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► **To cite this version:**

Hadrien Kurkjian, Yvan Castin, Alice Sinatra. Landau-Khalatnikov phonon damping in strongly interacting Fermi gases. 2016. hal-01350507v2

HAL Id: hal-01350507

<https://hal.science/hal-01350507v2>

Preprint submitted on 12 Oct 2016 (v2), last revised 3 Jan 2017 (v3)

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Landau-Khalatnikov phonon damping in strongly interacting Fermi gases

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PACS 03.75.Kk – Dynamic properties of condensates; collective and hydrodynamic excitations, superfluid flow

PACS 67.85.Lm – Degenerate Fermi gases

PACS 47.37.+q – Hydrodynamic aspects of superfluidity; quantum fluids

Abstract – We derive the phonon damping rate due to the four-phonon Landau-Khalatnikov process in low temperature strongly interacting Fermi gases using quantum hydrodynamics, correcting and extending the original calculation of Landau and Khalatnikov [ZhETF, **19** (1949) 637]. Our predictions can be tested in state-of-the-art experiments with cold atomic gases in the collisionless regime.

Introduction. – Phonons, sound waves, low energy normal modes or gapless collective excitations are ubiquitous in physics. In uniform weakly-excited quantum many-body systems with short-range interactions, they are described as quasiparticles characterized by a dispersion relation approximately linear at low wavenumber, $\omega_{\mathbf{q}} \sim cq$ with c the speed of sound, and by a damping rate much smaller than the eigenfrequency $\Gamma_{\mathbf{q}} \ll \omega_{\mathbf{q}}$. Phonon damping plays a central role in transport phenomena such as thermal conduction in dielectric solids, and in hydrodynamic properties such as temperature dependent viscosity and attenuation of sound in liquid helium [1, 2]. It is also crucial for macroscopic coherence properties, since it determines the intrinsic coherence time of bosonic and fermionic gases in the condensed or pair-condensed regime [3–5]. In the absence of impurities the damping of low-energy phonons is determined by phonon-phonon interactions that conserve energy and momentum and it crucially depends on the curvature of the phonon dispersion relation [6, 7]. For a concave dispersion relation, $1 \leftrightarrow 2$ Beliaev-Landau processes involving three phonons are not resonant and the $2 \leftrightarrow 2$ Landau-Khalatnikov process involving four quasiparticles dominates at low q .

In this paper we consider an unpolarized gas of spin-1/2 fermions prepared in thermal equilibrium at a temperature T below the critical temperature, where a macroscopic coherence between pairs of opposite spin fermions appears. Compared to other many-body fermionic systems, atomic gases offer the unique possibility to tune the interaction strength with an external magnetic field close

to a so-called Feshbach resonance. This allows experimentalists to explore the crossover between the Bose-Einstein Condensate (BEC) and Bardeen-Cooper-Schrieffer (BCS) regimes [8–16]. The dispersion relation of low energy excitations, describing the collective motion of the pair center of mass, has a phononic start at small wavenumbers [17–22] and changes from convex to concave in the BEC-BCS crossover, close to the strongly interacting unitary limit [20, 22]. Therefore, the damping caused by the $2 \leftrightarrow 2$ processes should be directly observable in cold Fermi gases, contrarily to helium-4 and weakly-interacting Bose gases where the convex dispersion relation supports Landau-Beliaev damping. On the theoretical side, the original study by Landau and Khalatnikov of the $2 \leftrightarrow 2$ damping rate [1] is limited to the case where one of the colliding phonons has a small wavenumber compared to the other and it performs as we shall see an unjustified approximation on the coupling amplitude. Here, we give the general expression of the phonon damping rate in the concave dispersion relation regime at low temperature, where it is dominated by the $2 \leftrightarrow 2$ processes, correcting and extending the original calculation of reference [1]. In the whole paper we restrict to the so-called collisionless regime where the phonon frequency times the typical collision time in the gas is much larger than one, $\omega_{\mathbf{q}}\tau_c \gg 1$ [23, 24]. This is in general the case in superfluid gases at low temperature¹.

¹One can estimate $\tau_c \simeq 1/\Gamma_{\mathbf{q}_{\text{th}}}$ where $\hbar c q_{\text{th}} = k_B T$. Then for excitation frequencies scaling as $k_B T$, as in eq. (14), the condition $\omega_{\mathbf{q}} \gg \Gamma_{\mathbf{q}}$, satisfied for a weakly-excited gas, implies $\omega_{\mathbf{q}} \gg \Gamma_{\mathbf{q}_{\text{th}}}$

Effective 2 \leftrightarrow 2 phonon coupling. – The theoretical framework we use is the irrotational quantum hydrodynamics of Landau and Khalatnikov [1] with the Hamiltonian ²

$$\hat{H} = \int d^3r \left[\frac{\hbar^2}{2m} \nabla \hat{\phi} \cdot \hat{\rho} \nabla \hat{\phi} + e_0(\hat{\rho}) \right] \quad (1)$$

where $e_0(\hat{\rho})$ is the ground state energy density, m is the mass of a particle, and the superfluid velocity field operator $\hat{\mathbf{v}}(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \hat{\phi}(\mathbf{r}, t)$ is the gradient of the phase field operator, canonically conjugated to the density field operator $[\hat{\rho}(\mathbf{r}, t), \hat{\phi}(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$. Assuming small deviations of $\hat{\rho}(\mathbf{r}, t)$ and $\hat{\phi}(\mathbf{r}, t)$ from their uniform spatial averages, one finds the normal modes Fourier components $\delta\hat{\rho}_{\mathbf{q}} \propto q^{1/2}(\hat{b}_{\mathbf{q}} + \hat{b}_{-\mathbf{q}}^\dagger)$ and $\delta\hat{\phi}_{\mathbf{q}} \propto -iq^{-1/2}(\hat{b}_{\mathbf{q}} - \hat{b}_{-\mathbf{q}}^\dagger)$, where the annihilation and creation operators of quasiparticles $\hat{b}_{\mathbf{q}}$ and $\hat{b}_{\mathbf{q}}^\dagger$ obey bosonic commutation relations. One inserts the expansion of $\hat{\rho}$ and $\hat{\phi}$ over these modes in the Hamiltonian (1), which results in the series $\hat{H} = E_0 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \dots$, where the index refers to the order in $\hat{b}_{\mathbf{q}}$ and $\hat{b}_{\mathbf{q}}^\dagger$.

