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► **To cite this version:**

| Giuseppe Iurato. Eye movement pre-algebra and visual semantic algebra . 2016. <hal-01346413>

HAL Id: hal-01346413

<https://hal.science/hal-01346413v1>

Preprint submitted on 18 Jul 2016

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Eye movement pre-algebra and visual semantic algebra

Their possible links within denotational mathematics framework, Husserlian phenomenological theory, and structuralism

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Abstract – We would like to propose a new denotational mathematics entity, i.e., the eye movement pre-algebra (EMpA), which may be considered as a pre-algebraic structure in a certain sense generating, according to universal algebra, Husserlian phenomenological theory and structuralism, another basic algebraic structure of denotational mathematics, said to be visual semantic algebra (VSA).

Keywords – *Eye movements, Visual-Spatial Skills, Algebra, Denotational Mathematics, Husserl’s Phenomenology, Structuralism*

I. INTRODUCTION

Basically, human visual function, the chief one among the physiological functions of human perception, has four main abilities: fixing eyes upon an object; pursuing a moving object; responding to a stimulus which appears in the external neighbourhood of visual field, through a movement of eyes toward the direction along which such a stimulus manifests; exploring visually environmental space in searching of objects and their details. These basic four skills spring out from the general cognitive capability of *visual-spatial attention* (VSA_t), one of the most important cognitive function of human psyche. Neurophysiology has showed that VSA_t always precedes any next eye movement, that is to say, the former implicitly guides and directs the latter in its explicit action of orientation, predisposing the related visual field, so influencing and conditioning many other higher cognitive functions (like memory, voluntary judgement, etc.) [1].

Human eye movements are classified into two main types, namely, *fixations* and *saccades*, respectively when eyes stop in a certain position, and when they suddenly and fastly move towards another position. The resulting sequence of fixations and saccades, is called a *scanpath*. A *smooth pursuit* refers instead to eyes slowly following an object in movement. The set of fixational eye movements includes the so-called *microsaccades*, which

are nothing but small, involuntary (i.e., unconscious) saccades that occur during attempted fixation of an object. From much time, it has been deemed that most information by eyes come mainly from fixations or smooth pursuits, but not by saccades. Instead, recent neurophysiology research has shown what primary role play saccades, included microsaccades, for the general visual perception [2].

In any case, the class of eye movements comprehends the following ones: *saccades*, *smooth pursuit movements*, *vergence movements*, and *vestibulo-ocular movements*, which will be briefly described, in their physiological essence, in the next section. Now, from a mathematical viewpoint, in this note, just we would like to consider a possible mathematical structure of universal algebra, that we shall call *eye movement pre-algebra* (in short, EMpA), formalizing these eye movements, in such a way to be closely related to another formal structure belonging to Yingxu Wang’s denotational mathematics framework, called *visual semantic algebra* (in short, VSA), through a suitable conceptual bridge casted by Husserlian phenomenology.

II. TYPES OF EYE MOVEMENTS AND THEIR FUNCTIONS: A BRIEF SURVEY

Human beings are epi-ontogenetically prepared, since their childbirth, to develop eye movements, in particular saccades. To be more precise, there are four basic types of human eye movements, i.e., *saccades*, *smooth pursuit movements*, *vergence movements*, and *vestibulo-ocular movements*. The functions of each type of eye movement are briefly introduced herein, closely following [3], [4].

Saccades (in short, *S*) are rapid, ballistic movements of the eyes that abruptly change the point of fixation. They range in amplitude from the small movements, made while reading for example, to much larger movements, made while gazing around a room for example. Saccades can be elicited voluntarily, but they also occur reflexively whenever the eyes are open, even when are fixed on a

target. For instance, the rapid eye movements that occur during an important phase of sleep are also saccades. After the onset of a target for a saccade (as, for example, the stimulus in the case of the movement of an already fixated target), it takes about 200 ms for eye movement to begin. During this delay, the position of the target with respect to the fovea is computed (that is, how far the eye has to move), and the difference between the initial and intended position, or “motor error”, is converted into a motor command that activates the extraocular muscles to move the eyes the correct distance in the appropriate direction.

