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# Computation and Experimental Measurements of the Magnetic Fields between Filamentary Circular Coils

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**Abstract.** MATLAB computations and experimental measurements of the magnetic fields between two filamentary circular (FC) coils are presented in this paper. In order to achieve accurate computations, the induced voltage and current in the secondary side of the coil are determined and validated with measured values. The findings are obtained by formulating the models for computing the total magnetic flux through the secondary coil and mutual inductance between the two FC coils. Based on a more efficient and general model available in the literature, the model for computing the magnetic fields between the coils is derived. The comparison of the superimposed computed magnetic fields results and measured values show that the model formulated and experimental measurements implemented in the laboratory are accurate.

*Keywords:* Computational electromagnetics, inductive charging systems, magnetic fields

## 1. Introduction

Inductive charging systems allow the transfer of electrical energy between its primary and secondary coils via a high frequency magnetic fields to an electrical device, which can then use that energy to either charge batteries or run the device [1, 2].

Air-cored coils are widely used in various electromagnetic applications [2]. They are preferred to the iron-cored type due to their design objectives of controllability and capability of a high electrical power transfer [1]. Nonetheless, the measure of the magnetic fields and mutual inductance between them are of importance to electrical engineers and physicists.

Several contributions have been made in the literature concerning the computations of the magnetic fields and mutual inductance between air-cored filamentary circular (FC) coils. In this view, mathematical models based on the application of Maxwell's formula, Neumann's integral approach, Biot-Savart law, Lorentz approach and magnetic vector potential approach have been presented in [1–7].

The objectives of this paper are as follows.

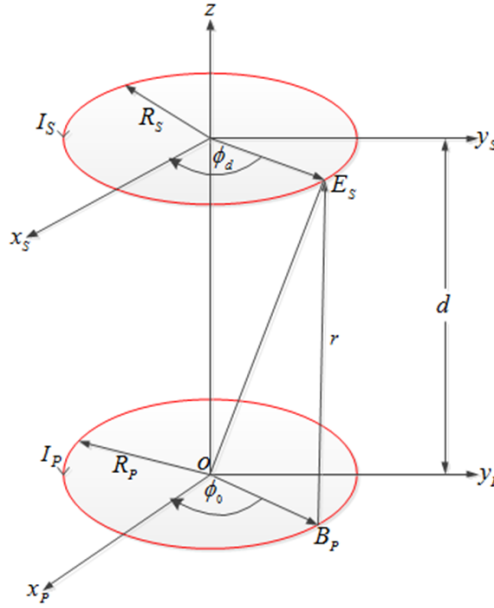
- To achieve accurate MATLAB computations for the voltage and current induced in the secondary side of the FC coil by formulating the models for computing the total magnetic flux through the secondary coil and mutual inductance between the two FC coils.
- To formulate the model for computing the magnetic fields between the coils based on a more efficient and general model given in [6].

- To present MATLAB computations and experimental measurements of the magnetic fields between the two FC coils as well as their distributions in 2-D and 3-D planes.

## 2. Formulated Models for Magnetic Flux, Mutual Inductance and Induced Voltage

The mathematical models for the computation of the total magnetic flux through the secondary FC coil, mutual inductance between the primary and secondary FC coils and induced voltage are presented in this section.

The model for computing the total magnetic flux through the secondary FC coil due to the current in the primary FC coil is formulated based on Fig. 1.



**Figure 1.** Coaxial filamentary circular coils

where  $R_P$  and  $R_S$  are the radii of the primary and secondary FC coils respectively,  $I_P$  and  $I_S$  are their primary and secondary currents,  $d$  is the separation distance between them,  $\phi_d$  is the angle between the positive  $x_S$  axis and the line segment pointing from its origin to the arbitrary point  $E_S$  and  $\phi_0$  is the angle between the positive  $x_P$  axis and the line segment pointing from its origin to the arbitrary point  $B_P$ .