In the concave dispersion relation regime considered in this paper, the direct phonon coupling due to \hat{H}_3 is not resonant, and the leading resonant coupling is a four-phonon process. An effective interaction Hamiltonian \hat{H}_{eff} coupling an initial Fock state of quasiparticles $|i\rangle$ of energy E_i to a final one $|f\rangle$ of same energy $E_f = E_i$, where two wavevectors \mathbf{q}_1 and \mathbf{q}_2 are annihilated and two other wavevectors \mathbf{q}_3 and \mathbf{q}_4 are created, can then be derived in second order perturbation theory by considering the direct coupling by \hat{H}_4 to first order and the indirect coupling (involving a non-resonant intermediate state $|\lambda\rangle$) by \hat{H}_3 to second order,

$$\langle f | \hat{H}_{\text{eff}} | i \rangle \simeq \langle f | \hat{H}_4 | i \rangle + \sum_{\lambda} \frac{\langle f | \hat{H}_3 | \lambda \rangle \langle \lambda | \hat{H}_3 | i \rangle}{E_i - E_{\lambda}} \equiv \mathcal{A}_{i \rightarrow f} \quad (2)$$

The reader will notice that, for the purely linear dispersion relation $\omega_{\mathbf{q}} = cq$ predicted by \hat{H}_2 , the denominators in (2) diverge for aligned wavevectors because the intermediate processes become resonant, for example $E_i - E_{\lambda} = \omega_1 + \omega_2 - \omega_{\mathbf{q}_1 + \mathbf{q}_2} = 0$, with the short-hand notation $\omega_i \equiv \omega_{\mathbf{q}_i}$. Following Landau and Khalatnikov, we regularize this divergence by including the actual curvature of the spectrum [20, 22] in the energy denominators³

$$\hbar\omega_{\mathbf{q}} \underset{q \rightarrow 0}{=} \hbar cq \left[1 + \frac{\gamma}{8} \left(\frac{\hbar q}{mc} \right)^2 + O \left(\frac{\hbar q}{mc} \right)^4 \right]. \quad (3)$$

which ensures the collisionless regime.

²In principle, this Hamiltonian has to be regularized by introducing an ultraviolet momentum cut-off or by discretizing the real space on a lattice as in reference [5]. This however does not play a role here.

³The quantum hydrodynamics Hamiltonian can be supplemented by terms leading to a curved dispersion relation [25]. Except in the energy denominator, this brings a negligible correction to the phonon damping rate at low temperature.

Here the speed of sound c is related to the gas density ρ and the ground state chemical potential μ by

$$mc^2 = \rho \frac{d\mu}{d\rho} \quad (4)$$

whereas the dimensionless curvature parameter $\gamma < 0$ must be measured or determined from a microscopic theory. By introducing the dimensionless and state-independent effective coupling amplitude \mathcal{A}_{eff} ,

$$\mathcal{A}_{i \rightarrow f} = \sqrt{n_{\mathbf{q}_1} n_{\mathbf{q}_2} (1 + n_{\mathbf{q}_3}) (1 + n_{\mathbf{q}_4})} \frac{4mc^2}{\rho L^3} \mathcal{A}_{\text{eff}} \quad (5)$$

where $n_{\mathbf{q}_i}$ are the phonon occupation numbers in the initial Fock state $|i\rangle$, and by considering in eq. (2) the six possible intermediate states $|\lambda\rangle$ where a virtual phonon is created and reabsorbed (or absorbed and recreated in the six corresponding finite temperature diagrams, in such a way that the temperature dependence disappears) we find

$$\begin{aligned} \mathcal{A}_{\text{eff}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4) &= \frac{1}{16} \sqrt{\frac{\hbar^4 \omega_1 \omega_2 \omega_3 \omega_4}{m^4 c^8}} \left(\Sigma_{\text{F}} \right. \\ &+ \frac{(\omega_1 + \omega_2)^2 A_{1234} + \omega_{\mathbf{q}_1 + \mathbf{q}_2}^2 B_{1234}}{(\omega_1 + \omega_2)^2 - \omega_{\mathbf{q}_1 + \mathbf{q}_2}^2} \\ &+ \frac{(\omega_1 - \omega_3)^2 A_{1324} + \omega_{\mathbf{q}_1 - \mathbf{q}_3}^2 B_{1324}}{(\omega_1 - \omega_3)^2 - \omega_{\mathbf{q}_1 - \mathbf{q}_3}^2} \\ &\left. + \frac{(\omega_1 - \omega_4)^2 A_{1423} + \omega_{\mathbf{q}_1 - \mathbf{q}_4}^2 B_{1423}}{(\omega_1 - \omega_4)^2 - \omega_{\mathbf{q}_1 - \mathbf{q}_4}^2} \right) \quad (6) \end{aligned}$$

We introduced the angle-dependent coefficients

$$\begin{aligned} A_{ijkl} &= (3\Lambda_F + u_{ij})(1 + u_{kl}) \\ &+ (3\Lambda_F + u_{kl})(1 + u_{ij}) + (1 + u_{ij})(1 + u_{kl}) \quad (7) \\ B_{ijkl} &= (3\Lambda_F + u_{ij})(3\Lambda_F + u_{kl}) \quad (8) \end{aligned}$$

with $u_{ij} = \mathbf{q}_i \cdot \mathbf{q}_j / q_i q_j$ and the thermodynamic quantities

$$\Sigma_{\text{F}} \equiv \frac{\rho^3}{mc^2} \frac{d^3\mu}{d\rho^3} \quad (9)$$

$$\Lambda_{\text{F}} \equiv \frac{\rho}{3} \frac{d^2\mu}{d\rho^2} \left(\frac{d\mu}{d\rho} \right)^{-1} \quad (10)$$

