

A MultiGrid finite difference solver for brittle fracture simulation using phase field method in heterogeneous materials

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Abstract

The prediction of brittle fracture initiation and propagation in heterogeneous structures is one of the main challenging problems in the field of solid failure mechanics. Nowadays with the development of X-Ray tomography, it is possible to account for the real micro structure in a computational model. Recently, a new phase field method proposed by Miehe et al. [1] has been developed for describing the brittle fracture propagation, and Nguyen et al. [2] introduced a modified algorithm to simplify the computation of strain tensor split, which makes the problem linear.

However, due to the complexity of the heterogeneous structure, its simulation requires dense grids for the very local description of the actual material topology. The efficiency of the MultiGrid finite difference method allows a computational cost that depends linearly on the number of unknowns. This offers great opportunities to simulate the brittle fracture in strongly heterogeneous materials.

According to previous work by Boffy et al. [3] and Gu et al. [4], an efficient MultiGrid solver simulating a 3D heterogeneous material through the solution the elastic equations has been built. The efficiency has been validated as the computing time and allocated memory remains small compared to traditional numerical methods. Based on this, the aim of the current work is to extend the model with the phase field method for brittle fracture.

Following the algorithmic framework proposed in [1, 2], the extension from the current MultiGrid finite difference model at each incremental step is direct and can be summarized as follows:

- (1) Split strain tensor: $\epsilon^\pm(\epsilon, d)$.
- (2) MultiGrid finite difference scheme: compute elastic Lamé equation according to $\sigma(\epsilon^\pm, d)$.

- (3) Compute history strain energy: $H(\epsilon^\pm, d)$.
- (4) MultiGrid finite difference scheme: compute phase field function: $d(H)$.

The advantage is that steps (2) and (4) can be solved through a MultiGrid finite difference scheme and are expected to be more efficient compared to the Finite Element scheme in [2].

Keywords: MultiGrid, heterogeneous, brittle fracture, phase field

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