To successfully achieve the formulation of the magnetic flux model, the following mathematical expressions are considered:

- The coil of radius  $R_P$  lies in the plane  $x_P O y_P (z_P = 0)$  with its centre point at  $O(0, 0, 0)$
- The parametric coordinates of the arbitrary point  $B_P$  on the primary coil are

$$x_P = R_P \cos \phi_0, \quad y_P = R_P \sin \phi_0, \quad z_P = 0 \quad (1)$$

where  $\phi_0 \in [0, 2\pi]$

- The differential element located at (1) is given by

$$d\vec{l}_P = R_P(-\vec{i} \sin \phi_0 + \vec{j} \cos \phi_0) d\phi_0 \quad (2)$$

(iv) The parametric coordinates of the arbitrary point  $E_S$  on the secondary coil are

$$x_S = -R_S \sin \phi_d, \quad y_S = R_S \cos \phi_d, \quad z_S = d \quad (3)$$

where  $\phi_d \in [0, 2\pi]$

(v) The differential element located at (3) is given by

$$d\vec{l}_S = R_S(-\vec{i} \cos \phi_d - \vec{j} \sin \phi_d)d\phi_d \quad (4)$$

The magnetic vector potential  $\vec{A}$  at point  $E_S(x_S, y_S, z_S)$  produced by a circular current coil,  $I_P$  of radius  $R_P$  is given by

$$\vec{A} = \frac{\mu_0 \mu_r}{4\pi} \int_{l_P} \frac{I_P d\vec{l}_P}{r} \quad (5)$$

where  $\mu_0$  is the permeability of free space,  $\mu_r$  is the relative permeability of air,  $I_P$  is the primary coil current and  $r$ , which is the distance between the arbitrary points  $B_P$  and  $E_S$  is given by

$$\begin{aligned} r &= |E_S - B_P| = \sqrt{(x_S - x_P)^2 + (y_S - y_P)^2 + (z_S - z_P)^2} \\ r &= \sqrt{R_S^2 + R_P^2 + d^2 + 2R_S R_P (\sin \phi_d \cos \phi_0 - \cos \phi_d \sin \phi_0)} \end{aligned} \quad (6)$$

According to Stokes' theorem, the magnetic flux through the secondary coil as a result of the current,  $I_P$  in the primary coil is given by

$$\begin{aligned} \Phi &= \int \int_{S_s} \vec{B} \cdot d\vec{S}_s = \int \int_{S_s} (\nabla \times \vec{A}) d\vec{S}_s \\ \Phi &= \int_{l_s} \vec{A} d\vec{l}_s \end{aligned} \quad (7)$$

where  $S_s$  and  $l_s$  are respectively the cross-section and the perimeter of the secondary FC coil. Thus, substituting (2), (4) and (5) into (7) result to the formulated model for computing the total magnetic flux through the secondary FC coil

$$\Phi_T = \frac{\mu_0 \mu_r N_P N_S I_P R_P R_S}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{\sin \phi_0 \cos \phi_d - \cos \phi_0 \sin \phi_d}{r} d\phi_d d\phi_0 \quad (8)$$

where  $r$  is given in (6),  $N_P$  and  $N_S$  are the number of turns for the FC coils.

Furthermore, the computation of the mutual inductance between the two FC coils is an important parameter for obtaining the voltage induced in the secondary side of the coil and by definition, it is given by

$$M = \frac{\Phi_T}{I_P} = \frac{\mu_0 \mu_r N_P N_S R_P R_S}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{\sin \phi_0 \cos \phi_d - \cos \phi_0 \sin \phi_d}{r} d\phi_d d\phi_0 \quad (9)$$

According to Faraday's law, the induced voltage in the secondary FC coil is equal to the rate of change of the total magnetic flux  $\Phi_T$  and is given by