From the coupling amplitude \mathcal{A}_{eff} we finally obtain the sought low-energy effective Hamiltonian for the 2 \leftrightarrow 2 phonon process in a cubic quantization volume of size L :

$$\hat{H}_{\text{eff}} = \frac{mc^2}{\rho L^3} \sum_{\substack{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4 \\ \mathbf{q}_1 + \mathbf{q}_2 = \mathbf{q}_3 + \mathbf{q}_4}} \mathcal{A}_{\text{eff}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4) \hat{b}_{\mathbf{q}_3}^\dagger \hat{b}_{\mathbf{q}_4}^\dagger \hat{b}_{\mathbf{q}_1} \hat{b}_{\mathbf{q}_2} \quad (11)$$

The phonon damping rate. – To calculate the phonon damping rate from the effective Hamiltonian (11) we use a master equation approach as in reference [26]: the phonon mode of momentum $\hbar\mathbf{q}$ is linearly coupled to the thermal reservoir containing all the other phonon

modes by an interaction Hamiltonian $\hat{H}_{\text{int}} = \hat{R}^\dagger \hat{b}_{\mathbf{q}} + \hat{R} \hat{b}_{\mathbf{q}}^\dagger$, where \hat{R}^\dagger collects terms in $\hat{b}_{\mathbf{q}_3}^\dagger \hat{b}_{\mathbf{q}_4}^\dagger \hat{b}_{\mathbf{q}_2}$ from \hat{H}_{eff} . The master equation gives an exponential relaxation of the average population of the mode \mathbf{q} towards its thermal equilibrium value with a rate

$$\Gamma_{\mathbf{q}} = \frac{(mc^2)^2}{4\pi^5 \hbar^2 \rho^2} \int d^3 q_2 d^3 q_3 |\mathcal{A}_{\text{eff}}(\mathbf{q}, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4)|^2 \delta(\omega_3 + \omega_4 - \omega_2 - \omega_{\mathbf{q}}) [\bar{n}_2(1 + \bar{n}_3 + \bar{n}_4) - \bar{n}_3 \bar{n}_4] \quad (12)$$

where $\mathbf{q}_4 = \mathbf{q} + \mathbf{q}_2 - \mathbf{q}_3$ due to momentum conservation and we have used the notation $\bar{n}_i \equiv \bar{n}_{\mathbf{q}_i}$ to indicate the phonon occupation numbers in thermal equilibrium. The result (6),(12) goes beyond that of Ref. [1] where the authors concentrated on a single diagram for the intermediate processes. This will have a significant impact on the behavior of the damping rate as a function of q as we will see. To go further analytically, we restrict to sufficiently low temperature

$$\epsilon \equiv \frac{k_B T}{mc^2} \ll 1 \quad (13)$$

so that only the (almost) linear region of the phonon dispersion relation is explored and the integral (12) is dominated by configurations in which the four involved wavevectors are almost aligned. By introducing the rescaled quantities

$$\tilde{q}_i = \frac{\hbar c q_i}{k_B T}; \quad \tilde{\theta}_i = \frac{\theta_i}{\epsilon |\gamma|^{1/2}} \quad (14)$$

with θ_i the angles between \mathbf{q}_i and \mathbf{q} and γ the curvature parameter in the spectrum (3), and by expanding the coupling amplitude (6) for $\epsilon \rightarrow 0$ at fixed \tilde{q}_i and $\tilde{\theta}_i$, we obtain the central result of this paper:

$$\frac{\hbar \Gamma_{\mathbf{q}}}{\epsilon_F} \underset{\epsilon \rightarrow 0}{\sim} \frac{K}{|\gamma|} \left(\frac{T}{T_F} \right)^7 \tilde{\Gamma}(\tilde{q}) \quad (15)$$

In this formula $\epsilon_F = k_B T_F = \hbar^2 k_F^2 / 2m$ is the Fermi energy, K is an interaction-dependent thermodynamic quantity

$$K = 2 \left(\frac{3}{4} \right)^6 \left(\frac{\epsilon_F}{mc^2} \right)^3 (1 + \Lambda_F)^4 \quad (16)$$

and the rescaled phonon damping rate $\tilde{\Gamma}(\tilde{q})$, shown in fig. 1, is a universal, monotonically increasing function of the dimensionless wavenumber. Explicitly

$$\begin{aligned} \tilde{\Gamma}(\tilde{q}) &= \int_0^\infty d\tilde{q}_2 \int_0^{\tilde{q} + \tilde{q}_2} d\tilde{q}_3 \frac{\tilde{q}_2^3 \tilde{q}_3^3 (\tilde{q} + \tilde{q}_2 - \tilde{q}_3)}{\tilde{q} |v|} \\ &\times \frac{[1 + f(\tilde{q}_2)] f(\tilde{q}_3) f(\tilde{q} + \tilde{q}_2 - \tilde{q}_3)}{f(\tilde{q})} \\ &\times \int_0^\pi d\phi \int_0^{\pi/2} d\alpha \sin \alpha \cos \alpha \Theta\left(-\frac{v}{u}\right) |\mathcal{A}_{\text{red}}|^2 \quad (17) \end{aligned}$$

where $\Theta(x \geq 0) = 1$, $\Theta(x < 0) = 0$ is the Heaviside function, $f(x) = 1/(e^x - 1)$ originates from the Bose law, u and

