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Landau-Khalatnikov phonon damping in strongly interacting Fermi gases

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We derive the phonon damping rate due to the four-phonon Landau-Khalatnikov process in low temperature strongly interacting Fermi gases using quantum hydrodynamics, correcting and extending the original calculation of Landau and Khalatnikov [ZhETF **19**, 637 (1949)]. Our predictions can be tested in state-of-the-art experiments with cold atomic gases in the collisionless regime.

PACS numbers:

Phonons, sound waves, low energy normal modes or gapless collective excitations are ubiquitous in physics. In uniform weakly-excited quantum many-body systems with short-range interactions, they are described as quasiparticles characterized by a dispersion relation approximately linear at low wavenumber, $\omega_{\mathbf{q}} \sim cq$ with c the speed of sound, and by a damping rate much smaller than the eigenfrequency $\Gamma_{\mathbf{q}} \ll \omega_{\mathbf{q}}$. Phonon damping plays a central role in transport phenomena, such as thermal conduction in dielectric solids, in hydrodynamic properties, such as temperature dependent viscosity and attenuation of sound in liquid helium [1, 2], and in macroscopic coherence properties, since it determines the intrinsic coherence time of bosonic and fermionic gases in the condensed or pair-condensed regime [3–5]. In the absence of impurities the damping of low-energy phonons is determined by phonon-phonon interactions that conserve energy and momentum and it crucially depends on the curvature of the phonon dispersion relation [6, 7]. For a concave dispersion relation, Beliaev-Landau processes involving three phonons are not resonant and the $2 \leftrightarrow 2$ Landau-Khalatnikov process involving four quasiparticles dominates at low q .

In this paper we consider an unpolarized gas of spin-1/2 fermions prepared in thermal equilibrium at a temperature T below the critical temperature, where a macroscopic coherence between pairs of opposite spin fermions appears. Compared to other many-body fermionic systems, atomic gases offer the unique possibility to tune the interaction strength with an external magnetic field close to a so-called Feshbach resonance, which allows experimentalists to explore the crossover between the Bose-Einstein Condensate (BEC) and Bardeen-Cooper-Schrieffer (BCS) regimes [8–16]. The dispersion relation of low energy excitations, describing the collective motion of the pair center of mass, has a phononic start at small wavenumbers [17–22] and changes from convex to concave in the BEC-BCS crossover, close to the strongly interacting unitary limit [20, 22]. Therefore, the damping caused by the $2 \leftrightarrow 2$ processes should be directly observable in cold Fermi gases, contrarily to helium-4 and weakly-interacting Bose gases where the convex dispersion relation supports Landau-Beliaev damping. On the theoretical side, the original study of

Landau and Khalatnikov [1] is limited to the case where one of the colliding phonons has a small wavenumber compared to the other and it performs as we shall see an unjustified approximation on the coupling amplitude. Here, we give the general expression of the phonon damping rate in the concave dispersion relation regime at low temperature, where it is dominated by the $2 \leftrightarrow 2$ processes, correcting and extending the original calculation of [1]. In the whole paper we restrict to the so-called collisionless regime where the phonon frequency times the typical collision time in the gas is much larger than one, $\omega_{\mathbf{q}}\tau_c \gg 1$ [23, 24]. This is in general the case in superfluid gases at low temperature [33].

The theoretical framework we use is the irrotational quantum hydrodynamics of Landau and Khalatnikov [1] with the Hamiltonian [34]

$$\hat{H} = \int d^3r \left[\frac{\hbar^2}{2m} \nabla \hat{\phi} \cdot \hat{\rho} \nabla \hat{\phi} + e_0(\hat{\rho}) \right] \quad (1)$$

where $e_0(\hat{\rho})$ is the ground state energy density, m is the mass of a particle, and the superfluid velocity field operator $\hat{\mathbf{v}}(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \hat{\phi}(\mathbf{r}, t)$ is the gradient of the phase field operator, canonically conjugated to the density field operator $[\hat{\rho}(\mathbf{r}, t), \hat{\phi}(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$. Assuming small deviations of $\hat{\rho}(\mathbf{r}, t)$ and $\hat{\phi}(\mathbf{r}, t)$ from their uniform spatial averages, one finds the normal modes Fourier components $\delta\hat{\rho}_{\mathbf{q}} \propto q^{1/2}(\hat{b}_{\mathbf{q}} + \hat{b}_{-\mathbf{q}}^\dagger)$ and $\delta\hat{\phi}_{\mathbf{q}} \propto -iq^{-1/2}(\hat{b}_{\mathbf{q}} - \hat{b}_{-\mathbf{q}}^\dagger)$, where the annihilation and creation operators of quasiparticles $\hat{b}_{\mathbf{q}}$ and $\hat{b}_{\mathbf{q}}^\dagger$ obey bosonic commutation relations. One inserts the expansion of $\hat{\rho}$ and $\hat{\phi}$ over these modes in the Hamiltonian (1), which results in the series $\hat{H} = E_0 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \dots$, where the index refers to the order in $\hat{b}_{\mathbf{q}}$ and $\hat{b}_{\mathbf{q}}^\dagger$.

