phi}_{1D^*}}{\pi/4} \right)^2.$$

3. Fully uncoupled and homogeneous laminate with a square symmetric behaviour (with a prescribed orientation of the main axis of orthotropy)

$$\Psi = \left(\frac{\| \| [B^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [E^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [C_1^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [C_2^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{\| \| [C_3^*] \| \|}{\| \| [Q] \| \|} \right)^2 + \left(\frac{R_{1A^*}}{R_1} \right)^2 + \left(\frac{\Phi_{0A^*} - \widehat{\Phi}_{0A^*}}{\pi/4} \right)^2.$$

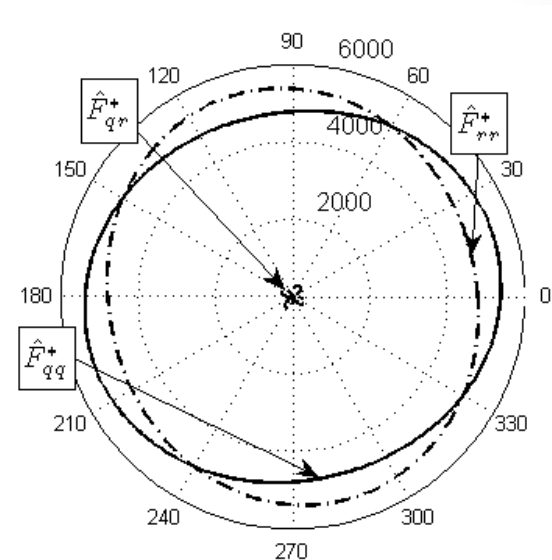
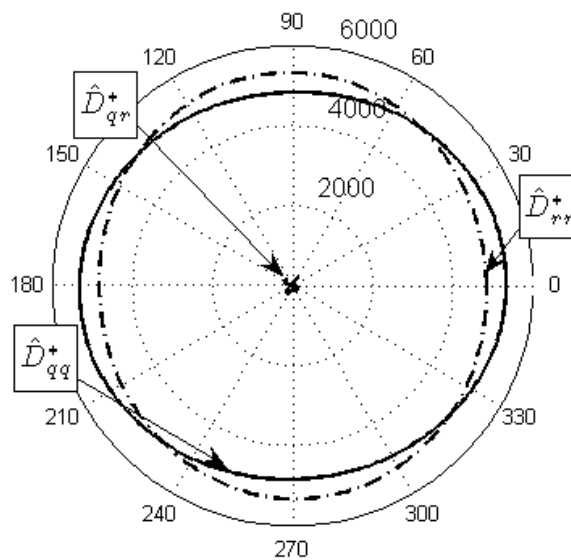
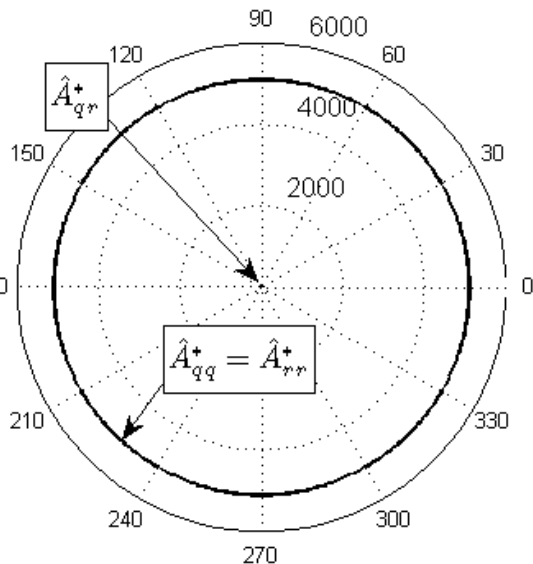
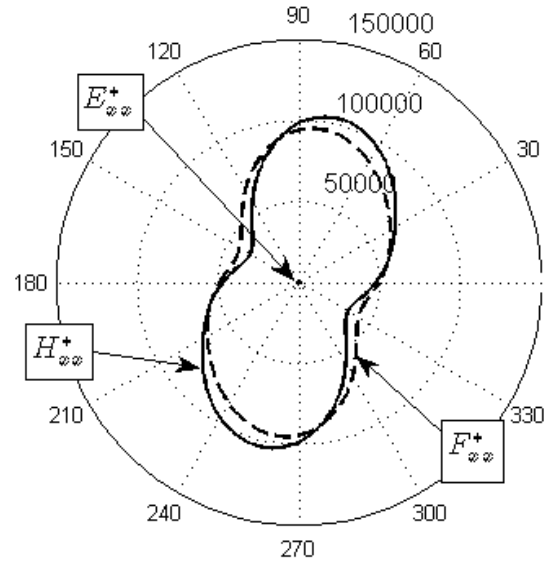
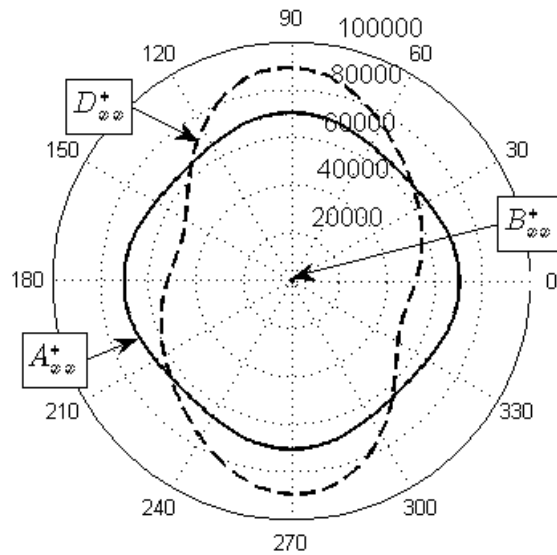
Designing the elastic symmetries of a laminate: numerical examples

Number of individuals	Number of generations	Crossover probability	Mutation probability	Selection operator
500	500	0.85	$1/N_{\text{ind}}$	Roulette wheel

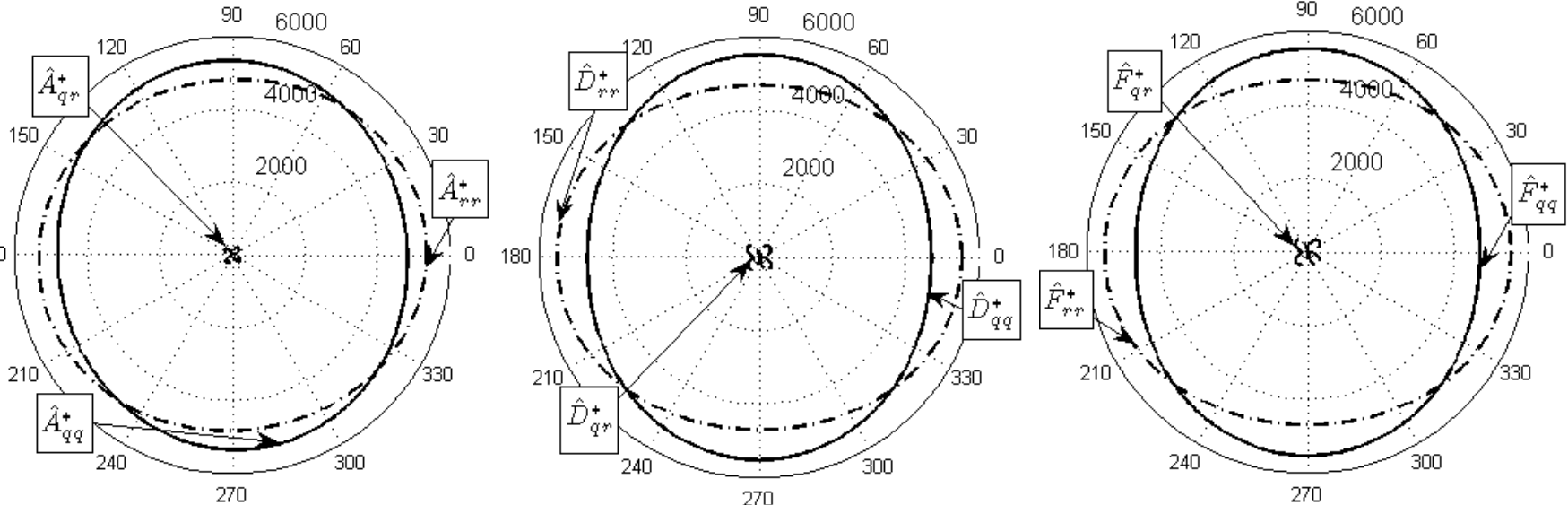
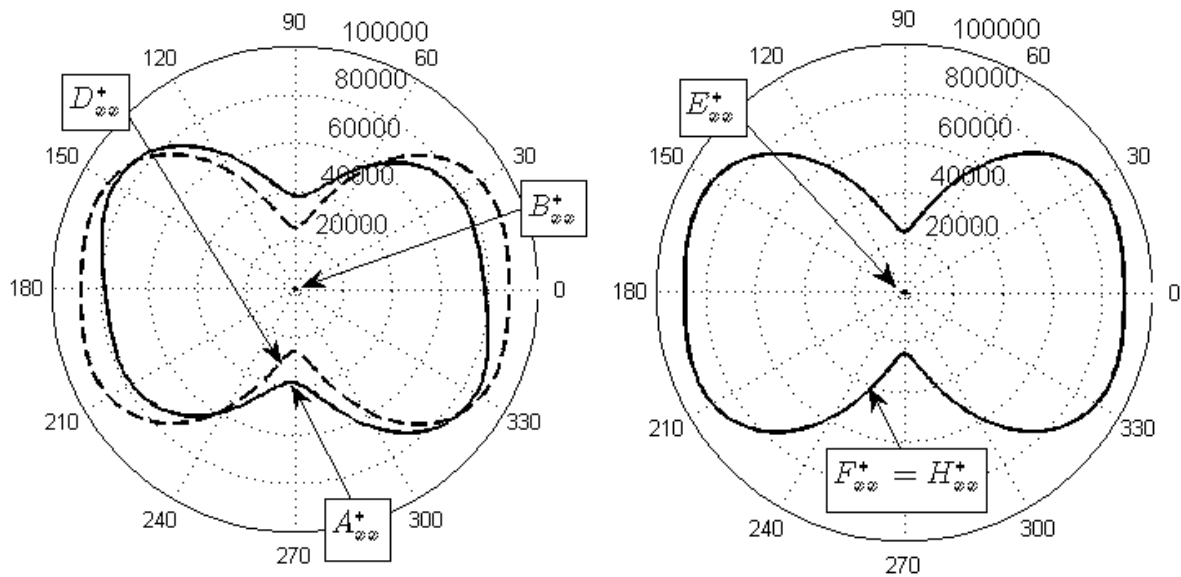
Case n.	Solution n.	Stacking sequence	Residual
1	1	[72/14/-63/-80/-48/70/3/37/-25/ -10/11/77/-19/62/11/-54/-88/-54/ 19/77]	6.0009×10^{-6}
	2	[-1/-14/88/51/-73/-88/-32/24/20/ 62/-49/22/-44/-81/72/-9/-75/73/16/ -15]	4.8924×10^{-5}
2	1	[-31/35/32/-31/-32/30/35/-70/-21/ -38/-70/59/43/-31/-34/21/36/-29/ -38/31]	1.7476×10^{-4}
	2	[35/-43/-25/-27/23/32/56/-40/30/56/ 41/-31/-37/12/-41/-38/33/47/25/ -31]	1.9490×10^{-4}
3	1	[78/-51/-2/10/41/89/-44/2/42/-82/ 71/-53/-19/17/-8/-87/87/62/-46/11]	6.6123×10^{-4}
	2	[-3/-75/69/90/-11/5/26/-74/6/-10/ 80/86/83/-66/-5/6/-89/-19/18/88]	8.1274×10^{-4}