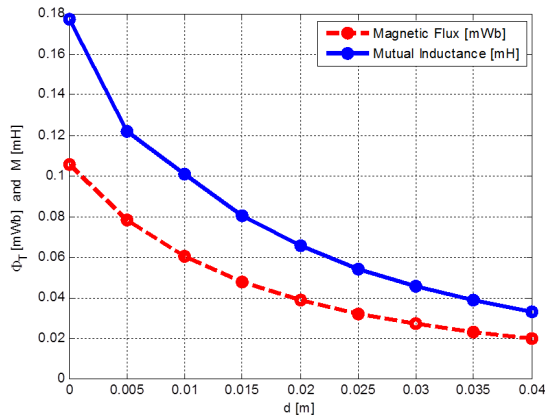
$$V = M\omega I_{P_{pk}} \cos(\omega t) \quad (10)$$

where  $I_{P_{pk}}$  is the primary coil peak current

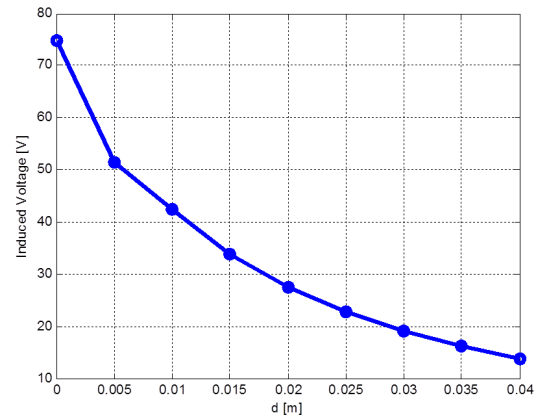
### 3. Results and Discussion

The variations of the total magnetic flux  $\Phi_T$ , mutual inductance  $M$  and induced voltage  $V$  versus the separation distance  $d$  between the two FC coils are shown in Figs. 2 and 3. These results are obtained based on the formulated models given in (8), (9) and (10) and the computation is achieved by using double quadrature function in MATLAB.

Based on (6), the value of the distance  $r$  between the arbitrary points  $B_P$  and  $E_S$  is dependent on the radii of the coils  $R_P$  and  $R_S$  as well as the distance  $d$  between the centres of the coils. Thus, as the values of  $R_P$ ,  $R_S$  and  $d$  are increased,  $r$  increases. For  $R_P = R_S = 0.052\text{ m}$ , the plots given in Figs. 2 and 3 show that as the value of  $d$  increases, the values of  $\Phi_T$ ,  $M$  and  $V$  decrease.



**Figure 2.** Variations of  $M$  and  $\Phi_T$  versus  $d$ .



**Figure 3.** Variation of  $V$  versus  $d$ .

Furthermore, these plots are obtained based on the parameters (i.e.,  $V_{rms} = 220\text{ V}$ ,  $I_P = 0.55\text{ A}_{rms}$ ,  $N_P = N_S = 30$ ,  $R_P = R_S = 0.052\text{ m}$ ,  $\mu_r = 1$ ,  $\mu_0 = 4\pi \times 10^{-7}\frac{\text{H}}{\text{m}}$ ) of the inductive charger, which is employed for experimental measurement in the laboratory. Also, with the aid of oscilloscope, it is found that the transfer of power from the primary side of the inductive charger to its secondary side is achieved with a high electromagnetic frequency of 80 kHz.

The primary and secondary sides of the inductive charger employed for experimental measurement are made up of coaxial FC coils. Thus, it is important to validate the computed induced voltage with measured values. With the use of Multimeter, a maximum voltage of 75 V is measured from the terminal of the plug-in connector and this is obtained when the separation distance between the two FC coils is approximately equal to zero (i.e., at  $d \approx 0$ ). Also, at  $d \approx 0$ , a computed induced voltage of 74.70  $V_{max}$  (see Fig. 3) is obtained and such outcome is in agreement with the measured value. In order to obtain the induced current,  $I_S$  in the secondary side of the FC coil, the inductive charger is used to power 60  $W_{max}$  LED lamp and by measurement, a peak  $I_S$  of 0.8 A is obtained.

In spite of the increment of  $d$ , the values of  $\Phi_T$ ,  $M$  and  $V$  can be increased if a higher frequency AC current  $I_P$  is supplied to the primary coil, the number of turns of the coils  $N_P$  and  $N_S$  are increased and a relative permeability of ferrimagnetic material (e.g. ferrite) is employed in the electromagnetic coupling structure of the inductive charger.

#### 4. Formulation of the Magnetic Fields Model for two Filamentary Circular Coils

According to [6], the final form of the magnetic fields in an arbitrary point  $E_S(x_S, y_S, z_S)$  produced by the primary coil of radius  $R_P$  carrying the current  $I_P$  is obtained as (see Fig. 4).