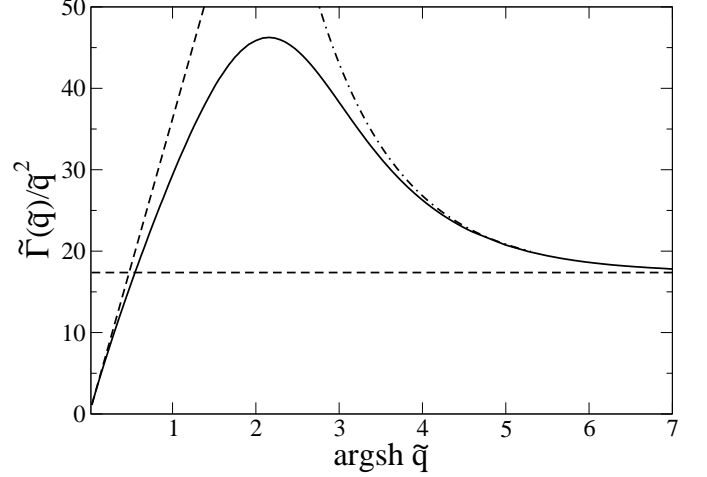


Fig. 1: Rescaled phonon damping rate due to $2 \leftrightarrow 2$ phonon processes in a superfluid spin-1/2 Fermi gas, as a function of the rescaled wavenumber $\tilde{q} = \hbar c q / k_B T$, or more precisely of its inverse hyperbolic sine. Due to these rescalings, see eq. (15), the result is universal and applies at sufficiently low temperature in the whole region of the BEC-BCS crossover where the dispersion relation is concave at low q . The dashed straight lines show the limiting behaviors (21) and (22) and the dot-dashed curve at large \tilde{q} is a fit of $\tilde{\Gamma}(\tilde{q})/\tilde{q}^2$ by an affine function of $1/\tilde{q}$.

v are the following functions of $\tilde{q}, \tilde{q}_2, \tilde{q}_3, \alpha = \arctan \theta_3 / \theta_2$, and of the relative azimuthal angle ϕ of \mathbf{q}_2 and \mathbf{q}_3 :

$$u = \frac{\tilde{q}(\tilde{q}_3 \sin^2 \alpha - \tilde{q}_2 \cos^2 \alpha) + \tilde{q}_2 \tilde{q}_3 (1 - \sin 2\alpha \cos \phi)}{\tilde{q} + \tilde{q}_2 - \tilde{q}_3} \quad (18)$$

$$v = \frac{1}{4} [\tilde{q}^3 + \tilde{q}_2^3 - \tilde{q}_3^3 - (\tilde{q} + \tilde{q}_2 - \tilde{q}_3)^3] \quad (19)$$

We introduced in eq. (17) the reduced coupling amplitude

$$\begin{aligned} \mathcal{A}_{\text{red}} &= \frac{1}{\tilde{q}_2 \left(\frac{\cos^2 \alpha}{(\tilde{q} + \tilde{q}_2)^2} - \frac{3u}{4v} \right)} - \frac{1}{\tilde{q}_3 \left(\frac{\sin^2 \alpha}{(\tilde{q} - \tilde{q}_3)^2} - \frac{3u}{4v} \right)} \\ &- \frac{1}{\frac{\tilde{q}_2 \cos^2 \alpha - \tilde{q}_3 \sin^2 \alpha + u}{(\tilde{q}_2 - \tilde{q}_3)^2} - \frac{3u}{4v} (\tilde{q} + \tilde{q}_2 - \tilde{q}_3)} \quad (20) \end{aligned}$$

The limiting behaviors of the normalized universal Landau-Khalatnikov damping rate (17)

$$\tilde{\Gamma}(\tilde{q}) \underset{\tilde{q} \rightarrow 0}{=} \frac{16\pi^5}{135} \tilde{q}^3 + O(\tilde{q}^4) \quad (21)$$

$$\tilde{\Gamma}(\tilde{q}) \underset{\tilde{q} \rightarrow \infty}{=} \frac{16\pi \zeta(5)}{3} \tilde{q}^2 + O(\tilde{q}) \quad (22)$$

are shown in fig. 1 as dashed straight lines. Here ζ is the Riemann zeta function. Equations (21)-(22) disagree with Eqs.(7.6) and (7.12) in reference [1], even for the order in \tilde{q} , our results being subleading by two orders. This is due to the fact that the diagrams neglected by Landau and Khalatnikov in the calculation of the effective amplitude \mathcal{A}_{eff} turn out to interfere destructively with the supposedly leading term that they keep.

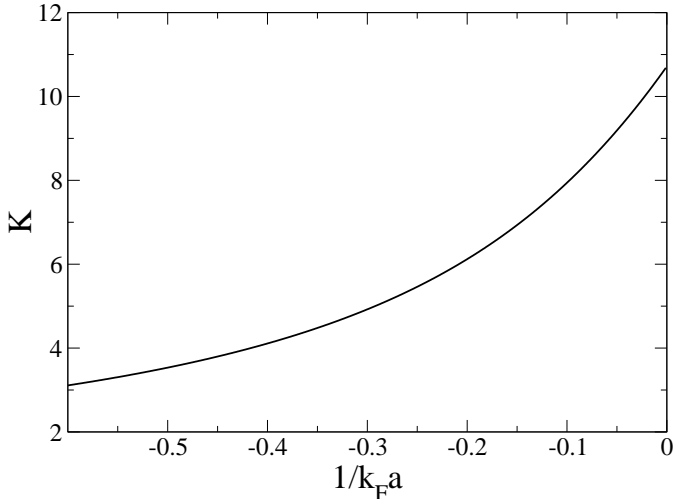


Fig. 2: Thermodynamic quantity (16) as a function of $1/k_F a$ where $k_F = (3\pi^2\rho)^{1/3}$ is the Fermi wavenumber and a the s -wave scattering length, from the measured zero-temperature equation of state of the spin-1/2 Fermi gas [14, 27].