$n = 20$ plies

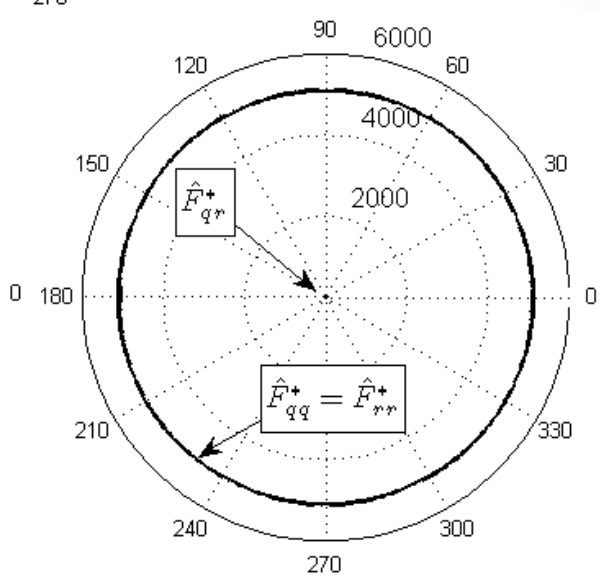
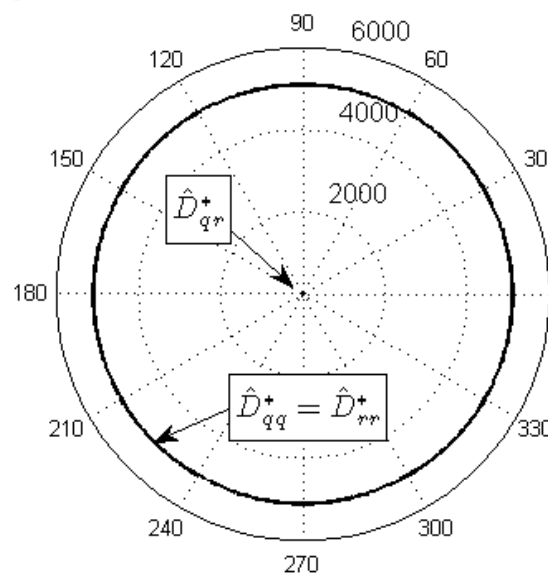
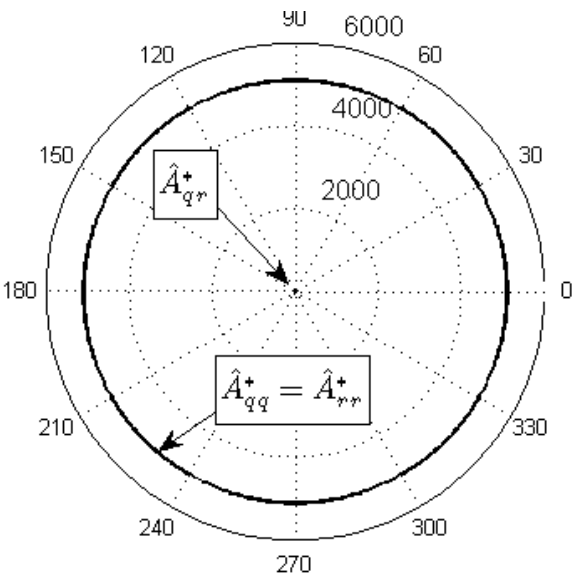
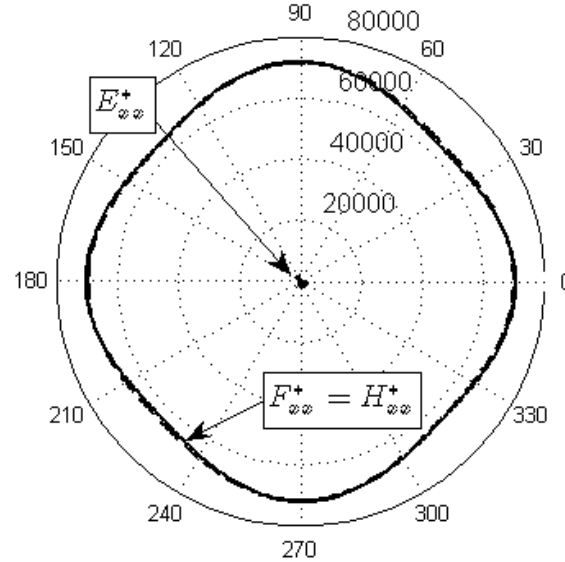
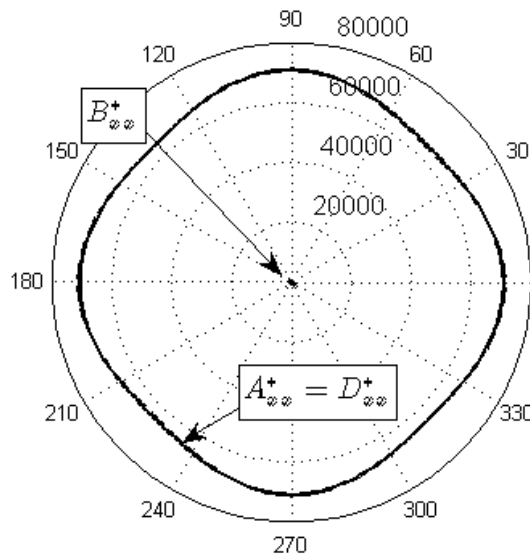
Case 1: fully uncoupled laminate with square symmetric membrane behaviour



Case 2: fully uncoupled laminate homogeneous in bending with an orthotropic behaviour



Case 3: fully uncoupled and homogeneous laminate with a square symmetric behaviour

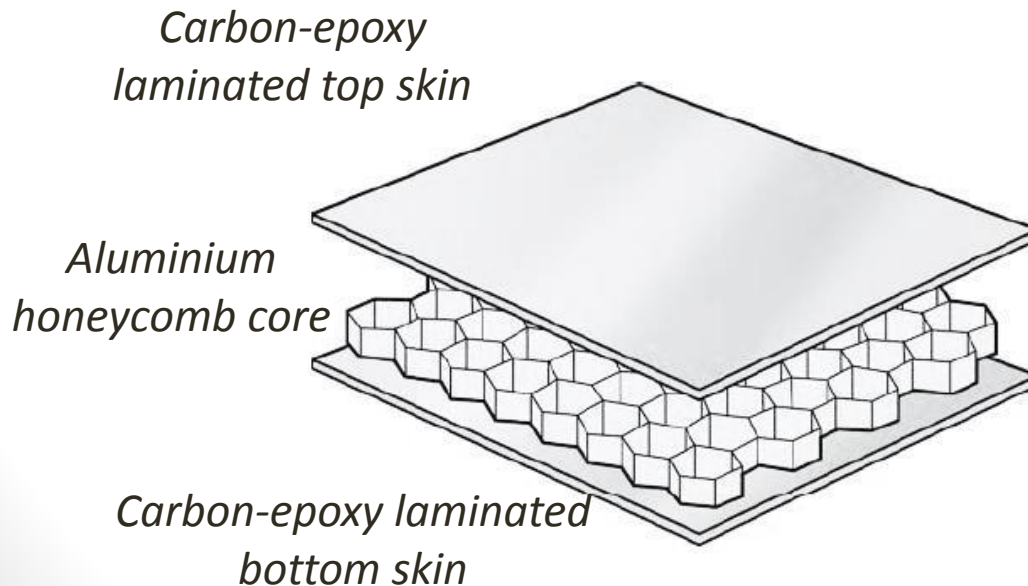


SIMULTANEOUS SHAPE AND MATERIAL OPTIMISATION OF SANDWICH PANELS WITH HONEYCOMB CORE FOR ADDITIVE MANUFACTURING

Shape and material optimisation of sandwich plates with honeycomb core: key-points

GOAL → Weight minimisation of a sandwich panel subject to constraints on:

1. local and global first buckling load
2. positive-definiteness of the stiffness tensor of the core
3. admissible values of the material parameters for the laminated skins
4. shape of the unit cell (due to additive manufacturing process)



A. Catapano, M. Montemurro, A multi-scale approach for the optimum design of sandwich plates with honeycomb core. Part I: homogenisation of core properties. *Composite Structures*, 118: 664-676, 2014

A. Catapano, M. Montemurro, A multi-scale approach for the optimum design of sandwich plates with honeycomb core. Part II: the optimisation strategy. *Composite Structures*, 118: 677-690, 2014

M. Montemurro, A. Catapano, D. Doroszewski. Simultaneous shape and material optimization of sandwich panels with honeycomb core for additive manufacturing. In: *18th International Conference on Composite Structures (ICCS18)*. Lisbon, Portugal; 2015.

Shape and material optimisation of sandwich plates with honeycomb core: the two-level strategy

Difficulties

- Design of anisotropic structures and materials
- Simultaneous shape and material optimisation of the structure
- Multi-scale problem (macro and meso)
- Different nature of design variables



TWO-LEVEL OPTIMISATION STRATEGY:

1. **Structural optimisation:** optimum shape of the unit cell of the core together with the material and geometric parameters of the laminated skins
2. **Lay-up design:** optimum skins lay-up giving the geometric and material parameters provided by the first-level problem

Shape and material optimisation of sandwich plates with honeycomb core: the two-level strategy

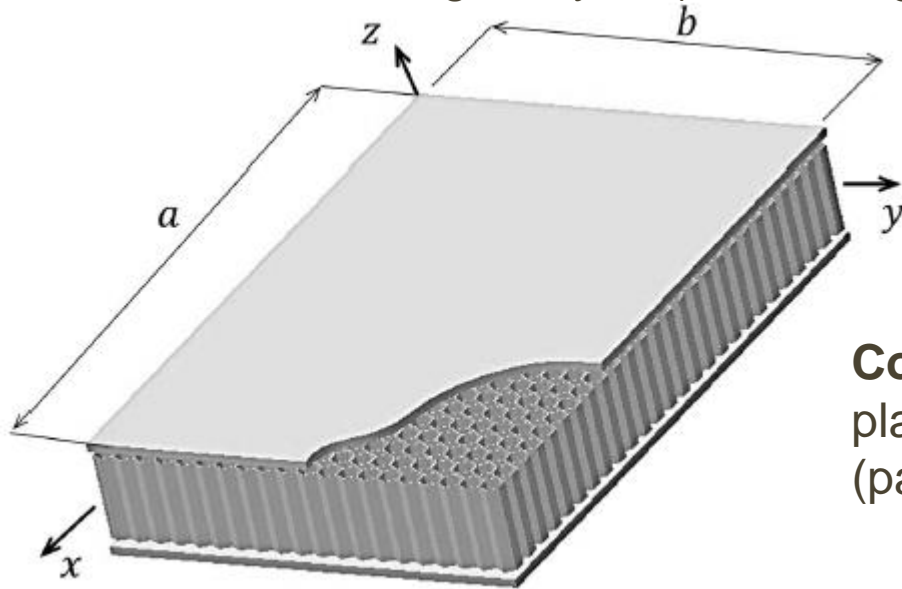
Remarks on the 1st level

- **Meso-scale model.** The core is modelled via the repetitive unit cell characterised by its geometric variables
- **Macro-scale model.** The sandwich plate is modelled as an heterogeneous continuum composed of:
 - two laminated skins, modelled as equivalent homogeneous anisotropic plates (the mechanical behaviour is described through a set of tensor invariants)
 - the core, considered as an equivalent homogeneous orthotropic solid (the mechanical behaviour depends upon the geometric parameters of the unit cell)

The link between these two scales is represented by the homogenisation phase of the honeycomb core properties

Shape and material optimisation of sandwich plates with honeycomb core : hypotheses

1. Honeycomb core:
 - linear, elastic behaviour for the material of the cell walls
 - perfect bonding for the wall-to-wall contact
 - the buckling of the cell walls due to shear stress is disregarded
2. Laminated skins:
 - Uncoupled, homogeneous and fully orthotropic behaviour
 - Identical skins
3. Linear buckling analysis (local and global)



Constant parameters: overall in-plane dimensions of the structure (parameters a and b)

Shape and material optimisation of sandwich plates with honeycomb core: fabrication of the core