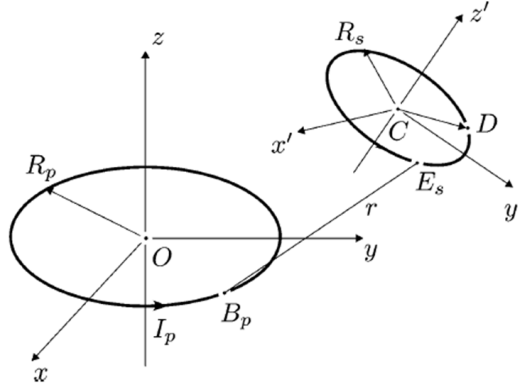
$$\begin{aligned} B_x(x_S, y_S, z_S) &= -\frac{\mu_0 I_P z_S x_S k}{8\pi\sqrt{R_P}(x_S^2 + y_S^2)^{5/4}} L_0 \\ B_y(x_S, y_S, z_S) &= -\frac{\mu_0 I_P z_S y_S k}{8\pi\sqrt{R_P}(x_S^2 + y_S^2)^{5/4}} L_0 \\ B_z(x_S, y_S, z_S) &= -\frac{\mu_0 I_P k}{8\pi\sqrt{R_P}(x_S^2 + y_S^2)^{3/4}} S_0 \end{aligned} \quad (11)$$

where

$$\begin{aligned} x_S &= x_C + R_S u_x \cos\phi + R_S v_x \sin\phi, \quad y_S = y_C + R_S u_y \cos\phi + R_S v_y \sin\phi, \quad z_S = z_C + R_S u_z \cos\phi + R_S v_z \sin\phi, \\ \vec{u} &= [u_x, u_y, u_z] = \left[-\frac{ab}{\ell L}, \frac{\ell}{L}, -\frac{bc}{\ell L}\right], \quad \vec{v} = [v_x, v_y, v_z] = \left[-\frac{c}{\ell}, 0, \frac{a}{\ell}\right], \quad \ell = \sqrt{a^2 + c^2}, \quad L = \sqrt{a^2 + b^2 + c^2}, \\ L_0 &= 2K(k) - \frac{2-k^2}{1-k^2} E(k), \quad S_0 = 2\sqrt{x_S^2 + y_S^2} K(k) - \frac{2\sqrt{x_S^2 + y_S^2} - (R_P + \sqrt{x_S^2 + y_S^2})k^2}{1-k^2} E(k), \\ k^2 &= \frac{4R_P\sqrt{x_S^2 + y_S^2}}{(R_P + \sqrt{x_S^2 + y_S^2})^2 + z_S^2} \end{aligned}$$

where

$x_S, y_S$  and  $z_S$  are the parametric coordinates of the secondary coil,  $\vec{u}$  is the unit vector lying in the secondary coil plane between points  $C$  and  $D$ ,  $\vec{v}$  is the cross product of the unit vector of the axis  $z'$  and  $\vec{u}$ ,  $S_0, L_0, L$  and  $\ell$  are dimensionless parameters while  $K(k)$  and  $E(k)$  are the complete elliptic integrals of the first and second kinds respectively.



**Figure 4.** Filamentary circular coils with angular and lateral misalignment.

Furthermore, according to [3], the following mathematical expressions must be considered for coaxial filamentary coils (FC) coils (i.e., the case without lateral and angular misalignment).

- The parameters ( $a, b$  and  $c$ ) which determine the positioning of the secondary coil plane are defined as  $a = 0, b = 0$  and  $c = 1$ .
- The centre of the secondary coil at point  $C$  becomes ( $x_C = 0, y_C = 0, z_C = h$ ). In this paper,  $h$  is the distance measured from the centre point between the two FC coils to any point along the symmetry axis of the coils.