Saccadic eye movements are said to be “ballistic” because the saccade-generating system cannot respond to subsequent changes in the position of the target during the course of the eye movement. If the target moves again during this time (which has the order of about 15-100 ms), the saccade will miss the target, and a second saccade must be made to correct the error. Saccades can also be voluntary, but are mainly made unconsciously.

Smooth pursuit movements (in short, *SPM*) are much slower tracking movements of the eyes designed to keep a moving stimulus on the fovea. Such movements are under voluntary control in the sense that the observer can choose whether or not to track a moving stimulus. Surprisingly, however, only highly trained observers can make a smooth pursuit movement in the absence of a moving target. Most people who try to move their eyes in a smooth fashion without a moving target simply make a saccade.

The smooth pursuit system can be tested by placing a subject inside a rotating cylinder with vertical stripes. In practice, the subject is more often seated in front of a screen on which a series of horizontally moving vertical bars is presented to conduct this “optokinetic test”. The eyes automatically follow a stripe until they reach the end of their excursion. There is then a quick saccade in the direction opposite to the movement, followed once again by smooth pursuit of a stripe. This alternating slow and fast movement of the eyes in response to such stimuli is called *optokinetic nystagmus*. Optokinetic nystagmus is a normal reflexive response of the eyes in response to large-scale movements of the visual scene and should not be confused with the *pathological nystagmus* that can result from certain kinds of brain injury (for example, damage to the vestibular system or the cerebellum).

Vergence movements (in short, *VM*) align the fovea of each eye with targets located at different distances from the observer. Unlike other types of eye movements in which the two eyes move in the same direction (conjugate eye movements), vergence movements are disconjugate (or disjunctive). They involve either a convergence or divergence of the lines of sight of each

eye to see an object that is nearer or farther away. Convergence is one of the three reflexive visual responses elicited by interest in a near object. The other components of the so-called *near reflex triad* are accommodation of the lens, which brings the object into focus, and *pupillary constriction*, which increases the depth of field and sharpens the image on the retina.

Vestibulo-ocular movements (in short, *VOM*) stabilize the eyes relative to the external world, thus compensating for head movements. These reflex responses prevent visual images from “slipping” on the surface of the retina as head position varies. The action of vestibulo-ocular movements can be appreciated by fixating an object and moving the head from side to side; the eyes automatically compensate for the head movement by moving the same distance but in the opposite direction, thus keeping the image of the object at more or less the same place on the retina. The vestibular system detects brief, transient changes in head position and produces rapid corrective eye movements.

Sensory information from the semicircular canals directs the eyes to move in a direction opposite to the head movement. While the vestibular system operates effectively to counteract rapid movements of the head, it is relatively insensitive to slow movements or to persistent rotation of the head. For example, if the vestibulo-ocular reflex is tested with continuous rotation and without visual cues about the movement of the image (i.e., with eyes closed or in the dark), the compensatory eye movements cease after only about 30 seconds of rotation. However, if the same test is performed with visual cues, eye movements persist. The compensatory eye movements in this case are due to the activation of the smooth pursuit system, which relies not on vestibular information but on visual cues indicating motion of the visual field.

Human visual attention is a cognitive process deployed to reduce the wide complexity of external visual scene analysis. To this purpose, a subset of the available visual information is selected by shifting the focus of attention across the visual scene towards the most salient objects, locations, and features. Visual attention is driven by two main mechanisms, said to be *overt* and *covert* attention. Overt attention relates to the process of actively directing a sense towards an object or an event. In terms of overt visual attention, this is mainly exhibited as moving the eye gaze towards an object or location in the visual space. Covert attention, instead, describes the mechanism of mentally shifting the focus of attention without moving the eyes¹. Overt visual search has been well

¹ Maybe, mirror neuron system might be involved in covert attention mechanism.

explored, while covert visual attention search is a largely unexplored topic [5].

On the other hand, covert attention is closely related to cognitive unconscious realm. Indeed, recent researches² show that in the first milliseconds of incipient perception, humans apprehend (above all, in covert attention) only in an unconscious fashion the given intentioned object, also providing (pre-)judgements on it [6]. So, we would like to formalize (through the so-called EMPA) these latter types of mental processes – which neurophysiologically are mainly supported by the above eye movements – involved in such an unconscious acquisition of the intentioned object, which will give rise then the ordinary visual, conscious perception just formalized by VSA.

