# THE RESEARCH ON THINKING OF VARIANT OF PYTHAGOREAN CHAPTER 

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#### Abstract

In this paper I consider the thinking of variant from the perspective of Pythagorean chapter in The Nine Chapters. Among the various elements to be considered I focus on the need to address five aspects to analysis the thinking of variation of Chinese classical works. The five aspects are solutions of a given problem, changes of a given problem, uses of a given problem, changes of a given diagram and changes of identity. Furthermore I shall analyze the thinking of variant from psychology of mathematics education in order to provide a reference for today's mathematics education research and practice.


## 1 Introduction

Variant teaching is a traditional and typical teaching methods in China. It has not only a wide range of experience, but also through the test of practice. Mathematics teaching mode in China is not "passive infusion" and "mechanical training", it is essentially a variable teaching. This thinking of variant has been reflected in the ancient classics in our country. This article sorts out the all thinking of variant about Pythagorean chapter in The Nine Chapters in order to provide some advice for today's mathematics education research and practice.

## 2 Variant method of section of Pythagorean in the Nine Chapters

Variant method is a method in our teaching and hiding in The Nine Chapters. For example, many solutions to a problem, variants of a problem, uses of a method, variants of a picture and identical variant. Now we show these methods of variant about the section of Pythagorean in The Nine Chapter.

The section of Pythagorean chapter consists of three parts: problems of three sides of triangle mutual obtained from the Pythagorean theorem, problems of the rectangle and the circle in the right-angle triangle respectively and the measurement problem by using the Pythagorean ratio and that of the Pythagorean numbers. There are 24 problems in the section of Pythagorean. It is mainly the application of the Pythagorean theorem from 1 to 14, the application of the Pythagorean scale from 17 to 24, how to solve Pythagorean number from 14 to 21 . There are eight types about Pythagorean number in this chapter, it is rare in the mathematical literature of the ancient world. The Pythagorean theorem is also called GaoShang theorem, it is called the Pythagorean theorem in the west. In ancient China the short rectangular is called hook, long rectangular edge called share, hypotenuse called string in the right triangle. 3000 years ago, mathematicians GaoShang in the Zhou Dynasty put forward the form of the Pythagorean theorem, furthermore people found and proved that the right angled triangle trilateral relations: two right angle sides of the square and the hypotenuse is equal to the sum of the squared, namely: if two right angle sides of a right triangle are respectively $a, b$, the hypotenuse is $c$, then $a^{2}+b^{2}=c^{2}$. It can be
transformed: $a^{2}=c^{2}-b^{2}, \quad b^{2}=c^{2}-a^{2}$.
The variant of contents of the Pythagorean theorem is used flexibly in the section of Pythagorean, the thinking of variant is embodied both each content and between the content. Identical variant also runs through many uses of a method and many variants of a picture. Now list is as follows:

Table 1. Thinking of variant of chapter Pythagorean Distribution

| Classification | Solutions ofa problem | Changes of a problem |
| :--- | :--- | :--- |
| Operation of <br> Pythagorean |  | (1)changes among $a, b, c ;$ <br> (2) changes among $\mathrm{c}+\mathrm{b}, \mathrm{c}-\mathrm{b}, \mathrm{c}+\mathrm{a}, \mathrm{c}-\mathrm{a} ;$ <br> (3)If $a, c-b$, then $b, c ;$ |
| Square of the <br> Pythagorean | Square of the Pythagorean | (4)If $\mathrm{c}, a$-b or $a+\mathrm{b}$ ) ,then $\mathrm{b}, a ;$ <br> (5)If $a, c+b$, then $b ;$ <br> (6)Ifc- $a, \mathrm{c}-\mathrm{b}$, then $a, \mathrm{~b}, \mathrm{c} ;$ |
| Circle of the <br> Pythagorean | Circle of the Pythagorean | (7)If c- $a, \mathrm{c}$-b,then $d$. |

### 2.1 Solutions of a problem

The formula that if hypotenuse and the Pythagorean difference then Pythagorean is used In solving the problem for households higher than wide, the formula that string is known to be associated with the sum of Pythagorean and the Pythagorean difference and hypotenuse constructed a quadratic equation to calculate the Pythagorean .

### 2.2 Changes of a problem

Three Pythagorean relationships were changed by different problems in the operation the Pythagorean.

### 2.3 Uses of a method

Two Pythagorean triangle are first constructed, and then use the similar principle to solve practical problems in the section of measurement.

The prominent variant is the derivation of Pythagorean general formula, namely generalized Pythagorean representation.

### 2.4 Identical variant

The sum and difference among Pythagorean chord are used to the deformation of the identities in order to solve three sides of right-angle triangle.

## 3 Thinking of variant is analyzed from the perspective of mathematical education psychology

The teaching of mathematics activity experience is a kind of teaching method. Mathematics activity experience is usually embedded in the dynamic process of mathematics, level is the basic feature of mathematical activity process. This level can be expressed as a series of steps, but also as a kind of activity strategy or experience. Therefore, the main teaching meaning of
process change is that students can solve the problem step by step and accumulate a variety of activities experience through the promotion of a level in the process of mathematical activities.

There are three developments of problem solving can be constructed the variants of Specific experience system: (1) a variety of changes of a problem, which includes both to pave the way for and all of the original problem extended (such as changing conditions and changing the conclusion and generalization); (2) a variety of solutions of a problem is that taking solving process of a problems as variant to link all the different solution; (3) the same method to solve a variety of problems is a particular method for a class of similar problems, which can produce some leads to reduction / inquiry strategy.

There are four process variant in the section of Pythagorean. (1)There are 7 cases in solving triangle by choosing either the number of known as triangle (edge) in the two right-angle sides, as well as the hypotenuse and difference; (2) There are 10 cases in solving triangle by choosing either the number of known as triangle (edge) in (1) and then choosing one condition in the product of two side; (3) There are 7 cases in solving triangle by choosing either the number of known as triangle (edge) in the sum and difference of sum and difference of chord and Pythagorean and then choosing one condition in the sum and difference of two; (4) There are 8 cases in solving triangle by choosing either the number of known as triangle (edge) in the sum and difference of sum and difference of hypotenuse and Pythagorean and then choosing one condition in the product of two.

## 4 Suggestions

We can also from the ancient classical mathematics to dig out more applicable to our teaching materials, such as changes of the graph in this chapter, changes of problem situation.

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