In the concave dispersion relation regime considered in this paper, the direct phonon coupling due to \hat{H}_3 is not resonant, and the leading resonant coupling is a four-phonon process. An effective interaction Hamiltonian \hat{H}_{eff} coupling an initial Fock state of quasiparticles $|i\rangle$ of energy E_i to a final one $|f\rangle$ of same energy $E_f = E_i$, where two wavevectors \mathbf{q}_1 and \mathbf{q}_2 are annihilated and two other wavevectors \mathbf{q}_3 and \mathbf{q}_4 are created, can then be derived in second order perturbation theory by considering the direct coupling by \hat{H}_4 to first order and the indirect coupling (involving a non-resonant intermediate

state $|\lambda\rangle$) by \hat{H}_3 to second order,

$$\langle f|\hat{H}_{\text{eff}}|i\rangle \simeq \langle f|\hat{H}_4|i\rangle + \sum_{\lambda} \frac{\langle f|\hat{H}_3|\lambda\rangle\langle\lambda|\hat{H}_3|i\rangle}{E_i - E_{\lambda}} \equiv \mathcal{A}_{i\rightarrow f} \quad (2)$$

The reader will notice that, for the purely linear dispersion relation $\omega_{\mathbf{q}} = cq$ predicted by \hat{H}_2 , the denominators in (2) diverge for aligned wavevectors because the intermediate processes become resonant, for example $E_i - E_{\lambda} = \omega_1 + \omega_2 - \omega_{\mathbf{q}_1 + \mathbf{q}_2} = 0$, with the short-hand notation $\omega_i \equiv \omega_{\mathbf{q}_i}$. Following Landau and Khalatnikov, we regularize this divergence by including the actual curvature of the spectrum [20, 22] in the energy denominators [35]

$$\hbar\omega_{\mathbf{q}} \underset{q\rightarrow 0}{=} \hbar cq \left[1 + \frac{\gamma}{8} \left(\frac{\hbar q}{mc} \right)^2 + O \left(\frac{\hbar q}{mc} \right)^4 \right]. \quad (3)$$

Here the speed of sound c is related to the ground state gas density ρ and chemical potential μ by

$$mc^2 = \rho \frac{d\mu}{d\rho} \quad (4)$$

whereas the dimensionless curvature parameter $\gamma < 0$ must be measured or determined from a microscopic theory. By introducing the dimensionless and state-independent effective coupling amplitude \mathcal{A}_{eff} ,

$$\mathcal{A}_{i\rightarrow f} = \sqrt{n_{\mathbf{q}_1} n_{\mathbf{q}_2} (1 + n_{\mathbf{q}_3})(1 + n_{\mathbf{q}_4})} \frac{4mc^2}{\rho L^3} \mathcal{A}_{\text{eff}} \quad (5)$$

where $n_{\mathbf{q}_i}$ are the phonon occupation numbers in the initial Fock state $|i\rangle$, and by considering the six possible intermediate states where a virtual phonon is created and reabsorbed (or absorbed and recreated in the six corresponding finite temperature diagrams) we find in a cubic quantization volume of size L :

$$\hat{H}_{\text{eff}} = \frac{mc^2}{\rho L^3} \sum_{\substack{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4 \\ \mathbf{q}_1 + \mathbf{q}_2 = \mathbf{q}_3 + \mathbf{q}_4}} \mathcal{A}_{\text{eff}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4) \hat{b}_{\mathbf{q}_3}^{\dagger} \hat{b}_{\mathbf{q}_4}^{\dagger} \hat{b}_{\mathbf{q}_1} \hat{b}_{\mathbf{q}_2} \quad (6)$$