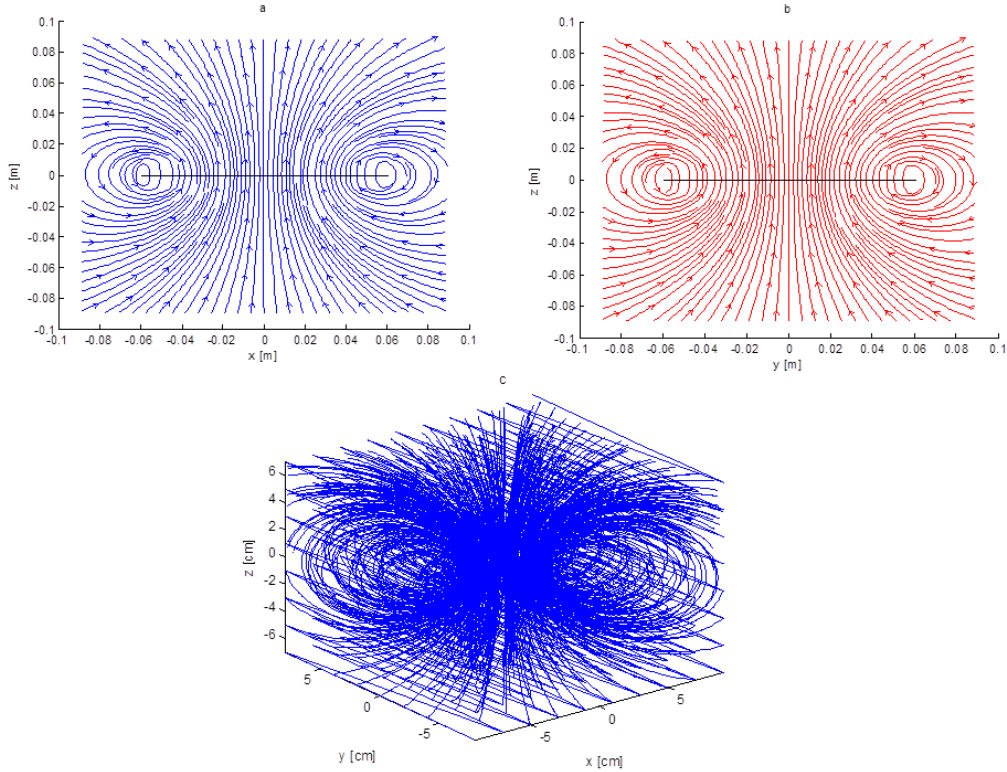
Based on the above information, the authors of this paper formulate the model for computing the total magnetic fields between two coaxial FC coils as

$$\begin{aligned}
B_x &= \left[ -\frac{\mu_0 \mu_r I_P N_P (z + \frac{d}{2}) x k}{8\pi \sqrt{R_P (x^2 + y^2)^{5/4}}} L_0 \right] + \left[ -\frac{\mu_0 \mu_r I_S N_S (z - \frac{d}{2}) x k}{8\pi \sqrt{R_S (x^2 + y^2)^{5/4}}} L_0 \right] \\
B_y &= \left[ -\frac{\mu_0 \mu_r I_P N_P (z + \frac{d}{2}) y k}{8\pi \sqrt{R_P (x^2 + y^2)^{5/4}}} L_0 \right] + \left[ -\frac{\mu_0 \mu_r I_S N_S (z - \frac{d}{2}) y k}{8\pi \sqrt{R_S (x^2 + y^2)^{5/4}}} L_0 \right] \\
B_z &= \left[ \frac{\mu_0 \mu_r I_P N_P k}{8\pi \sqrt{R_P (x^2 + y^2)^{3/4}}} S_0 \right] + \left[ \frac{\mu_0 \mu_r I_S N_S k}{8\pi \sqrt{R_S (x^2 + y^2)^{3/4}}} S_0 \right]
\end{aligned} \tag{12}$$

Since the radii of the two FC coils are equal (i.e.,  $R_P = R_S$ ),  $x = -R \sin \phi$ ,  $y = -R \cos \phi$ ,  $z = h$ ,  $L_0 = 2K(k) - \frac{2-k^2}{1-k^2}E(k)$ ,  $S_0 = 2\sqrt{x^2 + y^2}K(k) - \frac{2\sqrt{x^2 + y^2} - (R + \sqrt{x^2 + y^2})k^2}{1-k^2}E(k)$ ,  $k^2 = \frac{4R\sqrt{x^2 + y^2}}{(R + \sqrt{x^2 + y^2})^2 + z^2}$ . Also, the two coils are separated by a distance,  $d$  which is equal to their radii,  $R$ .