Physical discussion and observability. – We now discuss the main result (15) and the possibility to test it in state-of-the-art experiments with cold fermionic gases. In fig. 2 we use the measured equation of state of the spin-1/2 Fermi gas [14, 27] to plot the thermodynamic quantity K in eq. (16) as a function of $1/k_F a$. The curvature parameter γ appearing in the phonon dispersion relation (3) has not yet been measured in strongly interacting Fermi gases. We plot in fig. 3 the prediction of reference [22] which relies on the Random Phase Approximation and coincides with that of other approximate theories [18, 20]. We plot it separately so that, once an experimental value will be available, it can be used in eq. (15). To give a numerical example, we choose the interaction strength such that the coefficient of the quintic term in the expansion (3) vanishes, $1/k_F a \simeq -0.39$ according to reference [22]. From the measured equation of state, the value of the sound velocity is $c \simeq 0.43\hbar k_F/m$ and the parameter Λ_F obeys $1 + \Lambda_F \simeq 0.866$, leading to the thermodynamic constant $K \simeq 4.2$. The predicted value of the curvature parameter is $\gamma \simeq -0.30$ [22]. The temperature should be sufficiently low and the wavenumber q much smaller than $2\Delta/\hbar c$ to avoid the excitation of the fermionic branch [28, 29]. By choosing $T = 0.073 T_F$, larger than the temperature already achieved in reference [30], and $\tilde{q} \equiv \hbar c q/k_B T = 5/2$, one obtains $\epsilon \equiv k_B T/mc^2 \simeq 0.20$, $\hbar q/mc \simeq 0.50$, $q \simeq 0.21 k_F$, $\hbar c q/2\Delta \simeq 0.2$ and $\tilde{\Gamma} \simeq 265$. For the typical value $T_F = 1\mu\text{K}$ with ${}^6\text{Li}$ atoms, one obtains $2\pi/q \simeq 6\mu\text{m}$, $\omega_{\mathbf{q}}/2\pi \simeq 3.8\text{ kHz}$ and $c \simeq 2.2\text{ cm/s}$. Eq. (15) then predicts $\Gamma_{\mathbf{q}} \simeq 5\text{s}^{-1}$, that is a phonon lifetime $\Gamma_{\mathbf{q}}^{-1} \simeq 190\text{ ms}$ and a mode quality factor $\omega_{\mathbf{q}}/\Gamma_{\mathbf{q}} \simeq 4600$.⁴ Quality factors of this order of magnitude have been ob-

⁴As expected, this is in the collisionless regime since the angular frequency $\omega_{\mathbf{q}}$ is much larger than the thermalization rate, that we

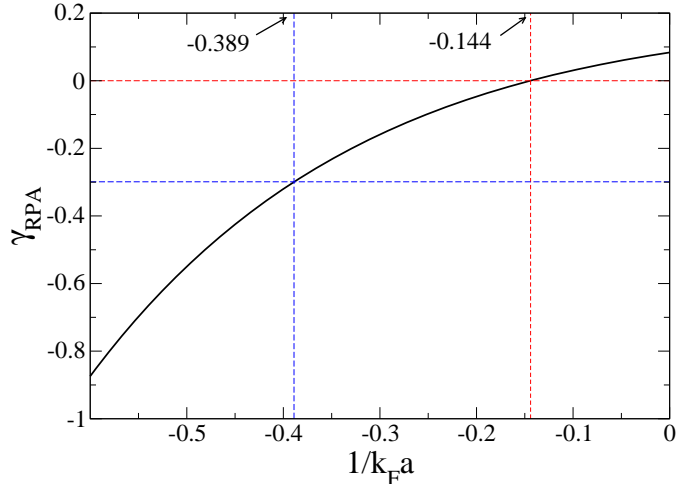


Fig. 3: Curvature parameter γ obtained with the RPA in reference [22] for the phonon dispersion relation (3), as a function of $1/k_F a$ where k_F is the Fermi wavenumber and a the s -wave scattering length. The dashed lines mark the two points where either the cubic ($1/k_F a = -0.144$) or the quintic ($1/k_F a = -0.389$) correction to the linear dispersion relation in eq. (3) vanishes.

served for the transverse monopole mode of an atomic Bose-Einstein condensate [31].

The possibility of trapping cold atoms in flat bottom potentials [32] opens the way to a direct test of our prediction in a spatially homogeneous system. In the box trapping potential, a Glauber coherent state of phonons in a standing-wave mode with a well-defined wavevector \mathbf{q} along a trap axis can be created by laser Bragg excitation of the condensate of pairs in the strongly interacting Fermi gas [33–35], on top of the preexisting background of thermal phonons. One simply matches the frequency difference and the wavevector difference of two laser standing waves to the eigenfrequency $\omega_{\mathbf{q}}$ and the wavevector \mathbf{q} of the desired phonon mode. The subsequent decay of the phonon coherent state can be monitored by measuring *in situ* the spatial modulation of the density at wavevector \mathbf{q} using the bosonizing imaging techniques of reference [13]. Note that in Bragg spectroscopy the density is usually determined after a time-of-flight which amounts to making a measurement in Fourier space. This is appropriate for $q \gg k_F$ where the scattered atoms separate from the Fermi sea of unscattered atoms. Here on the contrary $q \ll k_F$ and the measurement is best performed in real space. In weakly interacting Bose gases, damping times of a fraction of a second have been measured in experiments without being affected by extraneous damping mechanisms [31], and the Bragg technique has allowed to measure the Bogoliubov dispersion relation [36] and to observe the zero-temperature $1 \rightarrow 2$ Beliaev damping [37] with a successful

estimate by the damping rate at the typical thermal wavenumber $q_{\text{th}} = k_B T/\hbar c$: one finds $\tilde{\Gamma}(1) \simeq 26$ and $\Gamma_{\mathbf{q}_{\text{th}}}/\omega_{\mathbf{q}} \simeq 2 \times 10^{-5}$.

comparison of their q -dependence to theory.