Additive Manufacturing (AM) process for fabricating the honeycomb core

3D printing

Advantages

- able to reproduce complex shapes
- technological constraints linked to the AM process are less restrictive than those of other rapid prototyping processes

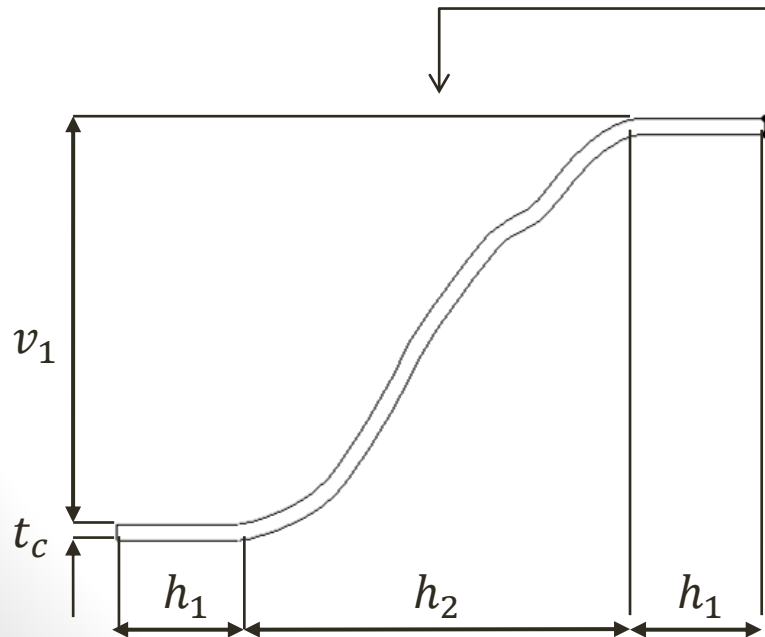
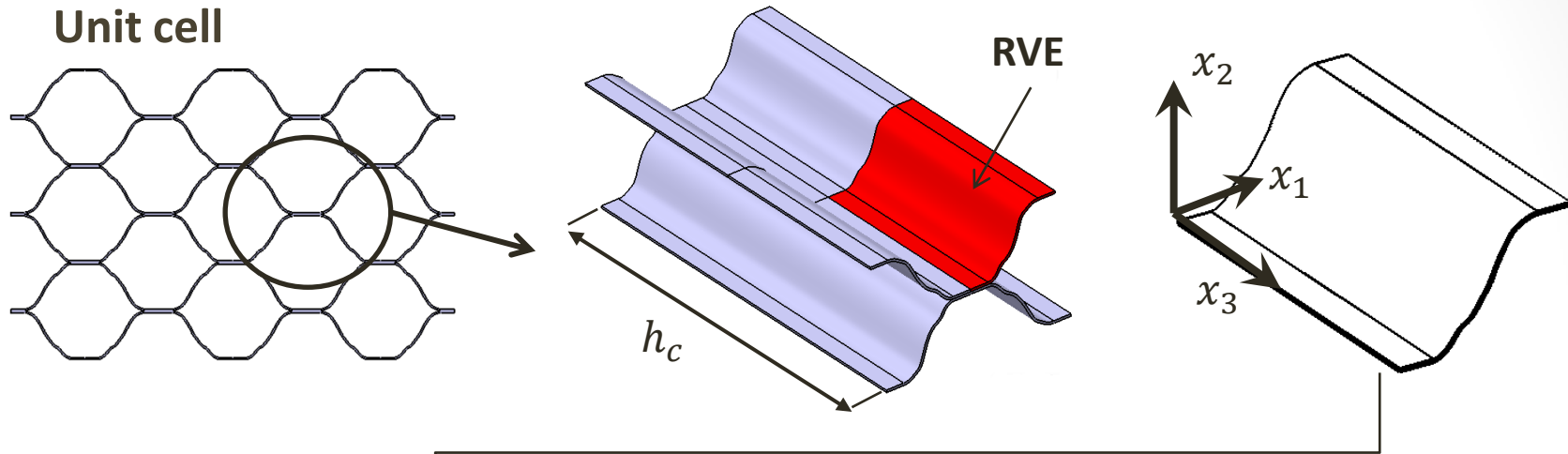
Drawbacks

- the support of the manufactured geometry must be mechanically or chemically removed
- the quality of functional surfaces is often “coarse” (in terms of roughness)
- the volume fraction of porosities within the material is rather considerable



First-level problem : geometrical design variables

Geometrical design variables : shape and geometric parameters of the unit cell and the skins thickness



“Global” design variables of the unit cell:

$$h_1, h_2, v_1, t_c, h_c$$

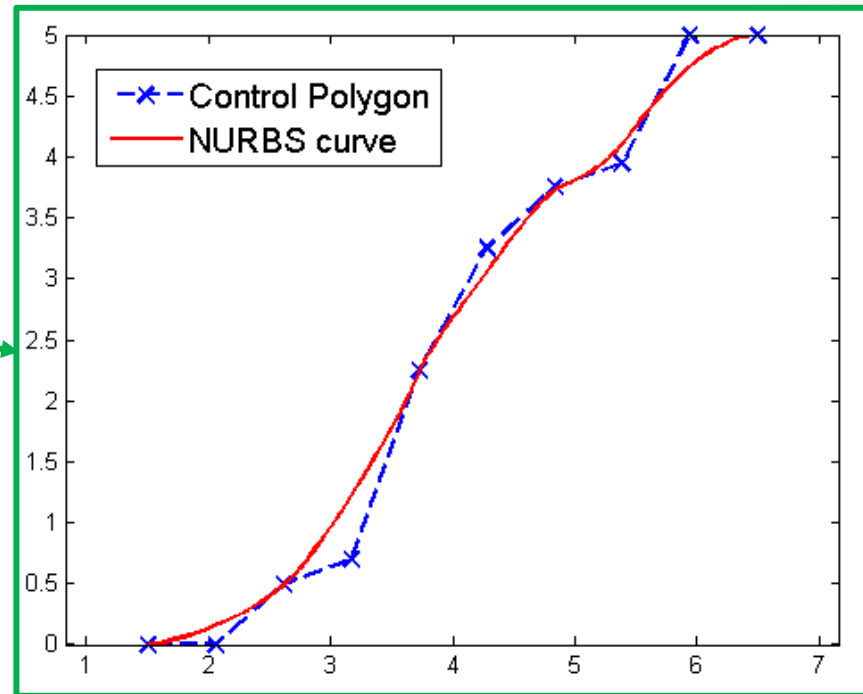
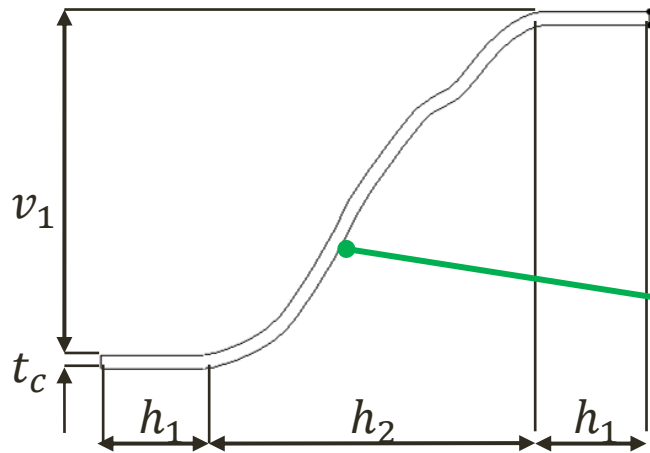
$$a_1 = 2h_1 + h_2, a_2 = v_1 + t_c, a_3 = \frac{h_c}{2}$$

Design variables of the skins:

$$h$$

First-level problem : geometrical design variables

“Local” design variables of the unit cell:



FIXED PARAMETERS

- Number of the “control” points, $N_p = 10$
- Degree of the NURBS basis, $p = 3$
- Knot vector,
 $u_k = \{0, 0, 0, 0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7, 1, 1, 1, 1\}$

VARIABLES

- Position of the points of the control polygon, (x_i, y_i) with $i = 1, \dots, N_p$
- Weights of the NURBS, Ω_i with $i = 1, \dots, N_p$

First-level problem : geometrical design variables

Vector of the geometrical design variables: $\mathbf{x}_g = \{h, h_1, h_2, v_1, t_c, h_c, x_i, y_i, \Omega_i\}$

Constraints on the geometrical design variables

1. Geometrical constraint on the shape of the unit cell (due to additive manufacturing process)

$$g_1(\mathbf{x}_g) = 2t_c - \min(r(s)) \leq 0 ,$$

+ Non self-intersecting constraint

$r(s)$: local radius of curvature of the NURBS curve

2. Constraints to ensure the positive definiteness of the stiffness matrix of the honeycomb core whose effective elastic properties depend on the geometric parameters of the unit cell

$$g_2(\mathbf{x}_g) = -E_1^c < 0 ,$$

$$g_3(\mathbf{x}_g) = -E_2^c < 0 ,$$

$$g_4(\mathbf{x}_g) = -E_3^c < 0 ,$$

$$g_5(\mathbf{x}_g) = -G_{12}^c < 0 ,$$

$$g_6(\mathbf{x}_g) = -G_{13}^c < 0 ,$$

$$g_7(\mathbf{x}_g) = -G_{23}^c < 0 ,$$

$$g_8(\mathbf{x}_g) = |\nu_{12}^c| - \sqrt{\frac{E_1^c}{E_2^c}} < 0 ,$$

$$g_9(\mathbf{x}_g) = |\nu_{13}^c| - \sqrt{\frac{E_1^c}{E_3^c}} < 0 ,$$

$$g_{10}(\mathbf{x}_g) = |\nu_{23}^c| - \sqrt{\frac{E_2^c}{E_3^c}} < 0 ,$$

$$g_{11}(\mathbf{x}_g) = 2\nu_{12}^c\nu_{13}^c\nu_{23}^c\frac{E_3^c}{E_1^c} + (\nu_{12}^c)^2\frac{E_2^c}{E_1^c} + (\nu_{23}^c)^2\frac{E_3^c}{E_2^c} + (\nu_{13}^c)^2\frac{E_3^c}{E_1^c} - 1 < 0 .$$

First-level problem : material design variables

Homogeneous, uncoupled laminate $\rightarrow A^* = D^* \ \& \ B \equiv 0$

Polar parameters: $T_0^{A^*}, T_1^{A^*}, K^{A^*}, R_{0K}^{A^*}, R_1^{A^*}, \Phi_1^{A^*}$

identical plies + definition = material design variables for an uncoupled, homogeneous, orthotropic laminate :

$$\begin{aligned} T_0^{A^*} &= T_0, \\ T_1^{A^*} &= T_1, \end{aligned}$$

$$R_{0K}^{A^*} = (-1)^{K^{A^*}} R_0^{A^*}$$

$$R_{0K}^{A^*}, R_1^{A^*}, \Phi_1^{A^*}$$

Vector of the material design variables: $\mathbf{x}_m = \{ \Phi_1^{A^*}, R_{0K}^{A^*}, R_1^{A^*} \}$

Feasibility constraints on the polar parameters (*Vannucci, 2013*)

$$g_{13}(\mathbf{x}_m) = 2 \left(\frac{R_1^{A^*}}{R_1} \right)^2 - 1 - \frac{R_{0K}^{A^*}}{R_0} \leq 0$$

First-level problem : design space

	Design variable	Type	Lower bound	Upper bound	Discretisation step
Skins	R_{0K}^{A*} [MPa]	continuous	-19710.0	19710.0	-
	R_1^{A*} [MPa]	continuous	0	21433.0	-
	Φ_1^{A*} [deg]	discrete	-90	90	1
	h [mm]	discrete	2.50	4.00	0.125 = h_{ply}
Repetitive unit cell	h_1 [mm]	discrete	1.00	4.00	0.1
	h_2 [mm]	discrete	2.00	5.00	0.1
	v_1 [mm]	discrete	2.00	5.00	0.1
	t_c [mm]	discrete	0.20	0.40	0.01
	h_c [mm]	discrete	20.00	60.00	1.00
	x_i [mm]	continuous	h_1	$h_1 + h_2$	-
	y_i [mm]	continuous	0.00	v_1	-
	Ω_i	discrete	0.01	1.00	0.01