with the effective on-shell coupling amplitude

$$\begin{aligned} \mathcal{A}_{\text{eff}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4) &= \frac{1}{16} \sqrt{\frac{\hbar^4 \omega_1 \omega_2 \omega_3 \omega_4}{m^4 c^8}} \left(\Sigma_{\text{F}} \right. \\ &+ \frac{(\omega_1 + \omega_2)^2 A_{1234} + \omega_{\mathbf{q}_1 + \mathbf{q}_2}^2 B_{1234}}{(\omega_1 + \omega_2)^2 - \omega_{\mathbf{q}_1 + \mathbf{q}_2}^2} \\ &+ \frac{(\omega_1 - \omega_3)^2 A_{1324} + \omega_{\mathbf{q}_1 - \mathbf{q}_3}^2 B_{1324}}{(\omega_1 - \omega_3)^2 - \omega_{\mathbf{q}_1 - \mathbf{q}_3}^2} \\ &\left. + \frac{(\omega_1 - \omega_4)^2 A_{1423} + \omega_{\mathbf{q}_1 - \mathbf{q}_4}^2 B_{1423}}{(\omega_1 - \omega_4)^2 - \omega_{\mathbf{q}_1 - \mathbf{q}_4}^2} \right) \quad (7) \end{aligned}$$

We introduced the angle-dependent coefficients

$$\begin{aligned} A_{ijkl} &= (3\Lambda_F + u_{ij})(1 + u_{kl}) \\ &+ (3\Lambda_F + u_{kl})(1 + u_{ij}) + (1 + u_{ij})(1 + u_{kl}) \quad (8) \end{aligned}$$

$$B_{ijkl} = (3\Lambda_F + u_{ij})(3\Lambda_F + u_{kl}) \quad (9)$$

with $u_{ij} = \mathbf{q}_i \cdot \mathbf{q}_j / q_i q_j$ and the thermodynamic quantities

$$\Sigma_{\text{F}} \equiv \frac{\rho^3}{mc^2} \frac{d^3\mu}{d\rho^3} \quad (10)$$

$$\Lambda_{\text{F}} \equiv \frac{\rho}{3} \frac{d^2\mu}{d\rho^2} \left(\frac{d\mu}{d\rho} \right)^{-1} \quad (11)$$

To calculate the phonon damping rate from the effective Hamiltonian (6) we use a master equation approach as in [25]: the phonon mode of momentum $\hbar\mathbf{q}$ is linearly coupled to the thermal reservoir containing all the other phonon modes by an interaction Hamiltonian $\hat{H}_{\text{int}} = \hat{R}^{\dagger} \hat{b}_{\mathbf{q}} + \hat{R} \hat{b}_{\mathbf{q}}^{\dagger}$, where \hat{R}^{\dagger} collects terms in $\hat{b}_{\mathbf{q}_3}^{\dagger} \hat{b}_{\mathbf{q}_4}^{\dagger} \hat{b}_{\mathbf{q}_2}$ from \hat{H}_{eff} . The master equation gives an exponential relaxation of the average population of the mode \mathbf{q} towards its thermal equilibrium value with a rate

$$\begin{aligned} \Gamma_{\mathbf{q}} &= \frac{(mc^2)^2}{4\pi^5 \hbar^2 \rho^2} \int d^3q_2 d^3q_3 |\mathcal{A}_{\text{eff}}(\mathbf{q}, \mathbf{q}_2; \mathbf{q}_3, \mathbf{q}_4)|^2 \\ &\delta(\omega_3 + \omega_4 - \omega_2 - \omega_{\mathbf{q}}) [\bar{n}_2(1 + \bar{n}_3 + \bar{n}_4) - \bar{n}_3 \bar{n}_4] \quad (12) \end{aligned}$$

where $\mathbf{q}_4 = \mathbf{q} + \mathbf{q}_2 - \mathbf{q}_3$ due to momentum conservation and we have used the notation $\bar{n}_i \equiv \bar{n}_{\mathbf{q}_i}$ to indicate the phonon occupation numbers in thermal equilibrium. The result (7)-(12) goes beyond that of Ref.[1] where the authors concentrated on a single diagram for the intermediate processes. This will have a significant impact on the behavior of the damping rate as a function of q as we will see. To go further analytically, we restrict to sufficiently low temperature

$$\epsilon \equiv \frac{k_B T}{mc^2} \ll 1 \quad (13)$$

so that only the (almost) linear region of the phonon dispersion relation is explored and the integral (12) is dominated by configurations in which the four involved wavevectors are almost aligned. By introducing the rescaled quantities