III. THE VISUAL SEMANTIC ALGEBRA (VSA)

The cognitive processes of pattern recognition may be modelled, within Y. Wang’s denotational mathematical framework, through a new mathematical structure known as *Visual Semantic Algebra* (VSA), where the cognitive informatics theories for pattern recognition are explored, such as cognitive principles of visual perception and the basic mechanisms of object and pattern recognition, as falling into the overt attention realm. Such a denotational mathematical means, that is to say, VSA, is developed to manipulate basic geometric shapes and figures, as well as their compositions, by a set of algebraic operations [7], [8].

The physical or environmental space can be formally modelled by Euclidean space, that is to say, the collection of all abstract points in the three Cartesian coordinates, equipped with the usual Pythagorean distance. Therefore, the spatial properties of abstract objects, shapes, and their interrelations, can be formally studied by Euclidean geometry, which can be classified into plane geometry and solid geometry: the former studies the geometric figures in a plane, while the latter studies the geometric objects known as solids in the three-dimensional space [7], [8].

In order to efficiently and formally model the abstract visual objects, their semantic representations, as well as their rigorous compositions and manipulations, a new denotational mathematics structure – the VSA – has just been introduced to this end, as well as for moulding pattern recognition and processing. VSA is a denotational mathematical structure which formally manipulates *visual objects* by algebraic operations on symbolic or semantic objects in geometric analyses and compositions. To be precise, if the basic geometric shapes (2-D), solids (3-D), figures (F) and spatial reference limits (L), are collectively called *visual objects*, or *geons*, let \mathcal{D} be a

finite non-empty set of geons, while let \mathcal{R} ($\subseteq \mathcal{D} \times \mathcal{D}$) be a finite non-empty set of *spatial relational operations*, then the *universal visual object environment*, say \mathcal{U}_{vis} , which forms the discourse’s environment of the abstract visual objects, is a formal structure of the type $\mathcal{U}_{vis} \stackrel{\text{def}}{=} (\mathcal{D}, \mathcal{R})$. Then, a *visual semantic algebra* (VSA) in the universal visual object environment \mathcal{U}_{vis} , is a formal structure of the type $VSA \stackrel{\text{def}}{=} (O, \star_{VSA})$, where $O \subseteq \mathcal{D}$ is a non-void finite set of geons, while $\star_{VSA} \subseteq \mathcal{R}$ is a non-empty finite set of algebraic operations on O , to be more precise, a set of *relational* and *behavioural operators* on O [7], [8].

IV. VISUAL SEMANTIC ALGEBRA (VSA) AND EYE MOVEMENT PRE-ALGEBRA (EMPA) IN THE HUSSERLIAN THEORY AND STRUCTURALISTIC FRAMEWORK

Let $VSA = (O, \star_{VSA})$ be a visual semantic algebra in the universal visual object environment $\mathcal{U}_{vis} = (\mathcal{D}, \mathcal{R})$. On the other hand, from a philosophical viewpoint, the term *object*, or *thing*, has a larger meaning. Indeed, from Kant onwards, we mainly distinguish a *thing in itself* (or the *general thing*) and a *sensible thing* of the external space-time. Then, a basic issue arose in regard to give or not a proper meaning to the being of the thing. Husserl gives a proper or intrinsic sense to the being of the thing, which is given to us with a transcendental modality going beyond the whole infinite set of apparitions of it, which is a unit that goes beyond these latter. So, the being of the thing is opposed to the being of the lived experiences of consciousness. Just in this regard, Husserl distinguishes an *immanent* perception and a *transcendental* perception of the thing [9], [10].

Husserl starts from the notion of *intentionality* means as the typical feature of human consciousness to relate with something which is *other* of the consciousness’ act itself, i.e., with something which goes beyond this latter. The former belongs to the transcendental perception, while the latter belongs to the immanent perception and gives rise to the set of all possible representations (imagines) of the object of transcendental realm. Therefore, since this typical relationship with the object does not fall into the consciousness field (due to the fact that it goes beyond – i.e., transcends – any act of consciousness directed on it), this relationship is outside every psychological discipline, that is to say, it does not have psychological nature but rather it has a logical-transcendental nature, in agreement with Kant according to whom the object *gives itself as such*, to the consciousness, according to another modality of being of consciousness (i.e., the logical-transcendental one) which is different from the psychological one. This modality of being may be explicated by means of the so-called *phenomenological reduction* (made by *epoché* plus *suspension of judgement*), which led to the absolute ego, neglecting every reality’s consideration [9], [10].