REMARK: the optimal value of skins thickness gives the optimal number of plies to be used during the second-level problem

First-level problem : problem formulation

Vector of design variables:

$$\mathbf{x} = \left\{ \Phi_1^{A*}, R_{0K}^{A*}, R_1^{A*}, h, h_1, h_2, v_1, t_c, h_c, x_i, y_i, \Omega_i \right\}$$

First-level optimisation problem :

$$\min_{\mathbf{x}} W(\mathbf{x})$$

subject to:

$$\lambda_{glob}^{ref} - \lambda_{glob}(\mathbf{x}) \leq 0 ,$$

$$\lambda_{loc}^{ref} - \lambda_{loc}(\mathbf{x}) \leq 0 ,$$

$$g_i(\mathbf{x}) \leq 0 , \text{ with } i = 1, 2, \dots, 13 .$$

→ Weight of the sandwich plate

→ Constraints on the first buckling load

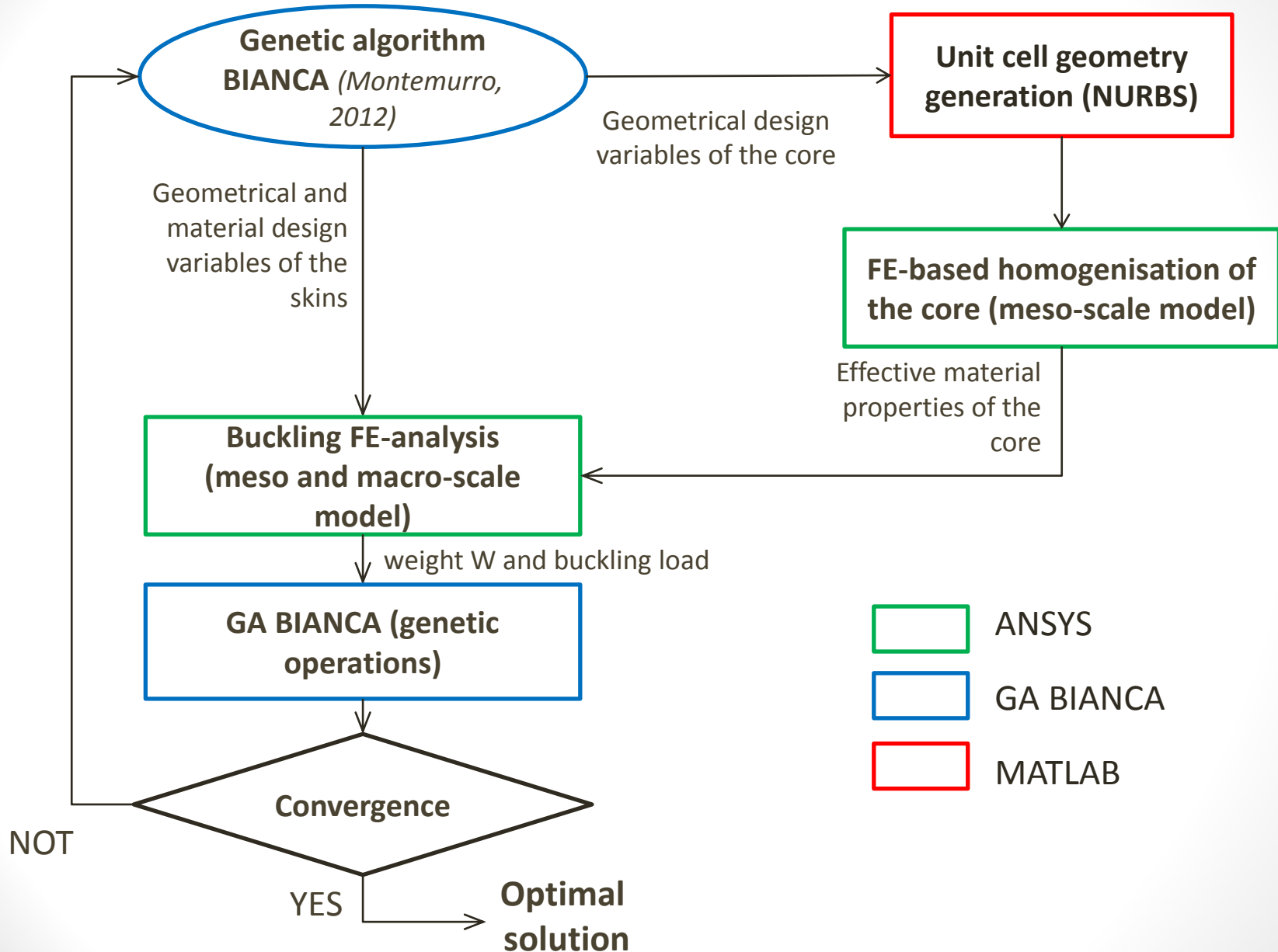
→ Constraints on geometrical and material design variables

Non-linear, non-convex problem in terms of both geometrical and material variables

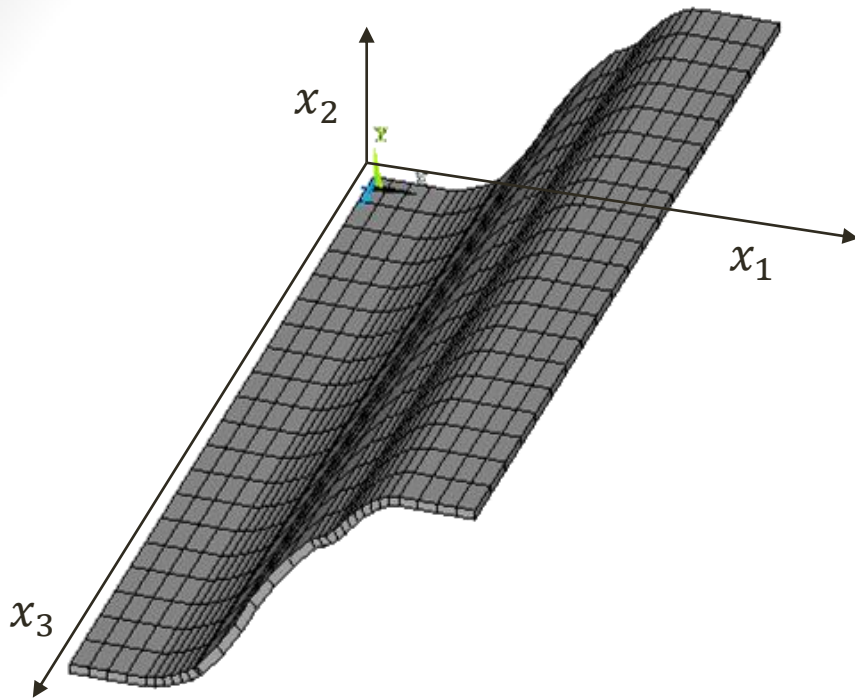
Total number of design variables = **39**

Total number of constraints = **15**

First-level problem : numerical strategy



First-level problem : meso-scale model, homogenisation



FE code: ANSYS

Elements: 20-node SOLID186

DOFs: 20312

Numerical homogenisation technique:
strain energy homogenisation
technique of periodic media

The generalised Hooke's law for the Repetitive Volume Element (RVE)

$$\begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_3 \\ \bar{\sigma}_4 \\ \bar{\sigma}_5 \\ \bar{\sigma}_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\epsilon}_3 \\ \bar{\epsilon}_4 \\ \bar{\epsilon}_5 \\ \bar{\epsilon}_6 \end{Bmatrix}$$

Volume averaged stress and strain components

First-level problem : meso-scale model, homogenisation

Expression of the components of \mathbf{C} :

$$C_{\alpha\beta} = \frac{\bar{\sigma}_\alpha}{\bar{\varepsilon}_\beta} \text{ with } \alpha, \beta = 1, 2, \dots, 6 \text{ and } \varepsilon_\gamma = 0 \text{ with } \gamma = 1, 2, \dots, 6, \gamma \neq \beta$$

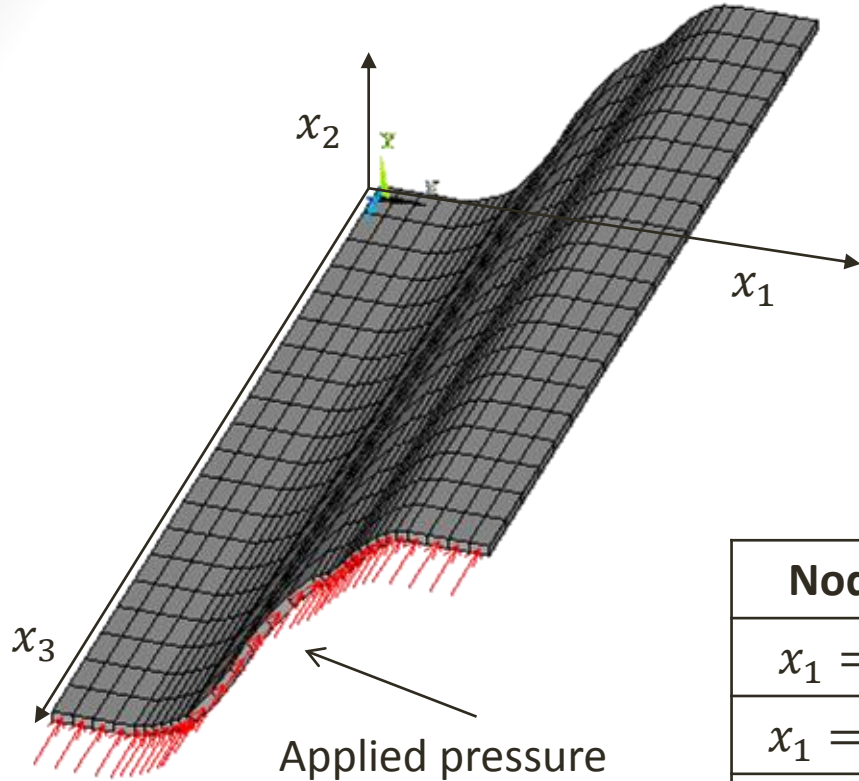
The **9** independent components of \mathbf{C} for the honeycomb core are determined through 6 static analyses on the FE model

1 st load case				2 nd load case				3 rd load case			
Nodes	U_1	U_2	U_3	Nodes	U_1	U_2	U_3	Nodes	U_1	U_2	U_3
$x_1 = 0$	0	free	free	$x_1 = 0$	0	free	free	$x_1 = 0$	0	free	free
$x_1 = a_1$	u	free	free	$x_1 = a_1$	0	free	free	$x_1 = a_1$	0	free	free
$x_2 = 0$	free	0	free	$x_2 = 0$	free	0	free	$x_2 = 0$	free	0	free
$x_2 = a_2$	free	0	free	$x_2 = a_2$	free	u	free	$x_2 = a_2$	free	0	free
$x_3 = 0$	free	free	0	$x_3 = 0$	free	free	0	$x_3 = 0$	free	free	0
$x_3 = a_3$	free	free	0	$x_3 = a_3$	free	free	0	$x_3 = a_3$	free	free	u

4 th load case				5 th load case				6 th load case			
Nodes	U_1	U_2	U_3	Nodes	U_1	U_2	U_3	Nodes	U_1	U_2	U_3
$x_1 = 0$	0	free	free	$x_1 = 0$	free	0	0	$x_1 = 0$	free	0	0
$x_1 = a_1$	0	free	free	$x_1 = a_1$	free	0	u	$x_1 = a_1$	free	u	0
$x_2 = 0$	0	free	0	$x_2 = 0$	free	0	free	$x_2 = 0$	0	free	0
$x_2 = a_2$	0	free	u	$x_2 = a_2$	free	0	free	$x_2 = a_2$	u	free	0
$x_3 = 0$	0	0	free	$x_3 = 0$	0	0	free	$x_3 = 0$	free	free	0
$x_3 = a_3$	0	u	free	$x_3 = a_3$	u	0	free	$x_3 = a_3$	free	free	0