$$\tilde{q}_i = \frac{\hbar c q_i}{k_B T}; \quad \tilde{\theta}_i = \frac{\theta_i}{\epsilon |\gamma|^{1/2}} \quad (14)$$

with θ_i the angles between \mathbf{q}_i and \mathbf{q} and γ the curvature parameter in the spectrum (3), and by expanding the coupling amplitude (7) for $\epsilon \rightarrow 0$ at fixed \tilde{q}_i and $\tilde{\theta}_i$, we obtain the central result of this paper:

$$\frac{\hbar \Gamma_{\mathbf{q}}}{\epsilon_F} \underset{\epsilon \rightarrow 0}{\sim} \frac{K}{|\gamma|} \left(\frac{T}{T_F} \right)^7 \tilde{\Gamma}(\tilde{q}) \quad (15)$$

In this formula $\epsilon_F = k_B T_F = \hbar^2 k_F^2 / 2m$ is the Fermi energy, K is an interaction-dependent thermodynamic quantity

$$K = 2 \left(\frac{3}{4} \right)^6 \left(\frac{\epsilon_F}{mc^2} \right)^3 (1 + \Lambda_F)^4 \quad (16)$$

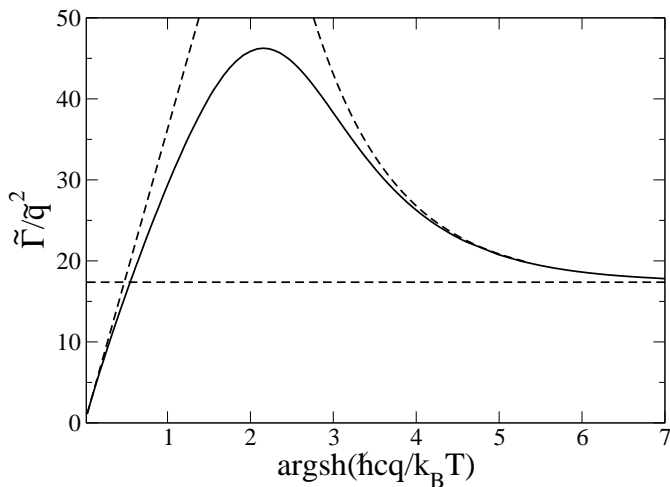


FIG. 1: Rescaled phonon damping rate due to $2 \leftrightarrow 2$ phonon processes in a spin-1/2 Fermi gas, as a function of the rescaled wavenumber $\tilde{q} = \hbar c q / k_B T$. Due to these rescalings, see Eq.(15), the result is universal and applies at sufficiently low temperature in the whole region of the BEC-BCS crossover where the dispersion relation is concave at low q . The dashed straight lines show the asymptotic behaviors (21) and (22) and the dashed curve at large \tilde{q} is a fit of $\tilde{\Gamma}/\tilde{q}^2$ by an affine function of $1/\tilde{q}$.

and the rescaled phonon damping rate $\tilde{\Gamma}(\tilde{q})$, shown in Fig.1, is a universal function of the dimensionless wavenumber. Explicitly

$$\begin{aligned} \tilde{\Gamma}(\tilde{q}) = & \int_0^\infty d\tilde{q}_2 \int_0^{\tilde{q}+\tilde{q}_2} d\tilde{q}_3 \frac{\tilde{q}_2^3 \tilde{q}_3^3 (\tilde{q} + \tilde{q}_2 - \tilde{q}_3)}{\tilde{q} |v|} \\ & \times \frac{[1 + f(\tilde{q}_2)] f(\tilde{q}_3) f(\tilde{q} + \tilde{q}_2 - \tilde{q}_3)}{f(\tilde{q})} \\ & \times \int_0^\pi d\phi \int_0^{\pi/2} d\alpha \sin \alpha \cos \alpha \Theta\left(-\frac{v}{u}\right) |\mathcal{A}_{\text{red}}|^2 \quad (17) \end{aligned}$$

where $\Theta(x \geq 0) = 1$, $\Theta(x < 0) = 0$ is the Heaviside function, $f(x) = 1/(e^x - 1)$ originates from the Bose law, u and v are the following functions of \tilde{q} , \tilde{q}_2 , \tilde{q}_3 , $\alpha = \arctan \theta_3/\theta_2$, and of the relative azimuthal angle ϕ of \mathbf{q}_2 and \mathbf{q}_3 :

$$u = \frac{\tilde{q}(\tilde{q}_3 \sin^2 \alpha - \tilde{q}_2 \cos^2 \alpha) + \tilde{q}_2 \tilde{q}_3 (1 - \sin 2\alpha \cos \phi)}{\tilde{q} + \tilde{q}_2 - \tilde{q}_3} \quad (18)$$

$$v = \frac{1}{4} [\tilde{q}^3 + \tilde{q}_2^3 - \tilde{q}_3^3 - (\tilde{q} + \tilde{q}_2 - \tilde{q}_3)^3] \quad (19)$$