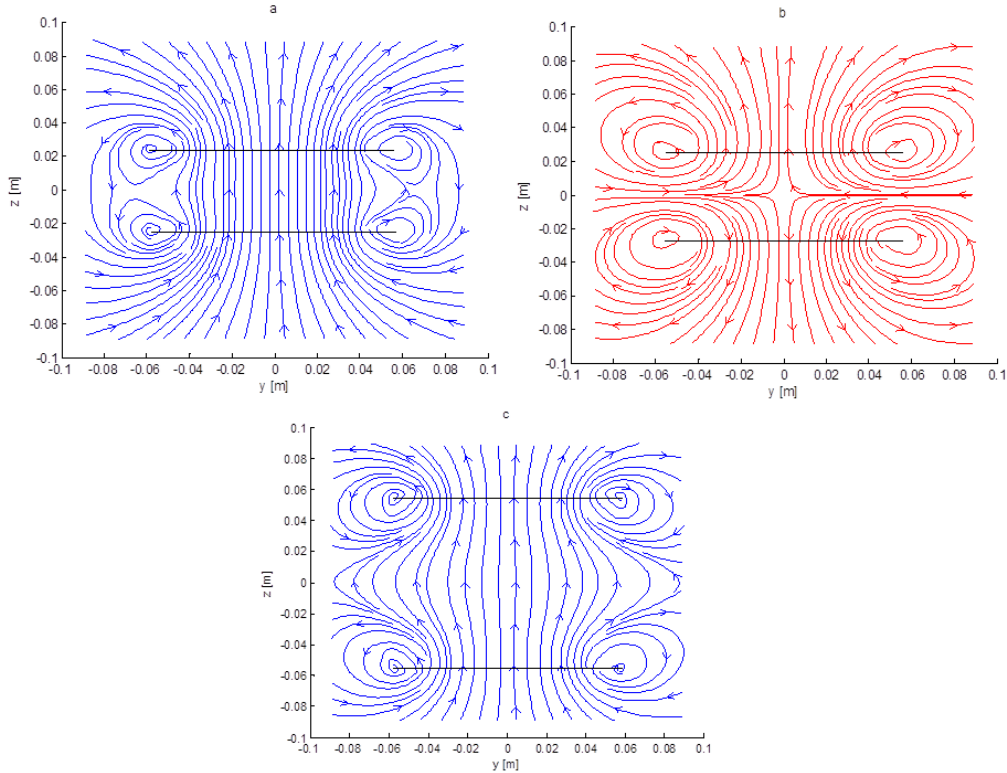
## 5. Results and Discussion

Figures 5 and 6 show the magnetic field lines due to a filamentary circular (FC) coil and between two FC coils respectively. These results are obtained by computing (12) in MATLAB using Quadrature function. The graphical representation of these field lines is very useful in visualizing the magnitude (or strength) and direction of the magnetic fields. The straight line on the plots (see Figs. 5 and 6) is simply drawn to show the physical presence of the coil.



**Figure 5.** Magnetic field lines due to a FC coil: (a) XZ (b) YZ and (c) XYZ planes.

The results obtained show that the magnetic fields lines in 2-D (i.e.,  $XZ$  and  $YZ$ ) planes are symmetrical (see Fig. 5). In addition, the 3-D (i.e.,  $XYZ$  planes) magnetic field lines are continuous, forming closed loops without beginning or end (see Fig. 5). The well spaced streamlines with direction arrows are obtained by using streamslice function in MATLAB. Figures 6 (a) and (b) show when the currents  $I_P$  and  $I_S$  of the FC coils flow in the same direction and opposite direction respectively while Fig. 6 (c) is obtained when the distance between the FC coils is equal to the radii,  $R_P$  and  $R_S$  (i.e., at  $d = 0.052 m$ ).