These BCs are imposed in order to satisfy the symmetries of the RVE and to generate a strain field having, for each static analysis, only one component of $\bar{\varepsilon}$ different from zero

First-level problem : meso-scale model, local buckling



FE code: ANSYS

Elements: 20-node SOLID186

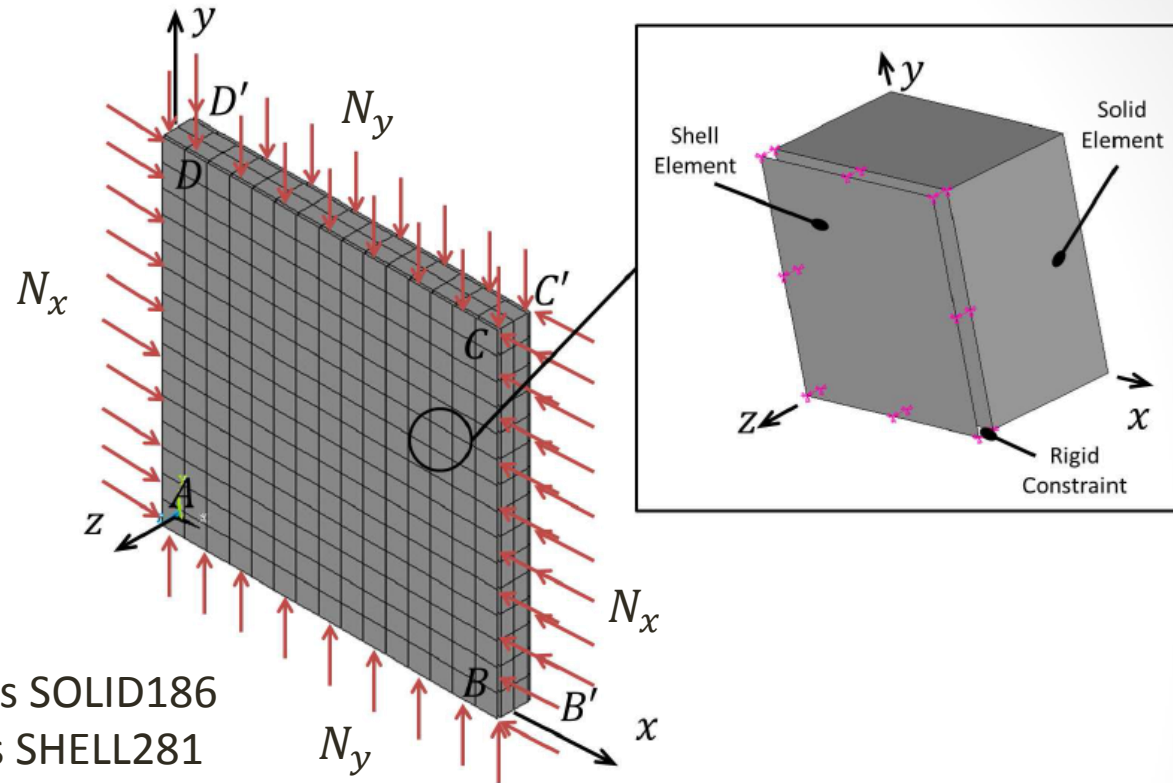
DOFs: 20312

Analysis: linear buckling

Displacement BCS

Nodes	U_1	U_2	U_3
$x_1 = 0$	0	free	free
$x_1 = a_1$	0	Free	free
$x_2 = 0$	free	free	free
$x_2 = a_2$	free	free	free
$x_3 = 0$	Free	free	0
$x_3 = a_3$	0	0	free

First-level problem : macro-scale model, global buckling



FE code: ANSYS

Core elements: 20-nodes SOLID186

Skins elements: 8-nodes SHELL281

Compatibility of the displacement field between skins and core: ANSYS

CERIG rigid constraints

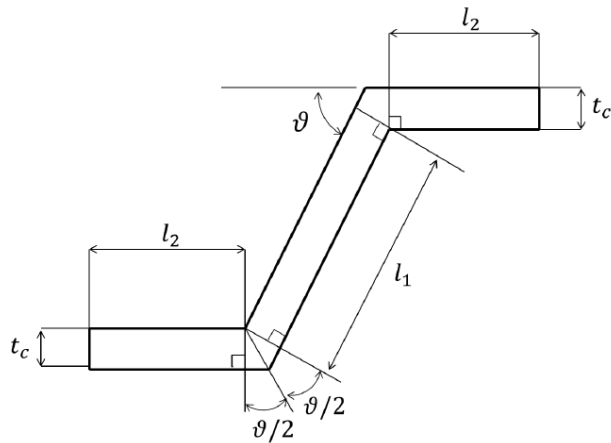
Analysis: linear buckling

DOFs: 12088

BCs of the FE model

Sides	Constraint
AB, A'B', CD, C'D'	$U_x = 0$ $U_z = 0$
BC, B'C', DA, D'A'	$U_y = 0$ $U_z = 0$

First-level problem : studied cases and results



Case	Sub-case	Load	Core
1	a	$N_y = 0,5N_x$	B-Spline
	b		NURBS
2	a	$N_y = N_x$	B-Spline
	b		NURBS

Reference Solution

a, b [mm]	1500.00
(R_{OK}^{A*}) [MPa]	-9855.21
(R_1^{A*}) [MPa]	5358.28
(Φ_1^{A*}) [deg]	0.0
h [mm]	4.00
ϑ [deg]	60.00
l_2 [mm]	2.75
l_1 [mm]	5.50
t_c [mm]	0.25
h_c [mm]	30
Skins Weight [Kg]	-
Core weight [Kg]	-
Panel weight [Kg]	40.26
Buckling load (Case 1) [N/mm]	1556.43
Buckling load (Case 2) [N/mm]	1283.50
Local buckling load [MPa]	500.66
Stacking sequence	N. of plies
$[45/0/45/45/ - 45/45/ - 45/0/0/45/ - 45/45/ - 45/ - 45/0/45]_s$	32

Genetic parameters:

Number of individuals	Number of generations	Crossover probability	Mutation probability	Selection operator
160	200	0.85	$1/N_{ind}$	Roulette wheel

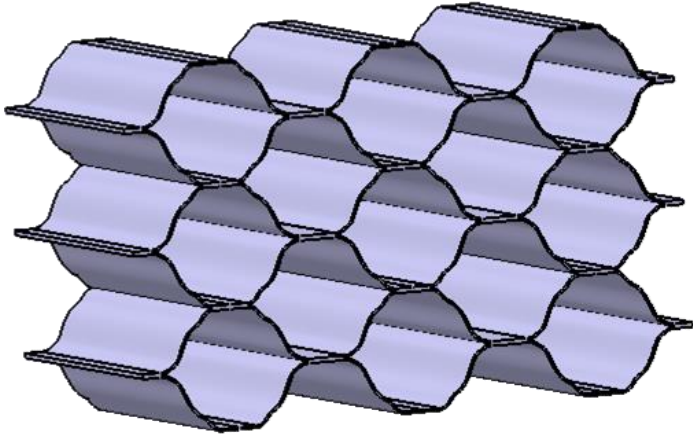
First-level problem: studied cases and results

	Solution case 1.a	Solution case 1.b	Solution case 2.a	Solution case 2.b
Φ_1^{A*} [deg]	83.00	47.00	44.00	49.00
R_{OK}^{A*} [MPa]	-6608.53	19555.90	19517.30	19517.30
R_1^{A*} [MPa]	9281.35	2891.26	4714.00	1613.24
h [mm]	2.50	2.50	2.50	2.50
h_1 [mm]	1.50	1.50	1.60	2.10
h_2 [mm]	5.00	5.00	5.00	4.90
v_1 [mm]	5.00	5.00	4.55	5.00
t_c [mm]	0.20	0.20	0.20	0.20
h_c [mm]	38.00	36.00	38.00	38.00
(x_1, y_1) [mm]	(1.50, 0.00)	(1.50, 0.00)	(1.60, 0.00)	(2.10, 0.00)
(x_2, y_2) [mm]	(2.05, 0.00)	(2.05, 0.00)	(2.15, 0.00)	(2.64, 0.00)
(x_3, y_3) [mm]	(2.61, 0.50)	(2.61, 0.50)	(2.71, 1.46)	(3.19, 1.55)
(x_4, y_4) [mm]	(3.17, 0.70)	(3.17, 0.70)	(3.27, 2.23)	(3.73, 2.20)
(x_5, y_5) [mm]	(3.72, 2.25)	(3.72, 2.25)	(3.82, 2.27)	(4.28, 3.40)
(x_6, y_6) [mm]	(4.28, 3.25)	(4.28, 3.25)	(4.38, 2.91)	(4.82, 3.45)
(x_7, y_7) [mm]	(4.83, 3.75)	(4.83, 3.75)	(4.93, 3.59)	(5.37, 3.60)
(x_8, y_8) [mm]	(5.39, 3.95)	(5.39, 3.95)	(5.49, 3.18)	(5.91, 4.30)
(x_9, y_9) [mm]	(5.94, 5.00)	(5.94, 5.00)	(6.04, 4.55)	(6.45, 5.00)
(x_{10}, y_{10}) [mm]	(6.50, 5.00)	(6.50, 5.00)	(6.60, 4.55)	(7.00, 5.00)
Ω_1	1.00	0.63	1.00	0.40
Ω_2	1.00	0.27	1.00	0.44
Ω_3	1.00	0.72	1.00	0.72
Ω_4	1.00	0.11	1.00	0.63
Ω_5	1.00	0.70	1.00	0.10
Ω_6	1.00	0.11	1.00	0.44
Ω_7	1.00	0.70	1.00	0.26
Ω_8	1.00	0.67	1.00	0.89
Ω_9	1.00	0.42	1.00	0.51
Ω_{10}	1.00	0.59	1.00	0.98
Panel weight [Kg]	29.35	28.63	29.98	28.94
Buckling load [N/mm]	1642.98	1574.91	1297.73	1284.69
Local buck. load [MPa]	684.88	585.57	664.59	534.68

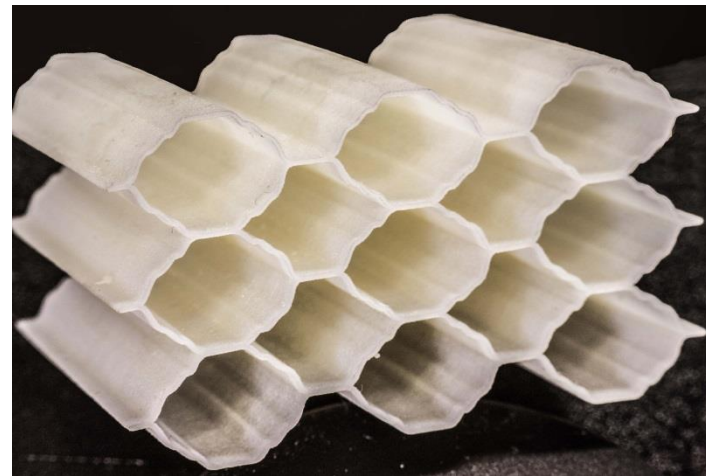
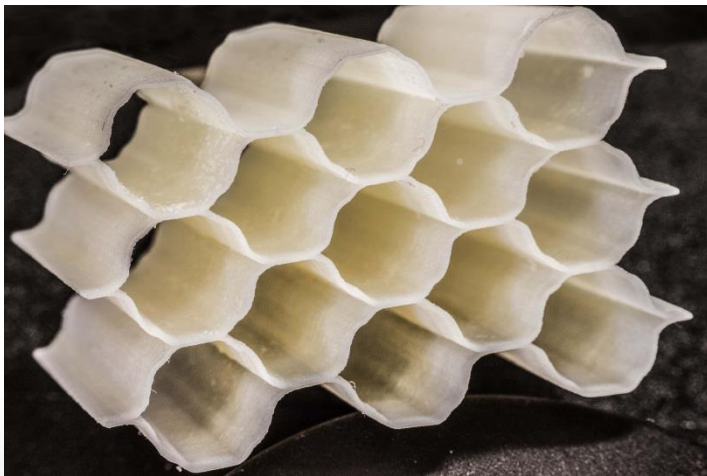
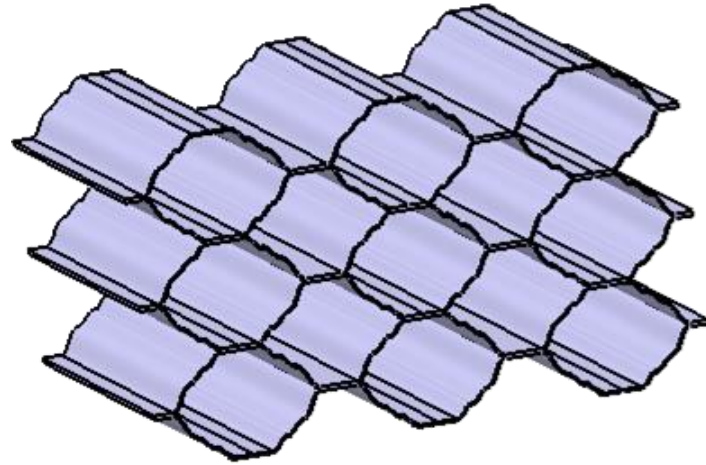
- W_{opt} about 29% lower than W_{ref}
- λ_{glob}^{opt} almost equal to λ_{glob}^{ref}
- λ_{loc}^{opt} about 17% greater than λ_{loc}^{ref}