We introduced in (17) the reduced coupling amplitude

$$\begin{aligned} \mathcal{A}_{\text{red}} = & \frac{1}{\tilde{q}_2 \left(\frac{\cos^2 \alpha}{(\tilde{q} + \tilde{q}_2)^2} - \frac{3u}{4v} \right)} - \frac{1}{\tilde{q}_3 \left(\frac{\sin^2 \alpha}{(\tilde{q} - \tilde{q}_3)^2} - \frac{3u}{4v} \right)} \\ & - \frac{1}{\frac{\tilde{q}_2 \cos^2 \alpha - \tilde{q}_3 \sin^2 \alpha + u}{(\tilde{q}_2 - \tilde{q}_3)^2} - \frac{3u}{4v} (\tilde{q} + \tilde{q}_2 - \tilde{q}_3)} \quad (20) \end{aligned}$$

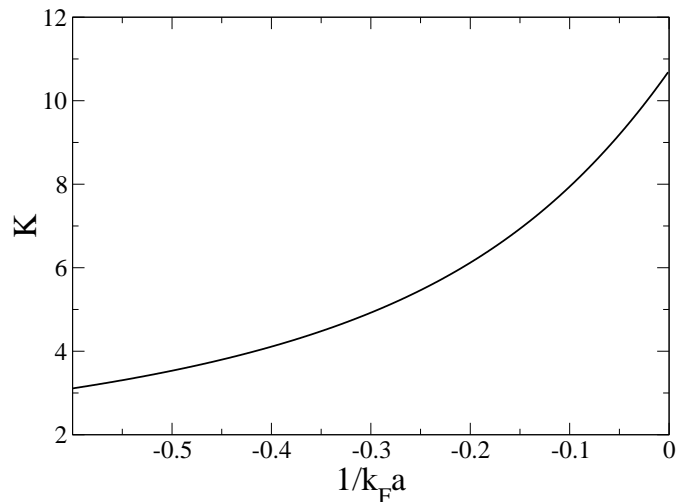


FIG. 2: Thermodynamic quantity (16) as a function of $1/k_F a$ where $k_F = (3\pi^2 \rho)^{1/3}$ is the Fermi wavenumber and a the s -wave scattering length, from the measured equation of state of the spin-1/2 Fermi gas [14, 26].

The limiting behaviors of the normalized universal Landau-Khalatnikov damping rate (17)

$$\tilde{\Gamma}(\tilde{q}) \underset{\tilde{q} \rightarrow 0}{\sim} \frac{16\pi^5}{135} \tilde{q}^3 + O(\tilde{q}^4) \quad (21)$$

$$\tilde{\Gamma}(\tilde{q}) \underset{\tilde{q} \rightarrow \infty}{\sim} \frac{16\pi\zeta(5)}{3} \tilde{q}^2 + O(\tilde{q}) \quad (22)$$

are shown in Fig.1 as dashed straight lines. Here ζ is the Riemann zeta function. Equations (21)-(22) disagree with Eqs.(7.6) and (7.12) in [1], even for the order in \tilde{q} , our results being subleading by two orders. This is due to the fact that the diagrams neglected by Landau and Khalatnikov in the calculation of the effective amplitude \mathcal{A}_{eff} turn out to interfere destructively with the supposedly leading term that they keep.