**Figure 6.** Magnetic field lines between two FC coils: (a)  $I_P$  and  $I_S$  flow in the same direction (b)  $I_P$  and  $I_S$  flow in opposite direction and (c) when  $d$  between the coils is increased.

The validation of the computed results based on the model formulated in (12) and experimental measurements implemented in the laboratory are shown in Figs. 7-13. Figures 7, 9 and 11 show the computed results for the magnetic field distribution,  $B_x$ ,  $B_y$  and  $B_z$  at  $d = 0.052 m$ ,  $d = 0.03 m$  and  $d = 0.01 m$  respectively. The results obtained show that the values of  $B_x$ ,  $B_y$  and  $B_z$  decrease as the separation distance  $d$  between the FC coils increases.

Figures 8, 10 and 12 show the validation of the computed and experimental measurements of the magnetic fields between the two FC coils. Experimental measurement is conducted only for  $B_z$  because  $B_x$  and  $B_y$  are symmetrical.

Universal robots (UR5) in connection with sensor, magnetometer and oscilloscope are used to obtain the magnetic fields results between the FC coils of the inductive charger employed for experimental measurements (see Fig. 13). UR5 is a machine that can be programmed to move a tool, and communicate with other machines using electrical signals. With the aid of Polyscope that is, the graphical user interface (GUI), it is easy to program the robot to move the sensor along a desired trajectory.

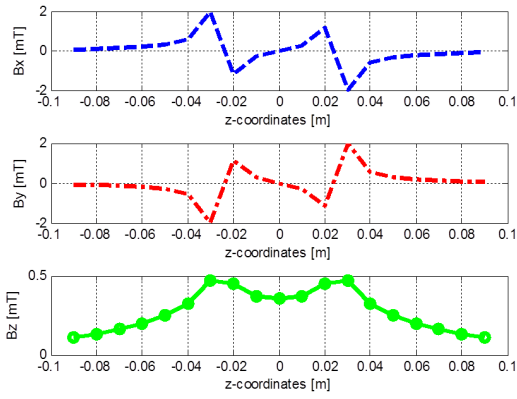


Figure 7.  $B_x$ ,  $B_y$  and  $B_z$  at  $d = 0.052$  m.

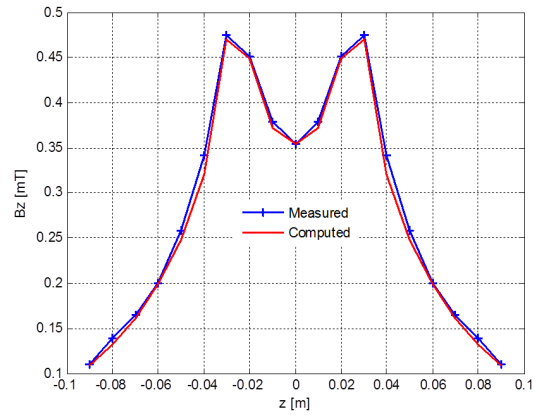


Figure 8. Validation of  $B_z$  at  $d = 0.052$  m.

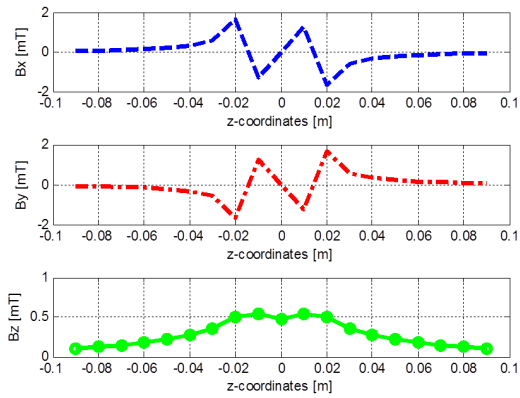


Figure 9.  $B_x$ ,  $B_y$  and  $B_z$  at  $d = 0.03$  m.