First-level problem: prototype of the optimum solutions

Case 1a



Case 2a



Second-level problem : the lay-up design

OBJECTIVE → find at least one stacking sequence meeting the material and geometrical parameters resulting from the first-level problem

- In the framework of the polar formalism, this problem can be stated in the form of an unconstrained minimisation problem

$$\min_{\delta} I(f_i(\delta)) \quad \text{with} \quad I(f_i(\delta)) = \sum_{i=1}^6 f_i(\delta)$$

$$f_1(\delta) = \left(\frac{|\Phi_0^{A^*}(\delta) - \Phi_1^{A^*}(\delta)|}{\pi/4} - K^{A^*(opt)} \right)^2, \quad f_2(\delta) = \left(\frac{R_0^{A^*}(\delta) - R_0^{A^*(opt)}}{R_0} \right)^2,$$

$$f_3(\delta) = \left(\frac{R_1^{A^*}(\delta) - R_1^{A^*(opt)}}{R_1} \right)^2, \quad f_4(\delta) = \left(\frac{|\Phi_1^{A^*}(\delta) - \Phi_1^{A^*(opt)}|}{\pi/4} \right)^2, \quad f_5(\delta) = \left(\frac{\|C(\delta)\|}{\|Q\|} \right)^2$$

$$f_6(\delta) = \left(\frac{\|B^*(\delta)\|}{\|Q\|} \right)^2,$$

- The **design variables** are the **plies orientations**

Second-level problem : numerical results

Genetic parameters:

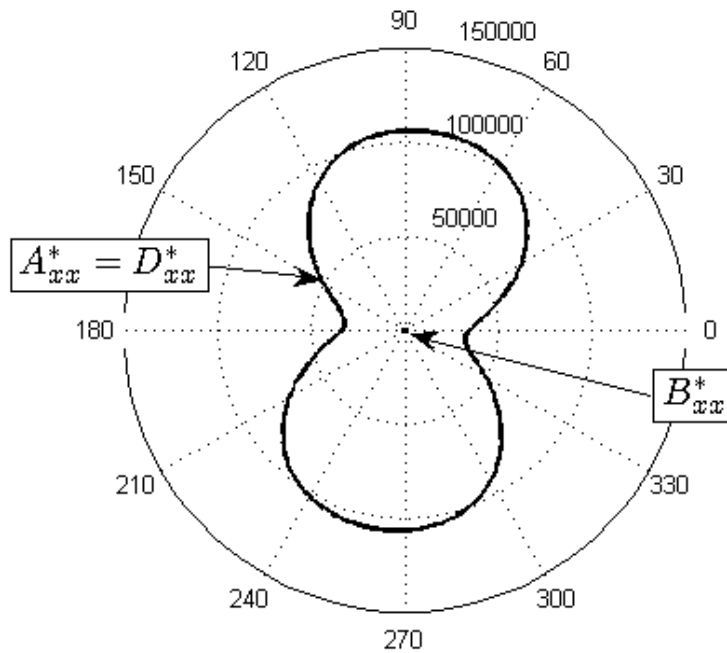
Number of individuals	Number of generations	Crossover probability	Mutation probability	Selection operator
500	500	0.85	$1/N_{\text{ind}}$	Roulette wheel

	Best stacking sequence	N. of plies	Residual
Reference Solution	$[45/0/45/45/ - 45/45/ - 45/0/0/45/ - 45/45/ - 45/ - 45/0/45]_s$	32	
Case 1.a	$[36/ - 55/68/ - 74/79/ - 71/48/57/ - 55/ - 87/44/50/ - 57/ - 49/26/ - 74/90/54/60/ - 60]$	20	2.20×10^{-4}
Case 1.b	$[47/47/ - 43/ - 43/47/ - 43/47/ - 43/47/ - 43/ - 43/ 47/47/47/47/ - 43/ - 43/47/47/ - 43]$	20	3.96×10^{-4}
Case 2.a	$[-45/44/43/ - 48/ - 44/44/ - 45/60/35/44/ - 45/ 43/ - 47/ - 48/45/ - 41/ - 43/43/ - 48/46]$	20	3.18×10^{-4}
Case 2.b	$[-40/50/49/ - 44/48/48/ - 40/ - 41/ - 38/55/ - 40/ - 43/46/48/ - 44/48/49/50/ - 36/ - 44]$	20	3.94×10^{-4}

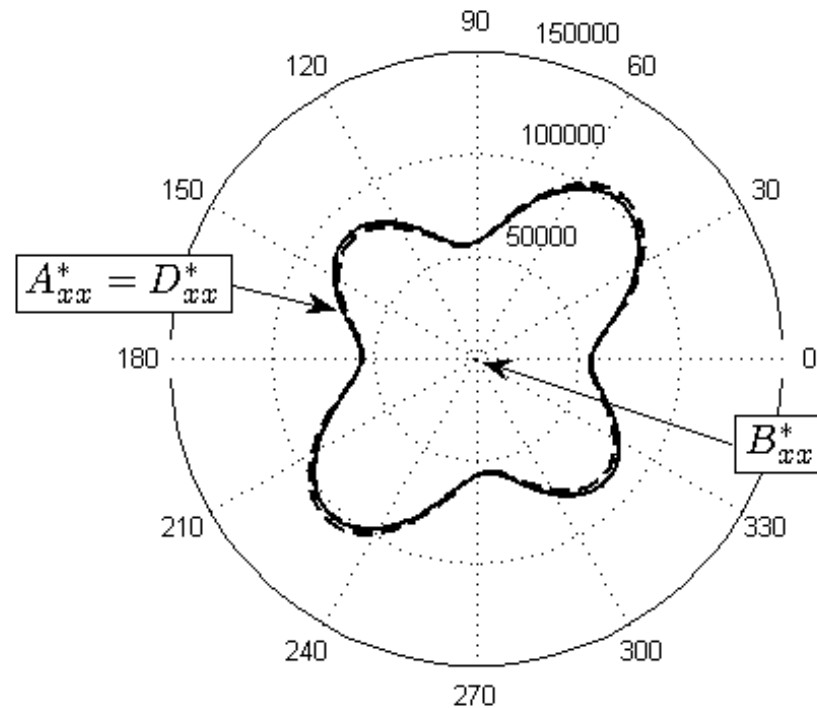
Second-level problem : numerical results

First component of the homogenised stiffness tensors of the laminate [MPa]

Case 1.a



Case 1.b



Optimal design of sandwich plates with honeycomb core : conclusive remarks

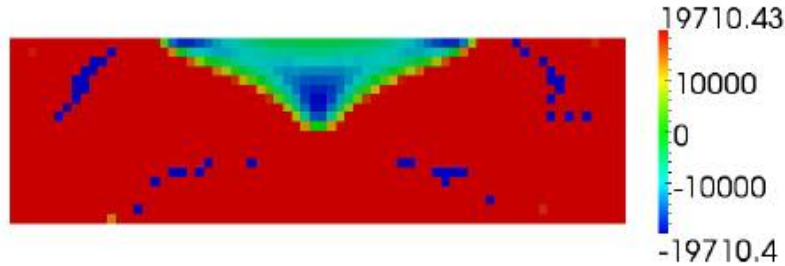
- ✓ General and complete mathematical formulation of the problem :
optimisation of every parameter characterising the structure
- ✓ Simultaneous shape and material optimisation at each scale
- ✓ Global search strategy
- ✓ Optimal non-standard configurations with enhanced mechanical
properties
- ✓ Weight saving up to ~29% with respect to the reference solution built
with the same material

MULTI-SCALE DESIGN AND OPTIMISATION OF VARIABLE ANGLE TOW LAMINATES

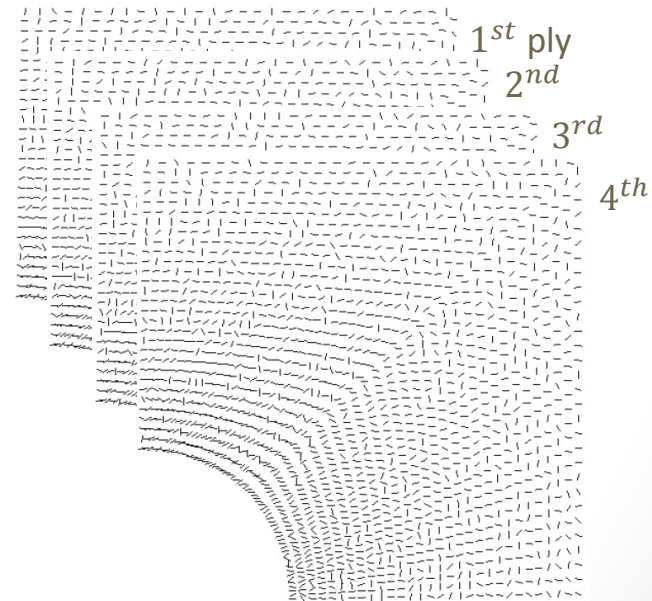
Introduction

Drawbacks of the first approach (Catapano, 2013)

- Discontinuous distribution of the polar parameters over the structure



- Discontinuous distribution of the local fibres orientation within each ply



- No rules for determining a proper fibre path

- Technological constraints (e.g. minimum radius of curvature of the tow) not taken into account

The bi-level multi-scale optimisation strategy: First-level problem

Major modifications

- Quasi-homogeneous orthotropic (locally, i.e. point-wise) laminates
- The effect of the laminate transverse shear stiffness is taken into account (FSDT framework)
- The point-wise variation of the laminate polar parameters is expressed through **B-spline and/or NURBS hyper-surfaces**
- **Strong convex hull property:** the feasibility constraints on the laminate polar parameters are checked only on the points of the *control net*

First-level problem: material design variables

B-spline hyper-surfaces

$$x(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) x^{(i,j)}$$

$$y(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) y^{(i,j)}$$

$$z(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) z^{(i,j)}$$

$$R_{0K}^{A*}(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) R_{0K}^{A*(i,j)}$$

$$R_1^{A*}(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) R_1^{A*(i,j)}$$

$$\Phi_1^{A*}(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) \Phi_1^{A*(i,j)}$$

$(n_c + 1) \times (m_c + 1)$: overall number of control points

$u, v \in [0,1]$: dimensionless parameters of the B-spline hyper-surface

p, q : degrees of the B-spline basis functions along u and v directions, respectively