² See, above all, the related works of John A. Bargh in [6], and references therein.

So, the *transcendental* perception concerns that object, or thing, placed into the ordinary space-time, but never presents into the consciousness in its full actuality. From this, it springs out the main feature of the thing to be *in itself*, which allows consciousness to return on the same object to be identified; from this, it follows a certain contingency of the thing, and not its necessity. Moreover, the object is given to the subjective phenomena only by successive approximation through a series of apparitions, adumbrations, and approaches, which refer to the overall transcendental unity (also in the sense of Gestalt theory) of the object. Then, the essence of the thing integrates in themselves and, at the same time, transcends, all the elements of this approximating series, which will led to the transcendental perception of the thing.

Instead, *immanent* perception is nothing but the well-known Cartesian cogito, or the transcendental ego, i.e., the reflected and indubitable consciousness (also said to be *transcendental consciousness*), whose objects are the various lived experiences stored (e.g., remembrances, imagines, desires, feelings, etc.), mainly characterized by their immediacy and absoluteness, of which is impossible to deny the existence. The immanent being is therefore absolute and sure, that is to say, it exists regardless other, is moreover self-consistent and self-perceived, while the transcendental being, i.e., the world of things or objects, is necessarily related to some consciousness. Thus, the transcendental consciousness is reached on the basis of a primary opposition between the immanent perception and the transcendental perception (i.e., the perceived), that is, between the (immanent) perception of the various states of consciousness and the (transcendental) perception of the external world, giving rise to an immediate unity or totality (Gestalt) [11].

But, the simple perception does not entail knowledge. This takes place only when the object of perception encounters the subjective (individual) lived experience, which is something that is already present inside us at a mere original state, then becoming (mental) thought by means of symbolic marking with its main representation function, which is the diachronic result of the own lived experience meant as a connected flow, or current, of past experiences (*Erlebnis*), which ‘catches’ the object. Each representation of any consciousness’ act is therefore the main feature of human thought, through which an arbitrary object (of transcendental perception) belongs to the consciousness field. So, the external world itself seems to exist in dependence on some consciousness. From this stance, it follows, among other thing, that typical idealistic view of Husserl, centred on the notion of transcendental Ego, which has an ontological priority as primary constitutive basis of lived experience [12].

Therefore, for Husserl, perception is that intentional act of consciousness in which are present, at the same time, the perceived object (of transcendental perception), and the perceiving subject (of immanent perception). In regard to immanent perception, then, we may distinguish between an objective aspect and a subjective aspect of *Erlebnis*. The objective aspect is said to be *noema*, and is the object as considered by the consciousness’ reflection in its various modes of being with which it is given to consciousness itself (e.g., the perceived, the remembered, the imagined, and so forth). But, it is distinct from the object itself of transcendental perception: for example, the object of transcendental perception of a tree, is different from the corresponding object of immanent perception, as the latter – i.e., the *noema* – is the set of all the properties, predicates, and modalities of being of the former (e.g., a green tree, a bare tree, an illuminated tree, a lonely tree, etc.) for which it exists for our immanent consciousness (to give rise the *regional ontologies*) but still not at a representational level, as this latter is reached only by means of the basic, dialectic opposition *noesis-noema* of the intentional contents of *Erlebnis*, just with the constitution of the *noesis*, with its flow of contents of lived experiences.

Moreover, *noema* also provides the so-called *givenness* of the experience, that is to say, it is, so to speak, the “threw down” of the human (internal and external) senses, phenomenologically reduced, that is to say, the manifestation of perceptual consciousness; it is nothing but the so-called *hyle*, the primary outcome of (covert) visualization that, thanks to noesis, will give rise to the intentional element of *Erlebnis*, clear and net in its (overt) visualization to consciousness. The *noema*, therefore, is a kind of “clot” of hyletic data (sensory matter). The *noesis*, instead, i.e., the subjective (or psychological) aspect of *Erlebnis*, is the set of all those (eidetic) acts of human consciousness – like, re-evoking, remembering, imagining, and so forth – which are able, so to speak, “to catch” the object of transcendental perception as *noema*; it gives rise to the so-called *eidetic sciences*: formal ontology, material ontology, and formal logic [13], [14].