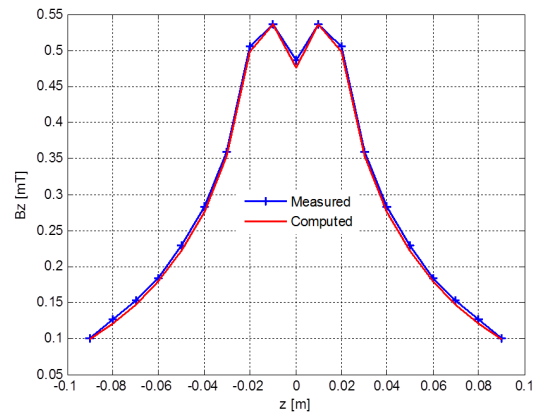


Figure 10. Validation of  $B_z$  at  $d = 0.03$  m.

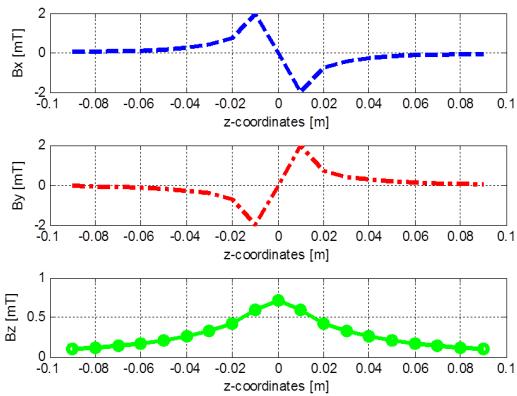


Figure 11.  $B_x$ ,  $B_y$  and  $B_z$  at  $d = 0.01$  m.

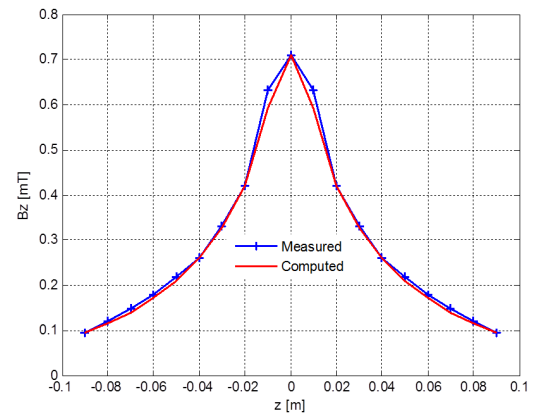
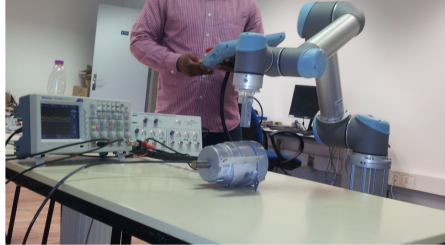


Figure 12. Validation of  $B_z$  at  $d = 0.01$  m.



**Figure 13.** Experimental set up for FC coils.

In the GUI, the motions of the sensor are given using a series of way points and a way point can be given by moving the robot to a certain position. So, the robot performs a task by moving through a sequence of way points. Figure 13 shows the UR5 holding the sensor of the magnetometer which is connected to the oscilloscope. With this experimental set up, the robot is programmed to move the sensor from the centre point (i.e.,  $z = h = 0$ ) between the two FC coils to a particular point (i.e.,  $z = h = 0.09 \text{ m}$ ), which is along the symmetric axis of the coils.

## 6. Conclusion

Based on the fact that the measure of the magnetic fields and mutual inductance between air-cored coils are of importance to electrical engineers and physicists, MATLAB computations and experimental measurements of the magnetic fields between two filamentary circular (FC) coils are presented in this paper.

Accurate MATLAB computation for the voltage and current induced in the secondary side of the FC coil is achieved by deriving the models for computing the total magnetic flux and the mutual inductance between the two coils. Also, the model for calculating the magnetic fields between two FC coils are formulated and validated with experimental measurement. The comparison of the results show that the model formulated and experimental measurements implemented in the laboratory are accurate.

The results obtained show that the values of  $\Phi_T$ ,  $M$  and  $V$  can be increased if a higher frequency AC current  $I_P$  is supplied to the primary coil, the number of turns of the coils  $N_P$  and  $N_S$  are increased and a relative permeability of ferrimagnetic material (e.g. ferrite) is employed in the electromagnetic coupling structure of the inductive charger.

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