To be complete, let us analyse in more detail the experimental proposal. In reference [32] the flat bottom trap has an elongated cylindric shape. For simplicity, we model it by an infinite square well potential in the three dimensions with widths $L_x = L_y \equiv L_\perp < L_z$ of rounded up values $L_\perp = 50\mu\text{m}$ and $L_z = 100\mu\text{m}$.⁵ The phonon mode functions are then products of sine functions $\frac{2^{3/2}}{V^{1/2}} \prod_{\alpha=x,y,z} \sin(q_\alpha r_\alpha)$, where $q_\alpha L_\alpha/\pi = n_\alpha \in \mathbb{N}^*$ and $V = L_\perp^2 L_z$ is the trap volume. During the short time interval $0 < t < \tau$, two retro-reflected far-off-resonant Bragg laser beams illuminate the trapped gas. They induce a conservative lightshift potential $W(\mathbf{r}, t) = W_0 |\mathcal{E}(\mathbf{r}, t)|^2$ where $\mathcal{E}(\mathbf{r}, t)$ is the reduced, dimensionless positive-frequency part of the laser electric field. In second quantized form this gives rise to the phonon-light coupling Hamiltonian

$$\hat{H}_W = \int d^3r W(\mathbf{r}, t) \delta\hat{\rho}(\mathbf{r}) \quad (23)$$

Following the values of the physical parameters given above, we take the phonon mode to be excited in the transverse ground state $n_x = n_y = 1$ with a wavenumber $q_z \simeq 0.5mc/\hbar$ along z , that is $n_z = 33$. To optimally excite this mode, we choose $\mathcal{E}(\mathbf{r}, t) = \sin(\mathbf{k}_1 \cdot \mathbf{r})e^{-i\omega_1 t} + i \cos(\mathbf{k}_2 \cdot \mathbf{r})e^{-i\omega_2 t}$. Here \mathbf{k}_i and ω_i , the wavevectors and the angular frequencies of the laser standing waves, obey $\mathbf{k}_2 = \mathbf{k}_1 + q_z \mathbf{e}_z$ and $\omega_2 = \omega_1 - \omega_{\mathbf{q}}$, with $\omega_{\mathbf{q}}$ the angular frequency of the phonon mode and \mathbf{e}_z the unit vector along z . In practice the wavevectors \mathbf{k}_1 and \mathbf{k}_2 are in the optical domain, with submicronic wavelength, so $k_1 \simeq k_2 \gg q$

⁵One may wonder if this is large enough for our infinite-system theory to apply. This will be the case if the typical spacing between the discrete values of $\omega_3 + \omega_4 - \omega_2 - \omega_{\mathbf{q}}$ in the argument of the Dirac distribution in eq. (12) is $\ll \Gamma_{\mathbf{q}}/2$. In the decay process of an unstable state, the energy is indeed conserved within $\pm \hbar\Gamma_{\mathbf{q}}/2$. We estimate the typical spacing by $1/\rho_{\text{states}}(\omega_{\mathbf{q}})$ with the density of states

$$\rho_{\text{states}}(\omega) = \sum_{\mathbf{q}_2, \mathbf{q}_3}^{\text{typ}} \delta(\omega_3 + \omega_4 - \omega_2 - \omega)$$

where the typical values of \mathbf{q}_2 and \mathbf{q}_3 are at an angle at most ϵ with respect to \mathbf{q} . Also we impose $q_2 < \langle q \rangle_{\text{th}}$ where $\hbar c \langle q \rangle_{\text{th}} = \frac{\pi^4}{30\zeta(3)} k_B T$ is the mean phonon thermal energy. The wavenumber q_3 is automatically limited by $q + q_2$ as in eq. (17). Replacing $\sum_{\mathbf{k}}$ by $V \int d^3k/(2\pi)^3$ in the thermodynamical limit, and taking the $\epsilon \rightarrow 0$ limit as in $\Gamma_{\mathbf{q}}$, we obtain $\rho_{\text{states}}(\omega_{\mathbf{q}})\Gamma_{\mathbf{q}}/2 \sim (\bar{L}/L_0)^6$ with $\bar{L}^3 = V$ the trap volume and

$$L_0 = \frac{4\pi\hbar}{3mc} \frac{\hbar k_F}{mc} \epsilon^{-7/3} \pi^{-1/3} (1 + \Lambda_F)^{-2/3} (\bar{\Gamma}I)^{-1/6}$$

where the integral

$$I = \int_0^{\frac{\pi^4}{30\zeta(3)}} \tilde{q}_2^2 d\tilde{q}_2 \int_0^{\tilde{q} + \tilde{q}_2} \tilde{q}_3^2 d\tilde{q}_3 \\ \times \int_0^1 R^3 dR \int_0^{\pi/2} \sin \alpha \cos \alpha d\alpha \int_0^\pi \frac{d\phi}{\pi} \delta(uR^2 + v)$$

is evaluated numerically, $I \simeq 2.445$. The thermodynamic limit is reached for $\bar{L} > L_0$. In our numerical example $\bar{L} \simeq 63\mu\text{m}$ is indeed larger than $L_0 \simeq 50\mu\text{m}$.