So, in the immanent perception, we have an immediate, unitary and total consciousness of a whole, a totality, fully structured, whose behavior is not determined by its composing (so-called *atomic*) elements (as in association psychology), but by a pre-constituted system of certain pre-existent structural laws, which are internal to the totality itself. This is one of the main principles of Gestalt theory, in its content very close to the unity of the thing of transcendental perception claimed by Husserl’s theory, as the essence of the thing integrates in itself, and at the same time transcends, the totality of all its apparitions. Whereupon, immanent perception, only when is put in

relationship with transcendental perception, gives rise to the basic pair *noema-noesis* of *Erlebnis*. Analogously, the transcendental consciousness is reached on the basis of a primary opposition between immanent perception and transcendental perception, where an object acquires its status of real existence only through the noetic lived experiences [11].

On the other hand, a *structure* may be defined as any set of relationships, well-defined and determined, so it may be understood either as a *form* in the sense of Gestalt psychology or as a *system* of relationships according to either Claude Lévi-Strauss' structural anthropology and mathematics. In the latter sense, a structure may be also considered as a totality self-regulating itself by means of a pre-constituted system of internal transformations or rules, always generating only elements belonging to the same structure, and running in such a manner to preserve the those basic laws of the structure, which are already implicitly (or unconsciously) present at a level of the incipit of the perception (i.e., the sensation), in the so-called *consciousness field*, which however spreads out of transcendental consciousness and holds together, in their synthetic totality, either immanent and transcendental perception; in it, from the hyletic data of transcendental perception, the natural object acquires its proper existential status only through noetic lived experiences, so reaching transcendental Ego, where its representations finally may take place [9-12, 15-16].

Therefore, in the incipit of the perception (i.e., in the sensation), it seems that every human being, so to speak, as early owns, unconsciously, *systems* of relations, meant as above, which then become certain *structures* by means of suitable explication processes triggered by the basic, dialectic opposition between immanent and transcendent perception; to be precise, by the unconscious acquisition of an implicitly pre-existent *system* of laws and rules at the level of the first perception, a *structure* may explicitly spring out from the primary, basic opposition between immanent and transcendental perception; a *relationship* between *objects* may then arise from the consequent, opposition noesis-noema inherent transcendental Ego, from which, finally, a *representation* is then possible.

To be more precise, we should distinguish between the notion of *system* and of *structure*. The former refers to a synthetic view of a totality of elements, that, seen from a functional stance, tries to catch laws and rules governing it, no matter its component parts; furthermore, the system is not the result of the simple sum of its elements. This notion is however closely related with the latter notion, that of structure, as Lévi-Strauss and Jean Piaget have pointed out. Indeed, even if the structure evokes the unity and cohesion of the parts, hence refers to the system, it is not the system itself, as such, but rather is the internal

order of the system together the group of transformations characterizing such a system. In this regard, the structure is still independent of the elements or constituents parts, as it is closely related with the system of relationships among them. This last standpoint, centered on the notion of structure³, mainly arises from cultural anthropology context that, above all with the works of A.R. Radcliffe-Brown and E.E. Evans-Pritchard, basically identify the structure with the system of relations which undergo it. This viewpoint will be then retaken by functionalism and, above all, by the structural anthropology of Lévi-Strauss, until up the many and interesting applications, in a wider context, of the general structuralism as for instance in linguistics, epistemology, sociology, human sciences, by Roman Jakobson, Piaget, Roland Barthes, Jacques Lacan, Noam Chomsky, Talcott Parsons, and so on [17].

Therefore, it seems [17] that the link between structure and system of relations, is the central core around which revolves any other possible definition of structure. Just structural linguistics claims that, if one wishes to define a certain entity, say *R*, then it should be considered from "outside" not from "inside", that is to say, considering all its possible relationships with the related (semantic⁴) *context* in which it relies or is embedded, not considering its intrinsic properties. So, a structure may be defined as a kind of transformational syntax of the system, that is to say, with the set of all rules of relationship, combination and permutation (i.e., a group⁵ of transformations) of its composing terms or elements. This mainly stems from the well-known 1940s studies of Lévi-Strauss on the systems of kinship, and was later retaken by Piaget. So, the early basis upon which to build up any structure is the preliminary acquisition of a certain system of relations, from which then to obtain a structure, as mentioned just above. It has been Lévi-Strauss to have pointed out that does not exist structure without a notion of relation or transformation, that a structure does not reduce to the system of relations to which it refers, and that it is the result of the action of the set of all the variants of such a system (or else, its representations).