$\{x^{(i,j)}, y^{(i,j)}, z^{(i,j)}\}$: Cartesian coordinates of the generic point of the control net (**fixed**)

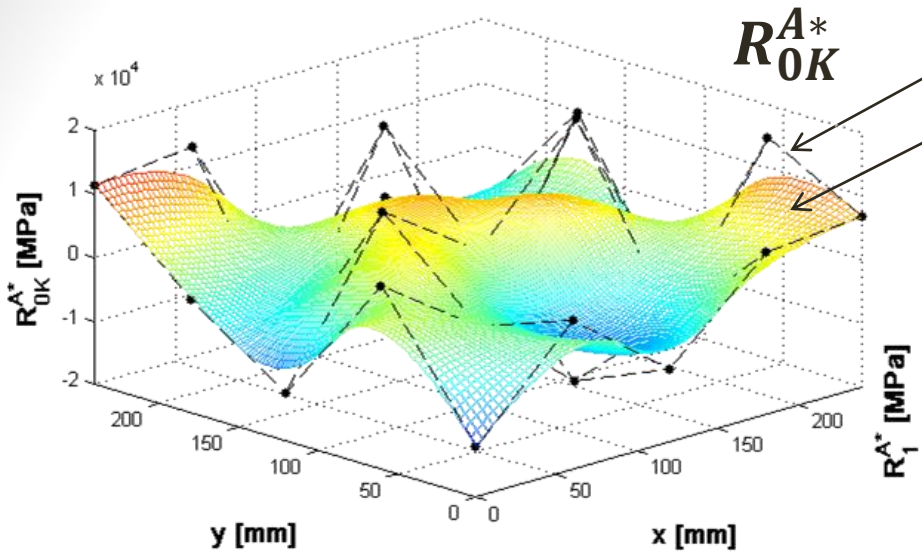
$\{R_{0K}^{A*(i,j)}, R_1^{A*(i,j)}, \Phi_1^{A*(i,j)}\}$: polar parameters linked to each control point (**material design variables**)

$$U = \left\{ \underbrace{0, \dots, 0}_{p+1}, u_{p+1}, \dots, u_{r-p-1}, \underbrace{1, \dots, 1}_{p+1} \right\}, r = n_c + p + 1$$

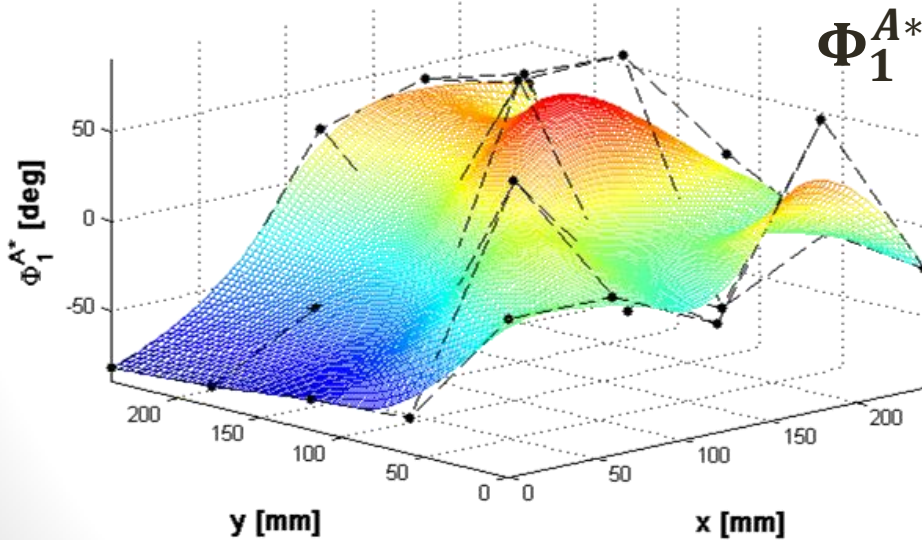
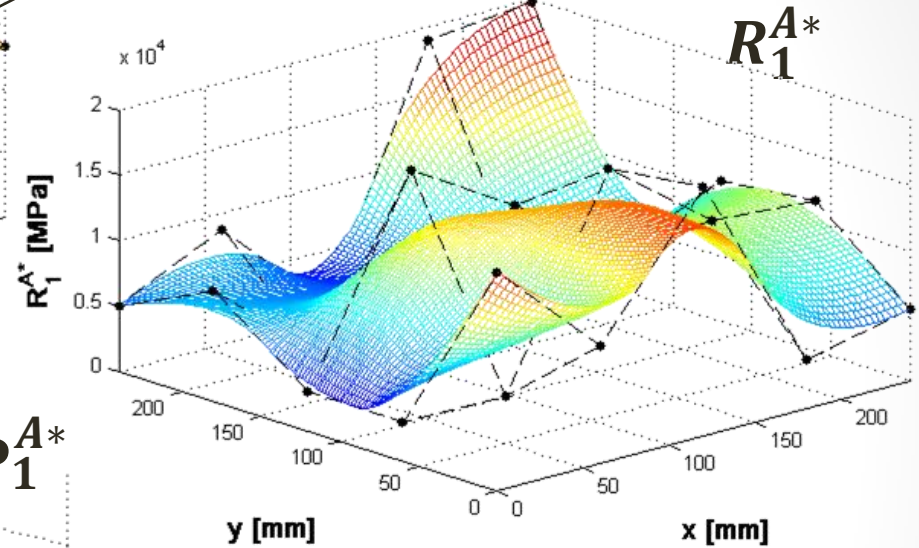
$$V = \left\{ \underbrace{0, \dots, 0}_{q+1}, v_{q+1}, \dots, v_{s-q-1}, \underbrace{1, \dots, 1}_{q+1} \right\}, s = m_c + q + 1$$

Knot vectors

First-level problem: material design variables



Control net
B-spline surface



The feasibility constraint is checked for each point of the *control net*

$$2 \left(\frac{R_1^{A*}}{R_1} \right)^2 - 1 - \frac{R_{0K}^{A*}}{R_0} \leq 0$$

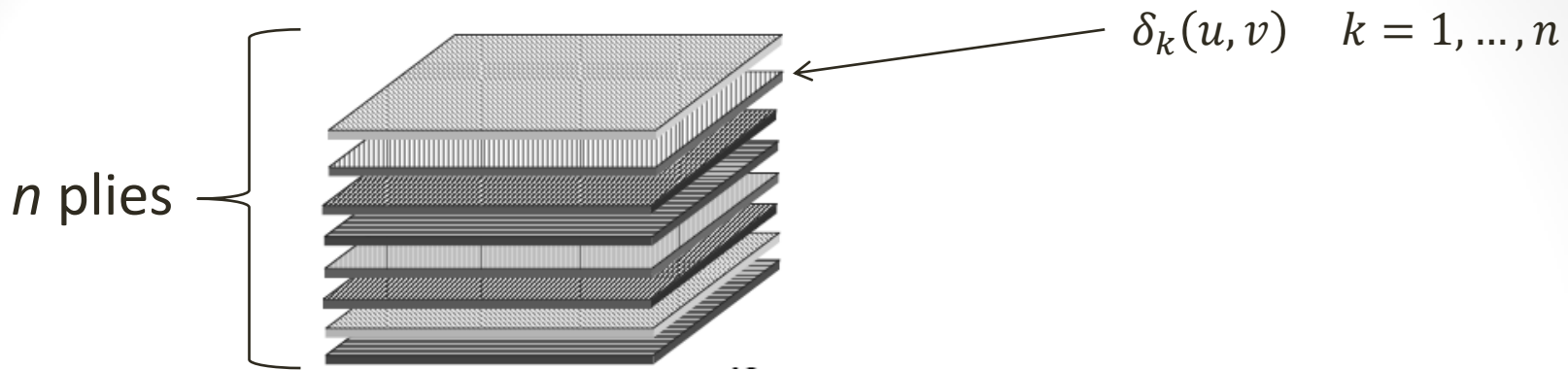
Overall number of material design variables: $3 \times (n_c + 1) \times (m_c + 1)$

The bi-level multi-scale optimisation strategy: Second-level problem

Major modifications

- Representation of the point-wise variation of the fibres orientation angle through B-spline surfaces (for each ply)
- Reconstruction of the fibres path: analogy with the problem of the *streamlines* (fluid mechanics)
- The technological constraint on the minimum radius of curvature of the tow is taken into account within the formulation of the second level problem

Second-level problem: design variables



$$x(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) x^{(i,j)}$$

$$y(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) y^{(i,j)}$$

$$z_k(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) z_k^{(i,j)}$$

$$\delta_k(u, v) = \sum_{i=0}^{n_c} \sum_{j=0}^{m_c} N_{i,p}(u) N_{j,q}(v) \delta_k^{(i,j)}$$

$\{x^{(i,j)}, y^{(i,j)}, z_k^{(i,j)}\}$: Cartesian coordinates of the generic point of the control net for the k -th layer (**fixed**)

$\delta_k^{(i,j)}$: orientation angle linked to each control point of the k -th layer (**design variables**)

Overall number of design variables: $n \times (n_c + 1) \times (m_c + 1)$

Second-level problem: determination of the fibres path

Analogy with the problem of the streamlines (fluid mechanics)

- Consider an angular field described in terms of a B-spline surface

$$x = x(u, v)$$

$$y = y(u, v)$$

$$\delta = \delta(u, v)$$

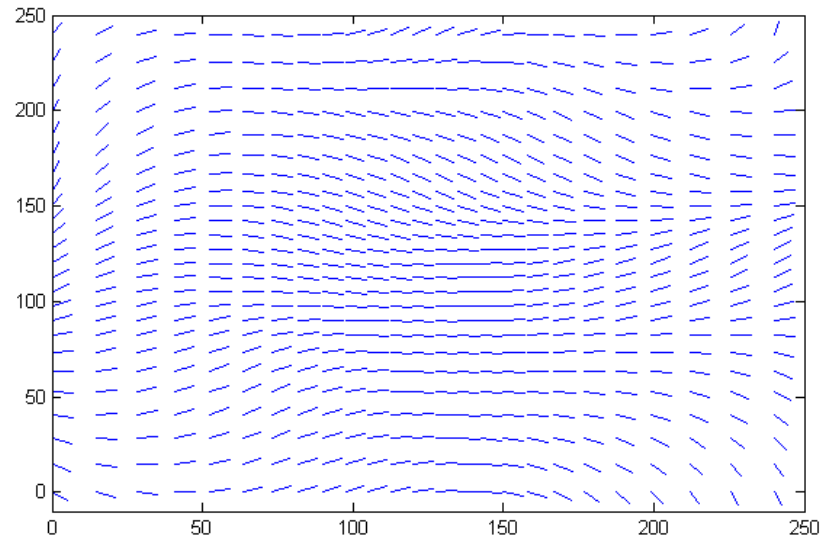
- The fibres path can be determined using the following relation:

$$\vec{\gamma}' \wedge \vec{t} = \vec{0} \Rightarrow \vec{\gamma}' = \lambda \vec{t}, \quad \lambda = 1$$

$$\vec{\gamma}' = \left\{ \begin{array}{l} \frac{dx_f}{d\xi} \\ \frac{dy_f}{d\xi} \end{array} \right\}; \quad \vec{t} = \left\{ \begin{array}{l} \cos \delta(u, v) \\ \sin \delta(u, v) \end{array} \right\}$$

$\vec{\gamma} = \left\{ \begin{array}{l} x_f(\xi) \\ y_f(\xi) \end{array} \right\}$: parametric equation of the path

ξ : curvilinear abscissa of the path



Second-level problem: curvature of the tow

Technological constraint on the minimum radius of curvature of the tow

- Definition of the curvature

$$\chi(\xi) = \frac{\|\vec{\gamma}' \wedge \vec{\gamma}''\|}{\|\vec{\gamma}'\|^3} = \left\| \vec{t} \wedge \frac{d\vec{t}}{d\xi} \right\|$$