and the condition $\mathbf{k}_2 = \mathbf{k}_1 + q_z \mathbf{e}_z$ is achieved by introducing a small angle between them. In the resulting optical potential

$$W(\mathbf{r}, t) = W_0 \left\{ 1 - \frac{1}{2} [\cos(2\mathbf{k}_1 \cdot \mathbf{r}) - \cos(2\mathbf{k}_2 \cdot \mathbf{r})] \right. \\ \left. + [\sin(q_z z) - \sin((\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r})] \sin(\omega_{\mathbf{q}} t) \right\} \quad (24)$$

the time-independent part is non-resonant and can be neglected. In the time dependent part, the second sine term excites a mode of wavevector $\mathbf{k}_1 + \mathbf{k}_2$ far from resonance and can also be omitted. One is left for $0 < t < \tau$ with the effective Bragg Hamiltonian⁶

$$\hat{H}_W^{\text{res}} \simeq 2\hbar\Omega(\hat{b}_{\mathbf{q}} + \hat{b}_{\mathbf{q}}^\dagger) \sin(\omega_{\mathbf{q}} t) \quad (25)$$

of Rabi frequency $\Omega = W_0[\omega_{\mathbf{q}}\rho V/(16\hbar mc^2)]^{1/2}$. Integrating the Heisenberg equations of motion for the phonon annihilation operator $\hat{b}_{\mathbf{q}}$ and taking the expectation value in the initial $t = 0$ thermal state⁷, one finds $\langle \hat{b}_{\mathbf{q}}(\tau) \rangle = \Omega[\tau e^{-i\omega_{\mathbf{q}}\tau} - \sin(\omega_{\mathbf{q}}\tau)/\omega_{\mathbf{q}}]$ where the second term, which is the off-resonant effect of the negative-frequency part of $\sin(\omega_{\mathbf{q}} t)$, is zeroed by the choice $\tau = \pi/\omega_{\mathbf{q}} \simeq 130\mu\text{s}$. At the end of the Bragg pulse, this gives a z -modulation of the gas density that will subsequently decay to zero due to scattering of thermal phonons. Since absorption imaging gives in general access to an integrated density, we give the density modulation integrated along x and y :

$$\bar{\delta\rho}(z, t) = \frac{\pi W_0 \rho L_\perp^2}{2mc^2} \sin(q_z z) \cos(\omega_{\mathbf{q}} t) e^{-\Gamma_{\mathbf{q}} t/2} \quad (26)$$

⁶As the lightshift potential $W_0 \sin(q_z z) \sin(\omega_{\mathbf{q}} t)$ is x - and y -independent, it actually couples to several transversally excited phonon modes during the Bragg pulse. Only the even x -parity and even y -parity states are populated, $n_x = 2s_x + 1$ and $n_y = 2s_y + 1$, $(s_x, s_y) \in \mathbb{N}^2$, with normalised amplitudes $c_{s_x, s_y} = 2L_\perp^{-2} \int_0^{L_\perp} dx dy \sin \frac{x\pi n_x}{L_\perp} \sin \frac{y\pi n_y}{L_\perp} = \frac{8\pi^{-2}}{n_x n_y}$. In the main text we have presented a monomode calculation assuming $|c_{0,0}|^2 = 1$ as if only the transversally fundamental mode was excited. In reality, the result in eq. (26) has to be weighted by $|c_{0,0}|^2$ and the transversally excited modes give additional time dependent contributions to the integrated density modulation that dephase in a time of the order of $t_\perp = qL_\perp^2/(2\pi^2 c) \simeq 6$ ms, that is $\simeq 140/\omega_{\mathbf{q}}$ and $\simeq 0.03/\Gamma_{\mathbf{q}}$. Since $\sum_{s_x, s_y} |c_{s_x, s_y}|^2$ converges rather rapidly to unity, the relevant n_x and n_y are indeed $\ll n_z$, and the angular frequency differences $\omega_{(n_x, n_y, n_z)} - \omega_{(1, 1, n_z)} \simeq t_\perp^{-1} [s_x(s_x + 1) + s_y(s_y + 1)]$, with the obvious notation $\omega_{\mathbf{q}} = \omega_{(1, 1, n_z)}$, etc. To summarize, eq. (26) becomes $\bar{\delta\rho}(z, t) = \frac{\pi W_0 \rho L_\perp^2}{2mc^2} \sin(q_z z) \{ |c_{0,0}|^2 \cos(\omega_{\mathbf{q}} t) + \text{Re}[f(t) \exp(-i\omega_{\mathbf{q}} t)] \} e^{-\Gamma_{\mathbf{q}} t/2}$ with the periodic function

$$f(t) = \sum_{(s_x, s_y) \in \mathbb{N}^{2*}} |c_{s_x, s_y}|^2 e^{-i[s_x(s_x+1) + s_y(s_y+1)]t/t_\perp}$$

Since $\omega_{\mathbf{q}} t_\perp \gg 1$, $f(t)$ plays the role of an envelope function and mainly its modulus matters. For our square well potential $|c_{0,0}|^2 \simeq 0.657$ and $|f(t)| < 0.343$. This shows that the contributions of the excited modes and their Hamiltonian dephasing are rather harmless and cannot imitate a complete damping of the phonon mode.

⁷A similar calculation, but with bosons and periodic boundary conditions, was done in reference [38].

Conclusion. – We have calculated the Landau-Khalatnikov low-temperature phonon damping rate in strongly interacting superfluid Fermi gases. Our expression, thanks to an appropriate rescaling of the wavenumber, takes a universal functional form, plotted in fig. 1, that applies in the whole region of the BEC-BCS crossover where the phononic dispersion relation has a concave start. This includes the strongly interacting regime and paves the way to an experimental observation with ultracold atomic gases. As the phonon dispersion relation has a convex rather than concave start in liquid helium [6, 7], such an observation would be unprecedented and would open a new era in the exploration of low-temperature dynamics of macroscopically coherent quantum many-body systems.