Now, intentionality is, so to speak, the "objectivating" function of consciousness, so phenomenological theory of intentionality is nothing but phenomenological theory of objectivity (and its structure and forms). Intentionality may be defined as a correlation of subjective (noetic) and extra-subjective (noematic) factors, two levels which, by themselves, make up a structured whole, i.e., a gestaltic

³ Historically, the functionalistic stance based on a certain notion of system of transformations, was already present, although *in nuce*, in the works of D'Arcy W. Thompson.

⁴ It should be much more complete to consider too the pragmatic context besides the semantics one; for an instance of pragmatic view of a formal structure, cf. [18].

⁵ In the sense of Universal Algebra; see [19].

form. Aron Gurwitsch has, for instance, associated this conception of intentionality with an equation of the type $P = f(x_i, x_e)$, where P refers to the perception while f refers to a relation between *internal* (x_i) and *external* (x_e) data, that is to say, to noematic properties of objects. In the Husserl's framework, P refers to the immanent perception, while x_i and x_e refer to the transcendental perception, so a formal relation f may take place only at the noetic stratum, i.e., at an higher level of abstraction, so having put into connection immanent perception with relations amongst noematic data which are already pre-existent but at a *proto-object* level, or a pre-linguistic level of perception [20].

Kevin Mulligan has distinguished between what he calls *external relations* on the one hand, and *internal relations* on the other. In order to explicate the use of the notion of "external relation," we may, for example, assume that a given stone a has mass-trope α and a certain other stone b has mass-trope β . The view that is rejected by Mulligan is the view that the statement "This stone a is heavier than b " is made true by a relational trope $\rho(a, b)$ that exists in addition to mass-tropes α and β . This external relation between a and b is a relation between things. It is supposed to be an entity that is borne by a as well as by b . Both Mulligan and Daniel Von Wachter wish to reject those theories that hold that $\rho(a, b)$ is the truthmaker of the statement "This stone a is heavier than b ". Mulligan also suggests that a given relation between things (to be more precise, an external relation, hence belonging at the *structure* level) obtains because a certain internal relation (hence belonging at the *system* level) between properties of these same things, already holds. So, the relation of "being heavier" that obtains between a and b , is made true by an internal relation between the mass-tropes α and β , undergoing the former. Furthermore, Mulligan holds too that internal relations between tropes⁶ are "irreducibly relational entities". Therefore, from an epistemological stance for Husserlian phenomenological theory, objectivities are the result of objectivating acts performing only at a very high layer within the (higher) stratified area of transcendental consciousness, hence belonging to transcendental Ego field, much well above perceptual proto-objects lying at the level of system of relations, pre-structural therefore [20-21].

Thus, we may identify two main levels, the first one (pre-linguistic, for proto-objects) concerning the *system* of relationships (Mulligan's internal relations), and the second one regarding the *structure* (Mulligan's external relations) built up upon this first level, coherently with above Lévi-Strauss ideas according to which *context* (i.e.,

semantics) is a basic component in determining the exact and correct setting of a structure [17]. The latter level then performs between immanent and transcendental perception, mainly running from noematic level to noetic one, until up to reach the highest strata of transcendental consciousness field, with the final rising of symbolic and abstract representations by means of *Erlebnis*. Now, Yingxu Wang's visual semantic algebra (VSA), as it has been defined in the previous section, may be considered just as the result of a possible formalization of most of this latter process of human understanding according to Husserl's phenomenology. Indeed, it formalizes the main steps of human visual perception (overt attention), from transcendental to immanent perception of the external world objects or things, till to their geometric (i.e., formal) representation (as geons), in the transcendental consciousness field, with respect to a given visual object environment, that, providing the right semantic context (provided by *Erlebnis*), will allow to define other formal structures, as the VSA, in terms of (geometric) relations among geons (just the elements of the formal structure VSA).