- It can be proved that the following relationship applies:

$$\chi(\xi) = \|\nabla \wedge \vec{t}\| = \vec{t} \cdot (\mathbf{J}^{-1} \tilde{\nabla} \delta)$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} : \text{Jacobian matrix}$$

$$\tilde{\nabla} \delta = \begin{Bmatrix} \frac{\partial \delta}{\partial u} \\ \frac{\partial \delta}{\partial v} \end{Bmatrix} : \text{gradient of the angular field (in terms of the parametric$$

coordinates of the B-spline surface)

- Expression of the technological constraint

$$r(\xi) = \frac{1}{\chi(\xi)}, \quad \min_{\xi} r(\xi) \geq r_{\text{adm}}$$

Second-level problem (with constraints): mathematical formulation

$$\min_{\delta_k^{(i,j)}} \sum_{l=1}^N f_l \left(\delta_k^{(i,j)} \right)$$

subject to

$$\min_{\xi} r(\xi) \geq r_{\text{adm}}$$

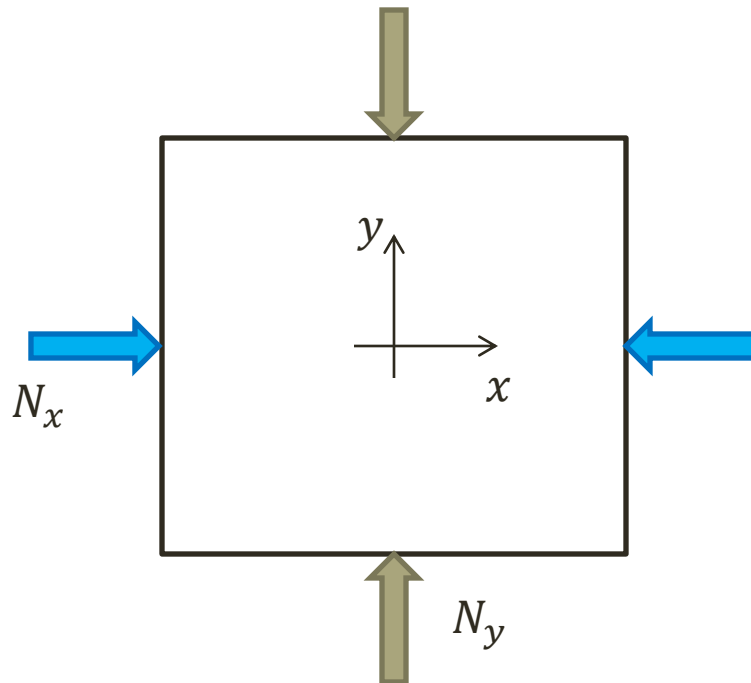
Remarks

- **Goal:** find at least one stacking sequence meeting the material and geometrical parameters resulting from the first-level problem (for each point P^l of the structure)
- $f_l \left(\delta_k^{(i,j)} \right)$ is a positive semi-definite function in terms of the polar parameters linked to every control point, while it is a non-convex function in terms of the *local* (i.e. point-wise) layers orientations
- The design variables are the orientation angles defined for each control point in every ply

On-going research and available results

Example: maximisation of the first buckling load of a VAT plate with a constraint on the number of plies ($n=24$)

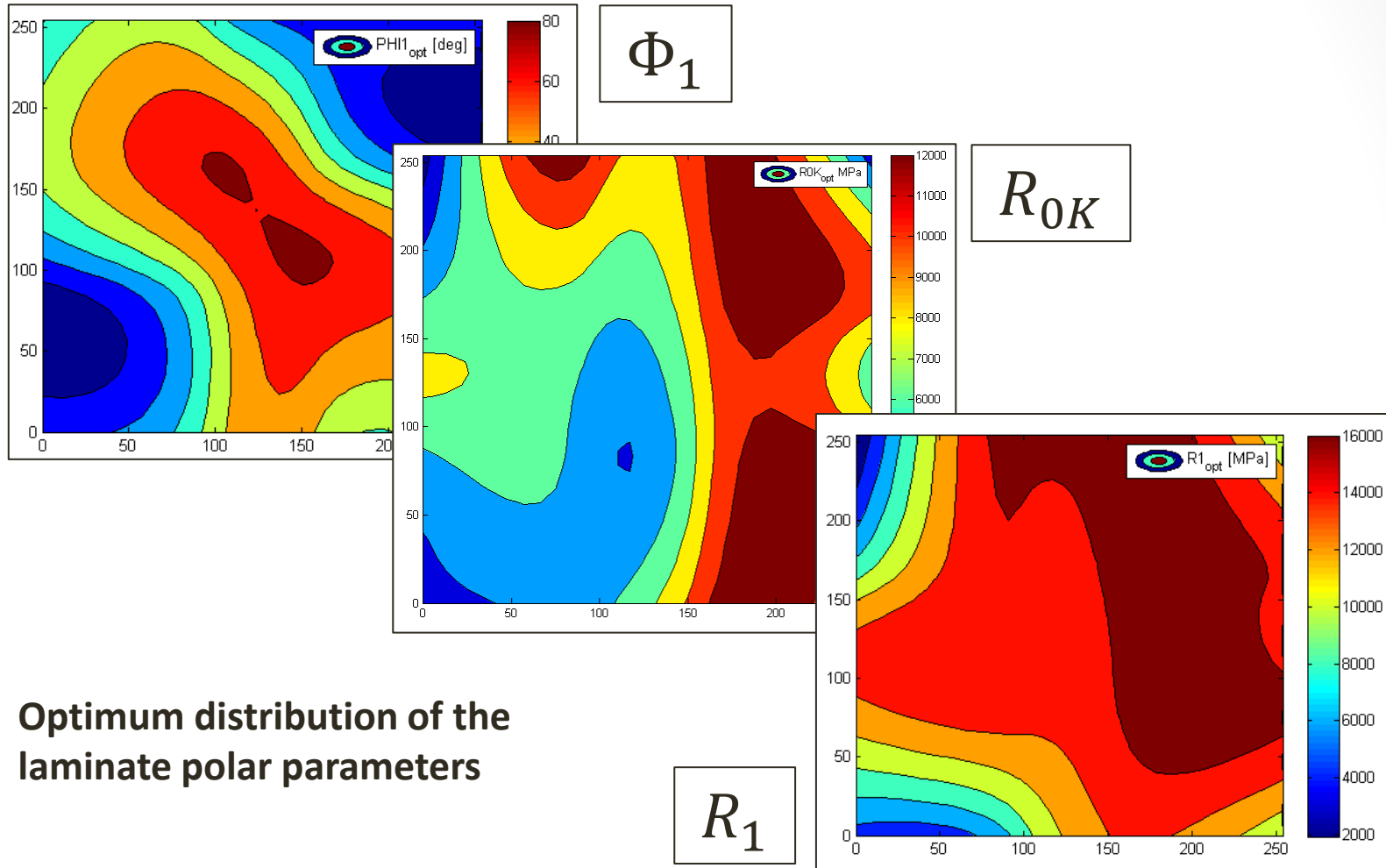
Reference solution	
Stacking sequence	$[0/-45/0/45/90/45/0/-45/90/45/90/-45]_S$
Buckling load	81,28 N/mm ($N_y = 0,5N_x$)



Optimum solution	
Buckling load	173,94 N/mm
Increment	114%

On-going research and available results

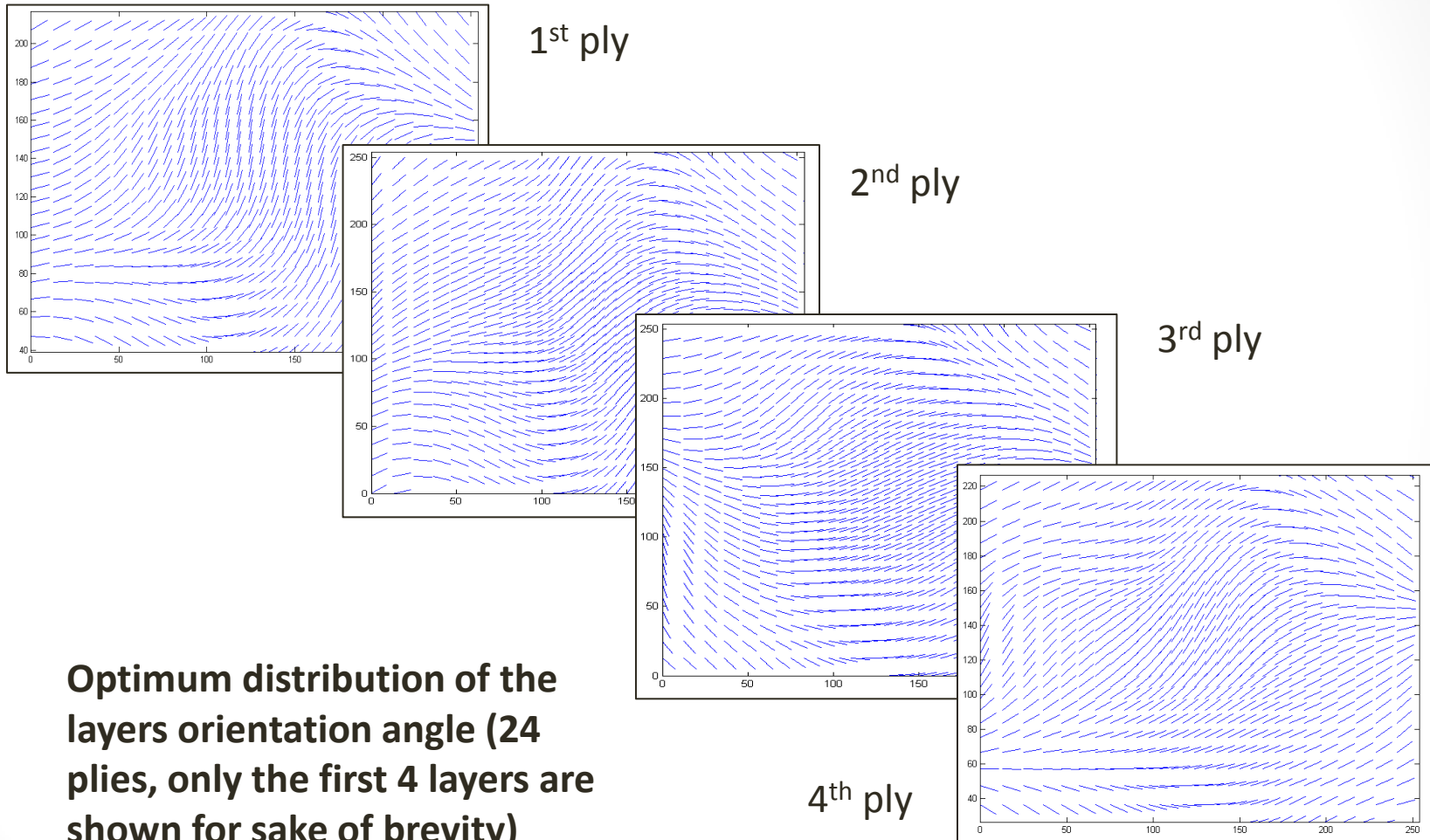
Optimum solution: first level problem



Optimum distribution of the laminate polar parameters

On-going research and available results

Optimum solution: second level problem



Multi-scale design and optimization of variable angle tow laminates: conclusive remarks and perspectives

- ✓ Understanding the potential and the technological restrictions linked to the automated fiber tow placement (AFP) process
- ✓ Formulating the design problem in the most general way
- ✓ Developing a multi-scale optimization strategy able to take into account, since the early stages of the design process, **all the manufacturability constraints linked to the AFP process**
- ✓ Validating the entire design process through experimental tests that will allow of characterizing the mechanical behavior of this new class of composite materials
- ✓ Analysis of the influence of:
 - heterogeneities due both to the local variation in the volume fraction of the reinforcement as well as to the presence of defects;
 - singularities at the free edges of the laminate to determine a link between the intensity of singularities and the breaking point interfaces;
 - anisotropy

on the evolution of damage taking into account both types of adhesive and cohesive failures

Thanks for your attention