REFERENCES

- [1] LANDAU L. and KHALATNIKOV I., *Zh. Eksp. Teor. Fiz.*, **19** (1949) 637.
- [2] KHALATNIKOV I. and CHERNIKOVA D., *Zh. Eksp. Teor. Fiz.*, **49** (1966) 1957.
- [3] JAKSCH D., GARDINER C. W., GHERI K. M. and ZOLLER P., *Phys. Rev. A*, **58** (1998) 1450.
- [4] SINATRA A., CASTIN Y. and WITKOWSKA E., *Phys. Rev. A*, **80** (2009) 033614.
- [5] KURKJIAN H., CASTIN Y. and SINATRA A., *C. R. Phys.*, **17** (2016) 789.
- [6] TUCKER M. A. H. and WYATT A. F. G., *Journal of Physics: Condensed Matter*, **4** (1992) 7745.
- [7] ADAMENKO I. N., KITSENKO Y. A., NEMCHENKO K. E. and WYATT A. F. G., *Phys. Rev. B*, **80** (2009) 014509.
- [8] O'HARA K. M., HEMMER S. L., GEHM M. E., GRANADE S. R. and THOMAS J. E., *Science*, **298** (2002) 2179.
- [9] BOURDEL T., CUBIZOLLES J., KHAYKOVICH L., MAGALHÃES K. M. F., KOKKELMANS S. J. J. M. F., SHLYAPNIKOV G. V. and SALOMON C., *Phys. Rev. Lett.*, **91** (2003) 020402.
- [10] BARTENSTEIN M., ALTMAYER A., RIEDL S., JOCHIM S., CHIN C., DENSLAG J. H. and GRIMM R., *Phys. Rev. Lett.*, **92** (2004) 120401.
- [11] BARTENSTEIN M., ALTMAYER A., RIEDL S., JOCHIM S., CHIN C., DENSLAG J. H. and GRIMM R., *Phys. Rev. Lett.*, **92** (2004) 203201.
- [12] ZWIERLEIN M. W., STAN C. A., SCHUNCK C. H., RAUPACH S. M. F., KERMAN A. J. and KETTERLE W., *Phys. Rev. Lett.*, **92** (2004) 120403.
- [13] ZWIERLEIN M. W., ABO-SHAER J. R., SCHIROTZEK A., SCHUNCK C. H. and KETTERLE W., *Nature*, **435** (2005) 1047.
- [14] NASCIMBÈNE S., NAVON N., JIANG K. J., CHEVY F. and SALOMON C., *Nature*, **463** (2010) 1057.
- [15] KU M. J. H., SOMMER A. T., CHEUK L. W. and ZWIERLEIN M. W., *Science*, **335** (2012) 563.
- [16] SIDORENKOV L. A., TEY M. K., GRIMM R., HOU Y.-H., PITAEVSKII L. and STRINGARI S., *Nature*, **498** (2013) 78.
- [17] ANDERSON P., *Phys. Rev.*, **112** (1958) 1900.
- [18] MARINI M., PISTOLESI F. and STRINATI G., *European Physical Journal B*, **1** (1998) 151.
- [19] COMBESCOT R., KAGAN M. Y. and STRINGARI S., *Phys. Rev. A*, **74** (2006) 042717.
- [20] KLIMIN S. N., TEMPERE J. and DEVREESE J. P. A., *Journal of Low Temperature Physics*, **165** (2011) 261.
- [21] RANDERIA M. and TAYLOR E., *Annual Review of Condensed Matter Physics*, **5** (2014) 209.
- [22] KURKJIAN H., CASTIN Y. and SINATRA A., *Phys. Rev. A*, **93** (2016) 013623.
- [23] STAMPER-KURN D. M., MIESNER H.-J., INOUE S., ANDREWS M. R. and KETTERLE W., *Phys. Rev. Lett.*, **81** (1998) 500.
- [24] BUGGLE C., PEDRI P., VON KLITZING W. and WALRAVEN J. T. M., *Phys. Rev. A*, **72** (2005) 043610.
- [25] BIGHIN G., SALASNICH L., MARCHETTI P. A. and TOIGO F., *Phys. Rev. A*, **92** (2015) 023638.
- [26] SINATRA A., CASTIN Y. and WITKOWSKA E., *Phys. Rev. A*, **75** (2007) 033616.
- [27] NAVON N., *Thermodynamique des gaz de bosons et fermions ultrafroids* Ph.D. thesis Université Pierre et Marie Curie, Paris (2011).
- [28] SCHIROTZEK A., SHIN Y.-I., SCHUNCK C. H. and KETTERLE W., *Phys. Rev. Lett.*, **101** (2008) 140403.
- [29] GAEBLER J. P., STEWART J. T., DRAKE T. E., JIN D. S., PERALI A., PIERI P. and STRINATI G. C., *Nature Phys.*, **6** (2010) 569.
- [30] HADZIBABIC Z., GUPTA S., STAN C. A., SCHUNCK C. H., ZWIERLEIN M. W., DIECKMANN K. and KETTERLE W., *Phys. Rev. Lett.*, **91** (2003) 160401.
- [31] CHEVY F., BRETIN V., ROSENBUSCH P., MADISON K. W. and DALIBARD J., *Phys. Rev. Lett.*, **88** (2002) 250402.
- [32] GAUNT A. L., SCHMIDTUTZ T. F., GOTLIBOVYCH I., SMITH R. P. and HADZIBABIC Z., *Phys. Rev. Lett.*, **110** (2013) 200406.
- [33] VEERAVALLI G., KUHNLE E., DYKE P. and VALE C. J., *Phys. Rev. Lett.*, **101** (2008) 250403.
- [34] KUHNLE E. D., HU H., LIU X.-J., DYKE P., MARK M., DRUMMOND P. D., HANNAFORD P. and VALE C. J., *Phys. Rev. Lett.*, **105** (2010) 070402.
- [35] LINGHAM M. G., FENECH K., HOINKA S. and VALE C. J., *Phys. Rev. Lett.*, **112** (2014) 100404.
- [36] STEINHAEUER J., OZERI R., KATZ N. and DAVIDSON N., *Phys. Rev. Lett.*, **88** (2002) 120407.
- [37] KATZ N., STEINHAEUER J., OZERI R. and DAVIDSON N., *Phys. Rev. Lett.*, **89** (2002) 220401.
- [38] SINATRA A., LOBO C. and CASTIN Y., *J. Phys. B*, **35** (2002) 3599.