All this, however, takes place within the transcendental consciousness field, starting from the interplay between immanent and transcendental perception, till to the basic, dialectic opposition noema-noesis which will give rise to single objects or things (whose set be \mathfrak{A}) and relations (whose set be \mathfrak{R}) amongst them, hence to the usual, many formal structures of the general type $(\mathfrak{A}, \mathfrak{R})$ of universal algebra (and of denotational mathematics), together their representations just enabled by *Erlebnis* and its contents which provide the right semantic context by means of the so-called *sedimentation process* of meanings for ideal or abstract objects, meant as meaning-constitution acts of intentional consciousness that, being this free from any psychological or subjective aspect, makes so possible a collective understanding of such abstract entities. All this is however carried out between the strata of both transcendental and immanent perception, even inside overt attention realm, this being just coherent with what Husserl says about mathematical entities, which are considered as the main result of intentionality [22, 23]. Nevertheless, and this is just the main aim of the present paper, we wish to claim on what there might be before one reaches this main stage of knowledge; this has been accomplished just laying out our argument within a well-determined epistemological framework, say \mathfrak{F} , outlined thanks to either Husserl's phenomenological theory and structuralism, framework that has allowed, among other things, to identify a preliminary stage (i.e., the *system* stage) prior to what has been just said above (i.e., the *structure* stage).

To be precise, within this epistemological framework, we have identified an early stage, belonging to the so-

⁶ Tropes are usually considered to have spatio-temporal nature [21], so they belong to transcendental perception. They may be considered as not yet structured elementary entities of phenomenal reality.

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called *system* stage, in which only relational entities exist at a pre-linguistic level for the so-called *proto-objects* to be acquired at a covert attention level and prior to overt attention of transcendental perception from which starts to spring out the next *structure* stage, as discussed just above. This early system stage, mainly acquired in an implicit fashion, is prior to the next structure stage which will be even more acquired at an explicit way until up to reach higher transcendental consciousness field, with its symbolic representations. So, what we wish to highlight in this paper, and at this point of the discussion made so far, is that just this system stage, of covert attention, is implicitly acquired thanks to the various eye movements described at the first sections of the present work, whose set therefore, say \mathfrak{E} , is what precedes the incipit of the next structure stage giving rise to $(\mathfrak{A}, \mathfrak{R})$. So, in terms of universal algebra, we might say that \mathfrak{E} , in a certain sense, generates $(\mathfrak{A}, \mathfrak{R})$ with respect to \mathfrak{F} , that is to say,

$$\langle \mathfrak{E} \rangle_{\mathfrak{F}} = (\mathfrak{A}, \mathfrak{R}).$$

Thus, we may say, in general, that \mathfrak{E} is a pre-algebra of the algebra (according to universal algebra [19]) $(\mathfrak{A}, \mathfrak{R})$, which, in our case, is said to be the *eye movement pre-algebra* ($EMpA$), and it is such that $\langle EMpA \rangle_{\mathfrak{F}} = VSA$, where

$$EMpA \stackrel{\text{def}}{=} \{S, SPM, VM, VOM\},$$

is a set defined according to what has been said in section II, about its composing elements (which are only relations). So, we intend to refer to the new fact according to which only a *system* of relationships, i.e., $EMpA$, occurring at an implicit (or unconscious) manner, is able to generate, within the framework \mathfrak{F} , the *structure* VSA , seen as an algebra to be explicitly meant in the sense of universal algebra [19]. But, all this must be understood only within the epistemological framework \mathfrak{F} as above worked out, with the contribution of Husserlian phenomenology and structuralism, since, in the current setting of universal algebra, every possible notion of “generation” of a generic algebraic structure, however does ever reference to another algebraic (sub)structure still of the type (set of elements, set of relations), not to a set of relations only, as in the case of above for $EMpA$. So, this new, formal notion of generation of an explicitly given structure, starting only from an implicitly given system of relations, has been possible exclusively making reference to both Husserlian phenomenology theory and structuralism, in establishing the above framework \mathfrak{F} with respect to which such a notion of generation has been worked out.

ACKNOWLEDGMENTS

My thank to Professor Yingxu Wang, for having always given me a chance to present, in a certain manner and in some place, my research studies on the foundations and applications of denotational mathematics.

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