We now discuss the main result (15) and the possibility to test it in state-of-the-art experiments with cold fermionic gases. In Fig.2 we use the measured equation of state of the spin-1/2 Fermi gas [14, 26] to plot the thermodynamic quantity K in (16) as a function of $1/k_F a$. The curvature parameter γ appearing in the phonon dispersion relation (3) has not yet been measured in strongly interacting Fermi gases. We plot in Fig.3 the prediction of [22] which relies on the Random Phase Approximation and coincides with that of other approximate theories [18, 20]. We plot it separately so that, once an experimental value will be available, it can be used in (15). To give a numerical example, we choose the interaction strength such that the coefficient of the quintic term in the expansion (3) vanishes, $1/k_F a \simeq -0.39$ according to [22]. The corresponding value of the curvature is $\gamma \simeq -0.30$ and the thermodynamic constant is $K \simeq 4.2$. The temperature should be sufficiently low and the wavenumber

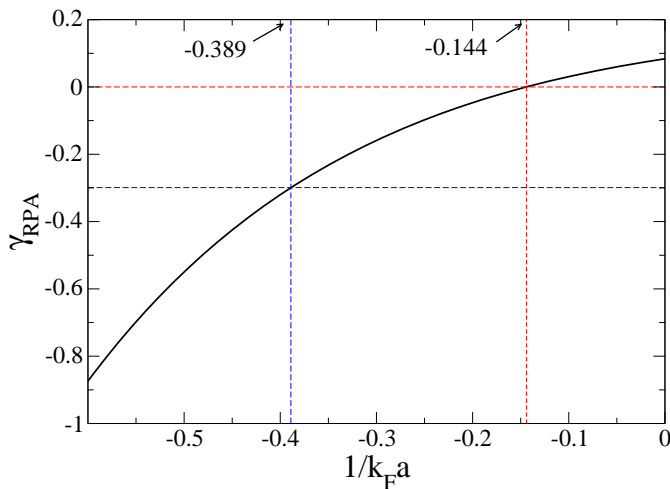


FIG. 3: Curvature parameter γ in the phonon dispersion relation (3) as predicted by [22], as a function of $1/k_F a$ where k_F is the Fermi wavenumber and a the s -wave scattering length. The dashed lines mark the two points where either the cubic ($1/k_F a = -0.144$) or the quintic ($1/k_F a = -0.389$) correction to the dispersion relation (3) vanishes.

q much smaller than $2\Delta/\hbar c$ to avoid the excitation of the fermionic branch [27, 28]. By choosing $T = 0.055 T_F$ [29] and $\tilde{q} = 10/3$, corresponding to $\epsilon = k_B T/mc^2 \simeq 0.15$, $\hbar q/mc \simeq 0.5$, and $\tilde{\Gamma} \simeq 504$, and for the typical value $T_F = 1\mu K$, one obtains $\Gamma_{\mathbf{q}} \simeq 1.4 s^{-1}$. A damping rate of this order should be measurable in state-of-the-art experiments in the time domain [30], and it may be accessible in the future also in the frequency domain [31]. As expected, it is d in the collisionless regime since the angular frequency $\omega_{\mathbf{q}}$ is much larger than the thermalization rate, that we estimate by the damping rate at the typical thermal wavenumber $q_{\text{th}} = k_B T/\hbar c$: one finds $\tilde{\Gamma}(1) \simeq 26$ and $\Gamma_{\mathbf{q}_{\text{th}}}/\omega_{\mathbf{q}} \simeq 3 \times 10^{-6}$.

In conclusion we have calculated the Landau-Khalatnikov low-temperature phonon damping rate in strongly interacting Fermi gases. Our expression, thanks to an appropriate rescaling of the wavenumber, takes a universal functional form that applies in the whole region of the BEC-BCS crossover where the phononic dispersion relation has a concave start. This includes the strongly interacting regime and opens the door to an experimental observation with ultracold atomic gases. As the phonon dispersion relation has a convex rather than concave start in liquid helium [6, 7], such an observation would be unprecedented and would open a new era in the exploration of low-temperature dynamics of macroscopically coherent quantum many-body systems.

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- [33] One can estimate $\tau_c \simeq 1/\Gamma_{\mathbf{q}^{\text{th}}}$ where $\hbar c q_{\text{th}} = k_B T$. Then for excitation frequencies scaling as $k_B T$, as in Eq.(14), the condition $\omega_{\mathbf{q}} \gg \Gamma_{\mathbf{q}}$, satisfied for a weakly-excited gas, implies $\omega_{\mathbf{q}} \gg \Gamma_{\mathbf{q}^{\text{th}}}$ which ensures the collisionless regime.
- [34] In principle, this Hamiltonian has to be regularized by introducing an ultraviolet momentum cut-off or by discretizing the real space on a lattice as in [5]. This however does not play a role here.
- [35] The quantum hydrodynamics Hamiltonian can be supplemented by terms leading to a curved dispersion relation [32]. Except in the energy denominator, this brings a negligible correction to the phonon damping rate at low temperature.